Review of 3-Phase AC Circuits

EE 340
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Advantages of 3-Phase Systems

• Can transmit more power for same amount of wire (twice as much as single phase)

• Torque produced by 3Φ machines is constant

• Three phase machines use less material for same power rating

• Three phase machines start more easily than single phase machines
Balanced 3-Phase Systems

- A balanced 3 phase (ϕ) system has
  - three voltage sources with equal magnitude, but with an angle shift of 120°
  - equal loads on each phase
  - equal impedance on the lines connecting the generators to the loads

- Bulk power systems are almost exclusively 3ϕ
- Single phase is used primarily only in low voltage, low power settings, such as residential and some commercial
3-Phase Voltage Source
Neutral Wire Sharing

(a) Diagram showing the sharing of neutral wire with currents $I_a$, $I_b$, and $I_c$. The neutral conductor is labeled with the currents $I_a + I_b + I_c$.

(b) Graph showing the currents $I_a$, $I_b$, and $I_c$ over a cycle. The graph includes peaks at $+I_{\text{max}}$ and $-I_{\text{max}}$. The phase angles are marked at 120, 240, 360, and 480 degrees.
Neutral Current

\[ I_n = I_a + I_b + I_c \]

\[ I_n = \frac{V}{Z} (1 \angle 0^\circ + 1 \angle -120^\circ + 1 \angle 120^\circ) = 0 \]
Phase and Line Voltages

\[ V_{an} = |V| \angle \alpha^\circ \]
\[ V_{bn} = |V| \angle \alpha^\circ - 120^\circ \]
\[ V_{cn} = |V| \angle \alpha^\circ + 120^\circ \]

(\( \alpha = 0 \) in this case)

\[ V_{ab} = V_{an} - V_{bn} = |V|(1 \angle \alpha - 1 \angle \alpha + 120^\circ) \]
\[ = \sqrt{3} |V| \angle \alpha + 30^\circ \]
\[ V_{bc} = \sqrt{3} |V| \angle \alpha - 90^\circ \]
\[ V_{ca} = \sqrt{3} |V| \angle \alpha + 150^\circ \]

Line to line voltages are also balanced
Example of Phase and Line Voltages
Y-Connected Load
Δ-Connected Load
Power in Balanced 3-Phase Circuits

- The real power, reactive power, apparent power, complex power and power factor are the same in each phase.

\[ P = 3V_p I \cos(\theta) = \sqrt{3}V_L I \cos(\theta) \]

\[ Q = 3V_p I \sin(\theta) = \sqrt{3}V_L I \sin(\theta) \]

\[ S = 3V_p I = \sqrt{3}V_L \]
Power measurement in 3-phase 4-wire circuit

- If the load is balanced, then $P_1 = P_2 = P_3$, hence, $P_T = 3P_1$
- If the load is unbalanced, $P_1 \neq P_2 \neq P_3$, hence, $P_T = P_1 + P_2 + P_3$
Power measurement in 3-phase 3-wire circuit: the 2-wattmeter method

- \( P_T = P_1 + P_2 \)
- The load can be Y-connected, \( \Delta \)-connected, balanced, or unbalanced.
- Any one of the 3 phases can be used as a reference.
- If the load is balanced, then \( P_1 = P_2 \), hence, \( P_T = 2P_1 \)
Measuring active power in a high power circuit
Per-Phase Analysis in Balanced 3-Phase Circuits

- Per phase analysis allows analysis of balanced 3Φ systems with the same effort as for a single phase system

- **To do per phase analysis**
  1. Convert all 3Φ load/sources to equivalent Y’s
  2. Solve phase “a” independent of the other phases
  3. Total system power $S = 3 V_a I_a$
  4. If desired, phase “b” and “c” values can be determined by inspection (i.e., ±120° degree phase shifts)
  5. If necessary, go back to original circuit to determine line-line values or internal 3Φ values.
Example of per-phase analysis

- Find the complex power supplied by each of the two sources.
To solve the circuit, write the KCL equation at a’
\[(V'_a - 1\angle 0)(-10j) + V'_a(3j) + (V'_a - \frac{1}{\sqrt{3}}\angle -30^\circ)(-10j) = 0\]

\[(10j + \frac{10}{\sqrt{3}}\angle 60^\circ) = V'_a(10j - 3j + 10j)\]

\[V'_a = 0.9\angle -10.9^\circ \text{ volts} \quad V'_b = 0.9\angle -130.9^\circ \text{ volts}\]

\[V'_c = 0.9\angle 109.1^\circ \text{ volts} \quad V'_{ab} = 1.56\angle 19.1^\circ \text{ volts}\]

\[S_{Y_{gen}} = 3V'_aI'_a = V'_a\left(\frac{V'_a - V'_a}{j0.1}\right)^* = 5.1 + j3.5 \text{ VA}\]

\[S_{\Delta_{gen}} = 3V''_a\left(\frac{V''_a - V'_a}{j0.1}\right)^* = -5.1 - j4.7 \text{ VA}\]
Example of 3-phase balanced circuit

4000 V 3-phase

\[ P_1 \]

\[ Q_L \] \[ Q_m \]

\[ Q_c \]

1800 kvar

3594 HP

\[ \eta = 93\% \]

\[ F_p = 90\% \]

\[ I_c \]

260 A

\[ J_L \]

420 A

\[ E_{LN} \]

2309 V

(a)

(b)

420 A

462 A

1800 kvar
Problems from Chap 2:

1, 2, 3, 4, 5, 6.