Computer Science 456/656 Fall 2013

Practice for the Second Examination, Thursday October 10, 2013

The entire practice examination is 245 points.

1. True or False. [5 points each]

   (a) ________ Every subset of a regular language is regular.

   (b) ________ The intersection of context-free languages is always a context-free language.

   (c) ________ If the Kleene closure of a language L is context-free, hen L must be a context-free language. ________ If a language L is generated by an unambiguous context-free grammar, then L must be accepted by some DPDA. ________

   (d) ______ The problem of whether a given string is generated by a given context-free grammar is decidable.

   (e) ______ Let L be the language over $\{a, b, c\}$ consisting of all strings which have more a's than b's and more b's than c's. There is some PDA that accepts L.

   (f) ______ The language $\{a^n b^n \mid n \geq 0\}$ is context-free.

   (g) ______ The language $\{a^n b^n c^n \mid n \geq 0\}$ is context-free.

   (h) ______ The language $\left\{a^i b^j c^k \mid j = i + k\right\}$ is context-free.

   (i) ______ The intersection of any three regular languages is context-free.

   (j) ________ There is a DPDA that accepts the language of all palindromes over the binary alphabet $\{0, 1\}$.

   (k) ________ There is a PDA that accepts the language of all palindromes over the binary alphabet $\{0, 1\}$.

   (l) ________ If a language has an unambiguous context-free grammar, then it is is accepted by some deterministic push-down automaton.

2. [20 points] Let G be the context-free grammar given below.

   (a) $S \to a$

   (b) $S \to wS$

   (c) $S \to iS$

   (d) $S \to iSeS$

   Prove that G is ambiguous by writing two different **leftmost** derivations for the string $iwiaea$. [If you simply show two different parse trees, you are not following instructions.]

3. [30 points] Give a context-free grammar for the language of all strings over $\{0, 1\}$ of the form 0^m1^n where $n \neq m$.

4. [30 points] The following context-free grammar G is ambiguous. Give an equivalent unambiguous grammar.

   - The terminal alphabet of G is $\{a, b, c, (,), +, -, *\}$.
   - G has only one variable, namely the start symbol E.
   - The productions of G are as follows:
     (a) $E \to E + E$
     (b) $E \to E - E$
     (c) $E \to E * E$
     (d) $E \to (E)$
     (e) $E \to a$
     (f) $E \to b$
     (g) $E \to c$

5. [30 points] Let L be the language generated by the Chomsky Normal Form (CNF) grammar given below.

   (a) $S \to a$
   (b) $E \to a$
   (c) $S \to LA$
   (d) $E \to LA$
   (e) $L \to ($
   (f) $A \to ER$
   (g) $R \to)$
   (h) $S \to PE$
   (i) $E \to PE$ tem $S \to EE$
   (j) $E \to EE$
   (k) $P \to EQ$
   (l) $Q \to +$

   Use the CYK algorithm to prove that the string $a(a + a)$ is a member of L.

6. [15 points] State the pumping lemma for context-free languages.

7. [30 points] Let $L = \{w \in \{a,b\}^* \mid \#_a(w) = 2\#_b(w)\}$, here $\#_a(w)$ denotes the number of instances of the symbol a in the string w. For example, $aaababaaabba \in L$, because that string has the twice as many a's as b's. Give a context-free grammar for L. Your grammar may be ambiguous.

8. [30 points]

1. $S \to \epsilon$
2. $S \to a_2 S_3 b_4 S_5$

	a	b	**eof**	S
0				
1			**halt**	
2				
3				
4				
5				

Complete the ACTION and GOTO tables of an LALR parser for the grammar given above. This grammar unambiguously generates the "balanced parentheses" language, where a represents a left parenthesis, and b represents a right parenthesis. Example strings include ϵ, ab, $aabb$, $abab$, and $aabbab$.