**Computer Science 456/656 Fall 2018 Practice Examination November 14, 2018**

**The entire examination is 615 points.**

1. True or False. [5 points each] T = true, F = false, and O = open, meaning that the answer is not known to science at this time.

   i ________ Every subset of a regular language is regular.

   ii ________ Let $L$ be the language over $\Sigma = \{a, b\}$ consisting of all strings of the form $a^m b^n$, where $m, n \geq 0$. Then $L$ is a regular language.

   iii ________ The complement of every regular language is regular.

   iv ________ The Kleene closure of every context-free language is context-free.

   v ________ If a language has an unambiguous context-free grammar, then it is is accepted by some deterministic push-down automaton.

   vi ________ If a language has an ambiguous context-free grammar, then it is is not accepted by any deterministic push-down automaton.

   vii ________ There is a PDA that accepts all valid C++ programs.

   viii ________ The intersection of any two regular languages is regular.

   ix ________ The language consisting of all base 7 numerals for positive integers $n$ such that $n \% 3 = 2$ is regular.

   x ________ The intersection of any two context-free languages is context-free.

   xi ________ Let $L$ be the language over $\Sigma = \{a, b\}$ consisting of all strings of the form $a^m b^n c^m$, where $m, n \geq 0$. Then $L$ is a context-free language.

   xii ________ Let $L$ be the language over $\Sigma = \{a, b\}$ consisting of all strings of the form $a^m b^n$, where $m \geq n$. Then $L$ is a context-free language.

   xiii ________ The complement of every context-free language is context-free.

   xiv ________ The union of any two context-free languages is context-free.

   xv ________ If a language has an context-free grammar, then it is is accepted by some push-down automaton.

   xvi ________ Every context-free language has an unambiguous context-free grammar.

   xvii ________ Every language that has an unambiguous context-free grammar is accepted by some DPDA.

   xviii ________ Every deterministic machine is a non-deterministic machine.

   xix ________ The language consisting of all base 2 numerals for integer powers of 2 is regular.

   xx ________ There is a DPDA that accepts the language of all palindromes over the binary alphabet $\{0, 1\}$.

   xxi ________ If a language has an unambiguous context-free grammar, then there must be some DPDA that accepts it.

   xxii ________ The language $\{a^n b^n c^n d^n \mid n \geq 0\}$ is recursive.

xxiii ______ The problem of whether a given context-free grammar generates all strings is decidable.

xxiv ______ The language $\left\{a^ib^jc^k \mid j \geq i+k\right\}$ is context-free.

xxv ______ If a language $L$ is undecidable, there is no machine that enumerates $L$ in canonical order.

xxvi Recall that if $\mathcal{L}$ is a class of languages, co-$\mathcal{L}$ is defined to be the class of all languages that are not in $\mathcal{L}$.

______ Let $\mathcal{RE}$ be the class of all recursively enumerable languages. If $L$ is in $\mathcal{RE}$ and also $L$ is in co-$\mathcal{RE}$, then $L$ must be decidable.

xxvii ______ Every bounded function is recursive.

xxviii ______ If $P$ is a mathematical proposition that can be stated using $n$ binary bits, and $P$ has a proof, then $P$ must have a proof whose length is $O(2^{2^n})$.

xxix ________ The intersection of any context-free language with any context-free language is context-free.

xxx ________ The complement of every recursive language is recursive.

xxxi ________ The complement of every recursively enumerable language is recursively enumerable.

xxxii ________ Every language which is generated by an general grammar is recursively enumerable.

xxxiii ________ The question of whether two context-free grammars generate the same language is undecidable.

xxxiv ________ Given any context-free grammar $G$ and any string $w \in L(G)$, there is always a unique leftmost derivation of $w$ using $G$.

xxxv ________ For any deterministic finite automaton, there is always a unique minimal non-deterministic finite automaton equivalent to it.

xxxvi ________ Using multi-processors and other advanced technology, it is possible to design a machine which decides the halting problem.

xxxvii ________ The question of whether two regular expressions are equivalent is decidable.

xxxviii ________ The intersection of any context-free language with any regular language is context-free.

xxxix ________ The halting problem is recursively enumerable.

xl ________ The complement of every context-free language is context-free.

xli ________ No language which has an ambiguous context-free grammar can be accepted by a DPDA.

xlii ________ The union of any two context-free languages is context-free.

xliii ________ The question of whether a given Turing Machine halts with empty input is decidable.

xliv ________ The class of languages accepted by non-deterministic Turing Machines is the same as the class of languages accepted by Turing Machines.

xlv ________ Let $F(0) = 1$, and let $F(n) = 2^{F(n-1)}$ for $n > 0$. Then $F$ is recursive.

xlvi ________ Every language which is accepted by some non-deterministic machine is accepted by some deterministic machine.

xlvii ________ The language of all regular expressions over the binary alphabet is a regular language.

xlviii ________ There cannot exist any computer program that can decide whether any two C++ programs are equivalent.

xlix ________ Every function that can be mathematically defined is recursive.

l ________ The language of all binary strings which are the binary numerals for multiples of 23 is regular.

li ________ The language of all binary strings which are the binary numerals for prime numbers is context-free.

lii ________ Every bounded function from integers to integers is Turing-computable. (We say that $f$ is *bounded* if there is some $B$ such that $|f(n)| \leq B$ for all $n$.)

liii ________ The language of all palindromes over $\{0, 1\}$ is inherently ambiguous.

liv ________ Every context-free grammar can be parsed by some deterministic top-down parser.

lv ________ Every context-free grammar can be parsed by some non-deterministic top-down parser.

lvi ________ Commercially available parsers cannot use the LALR technique, since most modern programming languages are not context-free.

lvii ________ The diagonal language is $\mathcal{RE}$.

lviii ________ The diagonal language is co-$\mathcal{RE}$.

2. Fill in the blanks.

   (a) [10 points] If there is an easy reduction from $L_1$ to $L_2$, then ________ is at least as hard as ________.

   (b) If a language is accepted by some Turing machine, it is ______________________ enumerable.

3. [30 points] State the Church-Turing thesis, and explain (in about 5 lines or less) why it is important.

4. [10 points] Consider the context-free grammar with start symbol $S$ and productions as follows:

   1 $S \to s$

   2 $S \to bLn$

   3 $S \to wS$

   4 $L \to \epsilon$

   5 $L \to SL$

Write a leftmost derivation of the string *bswsbwsnn*

5. [5 points] What class of machines accepts the class of context free languages?

6. [5 points] What class of machines accepts the class of recursively enumerable languages?

7. [10 points] What is the *canonical order* of a language?

8. [10 points] What does it mean to say that machines $M_1$ and $M_2$ are *equivalent*?

9. [20 points] What does it mean to say that a language $L$ is *decidable*?

10. [15 points] Let $\Sigma = \{0, 1\}$, the binary alphabet. We say a string $w$ over $\Sigma$ is *mostly positive* if $w$ has more 1's than 0's. Let $L$ be the set of mostly positive strings over $\Sigma$.

    Give a context-free grammar for $L$. **Very hard.**

11. [10 points] Give a Chomsky Normal Form grammar for the Dyck language.

12. [10 points] Consider the context-free grammar $G$, with start symbol $S$ and productions as follows:

    1 $S \to s$

    2 $S \to bLn$

    3 $S \to iS$

    4 $S \to iSeS$

    5 $L \to \epsilon$

    6 $L \to LS$

    Prove that $G$ is ambiguous by giving two different leftmost derivations for some string.

13. [30 points] Prove that every decidable language can be enumerated in canonical order by some machine.

14. [20 points] Let $G$ be the context-free grammar given below.

    1 $S \to a$

    2 $S \to wS$

    3 $S \to iS$

    4 $S \to iSeS$

    Prove that $G$ is ambiguous by writing two different **leftmost** derivations for the string *iwiaea*. [If you simply show two different parse trees, you are not following instructions.]

15. [20 points] Let $L$ be the language generated by the following context-free grammar.

1 $S \to a$

2 $S \to wS$

3 $S \to iS$

4 $S \to iSeS$

Write a Chomsky normal form grammar for $L$.

16. [30 points] Let $L = \left\{w \in \{a,b\}^* \mid \#_a(w) = 2\#_b(w)\right\}$, here $\#_a(w)$ denotes the number of instances of the symbol $a$ in the string $w$. For example, $aaababaaabba \in L$, because that string has the twice as many $a$'s as $b$'s. Give a context-free grammar for $L$. Your grammar may be ambiguous.

17. [10 points] what is a **reduction** of a language $L_1$ to a language $L_2$?

18. [50 points] Consider the language $L$ of all strings which would be acceptable as algebraic expressions involving variables and constants, where:

    - Every variable name is either $x$, $y$, or $z$.
    - Every constant is a natural number between 0 and 9
    - The only operators are addition, subtraction, and multiplication.
    - The symbol '$-$' is used only for subtraction. There is no negation.
    - There is no multiplication symbol. Multiplication is indicated by concatenating strings.
    - In multiplication of a constant by anything else, the constant must come first, and there can be at most one constant factor in any term.
    - Parentheses can be used.

    Here are some strings in the language $x(y+2z)$, $x-1-z$, $4(zx-2y)(x+z(x-1))$.

    Give an unambiguous context-free grammar for $L$ which is consistent with the usual semantics (as you learned in school) of such expressions.

19. (a) [10 points] State the pumping lemma for context-free languages accurately. If you have all the right words but in the wrong order, that means you truly do not understand the lemma, and you might get no partial credit at all.

    (b) [20 points] Let $L$ be the language consisting of all palindromes over the binary alphabet. What is the minimum pumping length of $L$?