Computer Science 456/656 Spring 2013

Practice for the First Examination, February 28, 2013

The entire practice examination is 455 points. The real exam will be much shorter.

1. True or False. [5 points each]

   (a) ________ Every subset of a regular language is regular.

   (b) ________ Let L be the language over $\Sigma = \{a, b\}$ consisting of all strings of the form $a^m b^n$, where $m, n \geq 0$. Then L is a regular language.

   (c) ________ The complement of every regular language is regular.

   (d) ________ The Kleene closure of every context-free language is context-free.

   (e) ________ If a language has an unambiguous context-free grammar, then it is is accepted by some deterministic push-down automaton.

   (f) ________ If a language has an ambiguous context-free grammar, then it is is not accepted by any deterministic push-down automaton.

   (g) ________ There is a PDA that accepts all valid C++ programs.

   (h) ________ The intersection of any two regular languages is regular.

   (i) ________ The language consisting of all base 7 numerals for positive integers n such that $n \% 3 = 2$ is regular.

   (j) ________ The intersection of any two context-free languages is context-free.

   (k) ________ Let L be the language over $\Sigma = \{a, b\}$ consisting of all strings of the form $a^m b^n c^m$, where $m, n \geq 0$. Then L is a context-free language.

   (l) ________ Let L be the language over $\Sigma = \{a, b\}$ consisting of all strings of the form $a^m b^n$, where $m \geq n$. Then L is a context-free language.

   (m) ________ The complement of every context-free language is context-free.

   (n) ________ The union of any two context-free languages is context-free.

   (o) ________ If a language has an context-free grammar, then it is is accepted by some push-down automaton.

   (p) ________ Every context-free language has an unambiguous context-free grammar.

   (q) ________ Every language that has an unambiguous context-free grammar is accepted by some DPDA.

   (r) ________ The intersection of any two context-free languages is context-free.

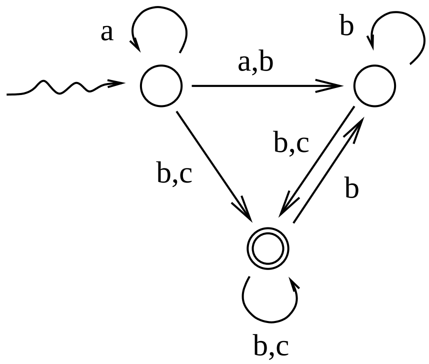
   (s) ________ Every deterministic machine is a non-deterministic machine.

   (t) ________ The language consisting of all base 2 numerals for integer powers of 2 is regular.

   (u) ________ There is a DPDA that accepts the language of all palindromes over the binary alphabet $\{0, 1\}$.

2. [25 points] Draw an NFA with five states which accepts the language described by the regular expression $(0+1)^*0(0+1)(0+1)(0+1)$

3. [25 points] Write a regular expression for the language accepted by the following NFA. If your answer is unnecessarily long by a wide margin, I might mark it wrong even if it's right.



Find a Regular Expression

4. [20 points] Let G be the context-free grammar given below.

   (a) $S \to a$
   (b) $S \to wS$
   (c) $S \to iS$
   (d) $S \to iSeS$

   Prove that G is ambiguous by writing two different **leftmost** derivations for the string $iwiaea$. [If you simply show two different parse trees, you are not following instructions.]

5. [30 points] Design a PDA that accepts the language $L = \{a^n bc^n \ : \ n \geq 0\}$.

6. [30 points] Give a context-free grammar for the language of all strings over $\{0, 1\}$ of the form $0^m 1^n$ where $n \neq m$.

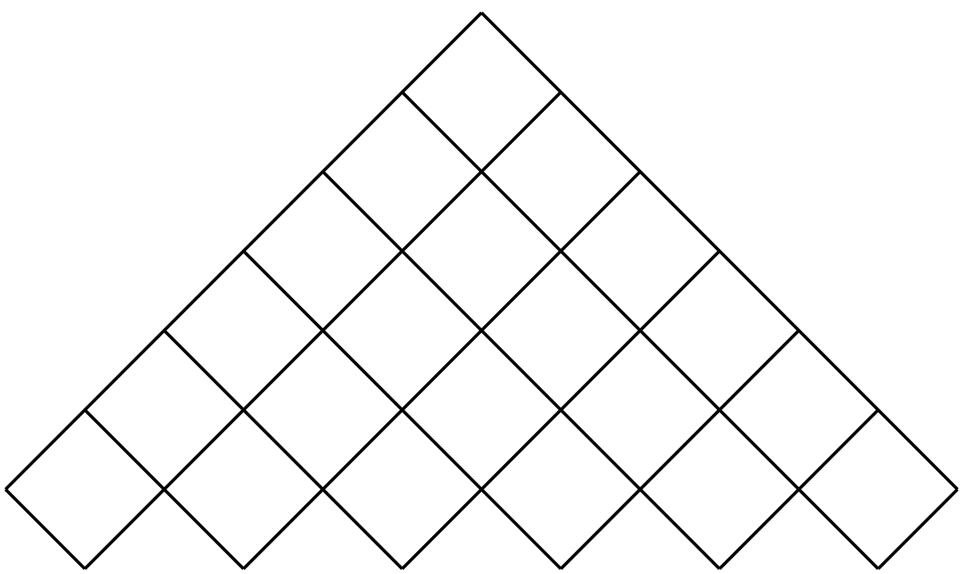
7. [30 points] The following context-free grammar G is ambiguous. Give an equivalent unambiguous grammar.

   - The terminal alphabet of G is $\{a, b, c, (,), +, -, *\}$.
   - G has only one variable, namely the start symbol E.
   - The productions of G are as follows:
     (a) $E \to E + E$
     (b) $E \to E - E$
     (c) $E \to E * E$
     (d) $E \to (E)$
     (e) $E \to a$
     (f) $E \to b$
     (g) $E \to c$

8. [30 points] Let L be the language generated by the Chomsky Normal Form (CNF) grammar given below.

   (a) $S \to a$
   (b) $E \to a$
   (c) $S \to LA$
   (d) $E \to LA$
   (e) $L \to ($
   (f) $A \to ER$
   (g) $R \to)$
   (h) $S \to PE$
   (i) $E \to PE$
   (j) $S \to EE$
   (k) $E \to EE$
   (l) $P \to EQ$
   (m) $Q \to +$

   Use the CYK algorithm to prove that the string $a(a+a)$ is a member of L. Use the figure below for your work.



9. [15 points] State the pumping lemma for regular languages.

10. [15 points] State the pumping lemma for context-free languages.

11. Let L be the language generated by the context-free grammar given in the first box. The second box contains a Chomsky Normal Form (CNF) grammar that also generates L.

(a) $S \to a$

(b) $S \to wS$

(c) $S \to iS$

(d) $S \to iSeS$

(a) $S \to WS$

(b) $S \to IS$

(c) $S \to AB$

(d) $A \to IS$

(e) $B \to ES$

(f) $S \to a$

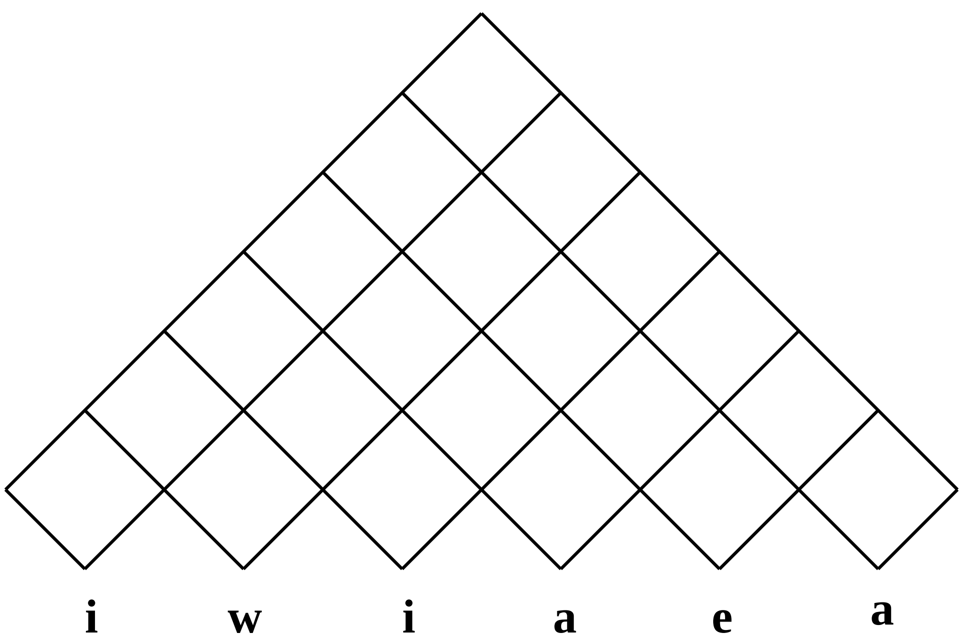
(g) $I \to i$

(h) $W \to w$

(i) $E \to e$

(a) [20 points] Use the CYK algorithm to prove that the string *iwiaea* is a member of L. Use the figure below for your work.

(b) [20 points] By looking at your work carefully, you can determine that the CNF grammar given above is ambiguous. Write two different parse trees for *iwiaea*, using the CNF grammar.



12. [30 points] Consider the NFA whose transition diagram is drawn below, where the input alphabet is $\{a, b, c\}$. Draw the transition diagram of an equivalent minimal DFA. Show your steps.

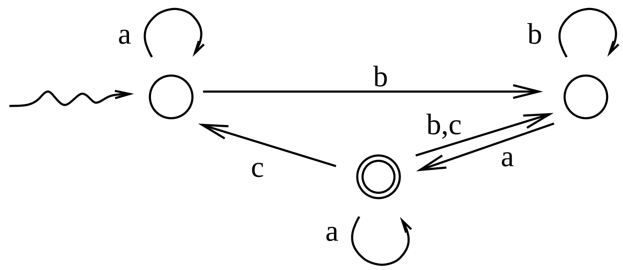


Figure 1: Find a minimal DFA equivalent to this NFA

13. [30 points] Let $L = \{w \in \{a,b\}^* \mid \#_a(w) = 2\#_b(w)\}$, here $\#_a(w)$ denotes the number of instances of the symbol a in the string w. For example, $aaababaaabba \in L$, because that string has the twice as many a's as b's. Give a context-free grammar for L. Your grammar may be ambiguous.

14. [30 points]

   1. $S \to \epsilon$
   2. $S \to a_2 S_3 b_4 S_5$

	a	b	**eof**	S
0				
1			**halt**	
2				
3				
4				
5				

Complete the ACTION and GOTO tables of an LALR parser for the grammar given above. This grammar unambiguously generates the "balanced parentheses" language, where a represents a left parenthesis, and b represents a right parenthesis. Example strings include ϵ, ab, $aabb$, $abab$, and $aabbab$.