1. (OS 2.22)  
Consider a discrete-time LTI system with impulse response \( h[n] \). If the input \( x[n] \) is a periodic sequence with period \( N \) (i.e. if \( x[n] = x[n + N] \)), show that the output \( y[n] \) is also a periodic sequence with period \( N \).

2. (OS 2.23 a,b,c) + additional systems  
For each of the following systems, determine whether the system is (1) stable, (2) causal, (3) linear, and (4) time-invariant.

(a) \( T(x[n]) = (\cos \pi n)x[n] \)
(b) \( T(x[n]) = x[n^2] \)
(c) \( T(x[n]) = x[n] \sum_{k=0}^{\infty} \delta[n - k] \)
(d) \( T(x[n]) = e^{x[n]} \)
(e) \( T(x[n]) = ax[n] + b \)

3. (OS 2.31)  
If the input and output of a causal LTI system satisfy the difference equation
\[
y[n] = ay[n - 1] + x[n],
\]
then the impulse response of the system must be \( h[n] = a^n u[n] \).

(a) For what values of \( a \) is this system stable?
(b) Consider a causal LTI system for which the input and output are related by the difference equation
\[
y[n] = ay[n - 1] + x[n] - a^N x[n - N],
\]
where \( N \) is a positive integer. Determine and sketch the impulse response of this system. 
\textit{Hint}: Use linearity and time-invariance to simplify the solution.
(c) Is the system in part (b) and FIR or an IIR system? Explain.
(d) For what values of \( a \) is the system in part (b) stable? Explain.

4. (OS 2.34 a,b,d)  
An LTI system has the frequency response
\[
H(e^{j\omega}) = \frac{1 - 1.25e^{-j\omega}}{1 - 0.8e^{-j\omega}} = 1 - \frac{0.45e^{-j\omega}}{1 - 0.8e^{-j\omega}}.
\]
(a) Specify the difference equation that is satisfied by the input \( x[n] \) and output \( y[n] \).
(b) Use one of the above forms of the frequency response to determine the impulse response \( h[n] \).
(d) If the input to the above system is \( x[n] = \cos(0.2\pi n) \), the output should be of the form \( y[n] = A\cos(0.2\pi n + \theta) \). What are \( A \) and \( \theta \).

5. (OS 2.74)  
The overall system in the dotted box in Figure P2.74 can be shown to be linear and time-invariant.
(a) Determine an expression for \( H(e^{j\omega}) \), the frequency response of the overall system from the input \( x[n] \) to the output \( y[n] \), in terms of \( H_1(e^{j\omega}) \), the frequency response of the internal LTI system. Remember that \((-1)^n = e^{j\pi n}\).

(b) Plot \( H(e^{j\omega}) \) for the case when the frequency response of the internal LTI system is

\[
H_1(e^{j\omega}) = \begin{cases} 
1, & |\omega| < \omega_c \\
0, & \omega_c < |\omega| \leq \pi
\end{cases}
\]

6. (a) Prove that

\[
\sum_{n=0}^{N} a^n = \frac{1 - a^{N+1}}{1 - a}
\]

(b) Prove that, for \(|a| < 1\),

\[
\sum_{n=0}^{\infty} a^n = \frac{1}{1 - a}
\]

(c) Find a closed form expressed for

\[
\sum_{n=N_1}^{N_2} a^n
\]

for any \(0 < N_1, N_2 < \infty\).

7. Use the convolution sum formula to find \( y[n] = h[n] * x[n] \) for

\[
h[n] = \begin{cases} 
1, & n \geq -3 \\
3^n, & n < -3
\end{cases}
\]

\[
x[n] = \begin{cases} 
(1/3)^n, & n \geq 3 \\
3^n, & n < 3
\end{cases}
\]