

Homework #2
Due Th. 9/19

1. Find the z-transform for each of the following sequences and indicate which also have a DTFT.

(a) $x[n] = 3\delta[n] + \delta[n - 2] + \delta[n + 2]$

(b) $x[n] = u[n] - u[n - 10]$

(c) $x[n] = \left(\frac{1}{3}\right)^n u(-n)$

(d) $x[n] = \left(\frac{1}{2}\right)^n u(n + 2) + (3)^n u(-n - 1)$

(e) $x[n] = \alpha^{|n|}$

(f) $x[n] = n \left(\frac{1}{2}\right)^n u[n - 2]$

2. Find the z-transform of the sequence $y[n] = \sum_{k=-\infty}^n x[k]$ in terms of the z-transform of $x[n]$.
3. Use the z-transform to perform the convolution of

$$h[n] = \begin{cases} \left(\frac{1}{2}\right)^n & 0 \leq n \leq 2 \\ 0 & \text{else} \end{cases}$$

$$x[n] = \delta[n] + \delta[n - 1] + 4\delta[n - 2]$$

4. Evaluate the convolution of the following sequences

$$h[n] = (0.5)^n u[n] \quad x[n] = 3^n u[-n]$$

5. Find the inverse of the following z-transforms and indicate which also have a DTFT

(a) $X(z) = 4 + 3(z^2 + z^{-2}) \quad 0 < |z| < \infty$

(b) $X(z) = \frac{1}{1 - \frac{1}{2}z^{-1}} + \frac{3}{1 - \frac{1}{3}z^{-1}} \quad |z| > \frac{1}{2}$

(c) $X(z) = \frac{1}{1 + 3z^{-1} + 2z^{-2}} \quad |z| > 2$

(d) $X(z) = \frac{1}{(1 - z^{-1})(1 - z^{-2})} + \quad |z| > 1$

6. A digital filter is described by the linear constant coefficient difference equation

$$y[n] = \frac{3}{4}y[n - 1] - \frac{1}{8}y[n - 2] + x[n].$$

The impulse response of the filter is measured with initial conditions

$$y[-1] = -1 \quad y[-2] = 1.$$

Determine the measured response of the filter and compare it with the zero state response.