ECG782: MULTIDIMENSIONAL DIGITAL SIGNAL PROCESSING

MORPHOLOGY
Mathematical Morphology
Erosion/Dilation
Opening/Closing
Grayscale Morphology
Morphological Operations
Connected Components
MORPHOLOGICAL IMAGE PROCESSING

- Filtering done on binary images
  - Images with two values [0,1], [0, 255], [black,white]
  - Typically, this image will be obtained by thresholding

\[
g(x, y) = \begin{cases} 
1 & f(x, y) > T \\
0 & f(x, y) \leq T
\end{cases}
\]

- Morphology is concerned with the structure and shape
- In morphology, a binary image is filtered with a structuring element \( s \) and results in a binary image

- Matlab Notes
MATHEMATICAL MORPHOLOGY

- Tool for image simplification while maintaining shape characteristics of objects
  - Image pre-processing
    - Noise filtering, shape simplification
  - Enhancing object structure
    - Skeletonizing, thinning, thickening, convex hull
  - Segmenting objects from background
  - Quantitative description of objects
    - Area, perimeter, moments
The language of mathematical morphology is set theory
- A set represents an object in an image

Example
- \( X = \{(1,0), (1,1), (1,2), \} \)
  \( \{(2,2), (0,3), (0,4)\} \)

Morphological transformation \( \Psi \)
- Relationship between image \( X \) and structuring element \( B \)
- Structuring element \( B \) is expressed with respect to a local origin \( O \)

Relationship computed as \( B \) is moved across the image in a raster scan
- Similar to filtering but with zero/one output
- Current pixel corresponds to origin \( O \)
Reflection
- $\hat{B} = \{ w | w = -b, \text{for } b \in B \}$
- Negative of coordinates (flip across origin)

Translation
- $\mathcal{B} = \{ c | c = b + z, \text{for } b \in B \}$
- Shift of set
The structuring element (SE) can be any shape

- This is a mask of “on” pixels within a rectangular container
- Typically, the SE is symmetric
Simple operations
- Does not require structuring element or raster scan

Extension of basic logic operators
- NOT, AND, OR, XOR

Often use for “masking”
BASIC MORPHOLOGY OPERATIONS

- Erosion
- Dilation
- Opening
- Closing
OUTLINE

- Mathematical Morphology
- Erosion/Dilation
- Opening/Closing
- Grayscale Morphology
- Morphological Operations
- Connected Components
**DILATION**

- Morphological combination of two sets using vector addition
  - $X \oplus B = \{ p \in E^2 : p = x + b, x \in X, b \in B \}$

- Output image is “on” anywhere the SE touches an “on” pixel
  - $A \oplus B = \{ z | (\hat{B})_z \cap A \neq \emptyset \}$
  - $A$ – image
  - $B$ – SE (notice flip)
  - $z$ – displacements (x,y locations)

*Figure 13.4: Learning 2015*
*Figure 9.6*

(a) Set $A$.
(b) Square structuring element (the dot denotes the origin).
(c) Dilation of $A$ by $B$, shown shaded.
(d) Elongated structuring element. (e) Dilation of $A$ using this element. The dotted border in (c) and (e) is the boundary of set $A$, shown only for reference.
DILATION PROPERTIES

- Use of $3 \times 3$ SE is an isotropic expansion
  - Called fill or grow operation

- Commutative and associative
  - $X \oplus B = B \oplus X \quad X \oplus (B \oplus D) = (X \oplus B) \oplus D$

- Can be used to fill small holes and gulfs in objects
  - Increases size of an object

- Not an invertible operation

Figure 13.5: Dilation as isotropic expansion. © Cengage Learning 2015.
EROSION

- Combine two sets using vector subtraction
  
  \[ X \ominus B = \{ p \in \mathbb{E}^2 : p = x + b \in X, \ b \in B \} \]

- Retain only pixels where the entire SE is overlapped
  
  \[ A \ominus B = \{ z | (B)_z \subseteq A \} \]
  
  - \( A \) – image
  - \( B \) – SE (notice no flip)
  - \( z \) – displacements (x,y locations)

- Not an invertible operation

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**Figure 13.7**: Erosion. © Cengage Learning 2015

**Figure 9.4**: (a) Set \( A \). (b) Square structuring element, \( B \). (c) Erosion of \( A \) by \( B \), shown shaded. (d) Elongated structuring element. (e) Erosion of \( A \) by \( B \) using this element. The dotted border in (c) and (e) is the boundary of set \( A \), shown only for reference.
Use of $3 \times 3$ SE is an isotropic reduction

- Called shrink or reduce operation

- Dual operation for dilation

- Can be used to get contours
  - Subtract erosion from original
OUTLINE

- Mathematical Morphology
- Erosion/Dilation
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- Grayscale Morphology
- Morphological Operations
- Connected Components
OPENING AND CLOSING

- Opening
  - Erosion followed by dilation (note they are not inverses)
    - $X \circ B = (X \ominus B) \oplus B$
  - Simplified, less detailed version
  - Removes small objects
  - Retains “size”
  - Idempotent
    - Repeated application does not change results

- Closing
  - Dilation followed by erosion
    - $X \cdot B = (X \oplus B) \ominus B$
  - Connects objects that are close
  - Fills small holes (gulfs)
  - Smooths object outline
  - Retains “size”
  - Idempotent
All pixels that fit inside when the SE is “rolled” on the inside of a boundary

\[ A \circ B = \bigcup \{(B)_z \mid (B)_z \subseteq A\} \]
CLOSING

- All pixels that fit inside when the SE is “rolled” on the outside of a boundary

**FIGURE 9.9** (a) Structuring element $B$ “rolling” on the outer boundary of set $A$. (b) The heavy line is the outer boundary of the closing. (c) Complete closing (shaded). We did not shade $A$ in (a) for clarity.
EXAMPLES

FIGURE 9.10
Morphological opening and closing. The structuring element is the small circle shown in various positions in (b). The SE was not shaded here for clarity. The dark dot is the center of the structuring element.
EXAMPLES II

**FIGURE 9.11**
(a) Noisy image.
(b) Structuring element.
(c) Eroded image.
(d) Opening of $A$.
(e) Dilation of the opening.
(f) Closing of the opening.
(Original image courtesy of the National Institute of Standards and Technology.)
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GRAYSCALE MORPHOLOGY

- Can extent binary morphology to grayscale images
  - Min operation – erosion
  - Max operation – dilation

- The structuring element not only specifies the neighborhood relationship
- It specifies the local intensity property

- Must consider image as a surface in 2D plane
**TOP SURFACE AND UMBRA**

- Top surface is the highest intensity in a set
  - \( T[A](x) = \max\{y, (x, y) \in A\} \)

- Umbra is the “shadow” points below top surface
  - \( U[f] = \{(x, y) \in F \times E, y \leq f(x)\} \)

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**Figure 13.12:** Top surface of the set \( A \) corresponds to maximal values of the function \( f(x_1, x_2) \). © Cengage Learning 2015.

**Figure 13.13:** Umbra of the top surface of a set is the whole subspace below it. © Cengage Learning 2015.
Dilation - Top surface of dilation of umbras

- $f \oplus k = T\{U[f] \oplus U[k]\}$
  - Left-side is grayscale dilation
  - Right-side is binary dilation

Erosion

- $f \ominus k = T\{U[f] \ominus U[k]\}$
OUTLINE

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OTHER MORPHOLOGICAL OPERATIONS

- Boundary extraction
  - \( \beta(A) = A - (A \ominus B) \)
  - Subtract erosion from original
  - Notice this is an edge extraction

- Convex hull \((H)\)
  - Smallest convex set that contains another set \(S\)
  - This is often done for a collection of 2D or 3D points
  - bwconvhull.m
MORE ON MORPHOLOGICAL OPERATIONS

- Definitions from Szeliski book
- Threshold operation
  \[ \theta(f, t) = \begin{cases} 
  1 & f \geq t \\ 
  0 & \text{else} 
  \end{cases} \]
- Structuring element
  - \( s \) – e.g. 3 x 3 box filter (1’s indicate included pixels in the mask)
  - \( S \) – number of “on” pixels in \( s \)
- Count of 1s in a structuring element
  - \( c = f \otimes s \)
- Correlation (filter) raster scan procedure

- Basic morphological operations can be extended to grayscale images
- Dilation
  - \( \text{dilate}(f, s) = \theta(c, 1) \)
  - Grows (thickens) 1 locations
- Erosion
  - \( \text{erode}(f, s) = \theta(c, S) \)
  - Shrink (thins) 1 locations
- Opening
  - \( \text{open}(f, s) = \text{dilate}(\text{erode}(f, s), s) \)
  - Generally smooth the contour of an object, breaks narrow isthmuses, and eliminates thin protrusions
- Closing
  - \( \text{close}(f, s) = \text{erode}(\text{dilate}(f, s), s) \)
  - Generally smooth the contour of an object, fuses narrow breaks/separations, eliminates small holes, and fills gaps in a contour
- Dilation - grows (thickens) 1 locations
- Erosion - shrink (thins) 1 locations
- Opening - generally smooth the contour of an object, breaks narrow isthmuses, and eliminates thin protrusions
- Closing - generally smooth the contour of an object, fuses narrow breaks/separations, eliminates small holes, and fills gaps in a contour
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CONNECTED COMPONENTS

- Semi-global image operation to provide consistent labels to similar regions
  - Based on adjacency concept
- Most efficient algorithms compute in two passes

- More computational formulations (iterative) exist from morphology
  - \( X_k = (X_{k-1} \oplus B) \cap A \)

Figure 3.23 Connected component computation: (a) original grayscale image; (b) horizontal runs (nodes) connected by vertical (graph) edges (dashed blue)—runs are pseudocolored with unique colors inherited from parent nodes; (c) re-coloring after merging adjacent segments.
Typically, only the “white” pixels will be considered objects
- Dark pixels are background and do not get counted

After labeling connected components, statistics from each region can be computed
- Statistics describe the region – e.g. area, centroid, perimeter, etc.

Matlab functions
- `bwconncomp.m`, `labelmatrix.m` (`bwlabel.m`) - label image
- `label2rgb.m` – color components for viewing
- `regionprops.m` – calculate region statistics
CONNECTED COMPONENT EXAMPLE

Grayscale image

Threshold image

Opened Image

Labeled image – 91 grains of rice