

# Adaptive background mixture models for real-time tracking

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# Improving Background Subtraction

- Adaptive Background Mixture Models for Real Time Tracking
  - Chris stauffer and W.E.L. Grimson
  - Over 4000 citations since 1999

# Motivation

- Robust background subtraction should handle lighting changes, repetitive motion from clutter and long term scene changes
- Standard method of adaptive background (Adv)
  - It is effective where object moves continuously
  - Background is visible a significant portion of the time

# Motivation

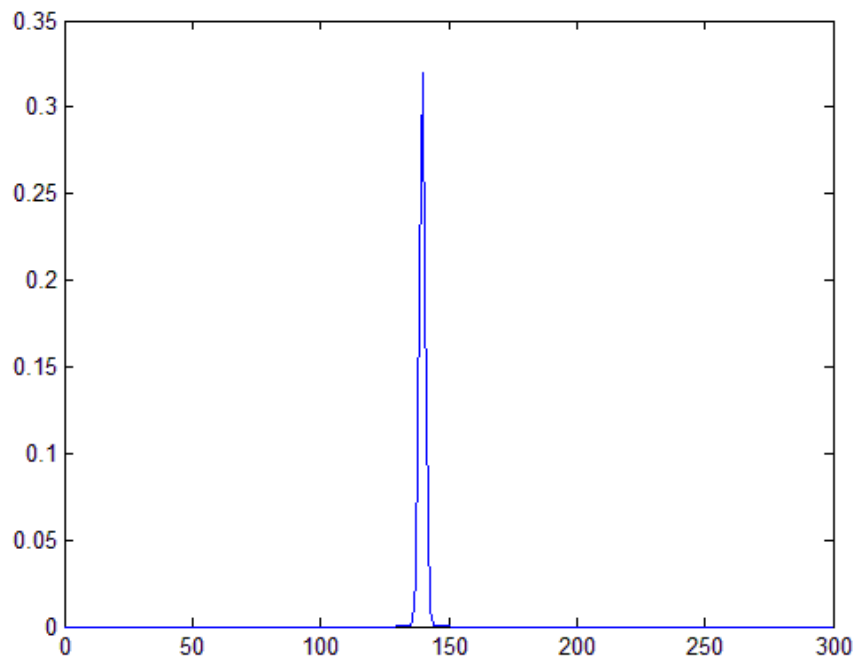
- Standard method of adaptive background  
(Disadv)
  - It is not robust to scenes with many moving objects particularly if they move very slowly
  - [http://www.youtube.com/watch?v=YA\\_lWWhePW8](http://www.youtube.com/watch?v=YA_lWWhePW8)
- Recovers slowly when background is uncovered
- It has single predetermined threshold for the entire scene

# Motivation

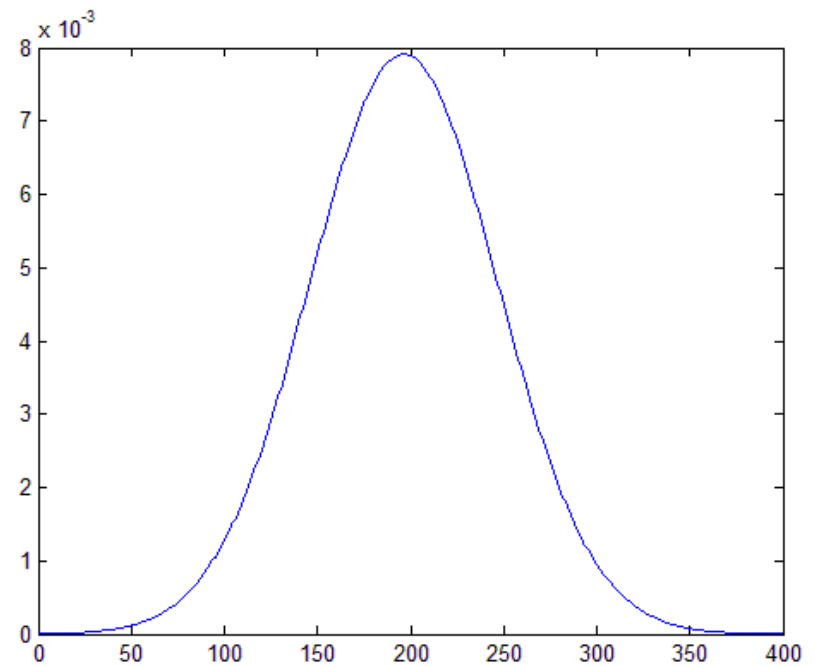
- Standard method of adaptive background  
(Disadv)
  - It can not handle bimodal backgrounds
  - Assume rainy situation
  - pixel intensity values of background for 16 frames :
    - 139,140,141,141,138,140,140,139,240,241,243,244,180,141,140,142
    - Modeling background for each 8 frame with Gaussian distribution
    - $\mu_1=139.75$ ,  $\delta_1=1.22$
    - $\mu_2=196.37$ ,  $\delta_2=50.43$

# Motivation

- 139,140,141,141,138,140,140,139,240,241,243,244,180,141,140,142

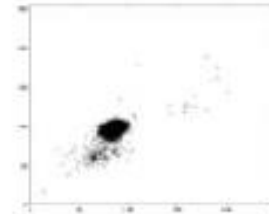
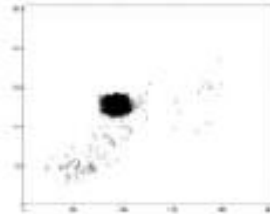


$\mu_1=139.75, \delta_1=1.22$



$\mu_2=196.37, \delta_2=50.43$

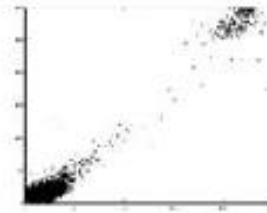
# Motivation



Differing threshold  
over time

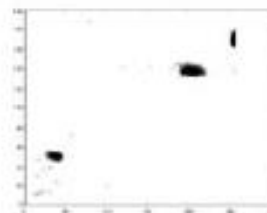
(a)

RG plots of a  
single pixel



Bimodal distribution  
over time

(b)



(c)

# Approach

- Modeling the values of a particular pixel as a mixture of Gaussians
- Based on the persistence and the variance of each Gaussians of the mixture, we determine which Gaussians may correspond to background color
- Pixel values that do not fit the background distributions are considered foreground



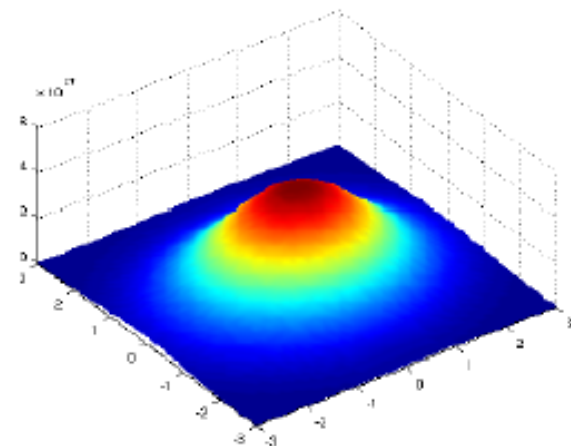
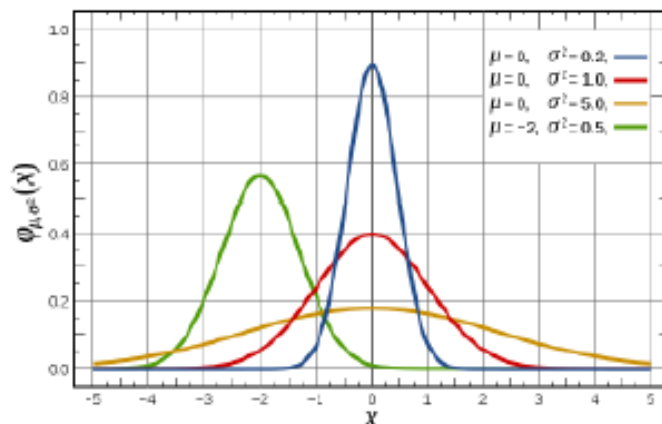
# Gaussian Distribution

- Univariate

$$\mathcal{N}(x|\mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

- Multivariate

$$\mathcal{N}(\mathbf{x}|\mu, \Sigma) = \frac{1}{(2\pi)^{D/2}} \frac{1}{|\Sigma|^{1/2}} e^{-\frac{1}{2}(\mathbf{x}-\mu)^T \Sigma^{-1}(\mathbf{x}-\mu)}$$



# Online Mixture Model

- History of a pixel is known up to current time  $t$

$$\{X_1, \dots, X_t\} = \{I(x_o, y_o, i): 1 \leq i \leq t\}$$

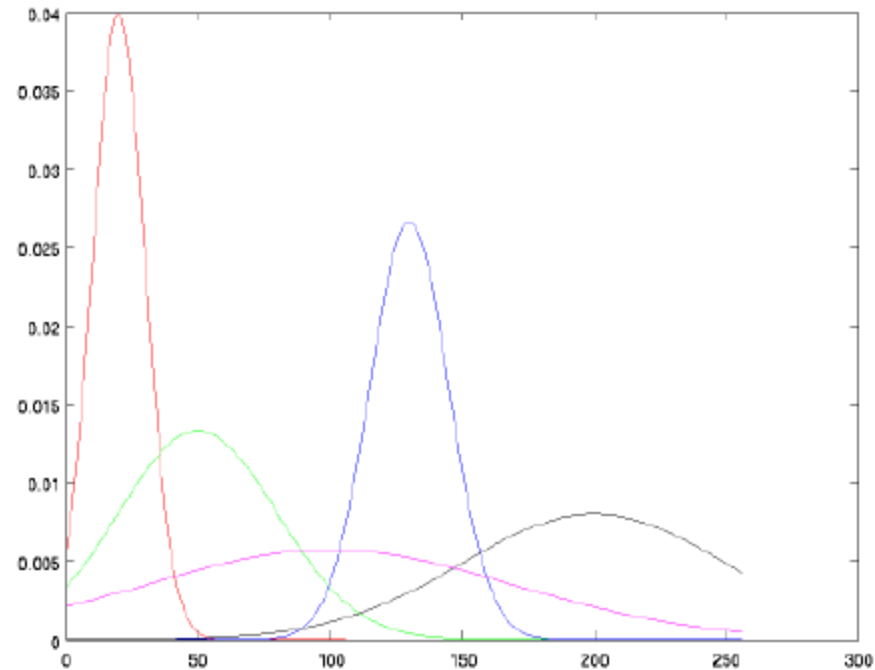
- Model the history as mixture of  $K$  Gaussian Distributions

$$P(X_t) = \sum_{i=1}^K w_{i,t} \mathcal{N}(X_t | u_{i,t}, \Sigma_{i,t})$$

$w_{i,t}$  - prior probability (weight) of Gaussians  $i$

# Mixture Model Example

- For grayscale image with  $K=5$



# Update the Mixture Model (Stage 1)

- Every new pixel value,  $X_{t+1}$ , is checked against the existing  $K$  Gaussian distributions until a match is found.
- A match is defined as a pixel value within 2.5 standard deviations of a distribution

## Stage 2

- Match

$$\mu_{i,t+1} = (1 - \rho)\mu_{i,t} + \rho X_{t+1}$$

$$\sigma_{i,t+1}^2 = (1 - \rho)\sigma_{i,t}^2 + \rho(X_{t+1} - \mu_{i,t})^2$$

- $\rho = \alpha \mathcal{N}(X_{t+1} | \mu_{i,t}, \sigma_{i,t}^2)$
- $\alpha$  – is a learning rate

- Prior weights of Gaussians are updated

$$w_{i,t+1} = (1 - \alpha)w_{i,t} + \alpha(M_{i,t+1})$$

$M_{i,t+1} = 1$  for matching Gaussian or  $M_{i,t+1} = 0$  for all others

# Stage 3

- **No Match**
  - If none of the K distributions match the current pixel value, the least probable distribution is go out.
  - A new distribution with the current value as its mean value, an initially high variance, and low prior weight, is enter .
  - The mean and variance remain unchanged.

# Background Model Estimation

- Gaussians are ordered by the value  $\omega/\sigma$
- We are interested in the Gaussian distributions which have the most **supporting evidence** = and the **least variance**. Why??
- For “background” distributions when a static, persistent object is visible, leading to high weight and relatively low variance.
- New object occludes the background object creation of a distribution or the increase in the variance of an existing distribution, so the variance of the moving object is expected to remain larger than a background pixel until the moving object stops

# Background Estimation Example

- First  $B$  distributions are selected as the background

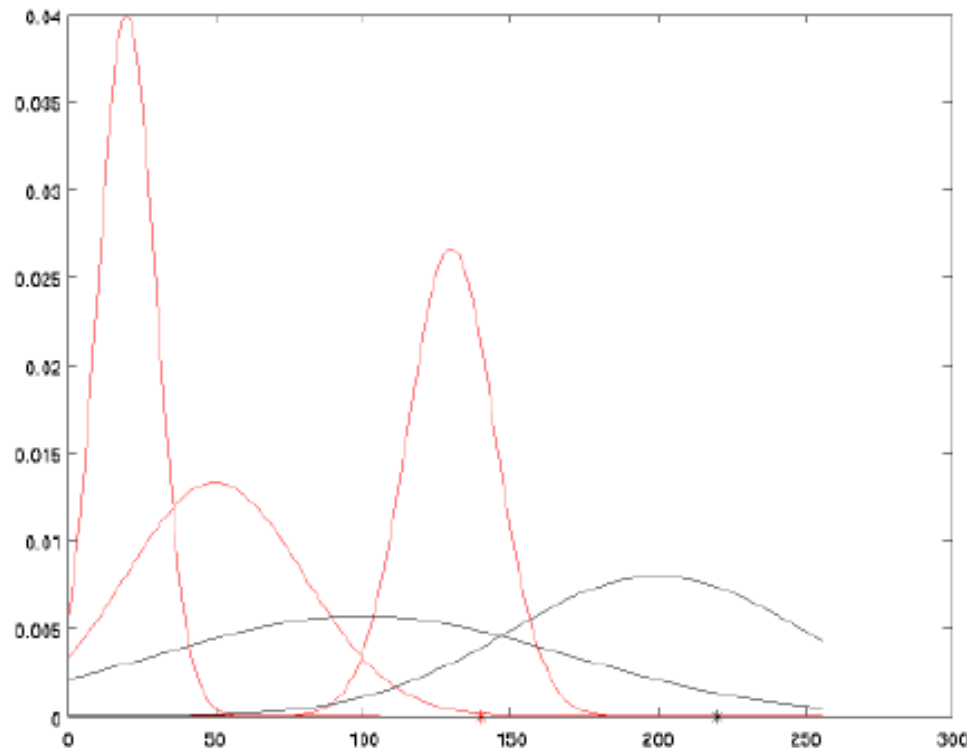
$$B = \operatorname{argmin}_b (\sum_{i=1}^b w_i > T)$$

- As we remember,  $W_i$  is portion of data that is accounted for by  $i$  Gaussian
- $T$  minimum portion of image expected to be background



# Background Estimation Example

- After background estimation red are backgrounds and blacks are foreground



# Discussion

- Advantageous
  - Different threshold for each pixel
  - Pixel-wise thresholds adapt over time
  - Objects are allowed to become part of the background without destroying the existing background model
  - Provides fast recovery
- Disadvantages
  - Can not handle sudden, drastic lighting changes
  - Must have good Gaussian initialization (median filtering)
  - There are number of parameters to tune

# Summary

- Simple background subtraction approaches such as frame diff, mean, and median filtering are fast
- Constant thresholds make them ill-suited for challenging real-world problems
- Adaptive background mixture model approach can handle challenging situations
- Bimodal backgrounds, long-term scene changes, and repetitive motion
- Improvements include upgrade the approach with temporal information or using region-based techniques

# Thank You

- Questions?

My Background subtraction implementation using GMM at OpenCV