10.21. The pole-zero plots are all shown in Figure S10.21.

(a) For $x[n] = \delta[n+5]$,

$$X(z) = z^5, \qquad \text{All } z.$$

The Fourier transform exists because the ROC includes the unit circle.

(b) For $x[n] = \delta[n-5]$,

$$X(z) = z^{-5}, \qquad \text{All } z \text{ except } 0.$$

The Fourier transform exists because the ROC includes the unit circle.

(c) For $x[n] = (-1)^n u[n]$,

$$\begin{aligned} X(z) &= \sum_{n=-\infty}^{\infty} x[n] z^{-n} \\ &= \sum_{n=0}^{\infty} (-1)^n z^{-n} \\ &= 1/(1+z^{-1}), \qquad |z| > 1 \end{aligned}$$

The Fourier transform does not exist because the ROC does not include the unit circle.

(d) For $x[n] = (1/2)^{n+1}u[n+3]$,

$$\begin{aligned} X(z) &= \sum_{n=-\infty}^{\infty} x[n]z^{-n} \\ &= \sum_{n=-3}^{\infty} (1/2)^{n+1} z^{-n} \\ &= \sum_{n=0}^{\infty} (1/2)^{n-2} z^{-n+3} \\ &= 4z^3/(1-(1/2)z^{-1}), \qquad |z| > 1/2 \end{aligned}$$

The Fourier transform exists because the ROC includes the unit circle.

(e) For $x[n] = (-1/3)^n u[-n-2]$,

$$\begin{aligned} X(z) &= \sum_{n=-\infty}^{\infty} x[n]z^{-n} \\ &= \sum_{n=-\infty}^{-2} (-1/3)^n z^{-n} \\ &= \sum_{n=2}^{\infty} (-1/3)^{-n} z^{n} \\ &= \sum_{n=0}^{\infty} (-1/3)^{-n-2} z^{n+2} \\ &= 9z^2/(1+3z), \qquad |z| < 1/3 \\ &= 3z/(1+(1/3)z^{-1}), \qquad |z| < 1/3 \end{aligned}$$

The Fourier transform does not exist because the ROC does not include the unit circle.

(f) For $x[n] = (1/4)^n u[-n+3]$,

$$\begin{aligned} X(z) &= \sum_{n=-\infty}^{\infty} x[n]z^{-n} \\ &= \sum_{n=-\infty}^{3} (1/4)^n z^{-n} \\ &= \sum_{n=-3}^{\infty} (1/4)^{-n} z^{n} \\ &= \sum_{n=0}^{\infty} (1/4)^{-n+3} z^{n-3} \\ &= (1/64)z^{-3}/(1-4z), \qquad |z| < 1/4 \\ &= (1/16)z^{-4}/(1-(1/4)z^{-1}), \qquad |z| < 1/4 \end{aligned}$$

The Fourier transform does not exist because the ROC does not include the unit circle.

(g) Consider $x_1[n] = 2^n u[-n]$.

$$\begin{aligned} X_1(z) &= \sum_{n=-\infty}^{\infty} x_1[n] z^{-n} \\ &= \sum_{n=-\infty}^{0} (2)^n z^{-n} \\ &= \sum_{n=0}^{\infty} (2)^{-n} z^n \\ &= 1/(1-(1/2)z), \qquad |z| < 2 \\ &= -2z^{-1}/(1-2z^{-1}), \qquad |z| < 2 \end{aligned}$$

Consider $x_2[n] = (1/4)^n u[n-1]$.

$$\begin{aligned} X_2(z) &= \sum_{n=-\infty}^{\infty} x_2[n] z^{-n} \\ &= \sum_{n=1}^{\infty} (1/4)^n z^{-n} \\ &= \sum_{n=0}^{\infty} (1/4)^{n+1} z^{-n-1} \\ &= (z^{-1}/4)[1/(1-(1/4)z^{-1})], \qquad |z| > 1/4 \end{aligned}$$

The z-transform of the overall sequence $x[n] = x_1[n] + x_2[n]$ is

$$X(z) = -\frac{2z^{-1}}{(1-2z^{-1})} + \frac{z^{-1}/4}{1-(1/4)z^{-1}}, \qquad (1/4) < |z| < 2.$$

The Fourier transform exists because the ROC includes the unit circle.

10.25. (a) The partial fraction expansion of $X(z)$ is

$$X(z) = -\frac{1}{1-\frac{1}{2}z^{-1}} + \frac{2}{1-z^{-1}}.$$

Since $x[n]$ is right-sided, the ROC has to be $|z| > 1$. Therefore, it follows that

$$x[n] = -\left(\frac{1}{2}\right)^2 u[n] + 2u[n].$$

(b) $X(z)$ may be rewritten as

$$X(z) = \frac{z^2}{(z-\frac{1}{2})(z-1)}.$$

Using partial fraction expansion, we may rewrite this as

$$\begin{aligned} X(z) &= 2z^2\left[-\frac{1}{z-\frac{1}{2}} + \frac{1}{z-1}\right] \\ &= 2z\left[-\frac{z}{z-\frac{1}{2}} + \frac{z}{z-1}\right] \end{aligned}$$

If $x[n]$ is right-sided, then the ROC for this signal is $|z| > 1$. Using this fact, we may find the inverse z-transform of the term within square brackets above to be $y[n] = -(1/2)^n u[n] + u[n]$. Note that $X(z) = 2zX(z)$. Therefore, $x[n] = 2y[n+1]$. This gives

$$x[n] = -2\left(\frac{1}{2}\right)^{n+1} u[n+1] + 2u[n+1].$$

Noting that $x[-1] = 0$, we may rewrite this as

$$x[n] = -\left(\frac{1}{2}\right)^n u[n] + 2u[n].$$

This is the answer that we obtained in part (a).

10.28. **(a)** Using eq. (10.3), we get

$$X(z) = 1 - 0.95z^{-6} = \frac{z^6 - 0.95}{z^6}.$$

(b) Therefore, $X(z)$ has six zeros lying on a circle of radius 0.95 (as shown in Figure S10.28) and 6 poles at $z = 0$.

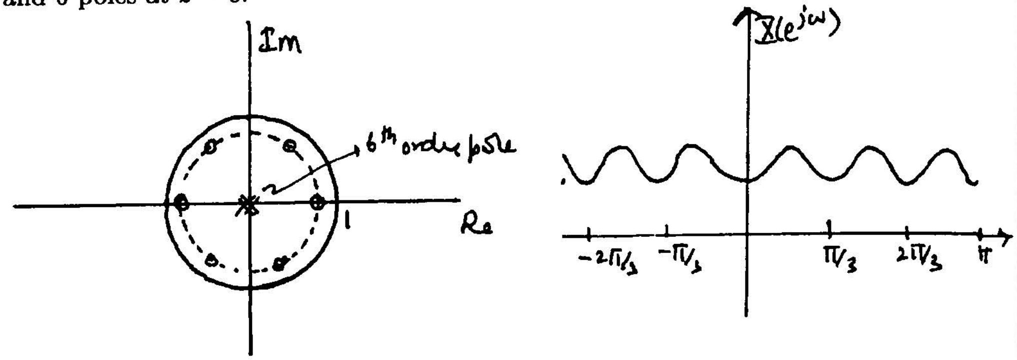

Figure S10.28

(c) The magnitude of the Fourier transform is as shown in Figure S10.28.

10.30. From the given information, we have

$$x_1[n] \overset{\mathcal{Z}}{\longleftrightarrow} X_1(z) = \frac{1}{1-\frac{1}{2}z^{-1}}, \qquad |z| > \frac{1}{2}$$

and

$$x_2[n] \overset{\mathcal{Z}}{\longleftrightarrow} X_2(z) = \frac{1}{1-\frac{1}{3}z^{-1}}, \qquad |z| > \frac{1}{3}.$$

Using the time shifting property, we get

$$x_1[n+3] \overset{\mathcal{Z}}{\longleftrightarrow} z^3 X_1(z), \qquad |z| > \frac{1}{2}.$$

Using the time reversal and shift properties, we get

$$x_2[-n+1] \overset{\mathcal{Z}}{\longleftrightarrow} z^{-1} X_2(z^{-1}), \qquad |z| < 3.$$

Now, using the convolution property, we get

$$y[n] = x_1[n+3] * x_2[-n+1] \overset{\mathcal{Z}}{\longleftrightarrow} Y(z) = z^2 X_1(z) X_2(z^{-1}), \qquad \frac{1}{2} < |z| < 3.$$

Therefore,

$$Y(z) = \frac{z^2}{(1-\frac{1}{2}z^{-1})(1-\frac{1}{3}z)}.$$

10.32. **(a)** We are given that $h[n] = a^n u[n]$ and $x[n] = u[n] - u[n-N]$. Therefore,

$$\begin{aligned} y[n] &= x[n] * h[n] \\ &= \sum_{k=-\infty}^{\infty} h[n-k]x[k] \\ &= \sum_{k=0}^{N-1} a^{n-k} u[n-k] \end{aligned}$$

Now, $y[n]$ may be evaluated to be

$$y[n] = \begin{cases} 0, & n < 0 \\ \sum_{k=0}^{n} a^n a^{-k}, & 0 \le n \le N-1 \\ \sum_{k=0}^{N-1} a^n a^{-k}, & n > N-1 \end{cases}.$$

Simplifying,

$$y[n] = \begin{cases} 0, & n < 0 \\ (a^n - a^{-1})/(1 - a^{-1}), & 0 \le n \le N-1 \\ a^n(1 - a^{-N})/(1 - a^{-1}), & n > N-1 \end{cases}.$$

(b) Using Table 10.2, we get

$$H(z) = \frac{1}{1 - az^{-1}}, \qquad |z| > |a|$$

and

$$X(z) = \frac{1 - z^{-N}}{1 - z^{-1}}, \qquad \text{All} z.$$

Therefore,

$$Y(z) = X(z)H(z) = \frac{1}{(1 - z^{-1})(1 - az^{-1})} - \frac{z^{-N}}{(1 - z^{-1})(1 - az^{-1})}.$$

The ROC is $|z| > |a|$. Consider

$$P(z) = \frac{1}{(1 - z^{-1})(1 - az^{-1})}$$

with ROC $|z| > |a|$. The partial fraction expansion of $P(z)$ is

$$P(z) = \frac{1/(1-a)}{1 - z^{-1}} + \frac{1/(1 - a^{-1})}{1 - az^{-1}}.$$

Therefore,

$$p[n] = \frac{1}{1-a} u[n] + \frac{1}{1 - a^{-1}} a^n u[n].$$

Now, note that

$$Y(z) = P(z)[1 - z^{-N}].$$

Therefore,

$$y[n] = p[n] - p[n-N] = \frac{1}{1-a}\{u[n] - u[n-N]\} + \frac{1}{1-a^{-1}}\{a^n u[n] - a^{n-N} u[n-N]\}.$$

This may be written as

$$y[n] = \begin{cases} 0, & n < 0 \\ (a^n - a^{-1})/(1 - a^{-1}), & 0 \le n \le N-1 \\ a^n(1 - a^{-N})/(1 - a^{-1}), & n > N-1 \end{cases} .$$

This is the same as the result of part (a).

10.34. **(a)** Taking the z-transform of both sides of the given difference equation and simplifying, we get

$$H(z) = \frac{Y(z)}{X(z)} = \frac{z^{-1}}{1 - z^{-1} - z^{-2}}.$$

The poles of $H(z)$ are at $z = (1/2) \pm (\sqrt{5}/2)$. $H(z)$ has a zero at $z = 0$. The pole-zero plot for $H(z)$ is as shown in Figure S10.34. Since $h[n]$ is causal, the ROC for $H(z)$ has to be $|z| > (1/2) + (\sqrt{5}/2)$.

(b) The partial fraction expansion of $H(z)$ is

$$H(z) = -\frac{1/\sqrt{5}}{1 - (\frac{1+\sqrt{5}}{2})z^{-1}} + \frac{1/\sqrt{5}}{1 - (\frac{1-\sqrt{5}}{2})z^{-1}}.$$

Therefore,

$$h[n] = -\frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^n u[n] + \frac{1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)^n u[n].$$

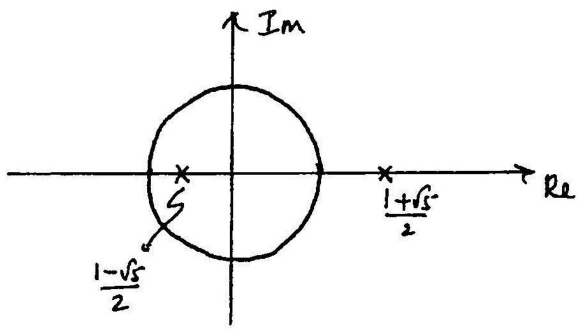

Figure S10.34

(c) Now assuming that the ROC is $(\sqrt{5}/2) - (1/2) < |z| < (1/2) + (\sqrt{5}/2)$, we get

$$h[n] = \frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^n u[-n-1] + \frac{1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)^n u[n].$$

10.37. (a) The block-diagram may be redrawn as shown in part (a) of the figure below. This may be treated as a cascade of the two systems shown within the dotted lines in Figure S10.37. These two systems may be interchanged as shown in part (b) of the figure Figure S10.37 without changing the system function of the overall system. From the figure below, it is clear that

$$y[n] = x[n] + \frac{9}{8}x[n-1] - \frac{1}{3}y[n-1] + \frac{2}{9}y[n-2].$$

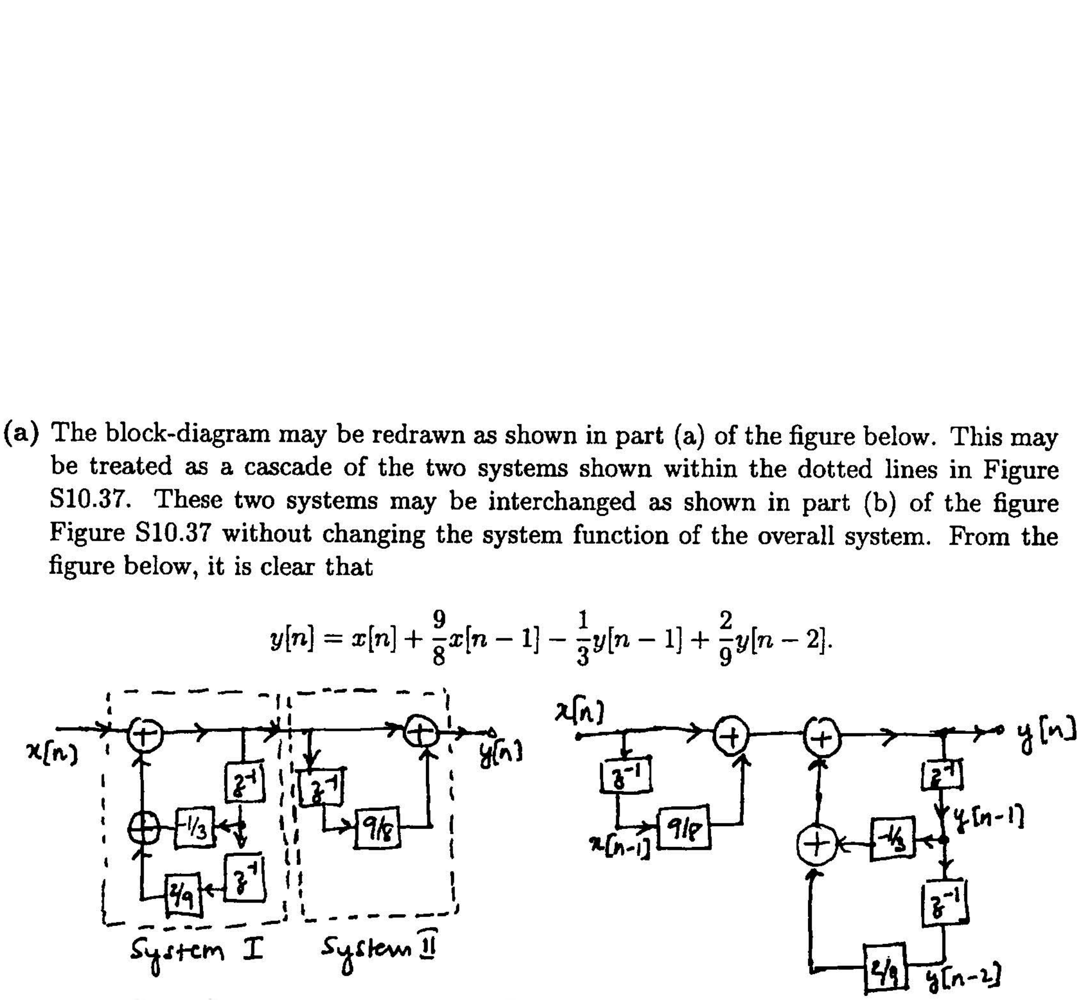

Figure S10.37

(b) Taking the z-transform of the above difference equation and simplifying, we get

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 + \frac{9}{8}z^{-1}}{1 + \frac{1}{3}z^{-1} - \frac{2}{9}z^{-2}} = \frac{1 + \frac{9}{8}z^{-1}}{(1 + \frac{2}{3}z^{-1})(1 - \frac{1}{3}z^{-1})}.$$

$H(z)$ has poles at $z = 1/3$ and $z = -2/3$. Since the system is causal, the ROC has to be $|z| > 2/3$. The ROC includes the unit circle and hence the system is stable.

10.42. (a) Taking the unilateral z-transform of both sides of the given difference equation, we get

$$\mathcal{Y}(z) + 3z^{-1}\mathcal{Y}(z) + 3y[-1] = \mathcal{X}(z).$$

Setting $\mathcal{X}(z) = 0$, we get

$$\mathcal{Y}(z) = \frac{-3}{1 + 3z^{-1}}.$$

The inverse unilateral z-transform gives the zero-input response

$$y_{zi}[n] = -3(-3)^n u[n] = (-3)^{n+1} u[n].$$

Now, since it is given that $x[n] = (1/2)^n u[n]$, we have

$$\mathcal{X}(z) = \frac{1}{1 - \frac{1}{2}z^{-1}}, \qquad |z| > 1/2.$$

Setting $y[-1]$ to be zero, we get

$$\mathcal{Y}(z) + 3z^{-1}\mathcal{Y}(z) = \frac{1}{1 - \frac{1}{2}z^{-1}}.$$

Therefore,

$$\mathcal{Y}(z) = \frac{1}{(1 - \frac{1}{2}z^{-1})(1 + 3z^{-1})}.$$

The partial fraction expansion of $\mathcal{Y}(z)$ is

$$\mathcal{Y}(z) = \frac{1/7}{1 - \frac{1}{2}z^{-1}} + \frac{6/7}{1 + 3z^{-1}}.$$

The inverse unilateral z-transform gives the zero-state response

$$y_{zs}[n] = \frac{1}{7}\left(\frac{1}{2}\right)^n u[n] + \frac{6}{7}(-3)^n u[n].$$

(b) Taking the unilateral z-transform of both sides of the given difference equation, we get

$$\mathcal{Y}(z) - \frac{1}{2}z^{-1}\mathcal{Y}(z) - \frac{1}{2}y[-1] = \mathcal{X}(z) - \frac{1}{2}z^{-1}\mathcal{X}(z).$$

Setting $\mathcal{X}(z) = 0$, we get

$$\mathcal{Y}(z) = 0.$$

The inverse unilateral z-transform gives the zero-input response

$$y_{zi}[n] = 0.$$

Now, since it is given that $x[n] = u[n]$, we have

$$\mathcal{X}(z) = \frac{1}{1 - z^{-1}}, \qquad |z| > 1.$$

Setting $y[-1]$ to be zero, we get

$$\mathcal{Y}(z) - \frac{1}{2}z^{-1}\mathcal{Y}(z) = \frac{1}{1 - z^{-1}} - \frac{(1/2)z^{-1}}{1 - z^{-1}}.$$

Therefore,

$$\mathcal{Y}(z) = \frac{1}{1 - z^{-1}}.$$

The inverse unilateral z-transform gives the zero-state response

$$y_{zs}[n] = u[n].$$

(c) Taking the unilateral z-transform of both sides of the given difference equation, we get

$$\mathcal{Y}(z) - \frac{1}{2}z^{-1}\mathcal{Y}(z) - \frac{1}{2}y[-1] = \mathcal{X}(z) - \frac{1}{2}z^{-1}\mathcal{X}(z).$$

Setting $\mathcal{X}(z) = 0$, we get

$$\mathcal{Y}(z) = \frac{1/2}{1 - \frac{1}{2}z^{-1}}.$$

The inverse unilateral z-transform gives the zero-input response

$$y_{zi}[n] = \left(\frac{1}{2}\right)^{n+1} u[n].$$

Since the input $x[n]$ is the same as the one used in the part (b), the zero-state response is still

$$y_{zs}[n] = u[n].$$