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EE360: Signals and System I

Schaum's Signals and Systems
Chapter 7
State Space Analysis

http://www.ee.unlv.edu/~b1morris/ee360/

Outline

- Concept of State
- DT State Space Representations
- DT State Equation Solutions
- CT State Space Representations
- CT State Equation Solutions

Introduction to State Space

- So far, we have studied LTI systems based on input-output relationships
  - Known as external description of a system
- Now will examine state space representation of systems
  - Known as internal description of systems

- Consists of two parts
  - State equations – set of equations relating state variables to inputs
  - Output equations – set of equations relating outputs to state variables and inputs

Advantages of State Space

- Provides new insight into system behavior
  - Use of matrix linear algebra
- Can handle multiple-input multiple-output (MIMO) systems
  - Generalize from single-input single-output
- Can be extended to non-linear and time-varying systems
  - General mathematical model
- State equations can be implemented efficiently on computers
  - Enables solving/simulation of complex systems

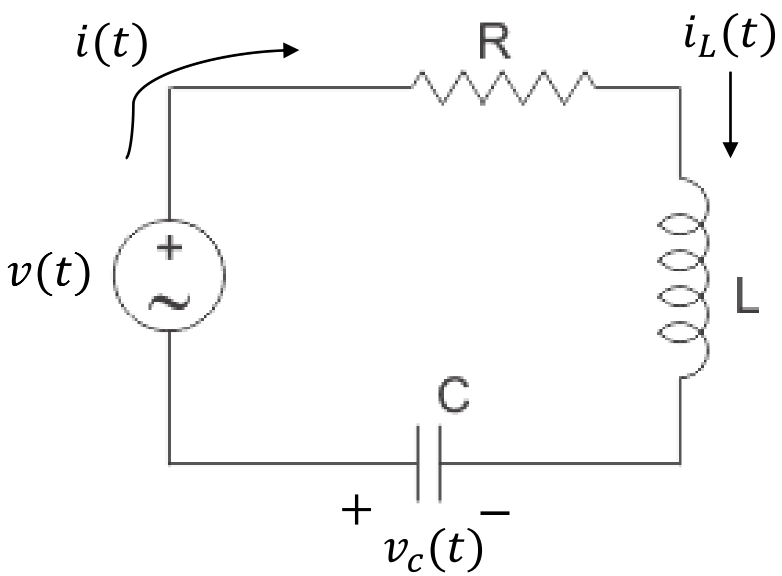
Definition of State

- State – state of a system @ time t_0 is defined as the *minimal information* that is *sufficient* to determine the <u>state</u> and <u>output</u> of a system for all times $t > t_0$ when the input is also known for $t > t_0$

- State variables (q_i) - variables that contain all state information (memory)

- Note: definition only applies to causal systems

Motivation Example



- Input: $v(t)$
- Output: $i(t)$
- State:
  - $v_L(t) = L\frac{di}{dt}$
  - $i_c(t) = C\frac{dv_c}{dt}$

- Knowing $x(t) = v(t)$ over $[-\infty, t]$ is sufficient to determine $y(t) = i(t)$ over the same interval

- If $x(t)$ is only known between $[t_0, t]$ then the output cannot be determined without knowledge of
  - Current through inductor
  - Voltage across capacitor
- Imagine being handed a circuit (system) that was in operation at time t_0
  - Initial condition problem

Selection of State Variables

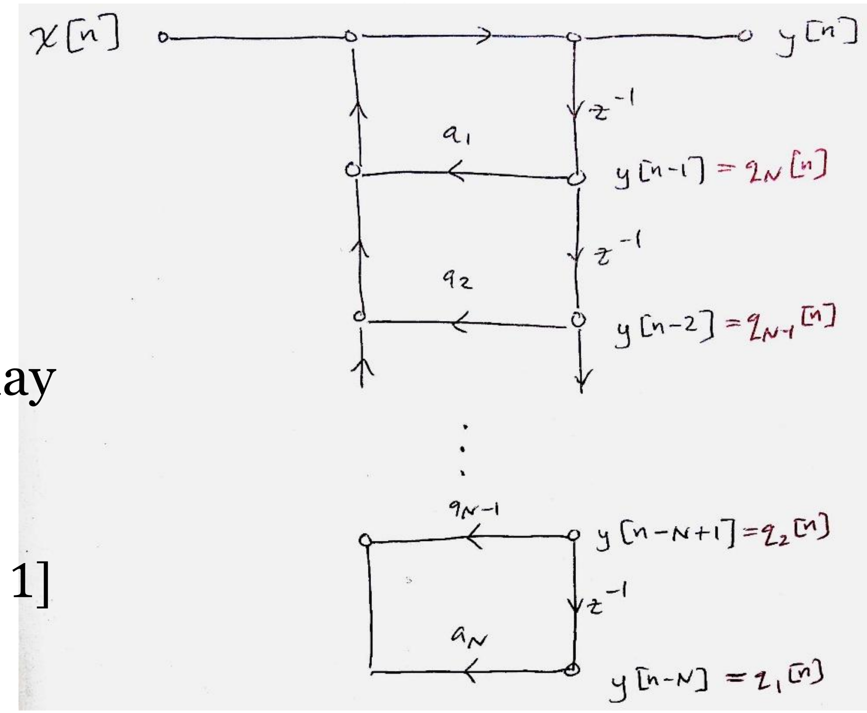
- Need to determine "memory elements" of a system

- DT: Select outputs of delay elements
- CT: Select outputs of integrators or energy-storing elements (capacitors, inductors)

- However, state-variable choice is not unique
  - Transformations of variables will result in same state space analysis

DT State Space Representation I

- Consider a single-input single-output (SISO) DT LTI system
  - $y[n] + a_1 y[n-1] + \cdots + a_N y[n-N] = x[n]$
- To uniquely determine a complete solution (output), requires N initial conditions
  - $y[-1], y[-2], \ldots, y[-N]$

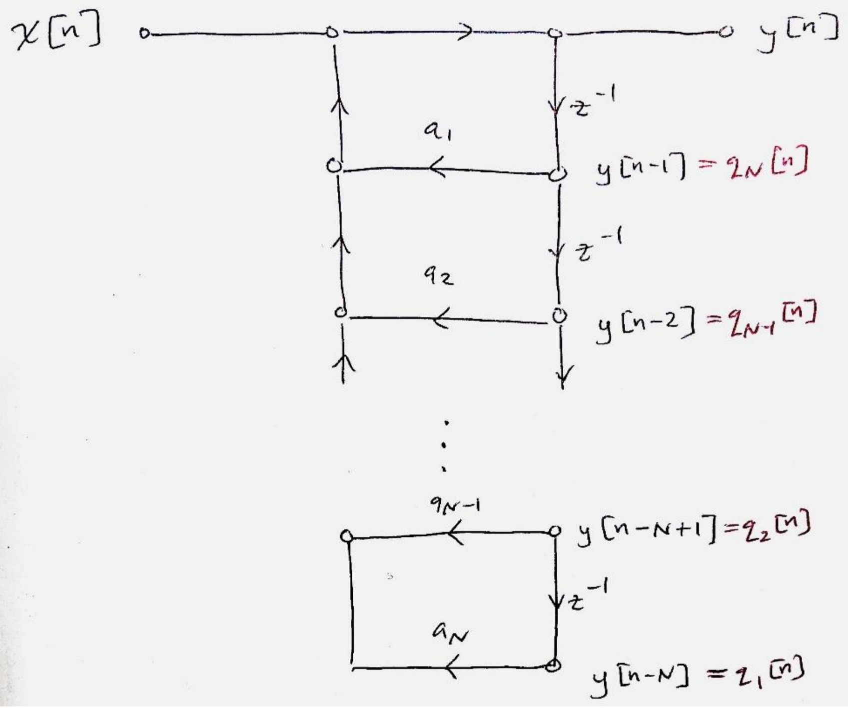


- Define state variables (outputs of delay elements)

N state vars:
  - $q_1[n] = y[n-N]$
  - $q_2[n] = y[n-(N-1)] = y[n-N+1]$
  - …
  - $q_N[n] = y[n-1]$

DT State Space Representation II

- Find next (step-ahead) state
  - By definition of delay or using signal flow graph



- $q_1[n+1] = y[n+1-N] = q_2[N]$
- $q_2[n+1] = y[n+1-N+1] = y[n-N+2] = q_3[n]$
- …
- $q_N[n+1] = y[n+1-1] = y[n] = -a_1 y[n-1] + \cdots + -a_N y[n-N]$ (recursive form)

- $q_N[n+1] = -a_1 q_N[n] - a_2 q_{N-1}[n] + \cdots + -a_N q_1[n]$

DT State Space Representation III

- These relationships can be compactly expressed in matrix form

$$\underbrace{\begin{bmatrix} q_1[n+1] \\ q_2[n+1] \\ \vdots \\ q_N[n+1] \end{bmatrix}}_{\underline{q}[n+1]} = \underbrace{\begin{bmatrix} 0 & 1 & 0 & 0 & \dots & 0 \\ 0 & 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 0 & 1 & \dots & 0 \\ & & \dots & & & \\ -a_N & -a_{N-1} & -a_{N-2} & -a_{N-3} & \dots & -a_1 \end{bmatrix}}_{\substack{\mathbf{A} \\ \text{system matrix}}} \underbrace{\begin{bmatrix} q_1[n] \\ q_2[n] \\ \vdots \\ q_N[n] \end{bmatrix}}_{\underline{q}[n]} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}}_{\mathbf{B}} x[n]$$

$$y[n] = \underbrace{\begin{bmatrix} -a_N & -a_{N-1} & \dots & -a_1 \end{bmatrix}}_{\mathbf{C}} \underbrace{\begin{bmatrix} q_1[n] \\ q_2[n] \\ \vdots \\ q_N[n] \end{bmatrix}}_{\underline{q}[n]} + \underbrace{\begin{bmatrix} 1 \end{bmatrix}}_{\mathbf{D}} x[n]$$

- State equation – next state from past state and input

  $\underline{q}[n+1] = \mathbf{A}\underline{q}[n] + \mathbf{B}\underline{x}[n]$

- Output equation – output based on state and input

  $\underline{y}[n] = \mathbf{C}\underline{q}[n] + \mathbf{D}\underline{x}[n]$

- Note: generalized form for MIMO systems with vector $\boldsymbol{y}[\boldsymbol{n}], \boldsymbol{x}[\boldsymbol{n}]$

DT State Space Representation IV

- Previous example:
  - Defined state variables as outputs of delay elements
  - Rewrote state relationships using a vectorized form of state $\underline{q}[n]$

- Goal: build state-equations given either a difference equation or block-diagram

- Note: previous example had no delayed input $x[n]$. How would delayed inputs change state space representation?
  - Consider DFII structure and develop state equations

Similarity Transformation

- Choice of state-variable is not unique

- Can have another choice of state variables as a transformation
- If $\underline{v}[n] = T\underline{q}[n]$
  - T is $N \times N$ non-singular transformation matrix
- Then, $\underline{q[n]} = T^{-1}\underline{v}[n]$

- You and your friend could have different (valid) state variable choices for same state space representation

Solution to DT State Equations

- Two approaches
  - Time-domain solution
  - Z-transform solution