

Homework #7  
Due Tu. 11/15

Note:

OW Oppenheim and Wilsky  
SSS Schaum's Signals and Systems  
SPR Schaum's Probability, Random Variables, and Random Processes

Be sure to show all your work for credit.

## 1. (SPR 3.55)

Consider an experiment of tossing a fair coin three times. Let  $(X, Y)$  be a bivariate r.v., where  $X$  denotes the number of heads in the first two tosses and  $Y$  the number of heads on the third toss.

- (a) Find the range of  $X$ .
- (b) Find the range of  $Y$ .
- (c) Find the range of  $(X, Y)$ .
- (d) Find (i)  $P(X \leq 2, Y \leq 1)$ ; (ii)  $P(X \leq 1, Y \leq 1)$ ; and (iii)  $P(X \leq 0, Y \leq 0)$ .

## 2. (SPR 3.57)

Let the joint pmf of  $(X, Y)$  be giving by

$$p_{XY}(x_i, y_j) = \begin{cases} k(x_i + y_j) & x_i = 1, 2, 3; y_j = 1, 2 \\ 0 & \text{else} \end{cases}$$

where  $k$  is a constant.

- (a) Find the value of  $k$ .
- (b) Find the marginal pmf's of  $X$  and  $Y$ .

## 3. (SPR 3.58)

The joint pdf of  $(X, Y)$  is given by

$$f_{XY}(x, y) = \begin{cases} ke^{-(x+2y)} & x > 0, y > 0 \\ 0 & \text{else} \end{cases}$$

where  $k$  is a constant.

- (a) Find the value of  $k$ .
- (b) Find  $P(X > 1, Y < 1)$ ,  $P(X < Y)$ , and  $P(X \leq 2)$ .

## 4. (SPR 3.63)

The joint pdf of  $(X, Y)$  is given by

$$f_{XY}(x, y) = \begin{cases} e^{-(x+y)} & x > 0, y > 0 \\ 0 & \text{else} \end{cases}$$

where  $k$  is a constant.

- (a) Are  $X$  and  $Y$  independent?
- (b) Find the conditional pdf's of  $X$  and  $Y$ .

5. (SPR 3.66)

Consider a bivariate r.v.  $(X, Y)$  with joint pdf

$$f_{XY}(x, y) = \frac{1}{2\pi\sigma^2} e^{-(x^2+y^2)/(2\sigma^2)} \quad -\infty < x, y < \infty.$$

Find  $P[(X, Y)|x^2 + y^2 \leq a^2]$ .

6. (SPR 4.87)

Let  $Y = 2X + 3$ . Find the pdf of  $Y$  if  $X$  is a uniform r.v. over  $(-1, 2)$ .

7. (SPR 4.92)

Let  $X$  denote the number of heads obtained when three independent tossings of a fair coin are made. Let  $Y = X^2$ . Find  $E[Y]$ .