

Homework #7  
Due Su 11/20

Note:

OW Oppenheim and Wilsky  
SSS Schaum's Signals and Systems  
SPR Schaum's Probability, Random Variables, and Random Processes

Be sure to show all your work for credit.

## 1. (SPR 3.55)

Consider an experiment of tossing a fair coin three times. Let  $(X, Y)$  be a bivariate r.v., where  $X$  denotes the number of heads in the first two tosses and  $Y$  the number of heads on the third toss.

- (a) Find the range of  $X$ .
- (b) Find the range of  $Y$ .
- (c) Find the range of  $(X, Y)$ .
- (d) Find (i)  $P(X \leq 2, Y \leq 1)$ ; (ii)  $P(X \leq 1, Y \leq 1)$ ; and (iii)  $P(X \leq 0, Y \leq 0)$ .

## 2. (SPR 3.57)

Let the joint pmf of  $(X, Y)$  be giving by

$$p_{XY}(x_i, y_j) = \begin{cases} k(x_i + y_j) & x_i = 1, 2, 3; y_j = 1, 2 \\ 0 & \text{else} \end{cases}$$

where  $k$  is a constant.

- (a) Find the value of  $k$ .
- (b) Find the marginal pmf's of  $X$  and  $Y$ .

## 3. (SPR 3.58)

The joint pdf of  $(X, Y)$  is given by

$$f_{XY}(x, y) = \begin{cases} ke^{-(x+2y)} & x > 0, y > 0 \\ 0 & \text{else} \end{cases}$$

where  $k$  is a constant.

- (a) Find the value of  $k$ .
- (b) Find  $P(X > 1, Y < 1)$ ,  $P(X < Y)$ , and  $P(X \leq 2)$ .

## 4. (SPR 3.59)

Let  $(X, Y)$  be a bivariate RV, where  $X \sim U[0, 1]$  and  $Y$  is an exponential RV with parameter 5, and  $X$  and  $Y$  are independent.

- (a) Find the joint pdf of  $(X, Y)$ .
- (b) Find  $P(Y \leq X)$ .

## 5. (SPR 3.63)

The joint pdf of  $(X, Y)$  is given by

$$f_{XY}(x, y) = \begin{cases} e^{-(x+y)} & x > 0, y > 0 \\ 0 & \text{else} \end{cases}.$$

- (a) Are  $X$  and  $Y$  independent?
- (b) Find the conditional pdf's of  $X$  and  $Y$ .

## 6. (SPR 3.65)

Consider the bivariate RV  $(X, Y)$  of Prob 3.14 (below)

$$p_{XY}(x_i, y_j) = \begin{cases} k(2x_i + y_j) & x_i = 1, 2; y_j = 1, 2 \\ 0 & \text{otherwise} \end{cases}.$$

- (a) Find the mean and variance of  $X$ .
- (b) Find the mean and variance of  $Y$ .
- (c) Find the covariance of  $X$  and  $Y$ .
- (d) Find the correlation coefficient of  $X$  and  $Y$ .

## 7. (SPR 3.66)

Consider a bivariate r.v.  $(X, Y)$  with joint pdf

$$f_{XY}(x, y) = \frac{1}{2\pi\sigma^2} e^{-(x^2+y^2)/(2\sigma^2)} \quad -\infty < x, y < \infty.$$

Find  $P[(X, Y)|x^2 + y^2 \leq a^2]$ .

## 8. (SPR 3.68)

The joint pdf of a bivariate RV  $(X, Y)$  is given by

$$f_{XY}(x, y) = \frac{1}{\sqrt{3}\pi} \exp \left[ -\frac{2}{3}(x^2 - xy + y^2) \right].$$

- (a) Find the means and variances of  $X$  and  $Y$ .
- (b) Find the correlation coefficient of  $X$  and  $Y$ .