

EE361: Signals and System II

Spring 2015

TTh 13:00-14:15 BEH 221

Probability Distributions

15/03/19

Big Idea: Probability Distribution

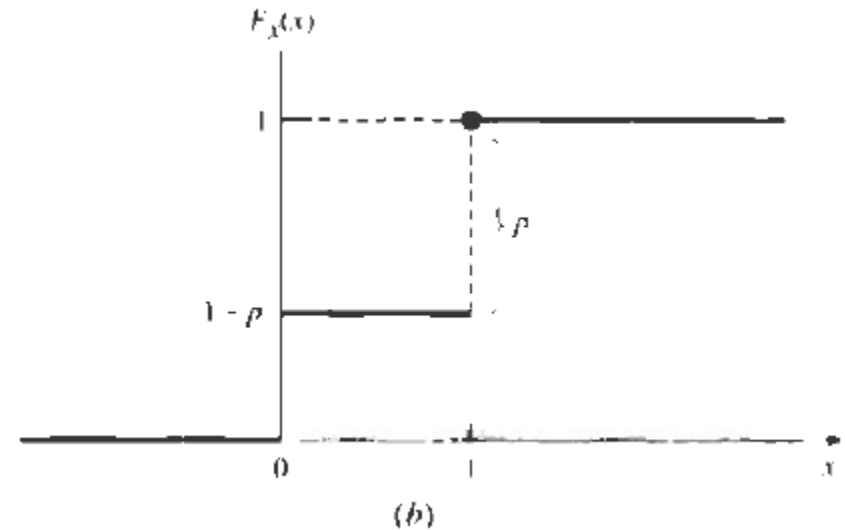
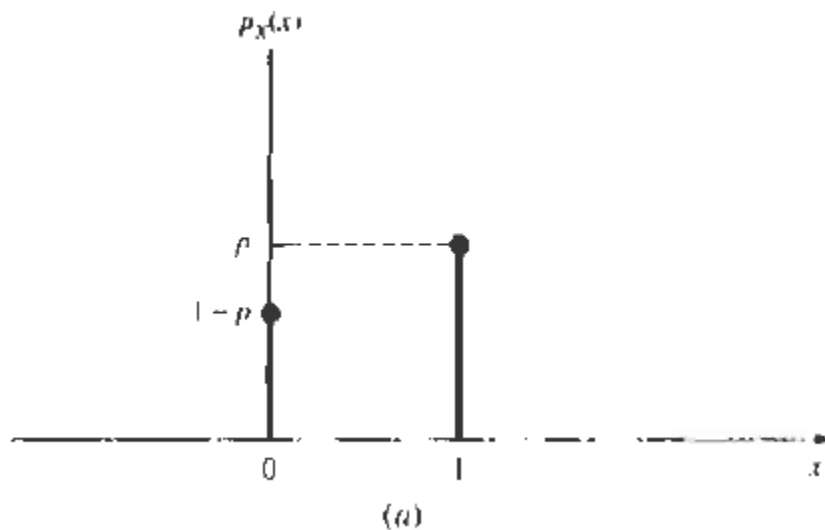
- Assign a probability to each of the possible outcomes of a random experiment
- Discrete
 - Probability mass function (pmf) – probability of each possible outcome
 - E.g. probability a roll of die will come up with a 3
- Continuous
 - Probability density function (pdf) – probability the outcome is within a range of values (interval)
 - E.g. probability that a 500 g package is between 490-510 g

Special Distributions

- Discrete
 - Bernoulli
 - Binomial
 - Geometric
 - Negative Binomial
 - Poisson
 - Uniform
- Continuous
 - Uniform
 - Exponential
 - Gamma
 - Normal

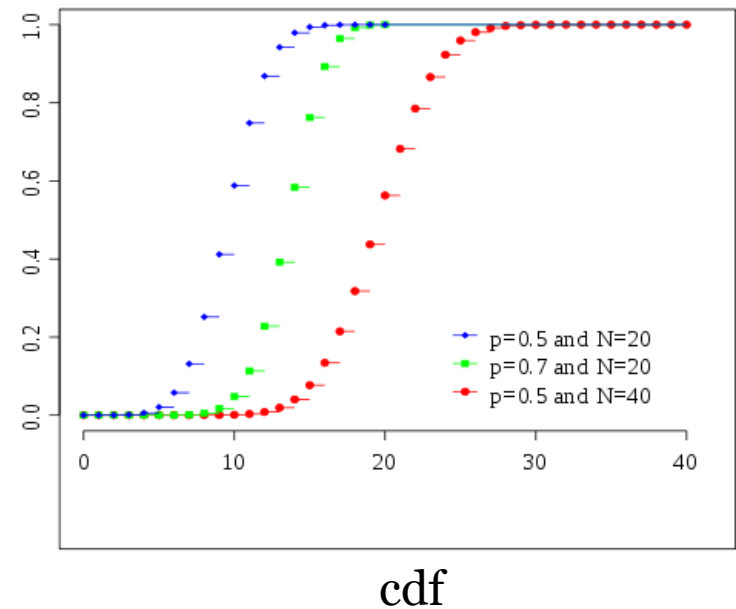
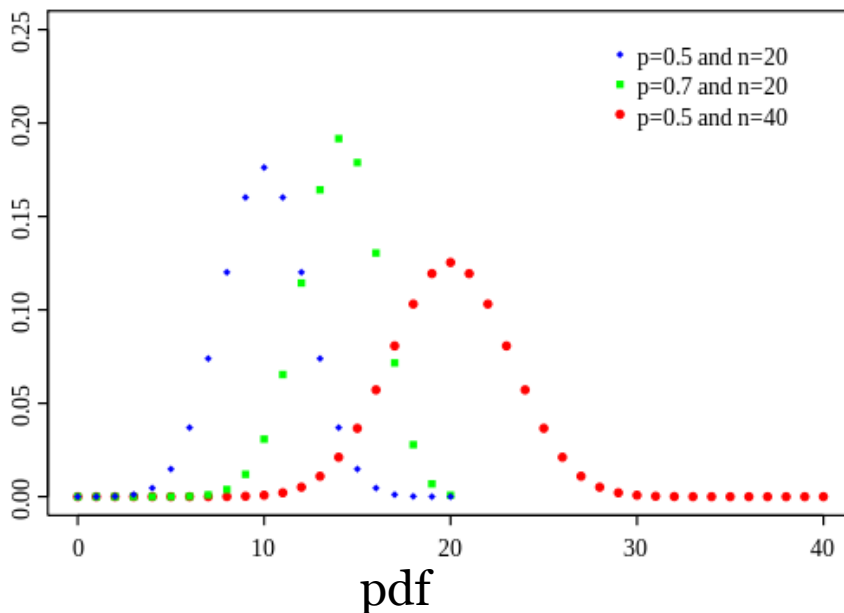
Bernoulli Distribution

- Binary RV with probability p of 1 (“success”)
- $p_X(k) = P(X = k) = p^k(1 - p)^{1-k}$



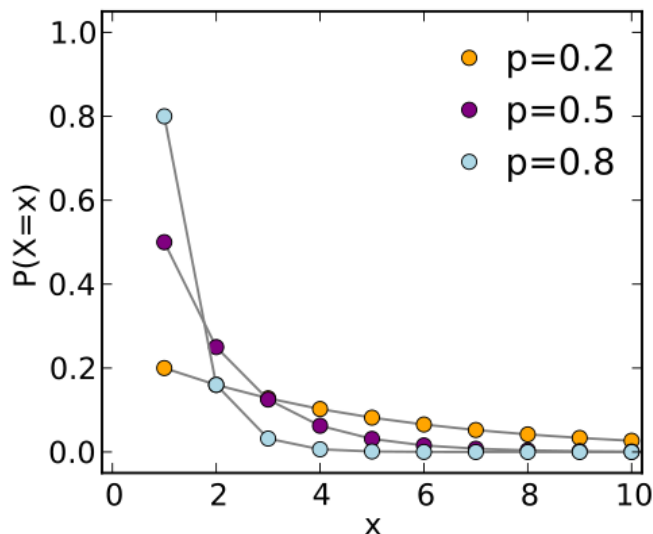
Binomial Distribution

- RV to count the number of successes with n independent Bernoulli trials
- $p_X(k) = P(X = k) = \binom{n}{k} p^k (1 - p)^{n-k}$
 - $\binom{n}{k}$ - n choose k ways to get k successes

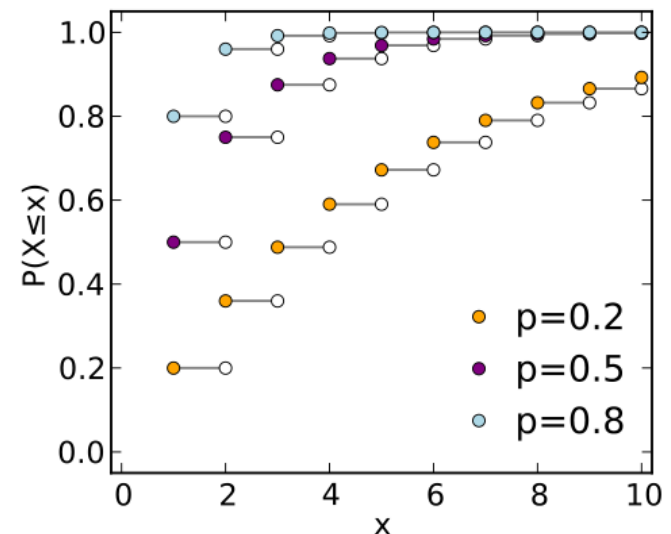


Geometric Distribution

- Sequence of Bernoulli trials observed until first success
- $p_X(x) = P(X = x) = (1 - p)^{x-1}p$



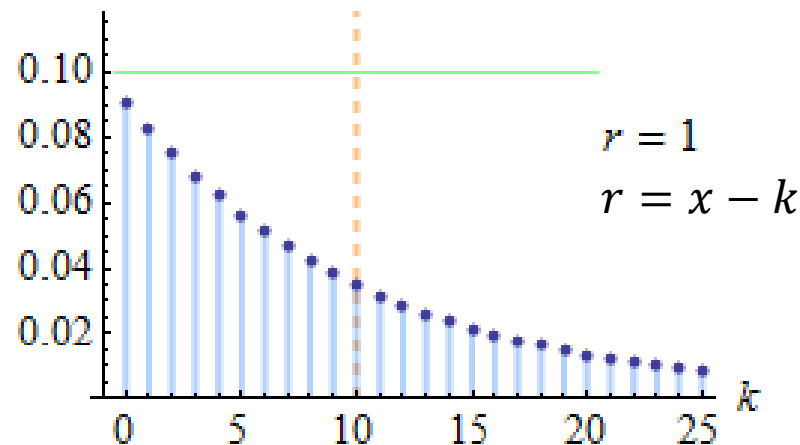
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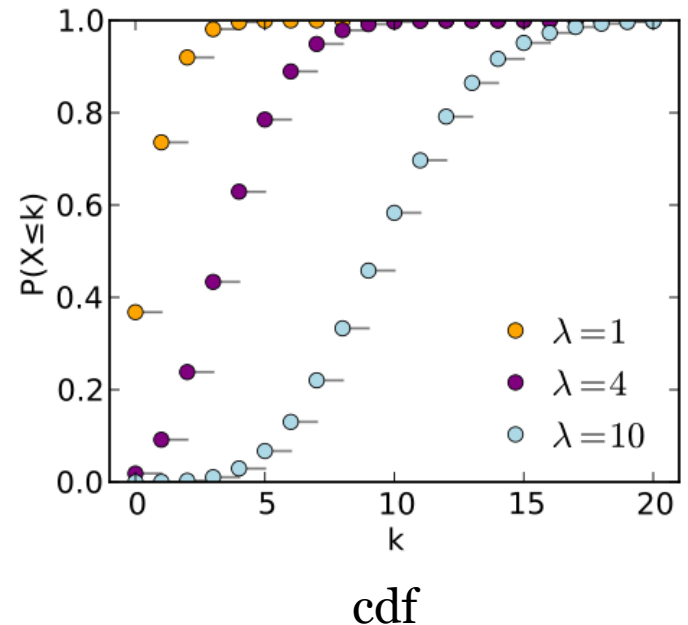
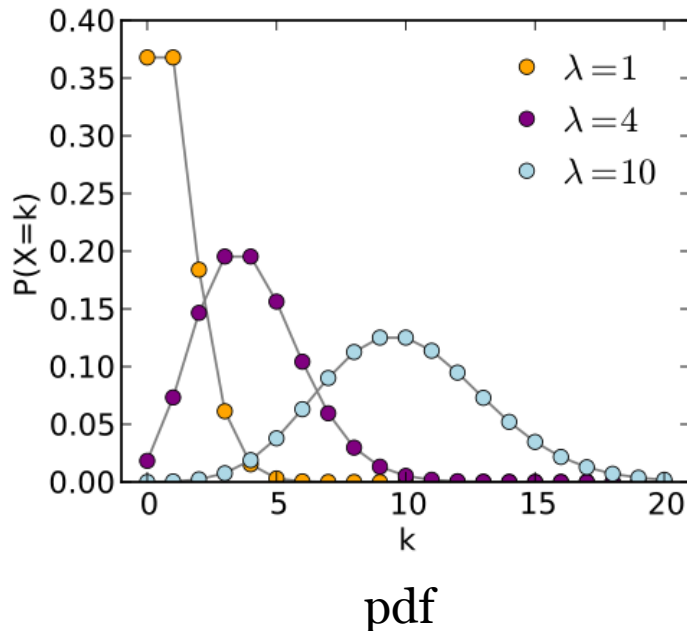
Negative Binomial Distribution

- Number of trials until k th success in sequence of Bernoulli trials
- $p_X(x) = P(X = x) = \binom{x-1}{k-1} p^k (1-p)^{x-k}$



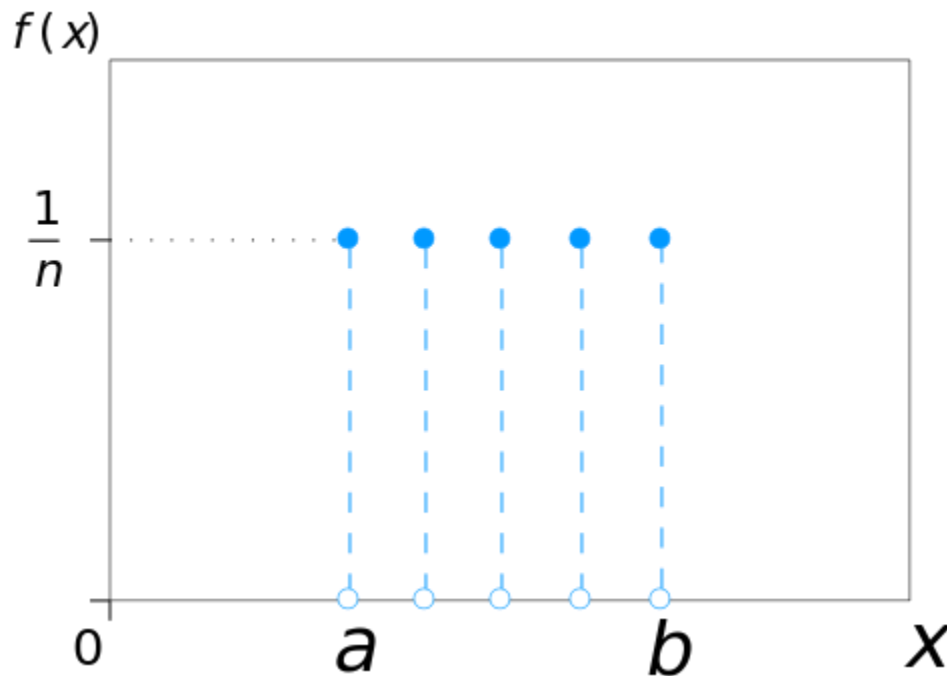
Poisson Distribution

- The number of events occurring in a fixed interval (time or space) given a known event average rate λ
- $p_X(k) = P(X = k) = e^{-\lambda} \frac{\lambda^k}{k!}$

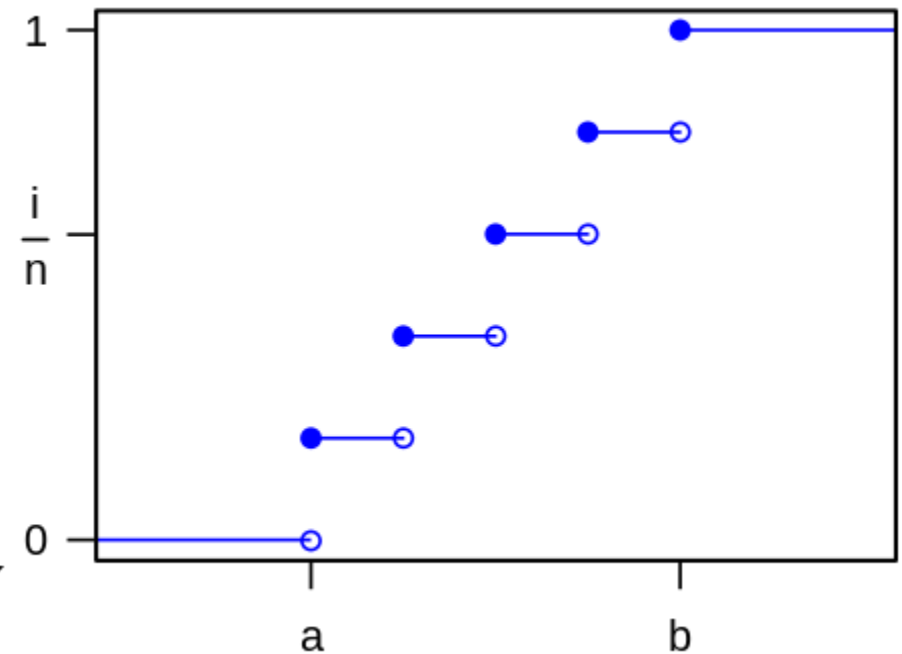


Discrete Uniform Distribution

- $p_X(x) = P(X = x) = \frac{1}{n}$



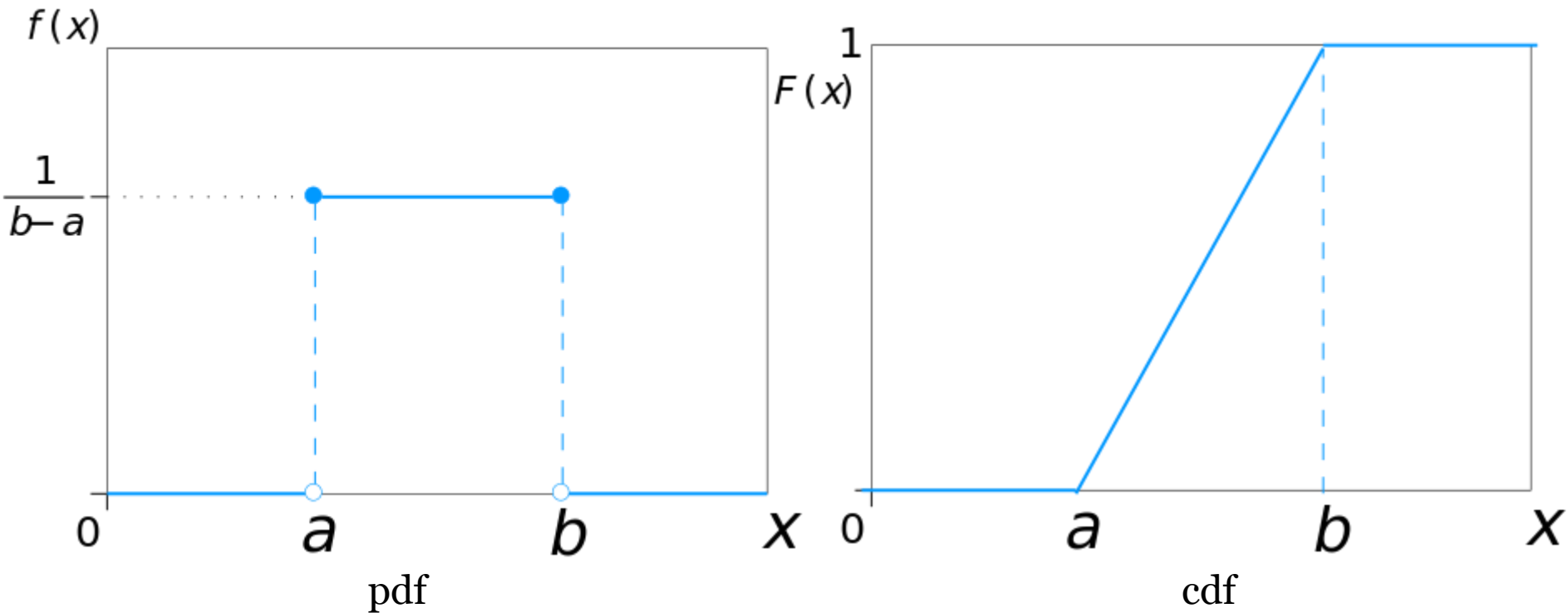
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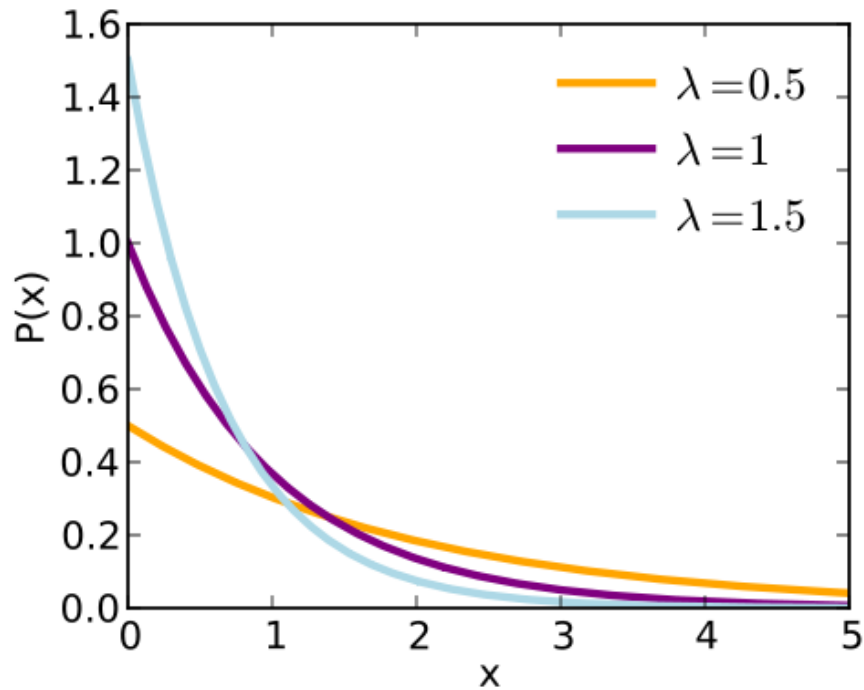
Continuous Uniform Distribution

- $f_X(x) = \begin{cases} \frac{1}{b-a} & a < x < b \\ 0 & \text{else} \end{cases}$

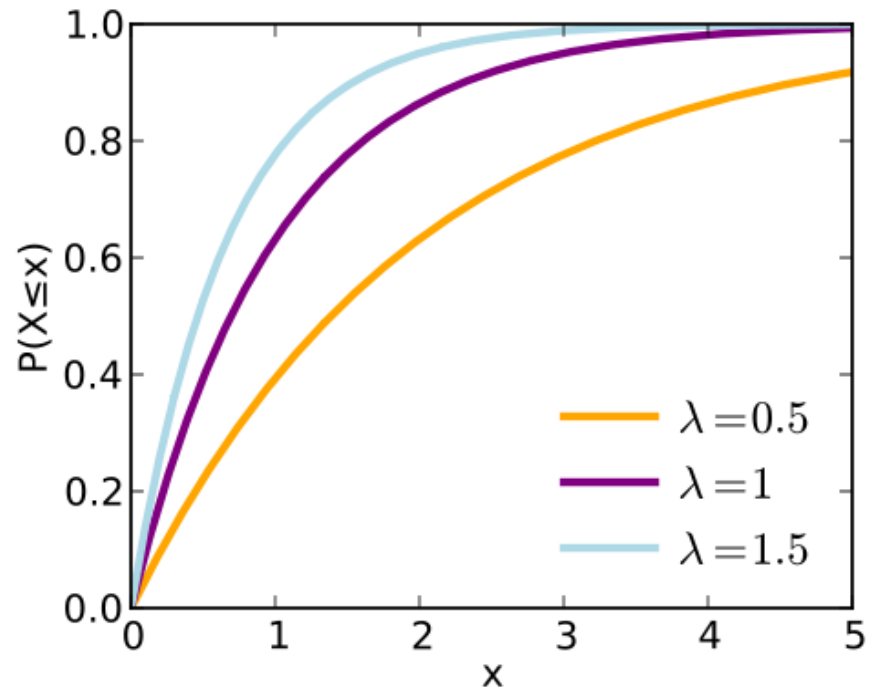


Exponential Distribution

- $f_X(x) = \lambda e^{-\lambda x} \quad x > 0$



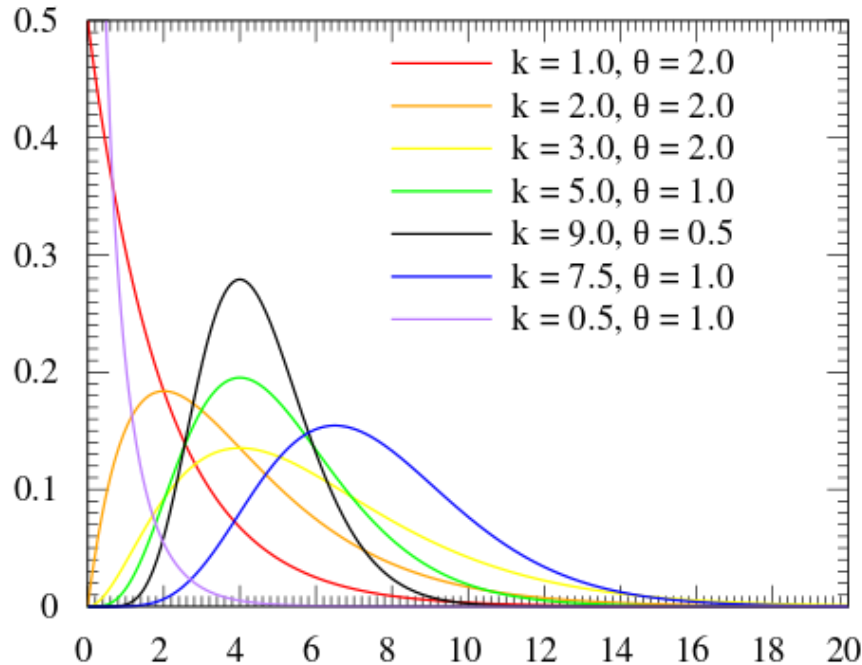
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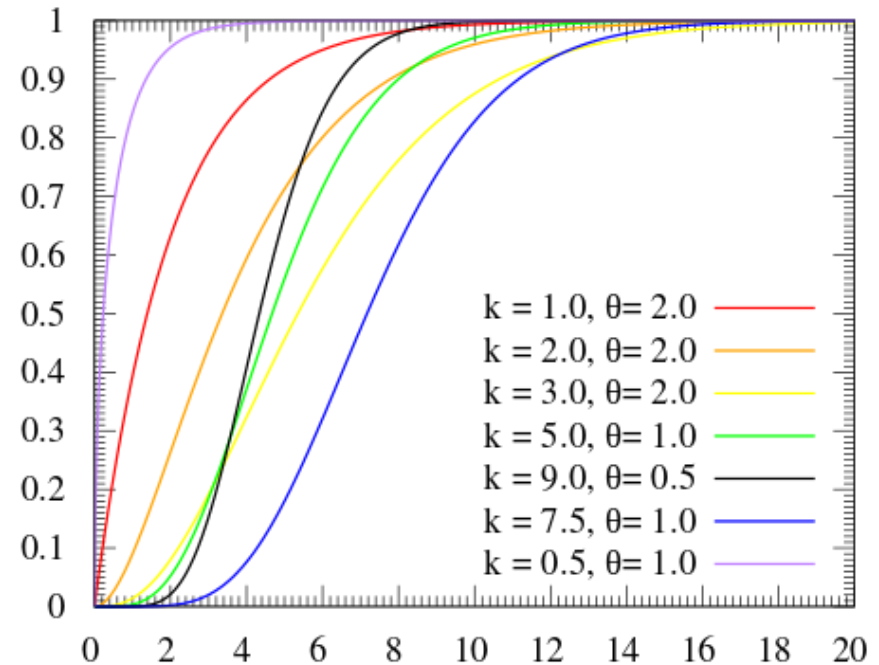
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Gamma Distribution

- $$f_X(x) = \frac{\lambda e^{-\lambda x} (\lambda x)^{\alpha-1}}{\Gamma(\alpha)} \quad x > 0$$



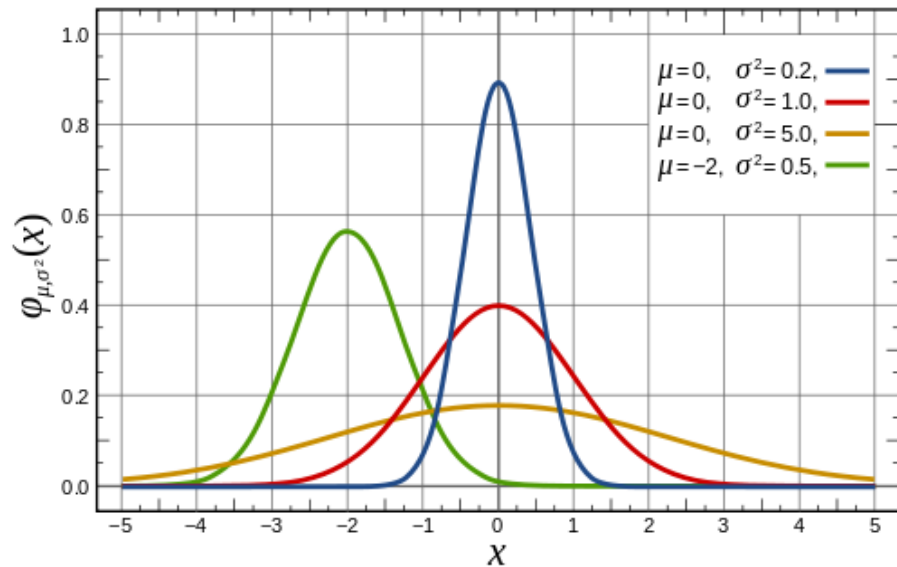
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 α – shape parameter $\theta = \frac{1}{\lambda}$ rate parameter (inverse scale)

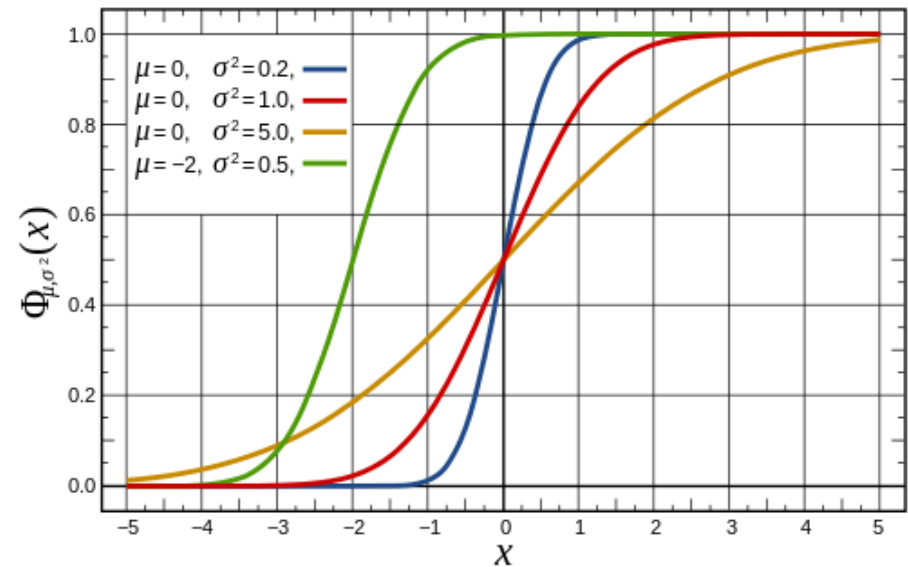
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Normal (Gaussian) Distribution

- $f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x-\mu)^2/2\sigma^2}$



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