Homework #2
Due Mo. 2/10

You must turn in your code as well as output files. Please generate a report that contains the code and output in a single readable format.

Visit the book website to download companion software, including all the example problems.


1. (KLT 3.1)
   Solution
   The 3 dB bandwidth is when power is 1/2, or gain is 1/√2. Therefore,
   \[
   20 \log_{10}(|H(\omega)|) = 3
   \]
   \[
   |H(\omega)| = \frac{1}{\sqrt{2}} = \sqrt{\frac{1}{2}[1 + \cos \omega]}
   \]
   \[
   \vdots
   \]
   \[
   \cos \omega = 0
   \]
   \[
   \omega_3 = \frac{\pi}{2}
   \]
   Since the sampling frequency \( f_s = 8 \text{ kHz} \), the bandwidth is \( f_3 = \frac{\omega_3 f_s}{2\pi} = 2 \text{ kHz} \).

2. (KLT 3.5)
   Solution
   \[
   y(n) = x(n) + x(n - L)
   \]
   \[
   Y(z) = X(z) + z^{-L}X(z)
   \]
   \[
   H(z) = \frac{Y(z)}{X(z)} = 1 + z^{-L}
   \]
   The zeros are located at \( z_L = -1 \). This has \( L \) equally spaced zeros on the unit circle
   \[
   z_l = e^{j \frac{2\pi l - \pi}{L}}, \quad l = 0, 1, ..., L - 1.
   \]
   The resulting comb filter for \( L = 8 \) has notches shifted by \( \pi/8 \) or 0.125 in normalized frequency. See Fig. 1 for the pole/zero locations and the magnitude and phase plots.

3. (KLT 3.9)
   Use \texttt{subplot(1,3,.)} to create a single Figure for (a) and (b).
   Solution
   See Fig. 2 for the pole/zero, magnitude, and phase plot. In order to plot the phase you need to use \texttt{unwrap.m} to take care of jumps that arise from the arctan function.
4. (KLT 3.11)

Notice the typo, the filter length should be \( L = 2M + 1 \).

**Solution**

There are various ways to attack this problem. Notice that a high pass filter is equivalent to
a low pass filter that has been shifted by $\pi$ radians.

$$H_{hp}(\omega) = H_{lp}(\omega - \pi) = \begin{cases} 1 & |\omega| \geq \omega_c \\ 0 & \text{else} \end{cases}.$$  

This requires you to design a LP filter with bandwidth of $1 - \omega_c$ to match the HP bandwidth after shifting. Therefore in this problem, design a LP filter with cutoff $\omega_{cl} = 0.4\pi$ and shift the resulting system to get the HP filter.

Shifting in the frequency domain results in multiplication in the time domain

$$h_{hp}(n) = e^{j\pi n} h_{lp}(n) = e^{j\pi n} \frac{\sin \omega_{cl} n}{\pi n}.$$  

The resulting FIR filter is obtained by truncating and shifting the results by $M$ to get a causal system

$$h(n) = h_{hp}(n - M), \quad n = 0, 1, \ldots, L - 1 = e^{j\pi (n-M)} \frac{\sin \omega_{cl} (n - M)}{\pi (n - M)}.$$  

The resulting filters for $M = 32$ and $M = 64$ is presented in Fig. 3. Notice the ringing is less for the larger filter and that the phase is linear in the passband of the filter.

![Figure 3](image-url)

5. (KLT 3.12)

Solution

Using the windows smooths the frequency response to remove the rippling. The Blackman window does more smoothing. See Fig. 4 for comparison between the different windowing schemes. Notice that the more samples results in sharper transition at cutoff frequency and larger slope in phase, and greater time delay.

6. (KLT 3.13)

Solution
(a)

(b)

(c)

(d)

Figure 4: (a)-(b) $M = 32$, (c)-(d) $M = 64$

(a)

(b)

Figure 5
This problem can be approached similarly to (3.11). The bandpass filter should be 1 minus
a LP filter and a HP filter. The results using Matlab are shown in Fig. 5. Notice again that
the windows affect the ripple and that the phase is linear within the passband of the filter.

7. (KLT 3.21)

Compare the resulting sounds using `soundsc.m`.

**Solution**

The resampling procedure changes the speech sound. There isn’t a noticeable difference
between the original 16 kHz and 12 kHz. As the sample frequency gets smaller, the speech
gets more distorted by losing high frequency components. It sounds like the speech is coming
from within a can with a hallow sound.

```plaintext
s = load('TIMIT_4.ASC');
fs = 16000;

% play sound
soundsc(s, fs);
```

```plaintext
for fp = [12000, 5000, 3000]
    x = fp/fs;
    [n,d]=rat(x);
    ss = resample(s, n, d);
    soundsc(ss, fp)
end
```