Homework #3
Due W 2/14

You must turn in your code as well as output files. Please generate a report that contains the code and output in a single readable format.

Visit the book website to download companion software, including all the example problems.

`http://www.wiley.com/WileyCDA/WileyTitle/productCd-1118414322.html`

1. (KLT 4.5)

   Solution

   Note that this transfer function can be re-written as

$$H(z) = \frac{(1+\sqrt{2}z^{-1}+z^{-2})(1+2z^{-1}+z^{-2})}{(1+0.8z^{-1}+0.64z^{-2})(1+\frac{13}{12}z^{-1}+\frac{1}{4}z^{-2})} = \frac{(1+\sqrt{2}z^{-1}+z^{-2})(1+2z^{-1}+z^{-2})}{(1+a_{11}^{-1}+a_{21}z^{-2})(1+a_{12}z^{-1}+a_{22}z^{-2})}.$$

   Solving for the roots results in

zeros	$\frac{1}{\sqrt{2}}(1+j)$	$\frac{1}{\sqrt{2}}(1-j)$	-1	-1
poles	$\frac{1}{3}$	$\frac{3}{4}$	$\frac{2}{5}(1+j\sqrt{3})$	$\frac{2}{5}(1-j\sqrt{3})$

   For stability, the poles must all be inside the unit circle. This is true for this system. This can also be verified by examining the second-order sections using the stability triangle. The results are presented below.

Criterion	Stage 1	Stage 2
$\lvert a_2\rvert < 1$	$\lvert a_{21}\rvert = 0.64 < 1$	$\lvert a_{22}\rvert = 0.25 < 1$
$\lvert a_1\rvert < 1 + a_2$	$\lvert a_{11}\rvert = 0.8 < (1 + a_{21} = 1.64)$	$\lvert a_{12}\rvert = 1.0833 < (1 + a_{22} = 1.25)$

2. (KLT 4.7)

   Solution

   (a) You should know how to do this.

   (b) There are various ways to solve this problem. Remember $H(\omega) = H(z)|_{z=e^{j\omega}}$.
   Method 1:

$$\begin{aligned} |H(\omega)|^2 = H(\omega)H^*(\omega) &= \frac{e^{-j\omega}-a}{1-ae^{-j\omega}}\frac{e^{j\omega}-a}{1-ae^{j\omega}} \\ &= \frac{1-ae^{-j\omega}-ae^{j\omega}+a^2}{1-ae^{j\omega}-ae^{-j\omega}+a^2} = \frac{1+a^2-2a\cos\omega}{1+a^2-2a\cos\omega} \\ &= 1 \end{aligned}$$

   Method 2:

$$H(\omega) = e^{-j\omega}\frac{1-ae^{j\omega}}{1-ae^{-j\omega}}$$
$$|H(\omega)| = |e^{-j\omega}|\frac{|1-ae^{j\omega}|}{|1-ae^{-j\omega}|}$$

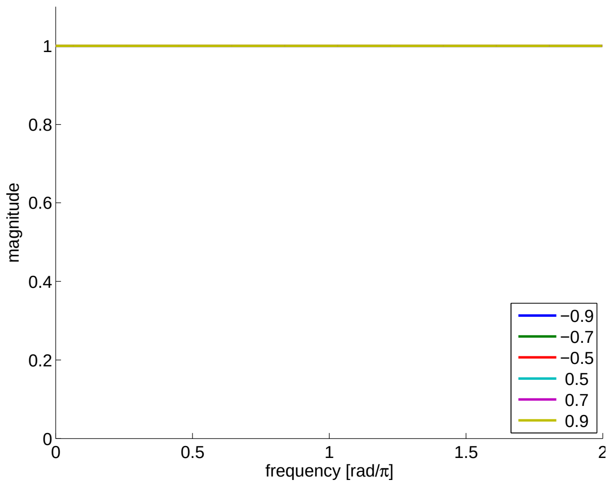
The term $|e^{-j\omega}| = 1$ because it is on the unit circle and the fraction numerator and denominator are complex conjugates of one another and therefore make the fraction 1.

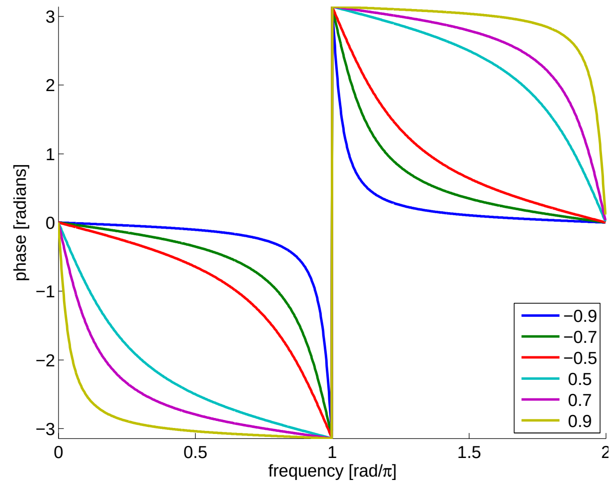
(c)

$$\angle H(\omega) = \angle \frac{e^{-j\omega} - a}{1 - ae^{-j\omega}} = -\omega - 2\arctan\left[\frac{r\sin(\omega - \theta)}{1 - r\cos(\omega - \theta)}\right]$$

when $a = re^{j\theta}$ is a complex number.

(d) The magnitude and phase plots are shown below.





3. (KLT 4.9)

All realizations should use DFII. Cascade realizations should account for numerical effects. Compare the result with Matlab's cascade. Only one parallel realization is required.

Solution

(a) Factoring the transfer function results in the following paring

	Stage 1	Stage 2
zeros	$0.8070 \pm j0.8439$	$-0.7236 \pm j0.8360$
poles	$0.5 \pm j0.5$	$0.5 \pm j0.2887$

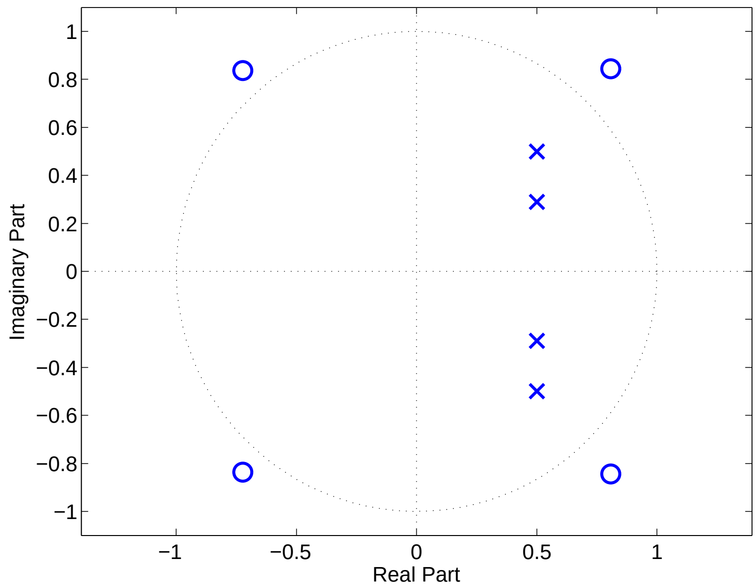
based off the pole zero locations

(b) Two different cascade realization can be found by different stage ordering techniques

```
[sos1, G] = tf2sos(b,a, 'up')
[sos2, G] = tf2sos(b,a, 'down')
```

This results in the following systems

```
sos1 =
    1.0000    1.4473    1.2225    1.0000   -1.0000    0.3333
    1.0000   -1.6139    1.3633    1.0000   -1.0000    0.5000

sos2 =
    1.0000   -1.6139    1.3633    1.0000   -1.0000    0.5000
    1.0000    1.4473    1.2225    1.0000   -1.0000    0.3333
```



The ordering in part (a) is from pole nearest the unit circle to furthest which is the `'down'` option for `tf2sos.m`. This is opposite the default ordering of pole nearest the origin to furthest (`'up'`).

(c) The parallel implementation can be found using the `residuez.m` function. However, since this only returns first order sections, this has to be transformed into DFII 2^{nd}-order.

```
[r, p, c] = residuez(b,a);
%make second order
sp(1,:) = [(r(1)*[1 -p(2)] + r(2)*[1 -p(1)]) conv([1 -p(1)], [1 -p(2)])];
sp(2,:) = [(r(3)*[1 -p(4)] + r(4)*[1 -p(3)]) conv([1 -p(3)], [1 -p(4)])]

sp =
   33.0000    -35.0000     1.0000    -1.0000        0.5000
  -51.0000    56.6667      1.0000    -1.0000        0.3333

c = 20
```

The rows of `sp` indicate the second order sections (2 terms for numerator and 3 terms for denominator) and `c` is a constant term. Notice the gain is factored into each section where in the cascade the gain is separated.

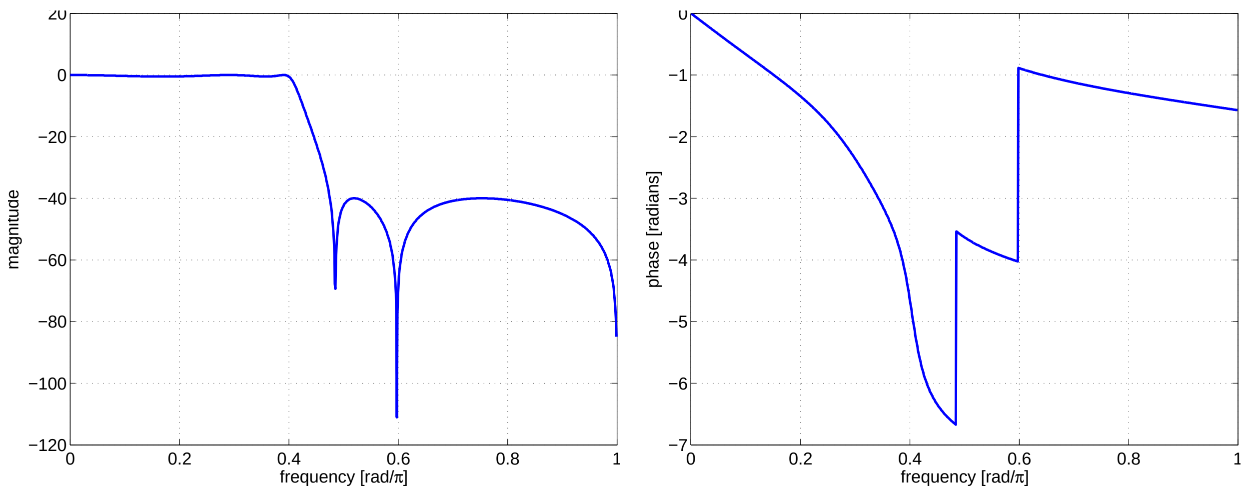
4. (KLT 4.10)

Please provide a 1×2 subplot of the magnitude and phase response as well as the a and b coefficient vectors.

Solution

```
f = 8000;
fp = 1600;
fs = 2000;
dp = 0.5;
ds = 40;
```

```
%determine order
[N, Wp] = ellipord(fp/f, fs/f, dp, ds);
[b,a] = ellip(N, dp, ds, Wp);

[H,w] = freqz(b,a,1024);

plot(w/pi, 20*log10(abs(H)), 'linewidth', 2);
xlabel('frequency [rad/\pi]'); ylabel('magnitude'); grid on;

plot(w/pi, unwrap(angle(H)), 'linewidth', 2);
xlabel('frequency [rad/\pi]'); ylabel('phase [radians]'); grid on;
```




```
b =
    0.0162   -0.0210    0.0156    0.0156   -0.0210    0.0162
a =
    1.0000   -3.7791    6.1915   -5.3939    2.4879   -0.4847
```

5. (KLT 4.12)

   Solution

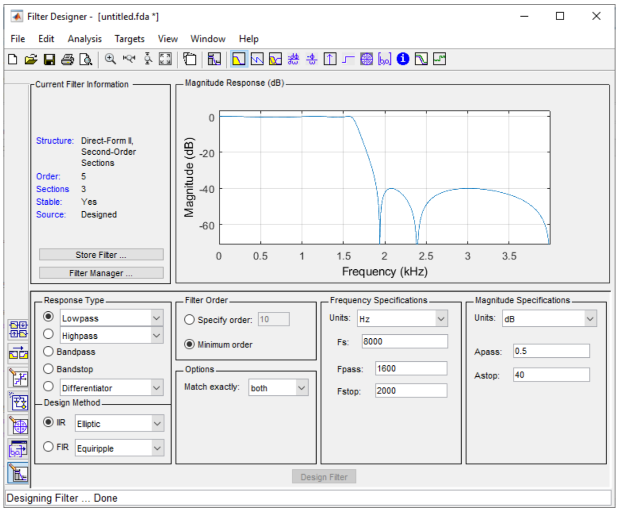
   When using `fdatool.m`, notice that the resulting filter is given in cascade form rather than DF (click the [**b,a**] button in the top right to see the coefficients for three stages). Filter quantization can be selected from the bottom left control panel. You should not see any difference with the 16-bit fixed point quantization. Quantization only becomes noticable at 6-bits.

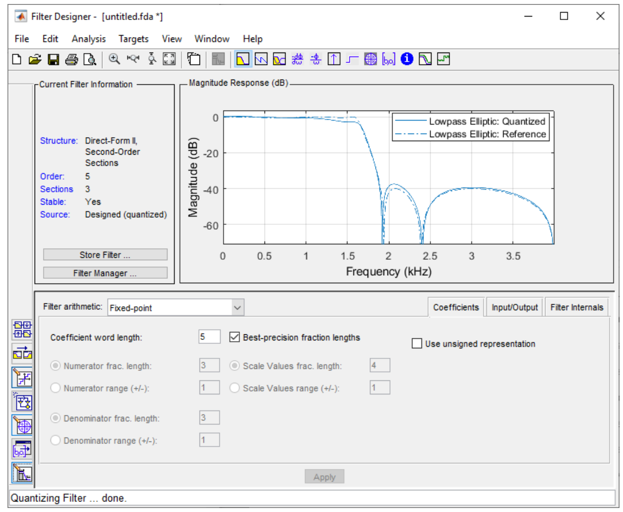
   Note: FDATool should be replaced by `filterDesigner.m`.

6. (KLT 4.14)

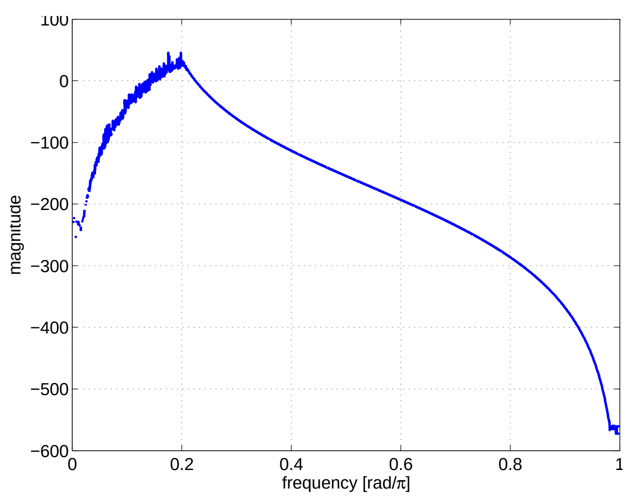
   Solution

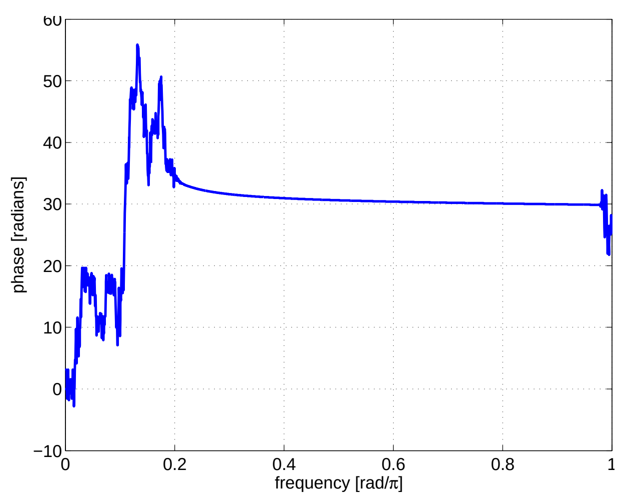
```
f = 8000;
fp = [450 650];
fs = [350 750];
```



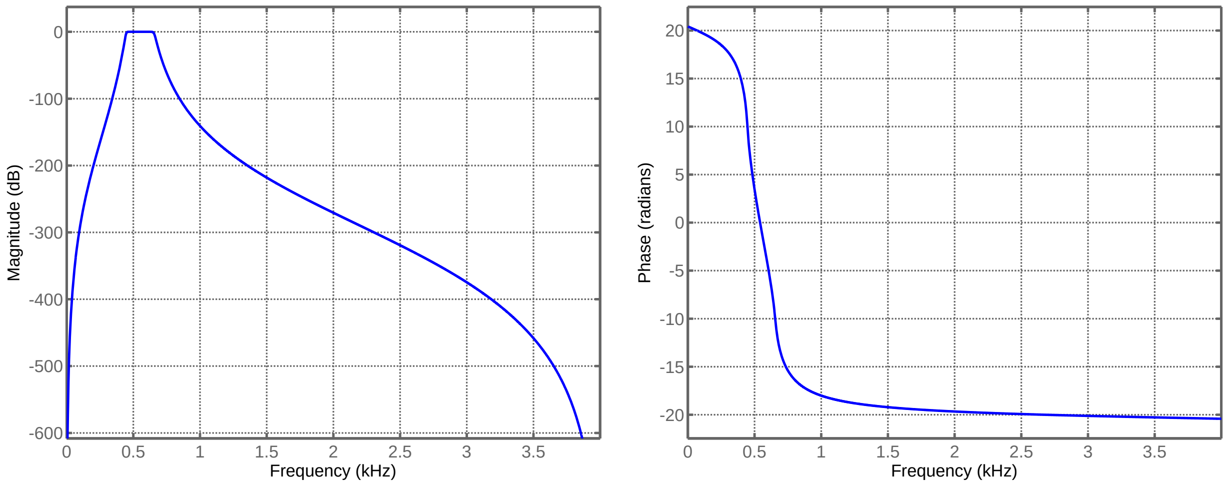



```
dp = 1;
ds = 60;

%determine order
[N, Wn] = buttord(2*fp/f, 2*fs/f, dp, ds)
[b,a] = butter(N, Wn), 'bandpass');

[H,w] = freqz(b,a,1024);

plot(w/pi, 20*log10(abs(H)), 'linewidth', 2);
xlabel('frequency [rad/\pi]'); ylabel('magnitude'); grid on;

plot(w/pi, unwrap(angle(H)), 'linewidth', 2);
xlabel('frequency [rad/\pi]'); ylabel('phase [radians]'); grid on;
```





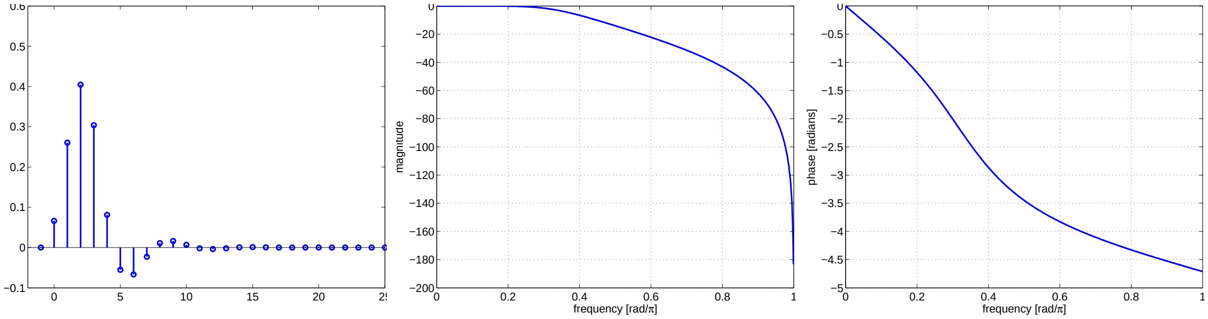
Notice that the design does not match the specifications. The gain is greater than 1 in the pass band so there is some problem in the design specifications. However, if you use the FDAtool you can do the design using `fdesign.m` from the signal processing toolbox as shown below



7. (KLT 4.18)

Solution

```
b = 0.0662*[1 3 3 1];
a = [1 -0.9356 0.5671 -0.1016];

%impulse input
x = zeros(100); x(10)=1;
y = filter(b,a,x);

stem([1:100]-10, y(1:100), 'linewidth',2);
set(gca, 'xlim', [-2 25]);

[H,w] = freqz(b,a,1048);
plot(w/pi, 20*log10(abs(H)), 'linewidth', 2);
plot(w/pi, unwrap(angle(H)), 'linewidth', 2);
```



Notice that the impulse response does die down to zero indicating stability. Even though this is an IIR design, practically, it has a finite length.