CLOUD STORAGE AND ONLINE BIN PACKING

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ABSTRACT

Cloud Storage and Online Bin Packing

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Cloud storage is the service provided by some corporations (such as Mozy and Carbonite) to store and backup computer files. We study the problem of allocating memory of servers in a data center based on online requests for storage. Over-the-net data backup has become increasingly easy and cheap due to cloud storage. Given an online sequence of storage requests and a cost associated with serving the request by allocating space on a certain server one seeks to select the minimum number of servers as to minimize total cost. We use two different algorithms and propose a third algorithm; we show that all algorithms perform well when the requests are random. The work here is related to "bin packing", a well studied problem in theoretical computer science. As an aside the thesis will survey some of the literature related to bin packing.

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I would also like to extend my appreciation towards my parents and my sister for being there for me through thick and thin and always encouraging me to strive for the best. Without their endless support I would never be able to reach to the place I'm standing today in my life. Last but not the least; I thank my friends, roommates for their support in the successful completion of this work.

CHAPTER 1

INTRODUCTION

In this chapter we present the problem of bin packing and applications of it. The packing problems which we study are theoretical, but serve as benchmarks to many algorithmic techniques. The ideas which originated in the study of the bin packing problem have helped shape computer science as we know it today. A well known packing problem which is one of the oldest and most thoroughly studied problems in computer science and combinatorial optimization is the bin packing problem which is a combinatorial NP-hard problem.

The importance of this problem is that it has spawned off whole areas of research, including the field of approximation algorithms. The bin packing optimization problem requires packing a set of objects into a finite number of bins of capacity V in a way that minimizes the number of bins used. We intend to concentrate on a number of problems which have many important applications in areas such as multi-processor scheduling, resource allocation, packet routing, paged computer memory systems, storage, multiprocessor scheduling, stock cutting, loading trucks with weight capacity, creating file backup in removable media and technology mapping in FPGA implementation of custom hardware and many others.

In this thesis we discuss theoretical problems which include the classical bin packing problem for general and restricted inputs. We

present this problem as well as its application in data server storage. The classical bin packing problem models a situation when a set of items (having scalar sizes, which can be seen as sizes of files) need to be stored on hard disks in the data centers. Without loss of generality we assume that all the hard drives have the same size, since buying them in bulk is much cheaper than buying them individually. We also assume that the total size of a hard drive is 1 (assuming that all sizes were scaled), and the hard drive represents a bin that needs to receive items to be stored. The goal is to minimize the number of bins (or disks) used for storing the files. Restricted inputs of this problem include the parametric case, where sizes of items are much smaller than the size of recipients. Other generalizations are variable-sized bin packing - where bins of several sizes are available - and cardinality constrained bin packing – where a limited number of items can be placed in each bin, irrespective of the fact that maybe more items can still fit in.

Thesis Overview: In chapter 2 we present basic notions related to bin packing and approximation algorithms. The K-Binary algorithm is presented in detail in Chapter 3, with suggestive examples. The implementation and comparative studies and a brief description of the code are presented in Chapter 4. Simulations of results are presented in Chapter 5. We finish with concluding remarks in Chapter 6.

CHAPTER 2

BIN PACKING

The classical one dimensional bin packing has long served as a proving ground for new approaches to the analysis of approximation algorithms. The classical bin packing problem was one of the first combinatorial optimization problems which were introduced in the early 1970's and can be formulated as follows:

Given a sequence of items $I = \{1,2,\ldots,n\}$ with sizes $s_1, s_2, \ldots, s_n \in (0,1]$, find a partition of the items into sets of size 1 (called *bins*) so that the number of sets in the partition is minimized [6]. Furthermore, the sum of the sizes of the pieces assigned to any bin may not exceed its capacity.

A bin is *empty* if no piece is assigned to it, otherwise it is *used*. We say that an item that belongs to a given bin (set) is "packed" into this bin. Since the goal of this problem is to minimize the number of bins used, we would like to find an algorithm which incurs cost which is within a constant factor of the minimum possible cost, no matter what the input is. Let us take a concrete example. Let us consider a number of seven items of sizes 0.2, 0.5, 0.4, 0.7, 0.1, 0.3 and 0.8 that need to be packed. The minimum (optimal) way of packing the items into bins is shown in Figure 1.

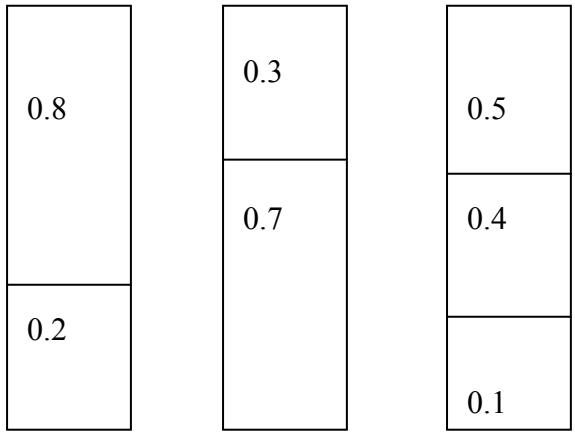


Figure 1: Example of Bin packing

For the classic bin packing problem one can have a cost within a constant factor r of the minimum number of required bins for $r < 3/2$ unless P=NP. This constant factor r is known as the asymptotic performance ratio or asymptotic performance guarantee, which leads to the usage of a standard quality measure for the performance of bin packing algorithms. We define the asymptotic performance ratio more precisely. For a given input sequence I, let $OPT(I)$ be the sum of the capacities of the bins used by algorithm A on I *also* OPT(I) is the cost of an optimal solution for input I, and A(I) is the cost algorithm A for this input [16]. The *asymptotic performance ratio* for an algorithm A is defined to be

$$R_A^{\infty} = \lim_{n \to \alpha} sup \sup_I \left\{ \frac{A(I)}{OPT(I)} \middle| OPT(I) = n \right\}$$

Approximation algorithms for the classical bin packing problem:

The first studies of the bin packing problem suggested. The natural and easy to implement algorithms, First Fit (FF), Next Fit (NF) and Best Fit (BF). These algorithms assume an arbitrary ordering of the input [5].

For the general bin packing algorithm we assume that the entire list and its item sizes are known before the packing begins. A common situation is where the items arrive in some order and must be assigned to some bin as soon as they arrive, without knowledge of the remaining items. A bin packing algorithm that can construct its packing under this on-line regime is called an *on-line bin packing* algorithm, for which items in the list must be processed in exactly the same order as they are given, one at a time. The on-line processing is difficult owing to the fact that unpredictable item sizes may appear. In general, the performance of an on-line bin-packing algorithm is substantially affected by the permutation of items in a given list. The Next-Fit and First Fit are two well-known and simplest on-line bin-packing algorithms where r (Next-Fit) = 2, while the r (First -Fit) = 1.7 [16].

2.1 Next Fit:

The simplest algorithm for classical one dimensional bin packing problem is Next Fit. Next Fit is a bounded space online algorithm in which the only partially filled bin that is open is the most recent one to be started. It uses one active bin into which it packs the input. Once the free space in this bin becomes too small to accommodate the

next item, a new active bin is opened and the previous active bin is never used again. This process continues until there are no more elements. This is the least efficient of the algorithms but it does have a practical benefit since at any time at most one bin is kept open. For instance, consider the conveyor belt placing items in boxes to be shipped to a single customer. One would not want to have to reverse the conveyor belt to go back to a partially empty box to fit in an item that the customer ordered. Next Fit has an approximation ratio of 2 and runs in linear time. Next Fit does the following steps:

Description: This heuristic places the next item in the currently open bin. If it does not fit the bin is closed and a new bin is opened [22].

Initialization:

Given a list of item weights L= $\{q_1, q_2, \ldots, q_n\}$.

Place item 1 in bin 1and remove L. let i=1. j=2.

Iterations:

1. If item j fits in bin I, place j in i. If not, then open a new bin i+1 and place j in bin i+1. Let i=i+1.
2. Remove item j from L. Let j = j+1.
3. While items remain in L, repeat from Step 1.

Let us take an example of bins of size 80, and the elements to be packed have the sizes of 26, 57, 18, 8, 45, 16, 22, 29, 5, 11, 8, 27, 54, 13, 17, 21, 63, 14, 16, 45, 6, 32, 57, 24, 18, 27, 54, 35, 12, 43, 36, 72, 14, 28, 3, 11, 46, 27, 42, 59, 26, 41, 15, 41, 68. We show in Figure 2 how these

items are packed into bins using the Next Fit algorithm. We write the bin number at the bottom, the bin size the left side, and we show how much each bin is actually filled. We color with blue the first item in each bin, with red the second item, with green the third item, with light blue the fourth item, and with cyan the fifth item.

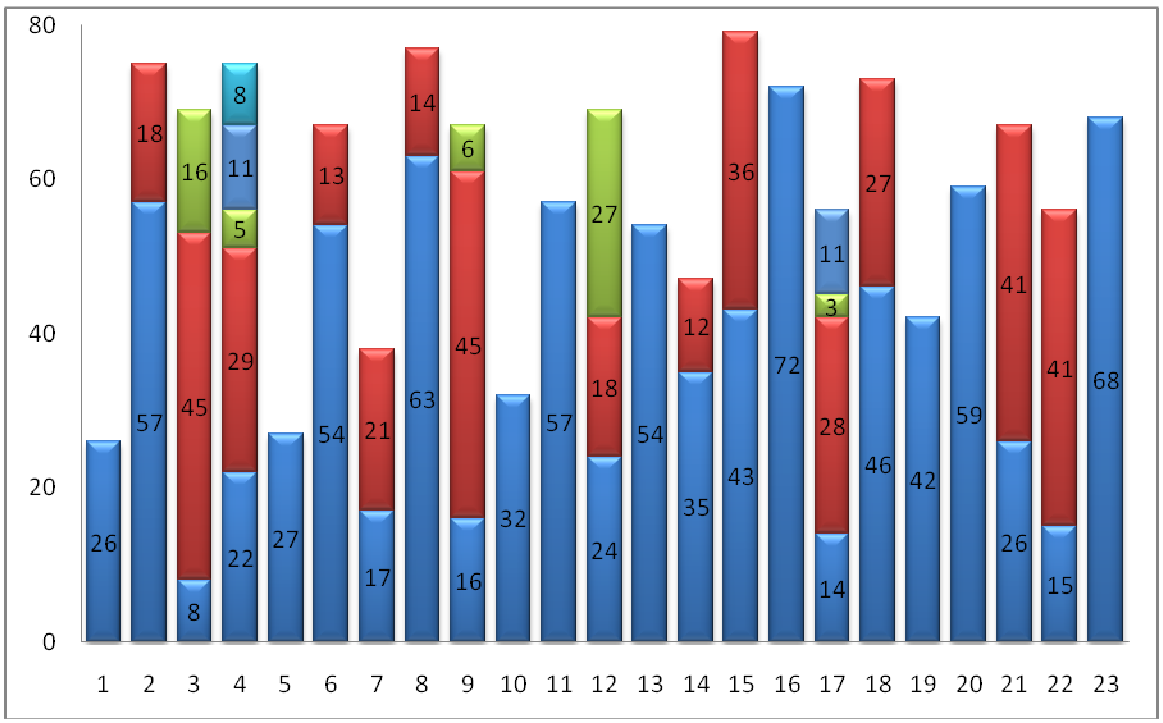


Figure 2: Chart showing items for Next Fit algorithm

Theorem 1:

Given an instance x of Minimum Bin Packing, Next Fit returns a solution with value m_{NF}(x) such that m_{NF}(x) / m^*(x) < 2 [19].

Proof: The functions of sum of the item sizes denoted by A (i.e., A= $\sum_{i=1}^{n} a_i$) is the value of the optimal solution. The number of bins used by Next Fit

is less than 2*A, since for each pair of consecutive bins, the sum of the sizes of items included in these two bins is greater than 1. The number of bins used in each feasible solution is at least the total size of items; we have that $m^*(x) \geq [A]$. It follows that $m_{NF}(x) < 2m^*(x)$.

Theorem 2: If M is the number of bins in the optimal solution, then Next Fit never uses more than 2M bins. There exist sequences that force Next Fit to use 2M-2 bins, thus 2M-2 is a lower bound for Next Fit. [20]

Proof: Let us consider any two adjacent bins. Sum of the items in the two bins must be greater than 1; otherwise Next Fit puts all items from the second bin into the first bin. Let A_1 be the total occupied space in the first bin, A_2 be the total occupied space in the second bin, so on. Thus, the total occupied space in ($A_1 + A_2$) is greater than 1. The same holds for A3+A4 etc.. Thus, at most half the space is wasted, and so Next Fit uses at most 2M bins.

For the lower bound, let us assume that the total number of items N is divisible by 4. Let us consider the sequence in which the item size $s_i = 0.5$ for *i* odd, and $s_i = 2/N$ if i is even. Then, the optimal solution puts all the items of size 0.5 in pairs using a total of N/4 bins. The rest of the items fit in a single bin, so the optimum number of bins is N/4 + 1. Next Fit would place the first and the second item together in one bin, the third and the fourth item together in the second bin, and so on, thus using a total of N/2 bins.

The weakness of the Next Fit algorithm is that assigns an item only to the last used bin. The algorithm First Fit tries to assign an item to any non-filled (open) bins.

2.2 First Fit

First Fit achieves a worse running time as it keeps all non-empty bins active and tries to pack every item in these bins before opening a new one. If no bin is found, it opens a new bin and puts the item in the new bin. So, the restriction of using a single bin is removed entirely and all partially filled bins are considered as possible destinations for the item to be packed. In this algorithm the rule followed is: First we place an item in the first, called lowest indexed, bin into which it will fit , i.e.., if there is any partially filled bin B_j with level(B_j)+s(a_i)$\leq$ 1, then we place a_i in the lowest indexed bin. Otherwise, we start a new bin with a_i as its first item.

The general class of such algorithms is called Any Fit algorithms. This class consists of all algorithms that open a new bin if there is no other option. First Fit always picks the first bin in the list of open bins where the item can fit. The algorithm works as follows:

Description: This keeps all infill bins open. It places the next item in the lowest numbered bin in which the item fits. If it does not fit in any bin, a new bin is opened.

Initialization:

G Given a list of item weights L= $\{q_1, q_2, \ldots, q_n\}$.

Place item 1 in bin 1and remove L. let i=1. j=2.

Iterations:

1. Find the lowest numbered bin I in which item j fits, and place j in i. If I does not fit in any bin, open a new bin and number it m+1, let m=m+1, and place j in bin m+1.
2. Remove item j from L. Let j=j + 1.
3. While items remain in L, repeat from Step 1.

Let us consider the following elements to be packed into bins of size 80: the size of the elements are 26, 57, 18, 8, 45, 16, 22, 29, 5, 11, 8, 27, 54, 13, 17, 21, 63, 14, 16, 45, 6, 32, 57, 24, 18, 27, 54, 35, 12, 43, 36, 72, 14, 28, 3, 11, 46, 27, 42, 59, 26, 41, 15, 41, and 68. We show in Figure 3 how these items are packed into bins using the First Fit algorithm. We write the bin number at the bottom, the bin size the left side, and we show how much each bin is actually filled. We color with blue the first item in each bin, with red the second item, with green the third item, with light blue the fourth item, with cyan the fifth item, and with orange the sixth item.

Theorem 3: First Fit never uses more than 2M bins, if M is the optimal.

Proof: Here At most one bin can be more than half empty: otherwise the contents of the second half-full bin would be placed in the first.

Theorem 4: If M is the optimal number of bins, then First Fit never uses more than 1.7M bins. On the other hand, there are sequences that force it to use at least 17/10 (M-1) bins.

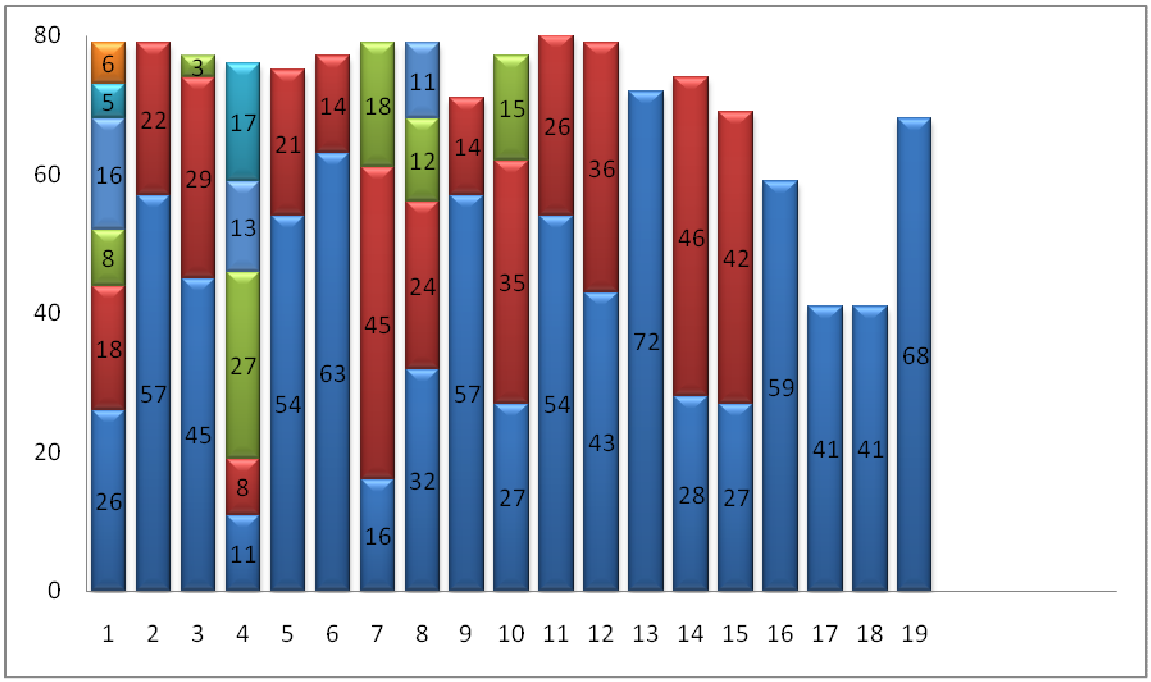


Figure 3: Chart showing items for First Fit algorithm

We show an example that forces First Fit to use 10/6 times optimal. For instance, consider the sequence of 6M items of size 1/7 + e; followed by 6M items of size 1/3 + e; followed by 6M items of size 1/2 + e, where e is a very small number. The optimal strategy is to pack each bin with one from each group, requiring a total of 6M bins. When First Fit is run, it packs all the small items (of size 1/7 + e) first into bins, a total of 6M/6=M bins. It then packs all the medium items (of size 1/3 + e) into bins, but requires 6M/2 = 3M bins, since only two such items fit into one bin. It then packs the large items (of size 1/2 +e) into bins, using a total of 6M bins since only one item can fit into a bin. Thus, in total First Fit uses M+3M+6M=10M bins [13].

First Fit achieves an approximation factor of 2 [15]. This is due to the observation that at any given time, it is impossible for 2 bins to be half full. The reason is that if at some point a bin was at most half full, meaning it has at least a space of $V/2$, the algorithm will not open a new bin for any item whose size is at most $V/2$. Only after the bin fills with more than $V/2$ or if an item with a size larger than $V/2$ arrives, the algorithm may open a new bin. Thus if we have B bins, at least $B-1$ bins are more than half full. Therefore $\sum_{i=1}^{n} a_i > \frac{B-1}{2}$ V. Because $\frac{\sum_{i=1}^{n} a_i}{V}$ is a lower bound of the optimum value OPT, we obtain that $B-1<2OPT$ and therefore $B \leq 2*OPT$ [13].

It is possible to construct the First Fit packing in time O (n*log n) using an appropriate data structure. This is the best possible for a comparison based implementation, since one can use the First Fit packing rule to sort. First Fit is an O (n)-space algorithm, since there are sequences in which all non-empty bins remain active until the end of the processing.

In addition to the performance ratio, we also consider the time and the space complexity of on-line bin-packing algorithms. Thus, Next Fit is an O (n)-time algorithm, whereas First Fit is an O (n log n)-time algorithm. For convenience, a non-empty bin during the processing is called “filled” if it is not intended to pack any more items and “active” if it is. Using the uniform cost criterion for space, we need one storage location for each active bin. As soon as a bin becomes filled, it is part of

the output of the algorithm used, and we do not count its storage location in defining the space complexity of the algorithm. Specifically, we use SA(n) to denote the maximum number of storage locations (for active bins) needed by algorithm A during the processing of the list L whose size is n, and refer to algorithm A as an SA(n)-space algorithm. Next Fit is an O (1)-space algorithm, since it involves only one active bin at all times, and First Fit is an O (n)-space algorithm, since in this case all non-empty bins remain active until the end of the processing [22].

2.3 Best Fit:

Best Fit is the best known algorithm for on-line bin packing which emerges as the winner among the various online algorithms: It is simple and behaves well in practice, and no other algorithm has a better both worst case and average uniform case. Best Fit (BF) picks (among the possible bins for the item) the one where the amount of free space is minimal. It picks the bin with the least amount of free space in which it can still hold the current element. The description of Best Fit algorithm follows:

Description:

This algorithm tries to choose the fullest bin possible with enough space each time an item is assigned. All unfilled bins are kept open until the end. It places the next item j in the bin whose current contents is the largest, but should not exceed Q-q_j. If it does not fit in any bin, new bin is opened [22].

Initialization:

Given a list of item weights L= $\{q_1, q_2, \ldots, q_n\}$.

Place item 1 in bin 1 and remove from L. let j=2, m=1.

Iterations:

1. Find the bin i whose remaining capacity is minimum but greater than q_j (if S_i are the items in bin i, $Q - \sum_{k \in S_i} q_k$ q_k is the remaining capacity of bin i) and place j in i. If j does not fit in any bin, open a new bin and number it m+1, place j in bin m+1 and let m=m+1
2. Remove item j from L. Let j=j + 1.
3. While items remain in L, repeat from Step 1.

Let us take an example of elements to be packed into bins of size 80: the size of the elements 26, 57, 18, 8, 45, 16, 22, 29, 5, 11, 8, 27, 54, 13, 17, 21, 6, 3, 14, 16, 45, 6, 32, 57, 24, 18, 27, 54, 3, 5, 12, 43, 36, 72, 14, 28, 3, 11, 46, 27, 42, 59, 26, 41, 15, 41, and 68. We show in Figure 4 how these items are packed into bins using the Best Fit algorithm. We write the bin number at the bottom, the bin size on the left side, and we show how much each bin is actually filled. We color with blue the first item in each bin, with red the second item, with green the third item, with light blue the fourth item, and with cyan the fifth item.

Best Fit can be easily implemented in *O* (N log N) time. Best Fit and First Fit never uses more than 1.7 times optimal.

We study the expected performance ratio, taking as the worst-case, the multi set of items L, and assuming that the elements of L are

inserted in random order. The lower bound acquires an approximation ratio of 1.08 and an upper bound of 1.5 [12].

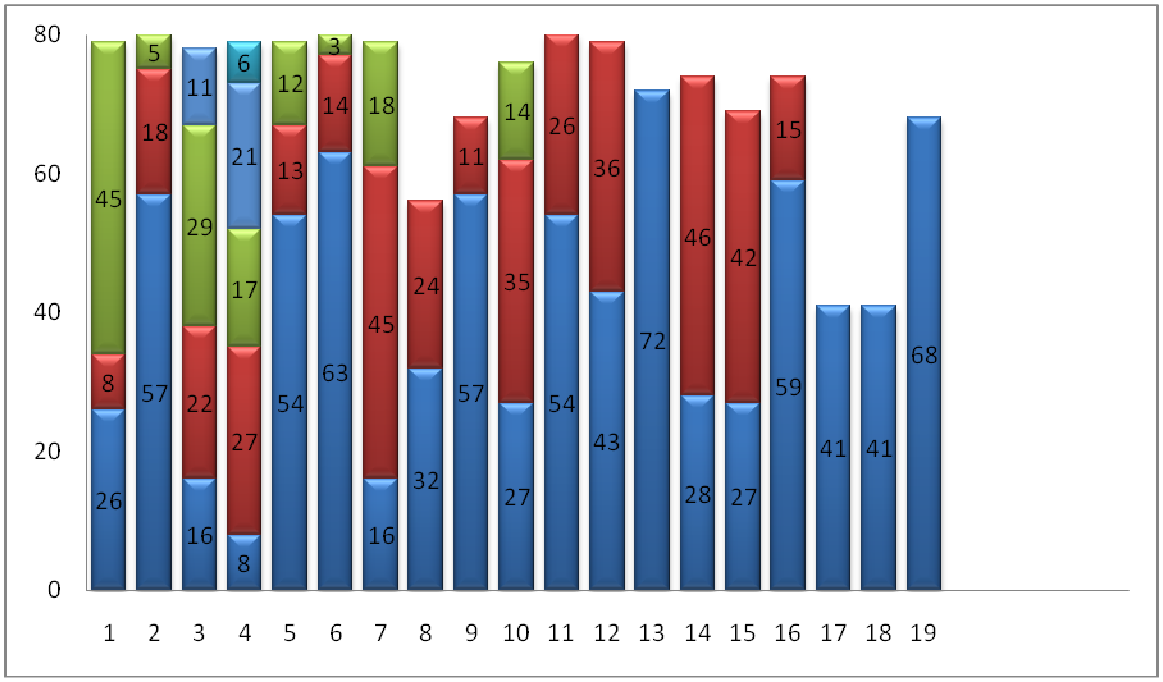


Figure 4: Chart showing items for Best Fit algorithm

Any on-line bin-packing algorithm has a performance ratio of at least 1.54 in which case Best Fit has an approximation ratio of 1.7 which is the same as First Fit and in the average uniform case the items generally draw in the interval[0,1] (then Best Fit has expected wasted space of $O(n/2(\log n))$. But the worst-case performance ratio and the uniform-distribution performance ratio are not quite satisfactory measures for evaluating online bin packing algorithms.

2.4 Offline Algorithms:

If no limit on re-packing is imposed then we perform offline algorithms. An offline algorithm simply repacks everything each time an item arrives. Packing large items is difficult with an online algorithm, especially when such items occur later in the sequence. We can circumvent this by sorting the input sequence and placing the large items first. There are three important offline algorithms for bin packing in which the inputs are not ordered arbitrarily but stored in a non decreasing order of item sizes which results in a new list. The algorithms Next Fit Decreasing, First Fit Decreasing and Best Fit Decreasing are defined in the same way as Next Fit, First Fit and Best Fit, only now the input is not ordered arbitrarily but is stored in non- increasing order of item sizes.

The approximation ratio of First Fit Decreasing is $\frac{11}{9}$. The approximation ratio for Next Fit Decreasing is 1.691 and was shown by Baker and Coffman. Gary and Johnson designed an algorithm called modified First Fit Decreasing that has an approximation ratio of $\frac{71}{60}$. This is the best currently known algorithm which has a relatively small running time in practice [21].

2.4.1 First Fit Decreasing

This algorithm first sorts items in non-increasing order with respect to their size and then processes items as the First Fit algorithm. For example, let us consider the following eight items of sizes 4, 1, 2, 5, 3, 2, 3, 6, 3, that need to be packed into bins of size 8. With the First Fit

Decreasing algorithm we sort the items into descending order first (see Figure 5). Then we use the First Fit algorithm to pack them into five bins (see Figure 6).

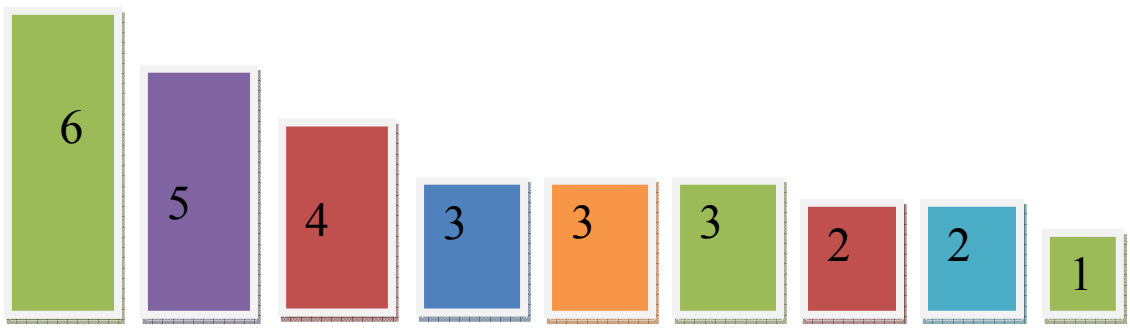


Figure 5: Non-increasing order of First Fit Decreasing

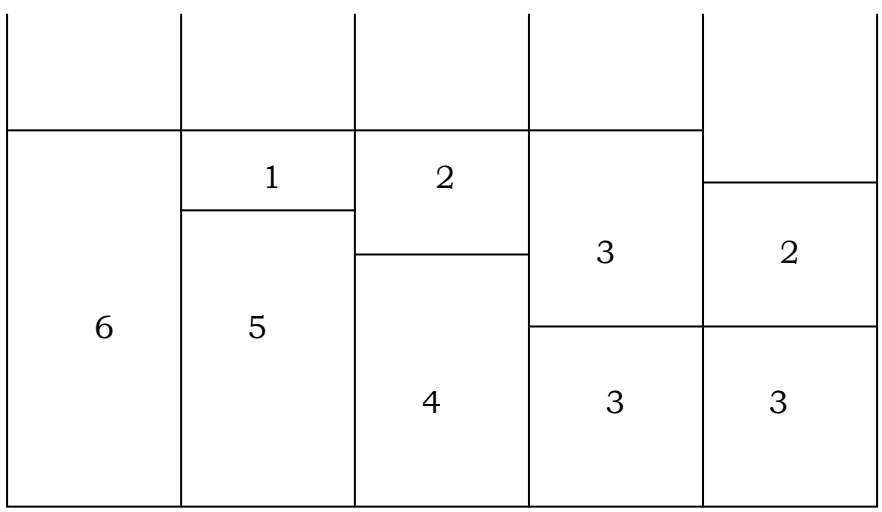


Figure 6: Example of First Fit Decreasing

Given an instance x of the minimum bin packing problem, the First Fit Decreasing algorithm finds a solution with measure m_{FFD}(x) such that

$$m_{FFD}(x) \leq 1.5m^*(x) + 1$$

We partition the ordered list of items $\{a_1, a_2, \ldots, a_n\}$ according to their sizes into the following four sets:

$$A = \{a_i \mid a_i > \tfrac{2}{3}\},$$

$$B = \{a_i \mid \tfrac{2}{3} \geq a_i > \tfrac{1}{2}\},$$

$$C = \{a_i \mid \tfrac{1}{2} \geq a_i > \tfrac{1}{3}\},$$

$$D = \{a_i \mid \tfrac{1}{3} \geq a_i\},$$

If there is at least one bin that contains only items belonging to D, then there is at most one bin i.e. the last opened one with total occupancy less than $\frac{2}{3}$. Thus, First Fit Decreasing finds an optimal solution if there is no bin that contains only items belonging to D. Let x be the items that are eliminated belonging to D. Since the value found by First Fit Decreasing for x and x is the same, it is sufficient to prove optimality of First Fit Decreasing for x. Since the First Fit Decreasing algorithm processes items in non-increasing order according to their weight, it packs the highest items in their respective bins that fit with it and do not share a bin with other items. This implies that the number of bins in optimal solution and the solution found by First Fit Decreasing are same. An apparently better algorithm is the Best Fit Decreasing algorithm.

2.4.2 Best Fit Decreasing

Like First Fit Decreasing, Best Fit Decreasing initially sorts items in non-increasing order with respect to their order and then processes them sequentially. The difference between the two algorithms is the rule used for choosing the bin in which new item a_i is inserted while trying to pack a_i. Best Fit Decreasing chooses a bin with the minimum empty space to be left after the item is packed into a bin. In this way, it tries to fit the items into bin by reducing the fragmentation of the bins. In some cases, the quality of solution found by Best Fit Decreasing may be worse than the quality of solution found by First Fit Decreasing. In other cases Best Fit Decreasing finds an optimal solution while First Fit Decreasing returns a non- optimal solution.

Next we present three algorithms in which the number of sets in the partition is minimized according to a set of given intervals.

2.5 Online algorithm based on partitioning:

Some types of algorithm are based on a non-uniform partitioning of interval space (0,1] into M sub-intervals and will run is O(n) time. It is known that no on-line algorithm can have an asymptotic worst case ratio <1.53. The best on-line algorithm given by Lee and Lee and called Harmonic has a worst-case ratio of 1.69. The performance ratio is better than the Best-Fit when the item sizes are uniformly distributed.

2.5.1 Harmonic Algorithm:

We are given a list of items L= $\{a_1, a_2, \ldots a_n\}$, each with item sizes $s(a_i)$ (0< $s(a_i) \leq 1$). The interval (0, 1] is partitioned into harmonic sub intervals $I_M = (0, \frac{1}{M}], \left(\frac{1}{M}, \frac{1}{M-1}\right], \ldots \left(\frac{1}{2}, 1\right]$ where M is a positive integer. Type 1 is a bin that contains items whose sizes are in the range $\left(\frac{1}{2}, 1\right]$, type 2 is a bin that contains items whose sizes are in the range $\left(\frac{1}{3}, \frac{1}{2}\right]$, type M is a bin that contains items whose sizes are in the range $\left(0, \frac{1}{M}\right]$ Each item is classified according to the size, i.e., if the item size is in the interval I_M, the item is called I_M -item or I_M -bin. For 1 ≤ j < k, at most j I_j-pieces can be packed in I_j –bin and is referred to as being filled if it has exactly j I_j– pieces, and unfilled otherwise. Let m_j denote the number of I_j -bins used by the algorithm, 1 ≤ j ≤ M. In Harmonic algorithm, we keep all unfilled bins active I_j-bin for each 1 ≤ j < M. The Harmonic algorithm is completely independent of the arriving order of items [6].

All elements will be packed by Harmonic Fit into I_j-bins as follows; Items of type i are packed i per bin for i = 1, . . . ,k-1, and corresponding weight is $1/i$ .we classify the bins into M j- 1 categories. Each category is designated to pack the same type of elements. It essentially performs item classification for each incoming item and then packs it by Next Fit into a corresponding bin. Now, items of type k are packed using Next Fit

When an item does not fit in a bin, the bin is at least $\frac{k-1}{k}$ full (size of each item ≤ $\frac{1}{k}$) and the weight of item of size x is $\frac{k}{k-1}x$. For the Next Fit, if the next small element does not fit into the opened bin, we close this bin and open a new bin. Elements of type k-1 pack k-1 per bin. The maximum wasted space is maximal for type 1 bin, as they pack 1 per bin.

The Algorithm Harmonic has the following steps:

Algorithm Harmonic

For a given value of N, M, and the set of items L= $\{a_1, a_2, \ldots a_n\}$

Step 1: Partition the interval (0, 1] for the given M into subintervals as given below:

$$(0,\frac{1}{M}],\left(\frac{1}{M},\frac{1}{M-1}\right],\ldots\left(\frac{1}{2},1\right]$$

Step 2: Assign each item a_i to the open bin of that type (the size of a_i fits into the corresponding subinterval).

Step 3: If an item does not fit into the corresponding bin, then close it and open a new one.

Step 4: Calculate the total number of bins used of each type.

Let us consider an example where M=5 and the sequence of items is 0.21, 0.41, 0.13, 0.59, 0.75, 0.64, 0.75, 0.83, 0.71, 0.81, 0.2, 0.95, 0.37, 0.67, 0.44, 0.82, 0.21, 0.84, 0.87, 0.81, 0.79, 0.59, 0.87, 0.41, 0.25, 0.36, 0.25, 0.17, 0.29, 0.19, 0.8, 0.05, 0.63, 0.33, 0.56, 0.18, 0.79, 0.16,

0.13, and 0.19. Thus the interval (0, 1] is partitioned into five subintervals are $(0,\frac{1}{5}]$, $(\frac{1}{5},\frac{1}{4}]$, $(\frac{1}{4},\frac{1}{3}]$, $(\frac{1}{3},\frac{1}{2}]$, $(\frac{1}{2},1]$. Bins are assigned to each interval as presented in the Table 1 for each partition. The total number of bins used is 27.

Partition 1- $(0,\frac{1}{5}]$

Bin 1	0.13	0.2	0.17	0.19	0.15	0.18
Bin 2	0.16	0.13	0.19			

Partition 2- $(\frac{1}{5},\frac{1}{4}]$

Bin 1	0.21	0.21	0.25	0.25

Partition 3 - $(\frac{1}{4},\frac{1}{3}]$

Bin1	0.29	0.23

Partition 4 - $(\frac{1}{3},\frac{1}{2}]$

Bin 1	0.41	0.37
Bin 2	0.44	0.41
Bin 3	0.36	

Partition 5 - $(\frac{1}{2},1]$

Bins	1	2	3	4	5	6	7	8	9	10
Items	0.59	0.75	0.64	0.75	0.83	0.71	0.81	0.95	0.67	0.82

11	12	13	14	15	16	17	18	19	20
0.84	0.87	0.81	0.79	0.59	0.87	0.8	0.63	0.56	0.79

Table 1: Distribution of items using Harmonic algorithm

We implemented the Harmonic algorithm as follows:

For k: = 1 to M do $m_k \leftarrow 0$

For i: = 1 to N do

If a_i is an I_j-piece, 1 ≤ j < M

then begin place a_i into the I_j-bin

if the I_j-bin is filled

then $m_k \leftarrow m_k + 1$ and get a new I_j-bin

end

else (Comment: a_i is an I_m piece)

begin

if there is room for a_i in the I_j -bin

then pack it

else $m_k \leftarrow m_k + 1$ and get a new I_j –bin

end

It can be performed in O (log M) time, for each item classification and there are only M active bins at any time, the algorithm runs in O (n 1ogM) time and uses M storage spaces for active bins. It is shown that, the worst-case performance ratio of the algorithm is not related to the partition number M. Therefore, M can be regarded as a constant, and hence we have an 0(I)-space and O (n)-time algorithm [6].

A crucial advantage of this algorithm is that each filled I_k-bin, 1 ≤ k < M, packs exactly k items, irrespective of the actual sizes of these items

in the interval I_k. A disadvantage of Harmonic is that items of type 1, that is, the items larger than 1/2, are packed one per bin, possibly wasting a lot of space in each single bin.

2.5.2 Epstein's Algorithm:

We define bounded space algorithm of each value of m > 1. For every m>1, changes on the lower bin are made. Epstein's algorithm works the same as the Harmonic algorithm except that in type M bins (the small type bins), at most M items are stored at any time. Whenever M items are stored in the small type bin, we close the bin and open a new one. For item classification, we partition the interval (0, 1] into sub-intervals. We use k–1 sub-intervals $I_M = \left(0,\frac{1}{M}\right],\left(\frac{1}{M},\frac{1}{M-1}\right],\ldots\left(\frac{1}{2},1\right]$. Each bin will contain only items from one sub-interval (type). The items in interval I_M are packed to a single bin. A bin which received the full amount of items (according to its type) is closed, therefore at most k − 1 bins are open simultaneously (one per interval, except for ($\frac{1}{2}$,1]. We pack each item according to its type and we note again that the exact size does not affect the packing process. Each bin will contain only items of one type. A bin that has received the full amount of items (according to its type) is closed [1]. The main difference between Harmonic algorithm and Epstein's algorithm is the way that small items are packed, no more than M items are allowed in any bin, irrespective of whether more items can still fit into that bin. The algorithm works as follows:

Algorithm Epstein

For a given value of N, M, and the set of items L= $\{a_1, a_2, \ldots a_n\}$

Step 1: Partition the interval (0, 1] for the given M into subintervals:

$$(0,\frac{1}{M}],\left(\frac{1}{M},\frac{1}{M-1}\right],\ldots\left(\frac{1}{2},1\right]$$

Step 2: Assign each item a_i to the open bin of that type (the size of a_i fits into the corresponding subinterval).

Step 3: If there are M items of type M into the small bin, then close it and open a new one.

Step 4: If an item does not fit into the corresponding bin, then close it and open a new one.

Step 5: Calculate the total number of bins used of each type.

Let us consider the following example, where M=5 and the sequence of items is 0.21, 0.41, 0.13, 0.59, 0.75, 0.64, 0.75, 0.83, 0.71, 0.81, 0.2, 0.95, 0.37, 0.67, 0.44, 0.82, 0.21, 0.84, 0.87, 0.81, 0.79, 0.59, 0.87, 0.41, 0.25, 0.36, 0.25, 0.17, 0.29, 0.19, 0.8, 0.05, 0.63, 0.33, 0.56, 0.18, 0.79, 0.16, 0.13, and 0.19. The interval (0, 1] is partitioned into the subintervals $(0,\frac{1}{5}]$, $(\frac{1}{5},\frac{1}{4}]$, $(\frac{1}{4},\frac{1}{3}]$, $(\frac{1}{3},\frac{1}{2}]$, $(\frac{1}{2},1]$. Bins are assigned to each interval as presented in Table 2 for each partition. The total number of bins formed is 27.

Partition 1- $(0,\frac{1}{5}]$

Bin1	0.13	0.2	0.17	0.19	0.05
Bin2	0.18	0.16	0.13	0.19	

Partition 2- $(\frac{1}{5},\frac{1}{4}]$

Bin1	0.21	0.21	0.25	0.25

Partition 3 - $(\frac{1}{4},\frac{1}{3}]$

Bin1	0.29	0.23

Partition 4 - $(\frac{1}{3},\frac{1}{2}]$

Bin1	0.41	0.37
Bin2	0.44	0.41
Bin3	0.36	

Partition 5 - $(\frac{1}{2},1]$

Bins	1	2	3	4	5	6	7	8	9	10
Items	0.59	0.75	0.64	0.75	0.83	0.71	0.81	0.95	0.67	0.82

11	12	13	14	15	16	17	18	19	20
0.84	0.87	0.81	0.79	0.59	0.87	0.8	0.63	0.56	0.79

Table 2: Distribution of items using Epstein algorithm

The Variable Harmonic algorithm proposed by Epstein considers the bins sizes to be $s_1 < \ldots s_m = 1$. The items are classified into intervals whose right endpoint is a critical size.

CHAPTER 3

K-BINARY ALGORITHM

K-Binary is one of the simplest unsupervised learning algorithms to group the bins according to the interval. To achieve this, the interval (0,1] is partitioned into sub intervals as follows (0, 1]= $\cup_{j=1}^{k}$ (I_k) where I_k= $(0, {}^{1}/_{2^{k-1}}]$, $({}^{1}/_{2^{k-1}}, {}^{1}/_{2^{k-2}}]$,... $({}^{1}/_{2^{k-m}}, 1]$ for k= 1, 2... n-1 where k is the number of partitions.

For example, if k=3 then the interval (0,1] is partitioned into three intervals $(0, {}^{1}/_{4}]$, $({}^{1}/_{4}, {}^{1}/_{2}]$, and $({}^{1}/_{2}, 1]$. Each item is classified according to its interval i.e., if the item size is in the interval I_k then the item is called an I_k -item. Items in sub-interval *k* are packed to a bin. A bin which received the full amount of items (according to its type) is closed and therefore new bin is opened. Each bin will contain only items from one sub-interval (type).

Algorithm k-Binary

For a given value of N, M, and the set of items L= $\{a_1, a_2, \ldots a_n\}$

Step 1: Partition the intervals for the given M value such that they are partitioned into given intervals as given below.

$\cup_{j=1}^{k}$ (I_k)

I_k= $(0, {}^{1}/_{2^{k-1}}]$, $({}^{1}/_{2^{k-1}}, {}^{1}/_{2^{k-2}}]$,... $({}^{1}/_{2^{k-m}}, 1]$ for k= 1,2... n-1

Step 2: Assign the items to bins according to each interval.

Step 3: Pack the items into each bin, if the next small element does not fit into the opened bin, then we close this bin and open a new one.

Step 4: Calculate the total number of bins formed for each interval.

K- Binary algorithm mainly depends on three factors:

1) The number of items.

2) The number of partitions.

3) Bins that are assigned according to the given interval.

Let us consider the following example where M=5 and the items to be packed are 0.2, 0.49, 0.85, 0.79, 0.53, 0.45, 0.31, 0.85, 0.56, 0.53, 0.85, 0.21, 0.1, 0.7, 0.76, 0.29, 0.54, 0.99, 0.31, 0.84, 0.8, 0.51, 0.15, 0.21, 0.47, 0.55, 0.69, 0.15, 0.44, 0.47, 0.15, 0.79, 0.9, 0.3, 0.24, 0.71, 0.46, 0.01, 0.69 and 0.16. The interval (0,1] is partitioned into five subintervals $(0,\frac{1}{16}]$, $(\frac{1}{16},\frac{1}{8}]$, $(\frac{1}{8},\frac{1}{4}]$, $(\frac{1}{4},\frac{1}{2}]$, $(\frac{1}{2},1]$. Bins are assigned to each interval as presented in Table 3. The total number of bins formed is 29.

Partition 1- $(0,\frac{1}{16}]$

Bin 1	0.1

Partition 2- $(\frac{1}{16},\frac{1}{8}]$

Bin 1	0.1

Partition 3 - $(\frac{1}{8},\frac{1}{4}]$

Bin 1	0.2	0.21	0.15	0.21	0.15
Bin 2	0.15	0.24	0.16		

Partition 4 - $(\frac{1}{4},\frac{1}{2}]$

Bin 1	0.49	0.45	
Bin 2	0.31	0.29	0.31
Bin 3	0.47	0.44	
Bin 4	0.47	0.3	
Bin 5	0.46		

Partition 5 - $(\frac{1}{2},1]$

Bins	1	2	3	4	5	6	7	8	9	10
Items	0.85	0.79	0.53	0.85	0.56	0.53	0.85	0.7	0.76	0.54

11	12	13	14	15	16	17	18	19	20
0.99	0.84	0.8	0.51	0.55	0.69	0.79	0.9	0.71	0.69

Table 3: Distribution of items using K- Binary algorithm

We implement the K- Binary algorithm as follows:

for i: = 1 to n do

begin

Case a_i in

If a_i is in I_k –piece

Pack a_i in empty bin of type NB_1

begin place a_i into I_k – bin

Pack a_1 in bins of type B_k using the Best Fit algorithm

if there exists a nonempty bin of type B_1 that does not contain an I_1-item

then pack a_i in such a bin

else pack a_i in an empty bin of type B_1

If a_i is the I_2 item to arrive, for some integer r≥ 1

then if there exists a bin of type B_1 containing at least one and at most an I_j-item

then pack a_i in such a bin

else if there exists a bin of type B_1 containing only an I_1-item

then pack a_1 in such a bin

else pack a_1 in an empty bin of type B_1

else harmonic pack (a_i, B_j)

end

It is obvious that each item takes O (1) time to pack. Therefore the K- algorithm is an *O* (n) - time algorithm and it needs *O* (n) space [10]. The main goal using K-means algorithm is to minimize the number of bins.

CHAPTER 4

IMPLEMENTATION AND COMPARISONS

In the thesis we implemented few methods in order to obtain the best possible solution for any given sequence of requests and any value of M. Different sequences of requests with different partitions produce different results depending on size and data used. As the requests get large, the number of bins generated by K-Binary algorithm compared to Harmonic (Lee & Lee) does not give good results because of the size we have chosen.

For generating random requests, instead of generating a random integer i in the interval 1 to 10 and considering the size of the request to be 1/random number, we generate requests of the type $\frac{1}{2^i}$. Thus, if the integers generated randomly are 5, 4, 7, 2, 5, 1 the data set becomes $\frac{1}{2^5}, \frac{1}{2^4}, \frac{1}{2^7}, \frac{1}{2^2}, \frac{1}{2^5}, \frac{1}{2^1}$. We compare the performance of the K-binary algorithm with the performance of Harmonic (proposed by Lee and Lee) and we obtained that in most of the cases, the K-Binary algorithm uses more bins that Harmonic. Hence we measure the efficiency of the implemented algorithms in a different way.

The algorithm begins with measuring the average number of bins of any type used. For a given input, let N be the total number of items and M be the total number of partitions. Consider TB_1 to be the total number of bins used by Harmonic (Lee & Lee), TB_2 to be the total

number of bins used by Epstein and TB_3 be the total number of bins used by the K-Binary algorithm. For each algorithm we will compute how well the algorithm balances the bins of any type. For an algorithm, let TB be the total number of bins. The next step is to compute how many bins of each type are used. Let NB_1, NB_2 ...NB_M be the number of bins of type 1, type 2... type M used by the algorithm on that input. Now, if the algorithm has a balanced distribution over the number of bins, then all NB_i should be roughly TB/M. For each input, after we compute the total number of bins the next step is to compute the sum based on the below formula:

$$(NB_1 - TB/M)^2 + (NB_2 - TB/M)^2 + (NB_3 - TB/M)^2 + \ldots (NB_M - TB/M)^2.$$

We will then average these numbers the same way we do with the total number of bins.

4.1 Concrete Examples

We present some concrete examples that will include a sequence of requests and the bins used by the k-Binary algorithm. We give an example in which it shows that the k- Binary algorithm works better than Epstein's algorithm and equally well as Lee and Lee's algorithm, and another example showing that Lee and Lee performs better compared to k- Binary algorithm. We also show how the proposed k-Binary algorithm works. Let us consider examples where N random requests will be fitted into M different types of bins. For N=100 and M=5,

in Table 4 we show how many bins of type 1 through 5 are used by each algorithm.

Bins	Lee & Lee	Epstein	K- Binary
NB_1	3	3	1
NB_2	2	2	1
NB_3	4	6	4
NB_4	10	11	14
NB_5	49	49	49
Total	68	71	69

Table 4: Example 1- comparison between Lee& Lee, Epstein & K-Binary

We compute the average sum for each algorithm as described before.

Average number of bins of Lee & Lee:

$(3 - 68/5)^2 + (3 - 68/5)^2 + (4 - 68/5)^2 + (10 – 68/5)^2 + (49 - 68/5)^2=23.61$

Average number of bins of Epstein:

$(3 - 71/5)^2 + (2 - 68/5)^2 + (6 - 68/5)^2 + (11 – 68/5)^2 + (49 - 68/5)^2=22.10$

Average number of bins of k-Binary:

$(1 - 69/5)^2 + (1 - 68/5)^2 + (4 - 68/5)^2 + (14 – 68/5)^2 + (69 - 68/5)^2=22.99$

Therefore our proposed k-Binary algorithm performs better than Lee and Lee's and Epstein's algorithms.

We take another example for same number of items and partitions. We get different data set as items are generated randomly. For N=100

and M=3, in Table 5 we show what number of bins of each of the types 1 through 5 are used by each algorithm.

Bins	Lee & Lee	Epstein	K- Binary
NB_1	2	4	1
NB_2	2	2	1
NB_3	3	3	2
NB_4	8	8	12
NB_5	49	49	49
Total	64	65	66

Table 5: Example 2- comparison between Lee& Lee, Epstein & K-Binary

Average sum obtained for Lee and Lee algorithm is 26.10, Epstein algorithm is 24.97 and K- Binary algorithm is 25.40. Therefore as seen from the two examples K- Binary algorithm performs better compared to other two. Now we take another example and find the average for the three algorithms for N=100 and M=5. Average obtained for Lee and Lee algorithm is 26.55, Epstein algorithm is 25.21 and K- Binary algorithm is 27.71. Hence from the above example lee and lee performs better compared to K- Binary algorithm.

Bins	Lee & Lee	Epstein	K-Binary
NB_1	3	5	1
NB_2	2	2	1
NB_3	3	3	3
NB_4	7	7	10
NB_5	50	50	50
Total	65	67	65

Table 6: Example 3- Comparison between Lee& Lee, Epstein & K-Binary

4.2 Pre Processing:

Input File:

We generate random value for the requests and we store them in the file input.dat. In the preprocessing step a set of N of items are obtained randomly (called random sequence) $a_1, a_2, \ldots, a_n$ with a_i in the range 0.01…0.99, to which we append another set of N values that are computed as (1- random sequence) (called computed sequence).At the beginning of the file we store the values of N and M, followed by the two sequences.

Description:

Input the value N and M.

Start the random number generator.

Generate N rational values in the range 0.01 to 0.99 that will represent

the sequence of requests generated in the file "input.dat" first the value of N, then M, then the sequence randomly generated

An example of such an input file is shown in Figure 7. where the first number represents N i.e. total number of items generated randomly and the second number M represents the number of partitions. Then we have the random sequence followed by the computed sequence.

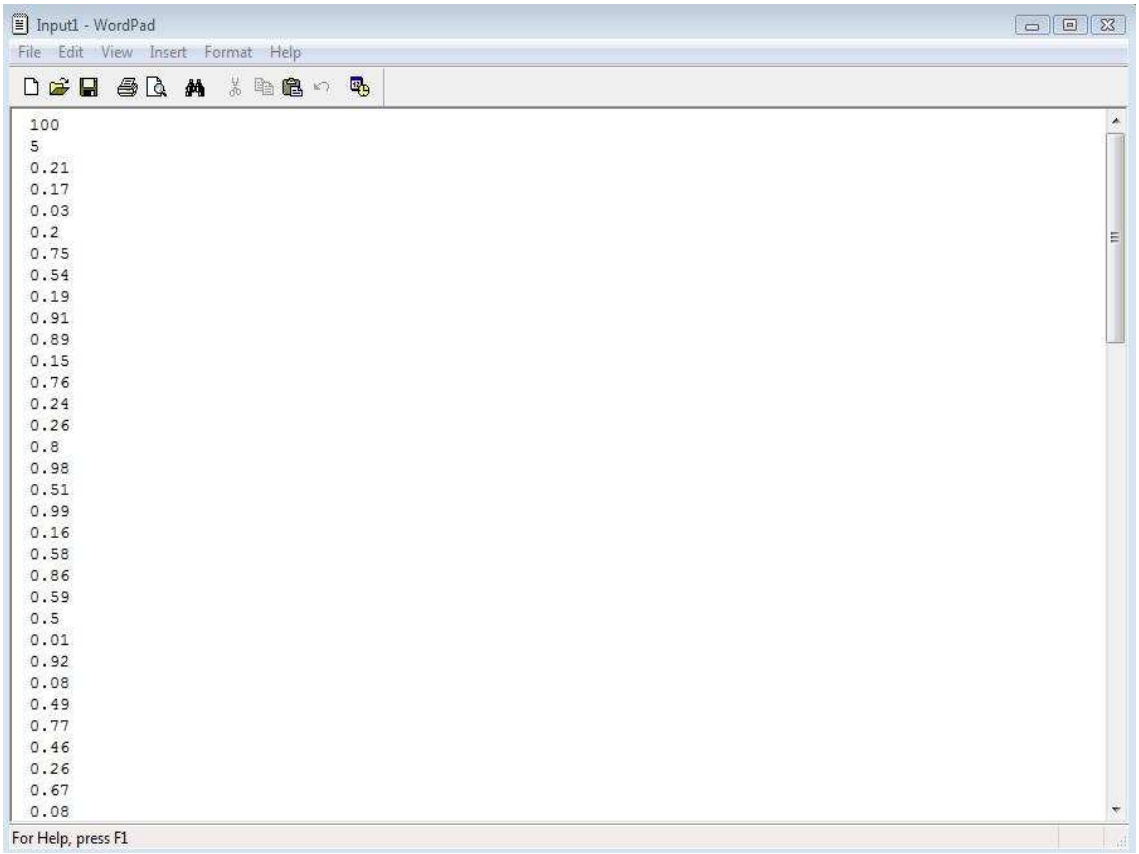


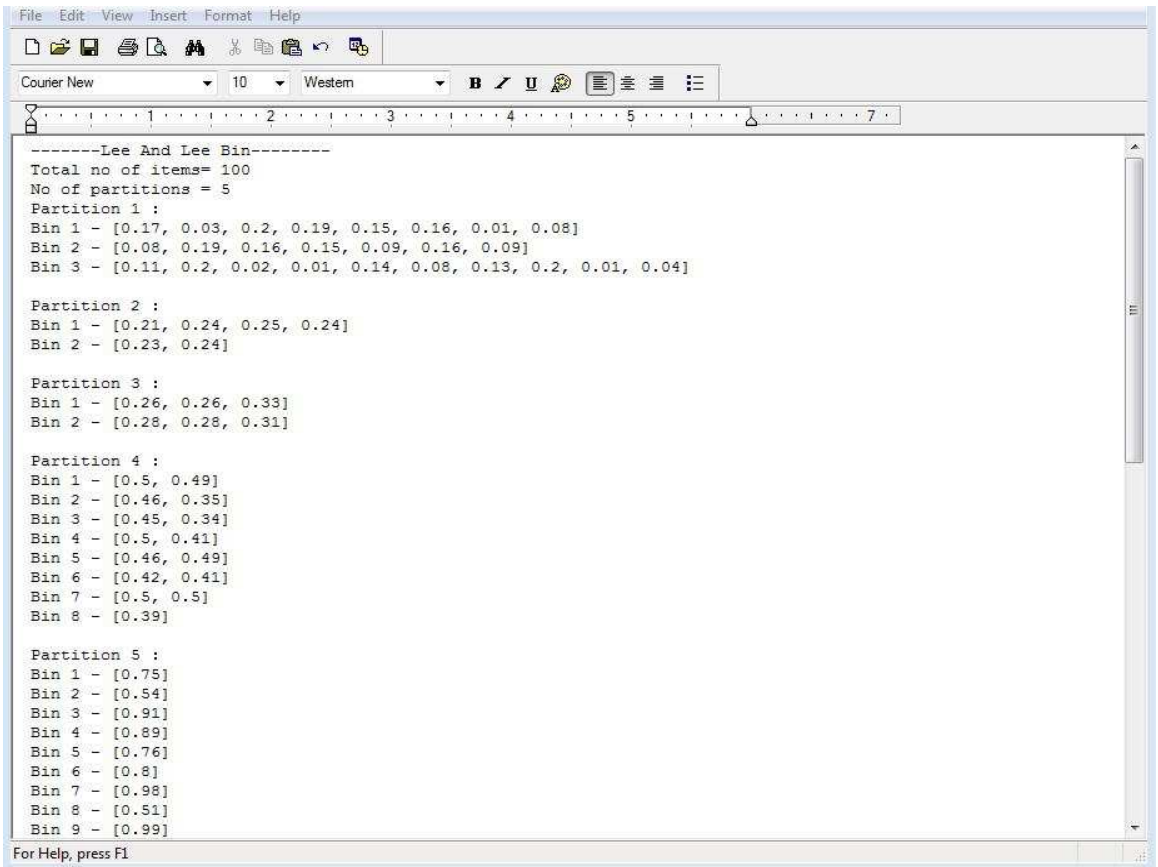
Figure 7: A Screenshot showing random number generation

4.3 Lee and Lee Algorithm:

Bins are packed according to the intervals described in Chapter 3.

Output: Display the number of bins needed and the content of the bins.

Description: Read from the file the value N, M and the sequence of requests $a_1, a_2 .. a_n$. Then compute the number of bins needed using the algorithm of Lee and Lee. Finally display the results. Each time for different input file and for different partitions the above steps are performed and the number of bins are generated differently as shown in Figure 8.



```
-------Lee And Lee Bin--------
Total no of items= 100
No of partitions = 5
Partition 1 :
Bin 1 - [0.17, 0.03, 0.2, 0.19, 0.15, 0.16, 0.01, 0.08]
Bin 2 - [0.08, 0.19, 0.16, 0.15, 0.09, 0.16, 0.09]
Bin 3 - [0.11, 0.2, 0.02, 0.01, 0.14, 0.08, 0.13, 0.2, 0.01, 0.04]

Partition 2 :
Bin 1 - [0.21, 0.24, 0.25, 0.24]
Bin 2 - [0.23, 0.24]

Partition 3 :
Bin 1 - [0.26, 0.26, 0.33]
Bin 2 - [0.28, 0.28, 0.31]

Partition 4 :
Bin 1 - [0.5, 0.49]
Bin 2 - [0.46, 0.35]
Bin 3 - [0.45, 0.34]
Bin 4 - [0.5, 0.41]
Bin 5 - [0.46, 0.49]
Bin 6 - [0.42, 0.41]
Bin 7 - [0.5, 0.5]
Bin 8 - [0.39]

Partition 5 :
Bin 1 - [0.75]
Bin 2 - [0.54]
Bin 3 - [0.91]
Bin 4 - [0.89]
Bin 5 - [0.76]
Bin 6 - [0.8]
Bin 7 - [0.98]
Bin 8 - [0.51]
Bin 9 - [0.99]
```

Figure 8: A Screenshot of output for Lee and Lee algorithm

For harmonic algorithm implemented by Lee and Lee and Epstein, intervals and corresponding bin types are defined in the Table 7

Interval	Bin Type
$I_1 = (0, \frac{1}{4}]$	NB_1
$I_2 = (\frac{1}{4}, \frac{1}{3}]$	NB_2
$I_3 = (\frac{1}{3}, \frac{1}{2}]$	NB_3
$I_4 = (\frac{1}{2}, 1]$	NB_4

Table 7: Intervals formed for Lee and Lee, Epstein

4.4 Epstein Algorithm:

In this algorithm, changes to the lowest indexed bin are made. For the lowest indexed bin only at most k-items are assigned (as described in Chapter 2).

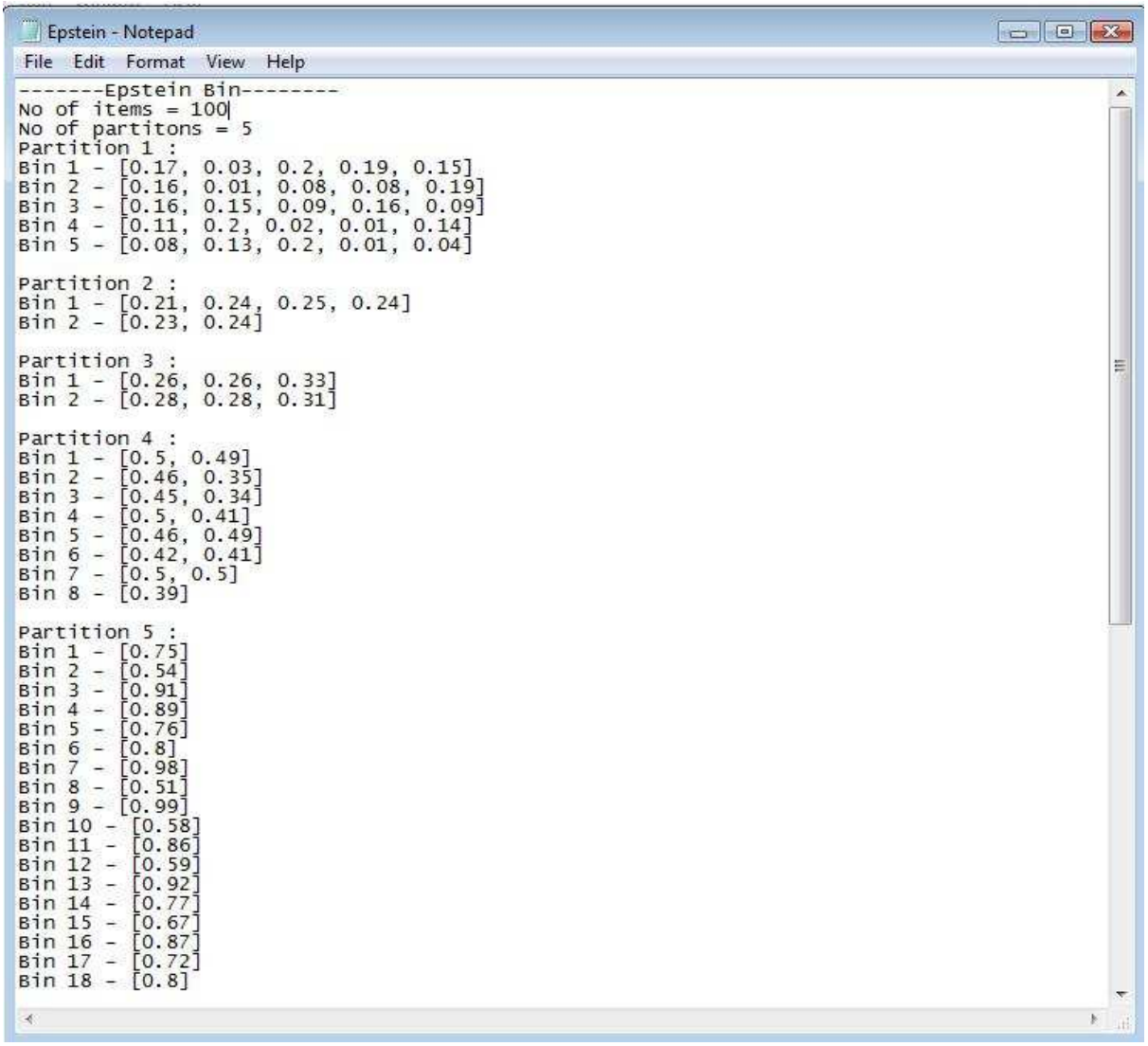
Input: the file "input.dat"

Output: display the number of bins needed and the content of the bins

Description:

Read from the file the value N, M and the sequence of requests $a_1, a_2 .. a_n$.

Then compute the number of bins needed using the algorithm of Epstein and display the results. Therefore total number of bins obtained by the algorithm is shown in the screenshot of Figure 9.



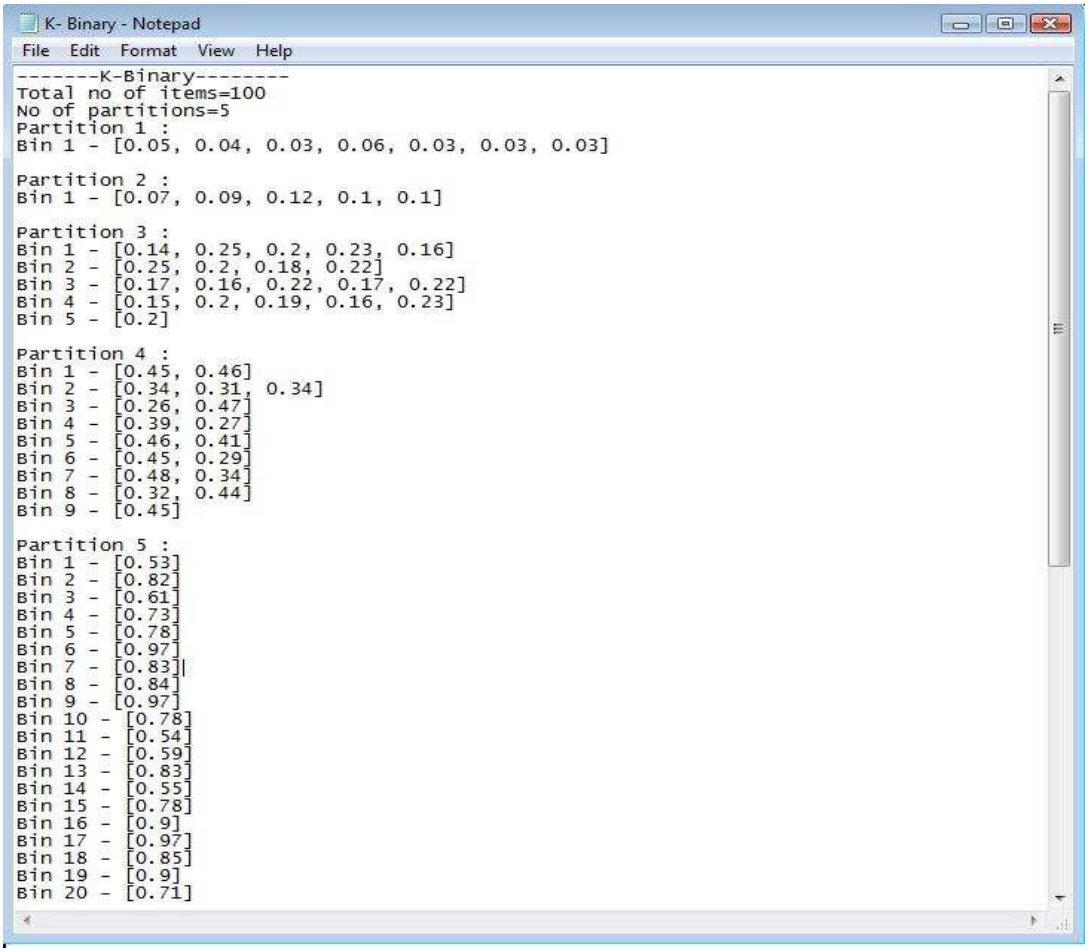
Epstein - Notepad

File Edit Format View Help

```
-------Epstein Bin--------
No of items = 100
No of partitons = 5
Partition 1 :
Bin 1 - [0.17, 0.03, 0.2, 0.19, 0.15]
Bin 2 - [0.16, 0.01, 0.08, 0.08, 0.19]
Bin 3 - [0.16, 0.15, 0.09, 0.16, 0.09]
Bin 4 - [0.11, 0.2, 0.02, 0.01, 0.14]
Bin 5 - [0.08, 0.13, 0.2, 0.01, 0.04]

Partition 2 :
Bin 1 - [0.21, 0.24, 0.25, 0.24]
Bin 2 - [0.23, 0.24]

Partition 3 :
Bin 1 - [0.26, 0.26, 0.33]
Bin 2 - [0.28, 0.28, 0.31]

Partition 4 :
Bin 1 - [0.5, 0.49]
Bin 2 - [0.46, 0.35]
Bin 3 - [0.45, 0.34]
Bin 4 - [0.5, 0.41]
Bin 5 - [0.46, 0.49]
Bin 6 - [0.42, 0.41]
Bin 7 - [0.5, 0.5]
Bin 8 - [0.39]

Partition 5 :
Bin 1 - [0.75]
Bin 2 - [0.54]
Bin 3 - [0.91]
Bin 4 - [0.89]
Bin 5 - [0.76]
Bin 6 - [0.8]
Bin 7 - [0.98]
Bin 8 - [0.51]
Bin 9 - [0.99]
Bin 10 - [0.58]
Bin 11 - [0.86]
Bin 12 - [0.59]
Bin 13 - [0.92]
Bin 14 - [0.77]
Bin 15 - [0.67]
Bin 16 - [0.87]
Bin 17 - [0.72]
Bin 18 - [0.8]
```

Figure 9: A Screenshot of output for Epstein algorithm

4.5 K- Binary Algorithm: In k-Binary algorithm bins are formed according to the interval and the number of bins obtained by the algorithm is shown in the screenshot of figure 10.



```
K- Binary - Notepad
File  Edit  Format  View  Help
-------K-Binary--------
Total no of items=100
No of partitions=5
Partition 1 :
Bin 1 - [0.05, 0.04, 0.03, 0.06, 0.03, 0.03, 0.03]

Partition 2 :
Bin 1 - [0.07, 0.09, 0.12, 0.1, 0.1]

Partition 3 :
Bin 1 - [0.14, 0.25, 0.2, 0.23, 0.16]
Bin 2 - [0.25, 0.2, 0.18, 0.22]
Bin 3 - [0.17, 0.16, 0.22, 0.17, 0.22]
Bin 4 - [0.15, 0.2, 0.19, 0.16, 0.23]
Bin 5 - [0.2]

Partition 4 :
Bin 1 - [0.45, 0.46]
Bin 2 - [0.34, 0.31, 0.34]
Bin 3 - [0.26, 0.47]
Bin 4 - [0.39, 0.27]
Bin 5 - [0.46, 0.41]
Bin 6 - [0.45, 0.29]
Bin 7 - [0.48, 0.34]
Bin 8 - [0.32, 0.44]
Bin 9 - [0.45]

Partition 5 :
Bin 1 - [0.53]
Bin 2 - [0.82]
Bin 3 - [0.61]
Bin 4 - [0.73]
Bin 5 - [0.78]
Bin 6 - [0.97]
Bin 7 - [0.83]
Bin 8 - [0.84]
Bin 9 - [0.97]
Bin 10 - [0.78]
Bin 11 - [0.54]
Bin 12 - [0.59]
Bin 13 - [0.83]
Bin 14 - [0.55]
Bin 15 - [0.78]
Bin 16 - [0.9]
Bin 17 - [0.97]
Bin 18 - [0.85]
Bin 19 - [0.9]
Bin 20 - [0.71]
```

Figure 10: A Screenshot of output for K-binary

Intervals and corresponding bin type are defined in the interval below for k-Binary algorithm.

Interval	Bin Type
$I_1 = (0, \frac{1}{8}]$	B_1
$I_1 = (\frac{1}{8}, \frac{1}{4}]$	B_2
$I_1 = (\frac{1}{4}, \frac{1}{2}]$	B_3
$I_1 = (\frac{1}{2}, 1]$	B_4

Table 8: Intervals formed for K-binary algorithm

4.6 Algorithm Implementation:

It is implemented in java and randomly generated items generated are stored in the file input.dat.

Below is a part of code for random generation of items

```
for (int i = 0;i<N;i++)
            {
            z = (int)Math.round(((double)Math.random()*1) * 100)/ 100.0;
              p.println(z);
              arr.add(z);
            }
            for(int i = 0;i<N;i++)
                  p.println(Math.round((1 - arr.get(i))*100)/ 100.0);


            System.out.println("File Created");
      }
}
```

Then we create the partition of items as shown with a part of code below and we name it as Abstract bin

```
public void createPartitions ()
  {
      partition = new ArrayList [NumberOfPartitions];
```

```
        for(int i = 0;i<NumberOfPartitions++)
        {
                partition[i] = new ArrayList<Bin>(1);
                partition[i].add(new Bin());
        }
    }
```

Once the partitions are created then we display the partition of items which is described in the code below

```
if(!partition[PartitionNumber].get(0).toString().equals(""))
                for(int i = 0;i<partition[PartitionNumber].size();i++)
        {
                        result += "Bin " + (i+1) +" - " +
partition[PartitionNumber].get(i).toString() + "\n";
                        NumberOfBins++;
                }
            return result;}
```

Once number of partitions is obtained then number of bins obtained by Lee and Lee is explained in the part of code below

```
if(flag)
                {
   if(!partition[NumberOfPartitions - end].get(partition[NumberOfPartitions -
end].size()-1).check(number))
    partition[NumberOfPartitions - end].add(new Bin());
    partition[NumberOfPartitions - end].get(partition[NumberOfPartitions -
end].size()-1).addElement(number);
 partition[NumberOfPartitions - end].set(partition[NumberOfPartitions –v
end].size()-1,partition[NumberOfPartitions -
nd].get(partition[NumberOfPartitions - end].size()-1));
                        break;
                }
                start = end;
                end -= 1;
        }
}
```

Then the items are assigned according to each interval is described in the

code below

```
  for(int i = 0;i<number_of_items;i++)
        {
```

```
        b.insert(items.get(i));
      }
     for(int i = 0;i<number_of_partitions;i++)
      {
        System.out.println ("Partition" + (i+1) + " : ");
        String output = b.display (i);

}

} }
```

Now the same method is used for algorithm implemented by Epstein the number of bins formed is explained in the code below

```
while (end != 0)
        {
                if(start == 0 && number <= 1.0/end)
                {
                        flag = true;
   if(partition[NumberOfPartitions - end].get(partition[NumberOfPartitions - end].size()-1).length() == NumberOfPartitions)
   partition[NumberOfPartitions - end].add(new Bin());
                }
                else if(number > (1.0/start) && number <= (1.0/end))
                {
                        flag = true;
        if(!partition[NumberOfPartitions - end].get(partition[NumberOfPartitions - end].size()-1).check(number))
        partition[NumberOfPartitions - end].add(new Bin());
                }
                if(flag)
                {
        partition[NumberOfPartitions - end].get(partition[NumberOfPartitions - end].size()-1).addElement(number);
  partition[NumberOfPartitions - end].set(partition[NumberOfPartitions - end].size()-1,partition[NumberOfPartitions - end].get(partition[NumberOfPartitions - end].size()-1));
                        break;
                }
                start = end;
                end -= 1;
        }
   }
}
```

Now for the K-Binary algorithm the number of bins obtained is described in the code below

```
while(end != 0)
        {
                if(start == 0 && number <= 1.0/Math.pow(2,(end-1)))
                {
                        flag = true;
                }
  else if(number > (1.0/Math.pow(2,(start-1))) && number <=
1.0/Math.pow(2,(end-1))))
                {
                flag = true;
                }
                if(flag)
                {
        if(!partition[NumberOfPartitions -
end].get(partition[NumberOfPartitions - end].size()-1).check(number))
        partition [NumberOfPartitions - end].add(new Bin());
        Partition [NumberOfPartitions - end].get(partition[NumberOfPartitions
- end].size()-1).addElement(number);
        partition[NumberOfPartitions - end].set(partition[NumberOfPartitions -
end].size()-1,partition[NumberOfPartitions -
end].get(partition[NumberOfPartitions - end].size()-1));
                        break;
                }
                start = end;
                end -= 1;
        }
        }
}
```

Thus, all these steps are implemented. The results obtained after partitioning the intervals and analysis over the results for different requests and graphs will be discussed in chapter 5 in the thesis.

CHAPTER 5

SIMULATION AND RESULTS

As discussed in Chapter 4, the K-Binary algorithm produces different results for different set of items. This chapter is divided in two sections. We first discuss the performance of the algorithm in terms of total number of bins for different runs. In the second part we discuss the performance of the algorithms based on the average values for the number of bins of each type.

5.1 Comparison based on Bins:

We discuss the performance of the K- binary algorithm based on the total number of bins. The bins formed vary for different algorithms. Table 9 provides the number of bins obtained for each algorithm according to the items specified. From Table 9 we notice that if the items generated are 20 then the necessary number of bins is more than half the items. i.e. bins formed are 13 for Lee and Lee, 14 for Epstein and K-binary results in 15, and so on. There are some cases when our proposed K-Binary algorithm requires less bins than Lee and Lee, and there are some cases where k-Binary algorithm requires more bins than Lee and Lee. No trend has been observed as for which number of items some algorithm is better.

Items	LL	E	O
20	13	14	15
40	27	28	28
60	41	41	40
80	53	55	53
100	66	68	67
150	99	100	98
200	132	130	131
250	162	168	163
300	194	200	193
350	225	233	230
400	260	267	260
450	288	299	293
500	322	330	323

Table 9: Total bins formed for three algorithms for different items

5.2 Comparisons based on Average:

We compute the average based on bins formed for each algorithm when the bins are separated into 5 partitions. We execute 50 different and we compute the total number of bins by summing the number of bins of each type as briefly described in Chapter 4. We also compute the average of the bins among all types and we show the values in Table 10.

N	Lee & Lee	Epstein	K-binary
20	5.017	4.0902	4.5152
40	9.5304	8.5226	8.8152
60	15.701	14.047	14.9744
80	20.756	19.0932	20.02
100	26.381	23.951	25.4988
150	40.0268	36.9672	38.9824
200	52.8476	49.39516	51.8582
250	66.8148	63.0702	65.9412
300	79.7872	75.40708	77.2372
350	93.5156	88.1338	92.3134
400	106.6986	102.793	107.5884
450	118.1946	112.2266	117.0962
500	132.7986	125.8654	131.5816

Table 10: Average values

Based on the average values from Table 10 of each algorithm we draw two graphs. For the first graph we consider relatively smaller number of items, for example 20, 40, 60, 80 and 100. The second graph is drawn for large number of items. The x axis shows the number of bins and the y axis show the average number of bins require by each

algorithm. We calculated the average with small or large sequences of requests and random values for the requests.

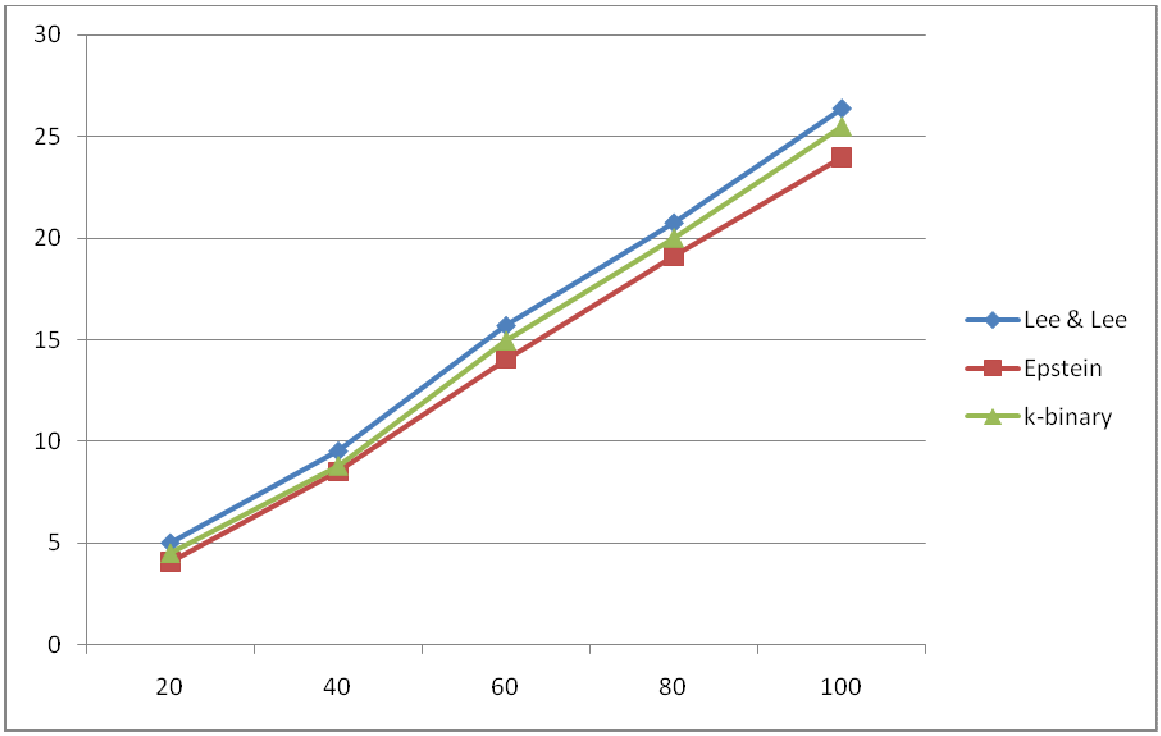


Figure 11. Trend of AVG_OFFSET smaller number of items

From Figures 11 and 12 we draw the following observations:

In figure 11, we note that for smaller items there is not much variation between the algorithms. The proposed K- Binary algorithm is as efficient as Lee and Lee's and Epstein's algorithms. From Figure 12, we note that for larger items the proposed K- Binary algorithm produces a lower average than Lee and Lee but higher average than Epstein. We can then conclude that the proposed K- binary algorithms behaves in average better than both previously proposed algorithms in terms of the variance

on the type of bins used. Lee and Lee have a high variation on the type of bins used, whereas Epstein uses more bins for packing the items than Lee and Lee.

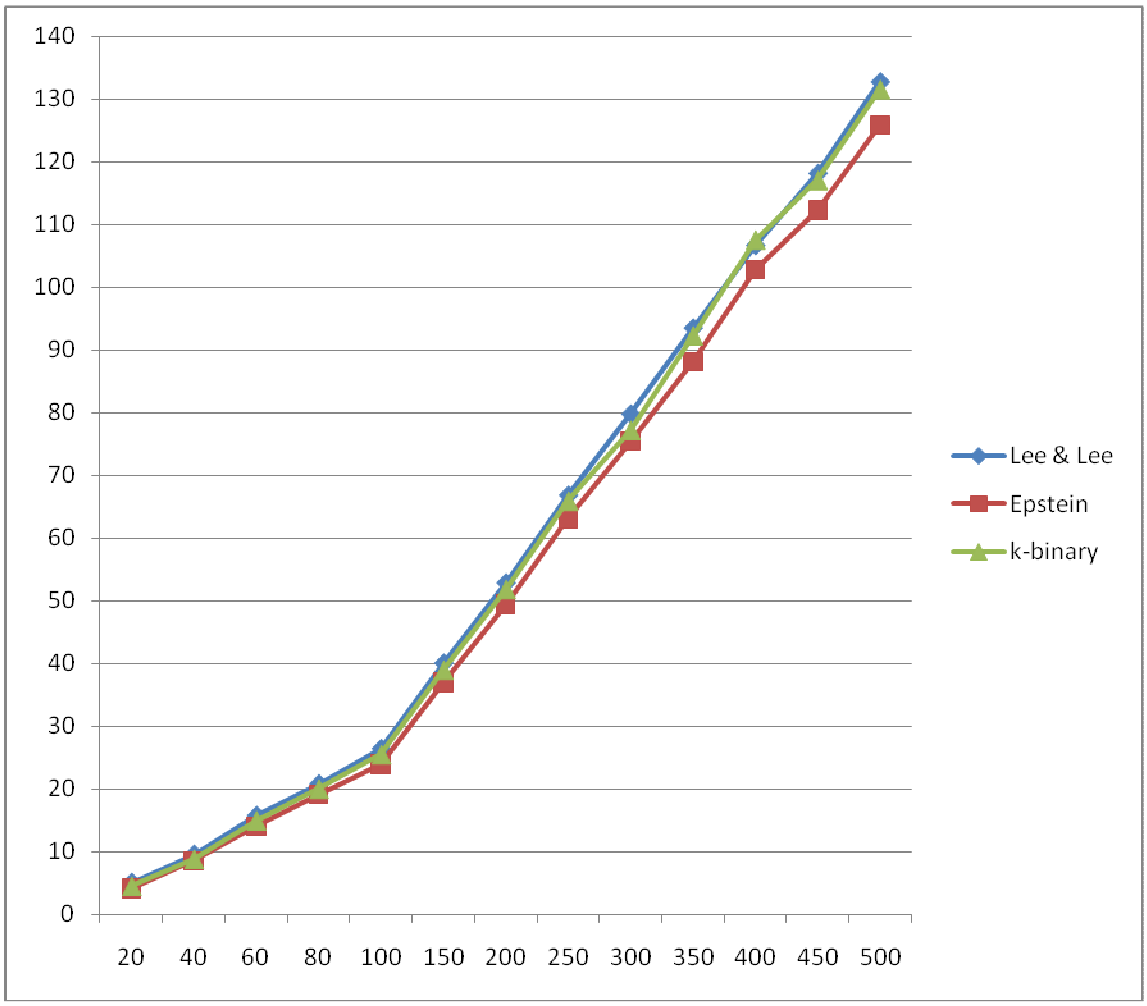


Figure 12. Trend of AVG_OFFSET large number of items

CHAPTER 6

CONCLUSION AND FUTURE WORK

Allocating memory in a data center for storage requests can be modeled as a classical bin packing problem where online requests for storage are served in minimum time and with the minimum cost. The cost for hardware components if cheaper is the components are bought in bulk; the maintenance is also easier. So we can assume that in a data center all the hard disks have the same capacity. The goal of bin packing problem is to minimize the number of bins used for serving a fixed number of requests. The algorithm proposed by Lee and Lee has a competitive ratio that is very close to the optimum, but has a high variation on the types of bins used, in the sense that for random requests it either uses a lot of small type bins or a lot of large type bins. The algorithm proposed by Epstein has a worse competitive ratio, i.e. uses more bins for packing the items than Lee and Lee, but it has a much smaller variation than Lee and Lee. We propose an algorithm, K-binary, that uses fewer bins than the algorithm proposed by Epstein, and a slightly more bins than the algorithm proposed by Lee and Lee. At the same time, the K-Binary algorithm works better than both previously proposed algorithms. We drew these conclusions after executing extensive simulations with small or large sequences of requests and random values for the requests.

This thesis focuses on serving the requests using one dispatcher. But modern computers have multiple processors, thus it is of future interest to provide algorithms with better competitive ratio than Lee and Lee that allow serving two or more requests at a time. For example, if two requests can be served at the same time, we can have bins that can be filled simultaneously by two dispatchers and allow only one dispatcher to fill a bin in the moment the bin is close to be filled, for example, it needs one more item to be filled.

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