

Three-Dimensional Embedding of Binary Trees

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Embeddings are considered

- for an efficient simulation of one parallel computer architecture by another
- an efficient method for using a parallel computer architecture to execute some standard computational process
- to give area-efficient patterns for printing circuits on VLSI chips or wafers

Embedding (Layout)

Guest graph $G = (V, E)$: T_h

Host graph $H = (V', E')$: $K_{a,b,c}$

Injection $f : V \rightarrow V'$

$load(x) =$

of vertices mapped by f to x

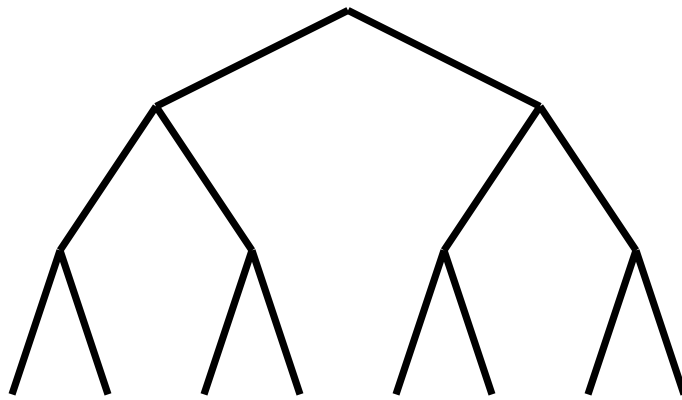
$congestion(x) =$

paths $f(uv)$ containing x as an internal vertex

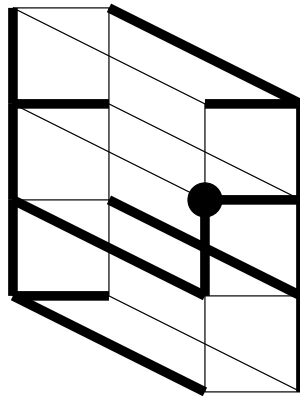
Our embeddings:

total congestion (load + congestion) = 1

An Embedding of T_3 into K_{224}

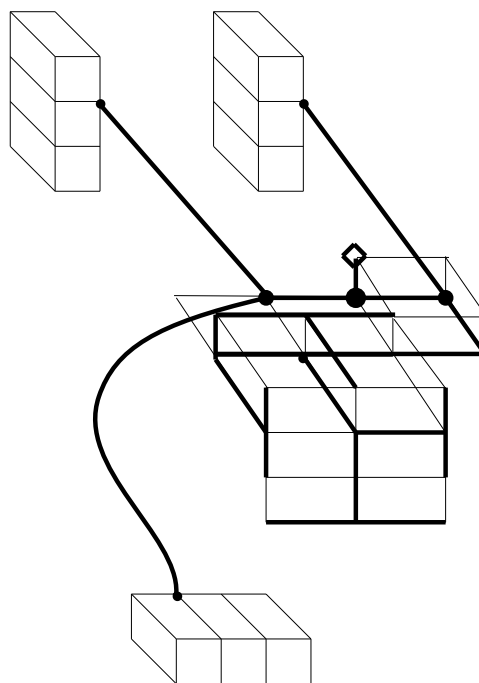
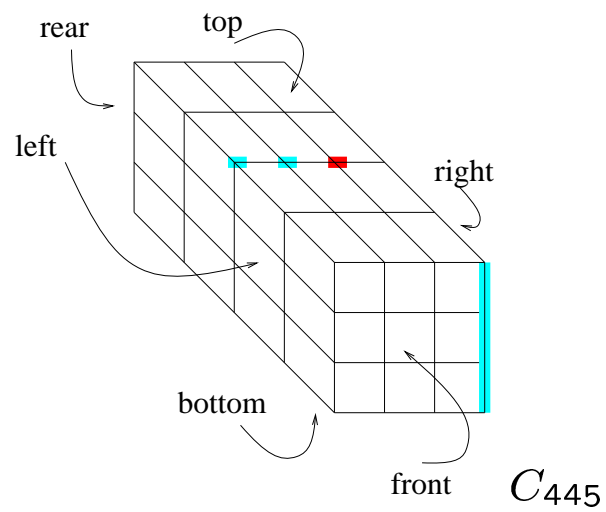


T_3

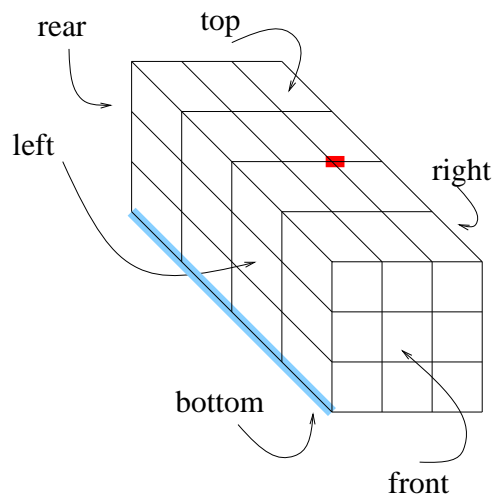


$2 \times 2 \times 4$ "module" holding T_3

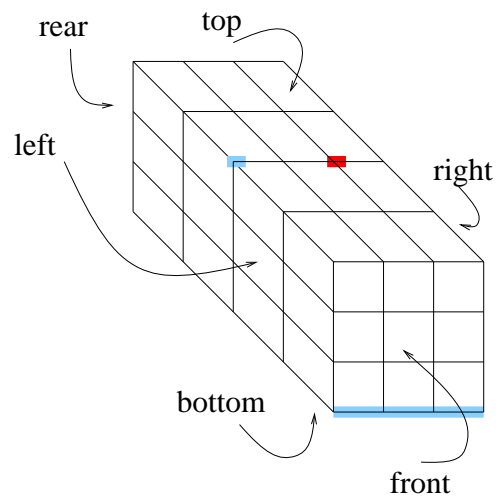
An Embedding of T_5^+ into K_{445}



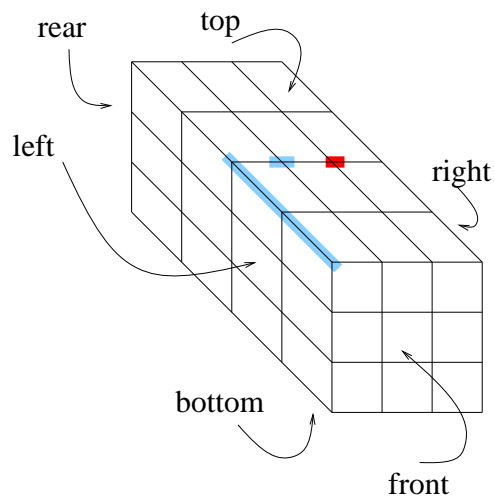
More K_{445} building blocks



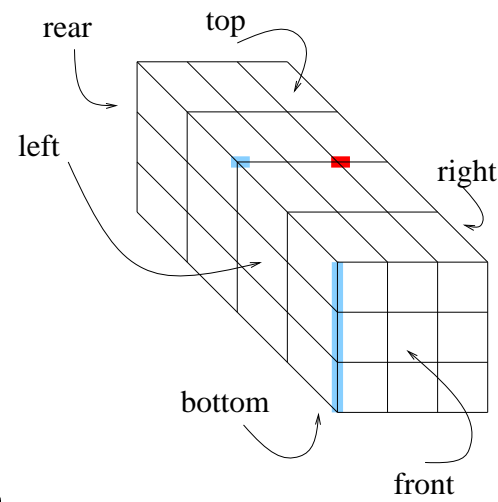
A_{445}



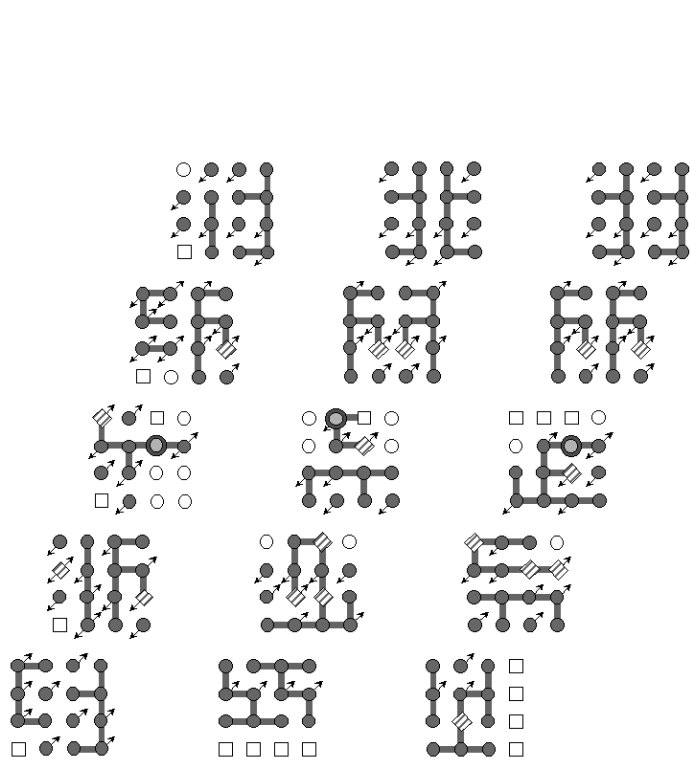
B_{445}



D_{445}



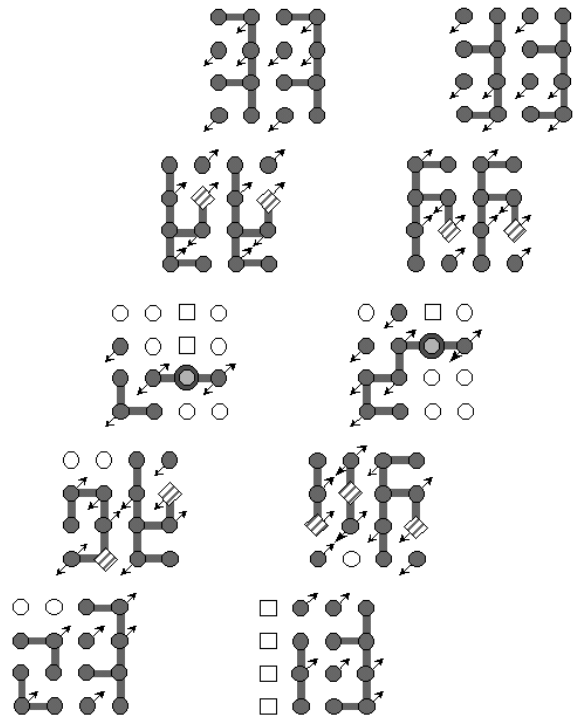
E_{445}



Piece A

Piece B

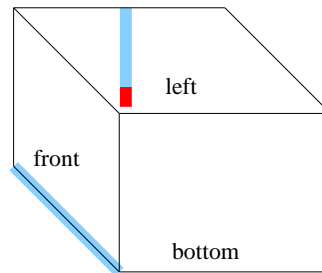
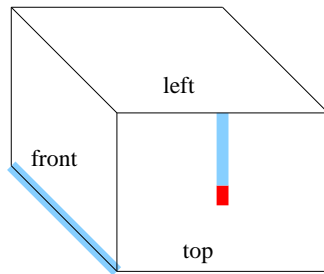
Piece C



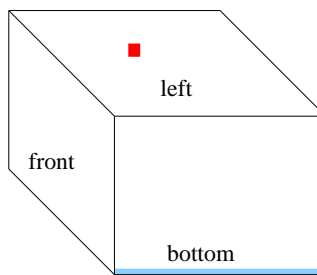
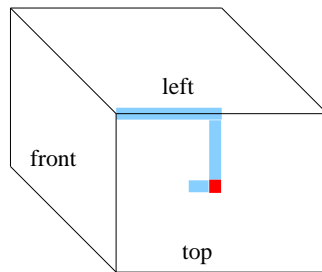
Piece D

Piece E

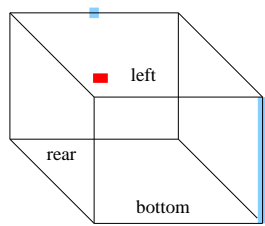
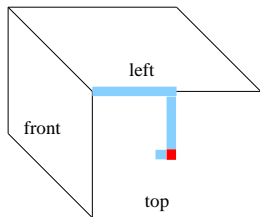
The recursive step



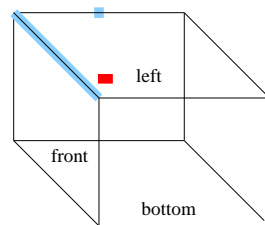
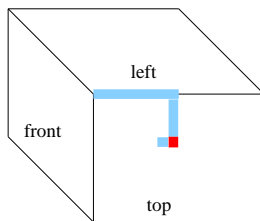
$$A_{b,c,2a} = C_{abc} + C_{abc}$$



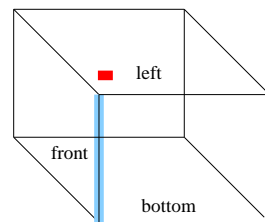
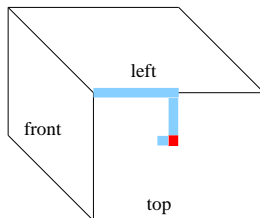
$$B_{b,c,2a} = D_{abc} + A_{abc}$$



$$C_{b,c,2a} = D_{abc} + B_{abc}$$



$$D_{b,c,2a} = D_{abc} + E_{abc}$$



$$E_{b,c,2a} = D_{abc} + B_{abc}$$

Expansion ratio is 1.25

We have embeddings of T_h^+ into $K_{a,b,c}$ with

$$a_h = b_{h-1}$$

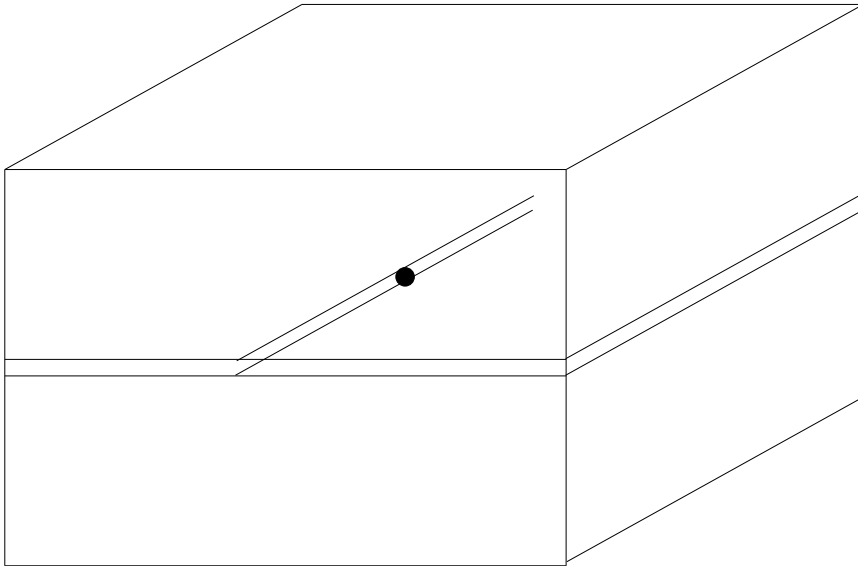
$$b_h = c_{h-1}$$

$$c_h = 2a_{h-1}$$

h	a	b	c
5	4	4	5
6	4	5	8
7	5	8	8
8	8	8	10
9	8	10	16
10	10	16	16
11	16	16	20

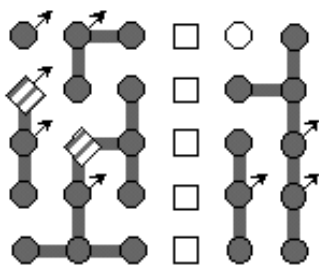
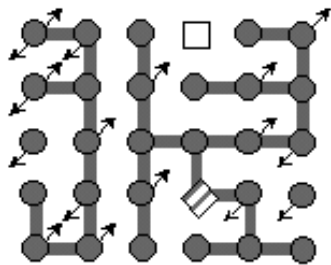
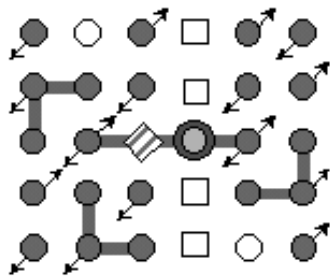
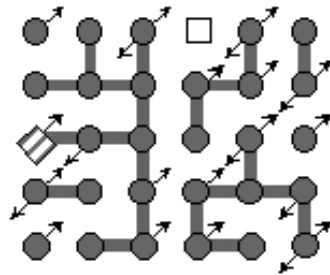
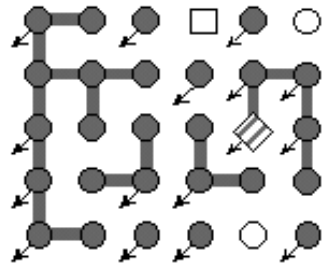
$K_{a,b,c}$ has exactly $\frac{5}{4}$ as many nodes as T_h^+ .

Layouts with a fixed number of Layers



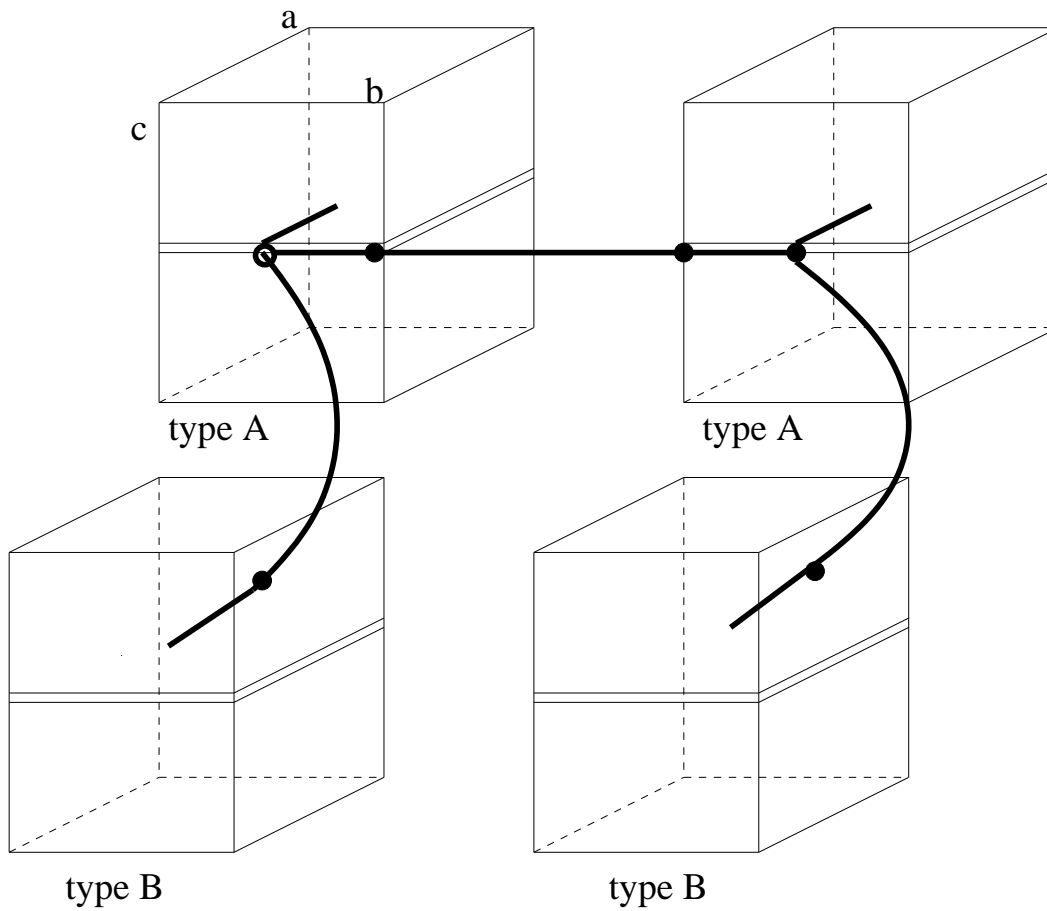
T_6 into a K_{566} :

.



Combined A and B

The recursive step



$$\text{Expansion ratio} : \frac{150}{128} = 1.172$$

Lower Bounds

2D grid: expansion ratio ≥ 1.122

based on the idea

number of nodes in a tree grows exponentially with
its height

number of nodes in a 2D grid at distance d is quadratic

analogous argument for 3D: expansion ratio ≥ 1.0015

How to improve embeddings

Likely, embeddings can be improved

- Have larger, more tightly, packed base cases in recursive constructions.
- Have recursive constructions which "interlock" intelligently.