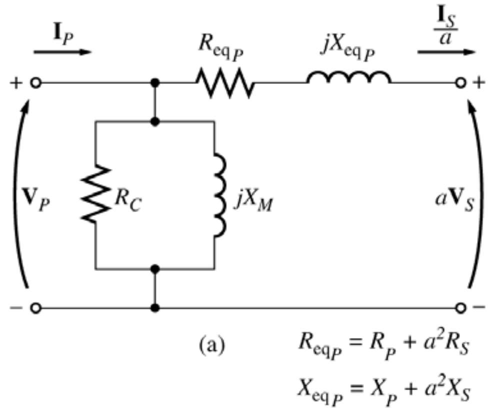
Chapter 3: *Transformers*

3-1. The secondary winding of a transformer has a terminal voltage of $v_s(t) = 282.8 \sin 377t$ V. The turns ratio of the transformer is 50:200 (a = 0.25). If the secondary current of the transformer is $i_s(t) = 7.07 \sin(377t - 36.87°)$ A, what is the primary current of this transformer? What are its voltage regulation and efficiency? The impedances of this transformer referred to the primary side are

$$R_{\text{eq}} = 0.05\ \Omega \qquad R_C = 75\ \Omega$$

$$X_{\text{eq}} = 0.225\ \Omega \qquad X_M = 20\ \Omega$$

SOLUTION The equivalent circuit of this transformer is shown below. (Since no particular equivalent circuit was specified, we are using the approximate equivalent circuit referred to the primary side.)



The secondary voltage and current are

$$\mathbf{V}_S = \frac{282.8}{\sqrt{2}}\angle 0°\text{ V} = 200\angle 0°\text{ V}$$

$$\mathbf{I}_S = \frac{7.07}{\sqrt{2}}\angle -36.87°\text{ A} = 5\angle -36.87°\text{ A}$$

The secondary voltage referred to the primary side is

$$\mathbf{V}_S' = a\mathbf{V}_S = 50\angle 0°\text{ V}$$

The secondary current referred to the primary side is

$$\mathbf{I}_S' = \frac{\mathbf{I}_S}{a} = 20\angle\text{-}36.87°\text{ A}$$

The primary circuit voltage is given by

$$\mathbf{V}_P = \mathbf{V}_S' + \mathbf{I}_S'\left(R_{\text{eq}} + jX_{\text{eq}}\right)$$

$$\mathbf{V}_P = 50\angle 0°\text{ V} + (20\angle -36.87°\text{ A})(0.05\,\Omega + j0.225\,\Omega) = 53.6\angle 3.2°\text{ V}$$

The excitation current of this transformer is

$$\mathbf{I}_{\text{EX}} = \mathbf{I}_C + \mathbf{I}_M = \frac{53.6\angle 3.2°\text{ V}}{75\,\Omega} + \frac{53.6\angle 3.2°\text{ V}}{j20\,\Omega} = 0.7145\angle 3.2° + 2.679\angle -86.8°$$

$$\mathbf{I}_{\text{EX}} = 2.77\angle -71.9°$$

Therefore, the total primary current of this transformer is

$$\mathbf{I}_P = \mathbf{I}_S' + \mathbf{I}_{\text{EX}} = 20\angle -36.87° + 2.77\angle -71.9° = 22.3\angle -41.0° \text{ A}$$

The voltage regulation of the transformer at this load is

$$\text{VR} = \frac{V_P - aV_S}{aV_S} \times 100\% = \frac{53.6 - 50}{50} \times 100\% = 7.2\%$$

The input power to this transformer is

$$P_{\text{IN}} = V_P I_P \cos\theta = (53.6 \text{ V})(22.3 \text{ A})\cos\left[3.2° - (-41.0°)\right]$$
$$P_{\text{IN}} = (53.6 \text{ V})(22.3 \text{ A})\cos 44.2° = 857 \text{ W}$$

The output power from this transformer is

$$P_{\text{OUT}} = V_S I_S \cos\theta = (200 \text{ V})(5 \text{ A})\cos(36.87°) = 800 \text{ W}$$

Therefore, the transformer's efficiency is

$$\eta = \frac{P_{\text{OUT}}}{P_{\text{IN}}} \times 100\% = \frac{800 \text{ W}}{857 \text{ W}} \times 100\% = 93.4\%$$

3-2. A 20-kVA 8000/277-V distribution transformer has the following resistances and reactances:

$R_P = 32\ \Omega$	$R_S = 0.05\ \Omega$
$X_P = 45\ \Omega$	$X_S = 0.06\ \Omega$
$R_C = 250\ \text{k}\Omega$	$X_M = 30\ \text{k}\Omega$

The excitation branch impedances are given referred to the high-voltage side of the transformer.

(a) Find the equivalent circuit of this transformer referred to the high-voltage side.

(b) Find the per-unit equivalent circuit of this transformer.

(c) Assume that this transformer is supplying rated load at 277 V and 0.8 PF lagging. What is this transformer's input voltage? What is its voltage regulation?

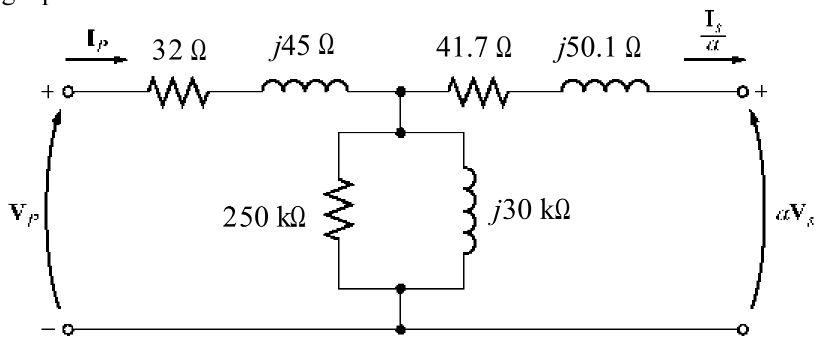
(d) What is the transformer's efficiency under the conditions of part *(c)*?

SOLUTION

(a) The turns ratio of this transformer is a = 8000/277 = 28.89. Therefore, the secondary impedances referred to the primary side are

$$R_S' = a^2 R_S = (28.89)^2 (0.05\ \Omega) = 41.7\ \Omega$$
$$X_S' = a^2 X_S = (28.89)^2 (0.06\ \Omega) = 50.1\ \Omega$$

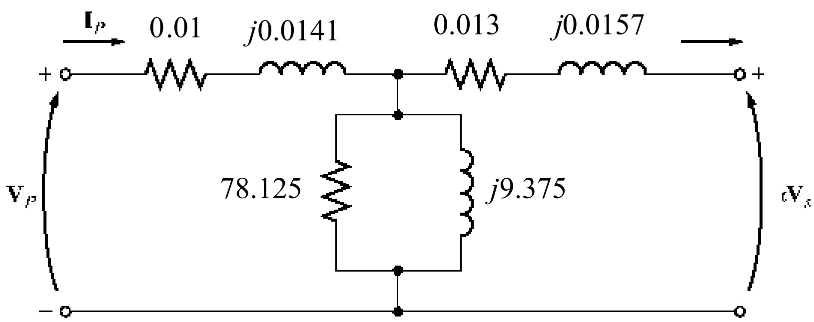
The resulting equivalent circuit is



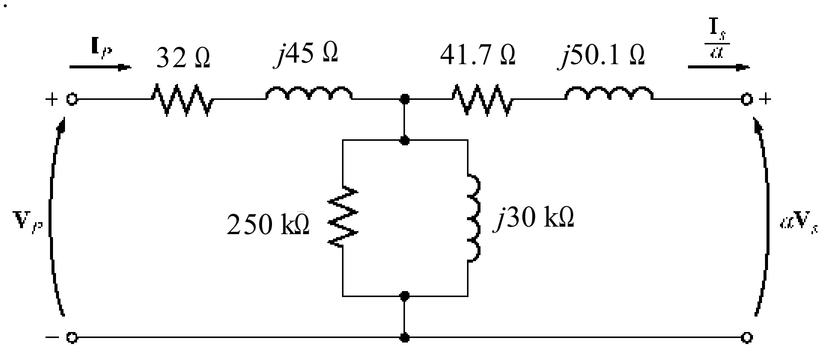
(b) The rated kVA of the transformer is 20 kVA, and the rated voltage on the primary side is 8000 V, so the rated current in the primary side is 20 kVA/8000 V = 2.5 A. Therefore, the base impedance on the primary side is

$$Z_{\text{base}} = \frac{V_{\text{base}}}{I_{\text{base}}} = \frac{8000 \text{ V}}{2.5 \text{ A}} = 3200\,\Omega$$

Since $Z_{\text{pu}} = Z_{\text{actual}} / Z_{\text{base}}$, the resulting per-unit equivalent circuit is as shown below:



(c) To simplify the calculations, use the simplified equivalent circuit referred to the primary side of the transformer:



The secondary current in this transformer is

$$\mathbf{I}_S = \frac{20 \text{ kVA}}{277 \text{ V}} \angle -36.87° \text{ A} = 72.2\angle -36.87° \text{ A}$$

The secondary current referred to the primary side is

$$\mathbf{I}_S' = \frac{\mathbf{I}_S}{a} = \frac{72.2\angle -36.87° \text{ A}}{28.89} = 2.50\angle -36.87° \text{ A}$$

Therefore, the primary voltage on the transformer is

$$\mathbf{V}_P = \mathbf{V}_S' + \left(R_{\text{EQ}} + jX_{\text{EQ}}\right)\mathbf{I}_S'$$
$$\mathbf{V}_P = 8000\angle 0° \text{ V} + (73.7 + j95.1)(2.50\angle -36.87° \text{ A}) = 8290\angle 0.55° \text{ V}$$

The voltage regulation of the transformer under these conditions is

$$\text{VR} = \frac{8290 - 8000}{8000} \times 100\% = 3.63\%$$

(d) Under the conditions of part *(c)*, the transformer's output power copper losses and core losses are:

$$P_{\text{OUT}} = S\cos\theta = (20\text{ kVA})(0.8) = 16\text{ kW}$$
$$P_{\text{CU}} = \left(I_S'\right)^2 R_{\text{EQ}} = (2.5)^2(73.7) = 461\text{ W}$$
$$P_{\text{core}} = \frac{V_S'^2}{R_C} = \frac{8290^2}{250{,}000} = 275\text{ W}$$

The efficiency of this transformer is

$$\eta = \frac{P_{\text{OUT}}}{P_{\text{OUT}} + P_{\text{CU}} + P_{\text{core}}} \times 100\% = \frac{16{,}000}{16{,}000 + 461 + 275} \times 100\% = 95.6\%$$

3-3. A 2000-VA 230/115-V transformer has been tested to determine its equivalent circuit. The results of the tests are shown below.

Open-circuit test	Short-circuit test
V_{OC} = 230 V	V_{SC} = 13.2 V
I_{OC} = 0.45 A	I_{SC} = 6.0 A
P_{OC} = 30 W	P_{SC} = 20.1 W

All data given were taken from the primary side of the transformer.

(a) Find the equivalent circuit of this transformer referred to the low-voltage side of the transformer.

(b) Find the transformer's voltage regulation at rated conditions and (1) 0.8 PF lagging, (2) 1.0 PF, (3) 0.8 PF leading.

(c) Determine the transformer's efficiency at rated conditions and 0.8 PF lagging.

SOLUTION

(a) OPEN CIRCUIT TEST:

$$\left|Y_{\text{EX}}\right| = \left|G_C - jB_M\right| = \frac{0.45\text{ A}}{230\text{ V}} = 0.001957\ \text{S}$$
$$\theta = \cos^{-1}\frac{P_{\text{OC}}}{V_{\text{OC}}I_{\text{OC}}} = \cos^{-1}\frac{30\text{ W}}{(230\text{ V})(0.45\text{ A})} = 73.15°$$
$$Y_{\text{EX}} = G_C - jB_M = 0.001957\angle -73.15°\text{ S} \;=\; 0.000567\text{-}j0.001873\text{ S}$$
$$R_C = \frac{1}{G_C} = 1763\,\Omega$$
$$X_M = \frac{1}{B_M} = 534\,\Omega$$

SHORT CIRCUIT TEST:

$$\left|Z_{\text{EQ}}\right| = \left|R_{\text{EQ}} + jX_{\text{EQ}}\right| = \frac{13.2\text{ V}}{6.0\text{ A}} = 2.20\,\Omega$$

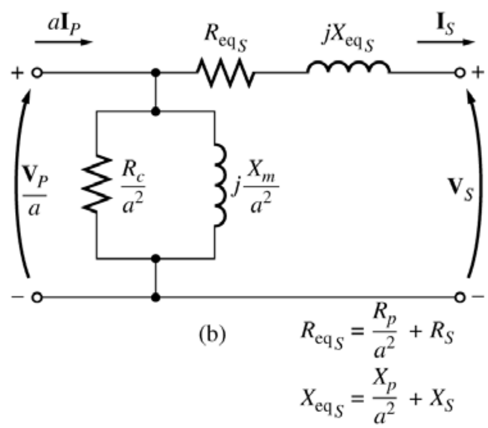
$$\theta = \cos^{-1}\frac{P_{\text{SC}}}{V_{\text{SC}}I_{\text{SC}}} = \cos^{-1}\frac{20.1\text{ W}}{(13.2\text{ V})(6\text{ A})} = 75.3°$$

$$Z_{\text{EQ}} = R_{\text{EQ}} + jX_{\text{EQ}} = 2.20\angle 75.3°\,\Omega = 0.558 + j2.128\,\Omega$$

$$R_{\text{EQ}} = 0.558\,\Omega$$

$$X_{\text{EQ}} = j2.128\,\Omega$$

To convert the equivalent circuit to the secondary side, divide each impedance by the square of the turns ratio (a = 230/115 = 2). The resulting equivalent circuit is shown below:



$R_{\text{EQ},S} = 0.140\ \Omega$ $\qquad$ $X_{\text{EQ},S} = j0.532\ \Omega$

$R_{C,S} = 441\ \Omega$ $\qquad$ $X_{M,S} = 134\ \Omega$

(b) To find the required voltage regulation, we will use the equivalent circuit of the transformer referred to the secondary side. The rated secondary current is

$$I_S = \frac{1000\text{ VA}}{115\text{ V}} = 8.70\text{ A}$$

We will now calculate the primary voltage referred to the secondary side and use the voltage regulation equation for each power factor.

(1) <u>0.8 PF Lagging:</u>

$$\mathbf{V}_P' = \mathbf{V}_S + Z_{\text{EQ}}\mathbf{I}_S = 115\angle 0°\text{ V} + (0.140 + j0.532\,\Omega)(8.7\angle\text{-}36.87°\text{ A})$$

$$\mathbf{V}_P' = 118.8\angle 1.4°\text{ V}$$

$$\text{VR} = \frac{118.8 - 115}{115} \times 100\% = 3.3\%$$

(2) <u>1.0 PF:</u>

$$\mathbf{V}_P' = \mathbf{V}_S + Z_{\text{EQ}}\mathbf{I}_S = 115\angle 0°\text{ V} + (0.140 + j0.532\,\Omega)(8.7\angle 0°\text{ A})$$

$$\mathbf{V}_P' = 116.3\angle 2.28°\text{ V}$$

$$\text{VR} = \frac{116.3 - 115}{115} \times 100\% = 1.1\%$$

(3) **<u>0.8 PF Leading:</u>**

$$\mathbf{V}_P' = \mathbf{V}_S + Z_{\text{EQ}}\mathbf{I}_S = 115\angle 0^\circ \text{ V} + (0.140 + j0.532\ \Omega)(8.7\angle 36.87^\circ \text{ A})$$

$$\mathbf{V}_P' = 113.3\angle 2.24^\circ \text{ V}$$

$$\text{VR} = \frac{113.3 - 115}{115} \times 100\% = -1.5\%$$

(c) At rated conditions and 0.8 PF lagging, the output power of this transformer is

$$P_{\text{OUT}} = V_S I_S \cos\theta = (115\text{ V})(8.7\text{ A})(0.8) = 800\text{ W}$$

The copper and core losses of this transformer are

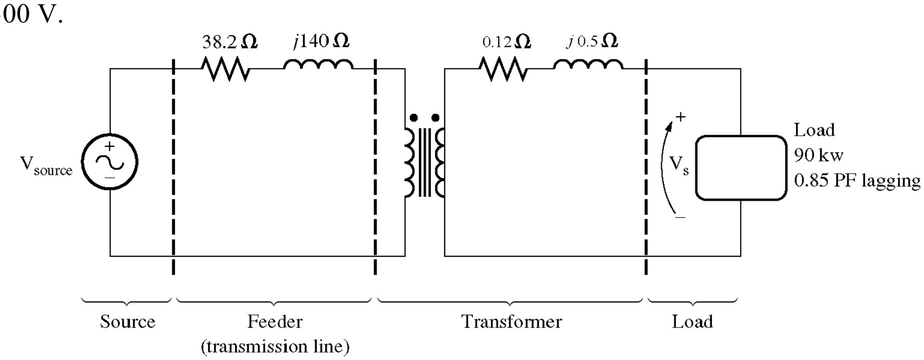
$$P_{\text{CU}} = I_S^2\, R_{\text{EQ},S} = (8.7\text{ A})^2 (0.140\ \Omega) = 10.6\text{ W}$$

$$P_{\text{core}} = \frac{\left(V_P'\right)^2}{R_C} = \frac{(118.8\text{ V})^2}{441\ \Omega} = 32.0\text{ W}$$

Therefore the efficiency of this transformer at these conditions is

$$\eta = \frac{P_{\text{OUT}}}{P_{\text{OUT}} + P_{\text{CU}} + P_{\text{core}}} \times 100\% = \frac{800\text{ W}}{800\text{ W} + 10.6\text{ W} + 32.0\text{ W}} = 94.9\%$$

3-4. A single-phase power system is shown in Figure P3-1. The power source feeds a 100-kVA 14/2.4-kV transformer through a feeder impedance of $38.2 + j140\ \Omega$. The transformer's equivalent series impedance referred to its low-voltage side is $0.12 + j0.5\ \Omega$. The load on the transformer is 90 kW at 0.85 PF lagging and 2300 V.



(a) What is the voltage at the power source of the system?

(b) What is the voltage regulation of the transformer?

(c) How efficient is the overall power system?

SOLUTION

To solve this problem, we will refer the circuit to the secondary (low-voltage) side. The feeder's impedance referred to the secondary side is

$$Z_{\text{line}}' = \left(\frac{2.4\text{ kV}}{14\text{ kV}}\right)^2 (38.2\ \Omega + j140\ \Omega) = 1.12 + j4.11\ \Omega$$

The secondary current $\mathbf{I}_S$ is given by

$$I_S = \frac{90\text{ kW}}{(2300\text{ V})(0.9)} = 43.48\text{ A}$$

$$\mathbf{I}_S = 43.48\angle -25.8°\text{ A}$$

(a) The voltage at the power source of this system (referred to the secondary side) is

$$\mathbf{V}_{\text{source}}' = \mathbf{V}_S + \mathbf{I}_S Z_{\text{line}}' + \mathbf{I}_S Z_{\text{EQ}}$$

$$\mathbf{V}_{\text{source}}' = 2300\angle 0°\text{ V} + (43.48\angle -25.8°\text{ A})(1.12 + j4.11\ \Omega) + (43.48\angle -25.8°\text{ A})(0.12 + j0.5\ \Omega)$$

$$\mathbf{V}_{\text{source}}' = 2441\angle 3.7°\text{ V}$$

Therefore, the voltage at the power source is

$$\mathbf{V}_{\text{source}} = (2441\angle 3.7°\text{ V})\frac{14\text{ kV}}{2.4\text{ kV}} = 14.24\angle 3.7°\text{ kV}$$

(b) To find the voltage regulation of the transformer, we must find the voltage at the primary side of the transformer (referred to the secondary side) under full load conditions:

$$\mathbf{V}_P' = \mathbf{V}_S + \mathbf{I}_S Z_{\text{EQ}}$$

$$\mathbf{V}_P' = 2300\angle 0°\text{ V} + (43.48\angle -25.8°\text{ A})(0.12 + j0.5\ \Omega) = 2314\angle 0.43°\text{ V}$$

There is a voltage drop of 14 V under these load conditions. Therefore the voltage regulation of the transformer is

$$\text{VR} = \frac{2314 - 2300}{2300}\times 100\% = 0.6\%$$

(c) The power supplied to the load is P_{OUT} = 90 kW. The power supplied by the source is

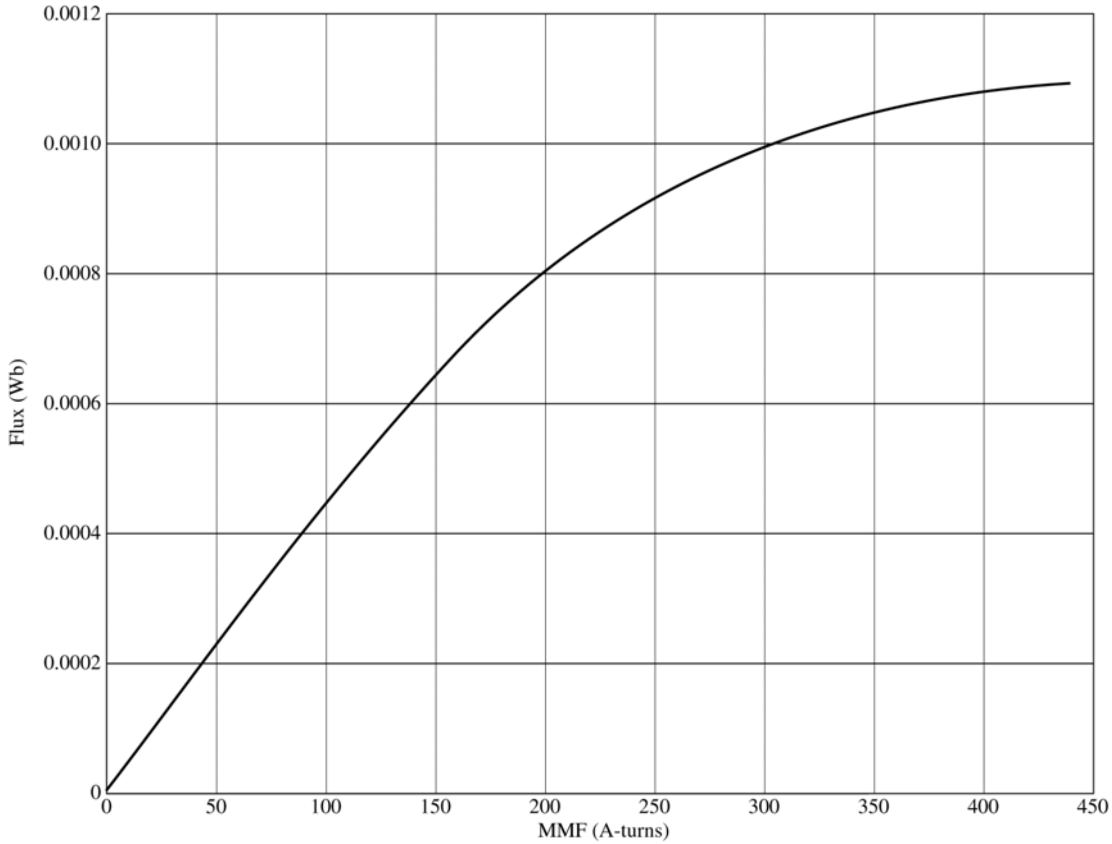
$$P_{\text{IN}} = V_{\text{source}}' I_S \cos\theta = (2441\text{ V})(43.48\text{ A})\cos 29.5° = 92.37\text{ kW}$$

Therefore, the efficiency of the power system is

$$\eta = \frac{P_{\text{OUT}}}{P_{\text{IN}}}\times 100\% = \frac{90\text{ kW}}{92.37\text{ kW}}\times 100\% = 97.4\%$$

3-5. When travelers from the USA and Canada visit Europe, they encounter a different power distribution system. Wall voltages in North America are 120 V rms at 60 Hz, while typical wall voltages in Europe are 220-240 V at 50 Hz. Many travelers carry small step-up / step-down transformers so that they can use their appliances in the countries that they are visiting. A typical transformer might be rated at 1-kVA and 120/240 V. It has **500**[1] turns of wire on the 120-V side and **1000** turns of wire on the 240-V side. The magnetization curve for this transformer is shown in Figure P3-2, and can be found in file `p32.mag` at this book's Web site.

[1] Note that this turns ratio was backwards in the first printing of the text. This error should be corrected in all subsequent printings.



(a) Suppose that this transformer is connected to a 120-V, 60 Hz power source with no load connected to the 240-V side. Sketch the magnetization current that would flow in the transformer. (Use MATLAB to plot the current accurately, if it is available.) What is the rms amplitude of the magnetization current? What percentage of full-load current is the magnetization current?

(b) Now suppose that this transformer is connected to a 240-V, 50 Hz power source with no load connected to the 120-V side. Sketch the magnetization current that would flow in the transformer. (Use MATLAB to plot the current accurately, if it is available.) What is the rms amplitude of the magnetization current? What percentage of full-load current is the magnetization current?

(c) In which case is the magnetization current a higher percentage of full-load current? Why?

SOLUTION

(a) When this transformer is connected to a 120-V 60 Hz source, the flux in the core will be given by the equation

$$\phi(t) = -\frac{V_M}{\omega N_P}\cos\omega t \tag{3-104}$$

The magnetization current required for any given flux level can be found from Figure P3-2, or alternately from the equivalent table in file `p32.mag`. The MATLAB program shown below calculates the flux level at each time, the corresponding magnetization current, and the rms value of the magnetization current.

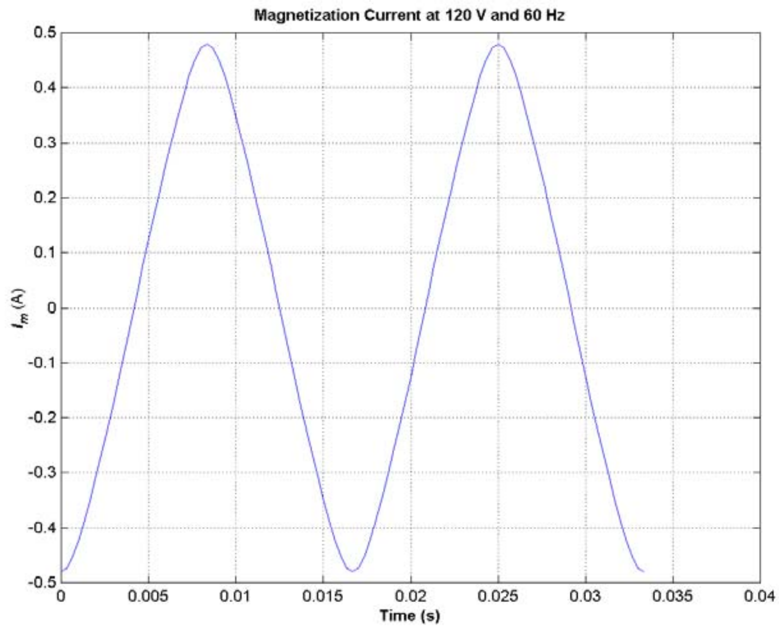
```
% M-file: prob3_5a.m
% M-file to calculate and plot the magnetization
% current of a 120/240 transformer operating at
```

```
% 120 volts and 60 Hz.  This program also
% calculates the rms value of the mag. current.

% Load the magnetization curve.  It is in two
% columns, with the first column being mmf and
% the second column being flux.
load p32.mag;
mmf_data = p32(:,1);
flux_data = p32(:,2);

% Initialize values
S = 1000;                   % Apparent power (VA)
Vrms = 120;                 % Rms voltage (V)
VM = Vrms * sqrt(2);        % Max voltage (V)
NP = 500;                   % Primary turns

% Calculate angular velocity for 60 Hz
freq = 60;                  % Freq (Hz)
w = 2 * pi * freq;

% Calculate flux versus time
time = 0:1/3000:1/30;       % 0 to 1/30 sec
flux = -VM/(w*NP) * cos(w .* time);

% Calculate the mmf corresponding to a given flux
% using the MATLAB interpolation function.
mmf = interp1(flux_data,mmf_data,flux);

% Calculate the magnetization current
im = mmf / NP;

% Calculate the rms value of the current
irms = sqrt(sum(im.^2)/length(im));
disp(['The rms current at 120 V and 60 Hz is ', num2str(irms)]);

% Calculate the full-load current
i_fl = S / Vrms;

% Calculate the percentage of full-load current
percnt = irms / i_fl * 100;
disp(['The magnetization current is ' num2str(percnt) ...
      '% of full-load current.']);

% Plot the magnetization current.
figure(1)
plot(time,im);
title ('\bfMagnetization Current at 120 V and 60 Hz');
xlabel ('\bfTime (s)');
ylabel ('\bf\itI_{m} \rm(A)');
axis([0 0.04 -0.5 0.5]);
grid on;
```

When this program is executed, the results are

```
» prob3_5a
The rms current at 120 V and 60 Hz is 0.31863
The magnetization current is 3.8236% of full-load current.
```

The rms magnetization current is 0.318 A. Since the full-load current is 1000 VA / 120 V = 8.33 A, the magnetization current is 3.82% of the full-load current. The resulting plot is



(b) When this transformer is connected to a 240-V 50 Hz source, the flux in the core will be given by the equation

$$\phi(t) = -\frac{V_M}{\omega N_P} \cos \omega t \tag{3-104}$$

The magnetization current required for any given flux level can be found from Figure P3-2, or alternately from the equivalent table in file `p32.mag`. The MATLAB program shown below calculates the flux level at each time, the corresponding magnetization current, and the rms value of the magnetization current.

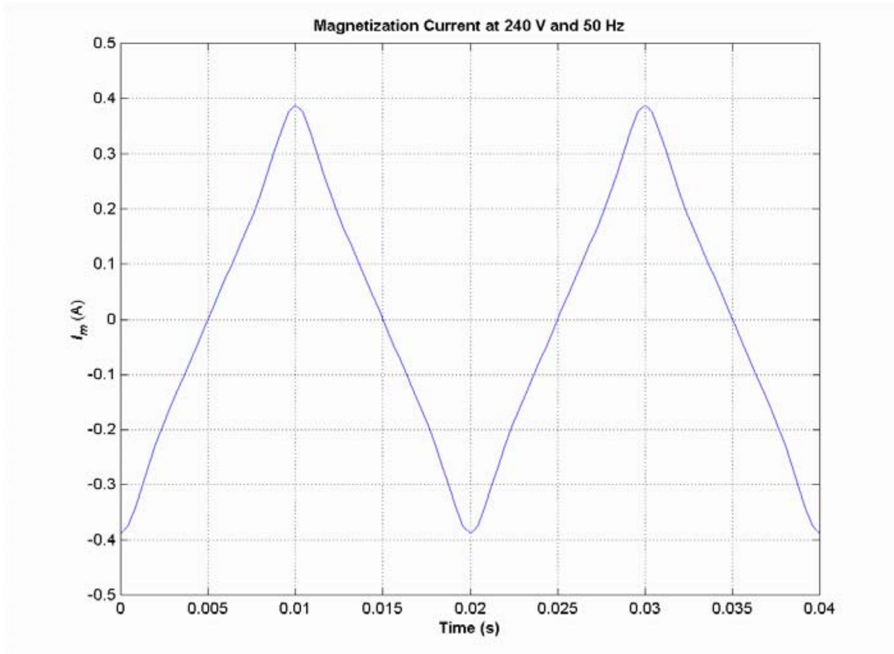
```
% M-file: prob3_5b.m
% M-file to calculate and plot the magnetization
% current of a 120/240 transformer operating at
% 240 volts and 50 Hz.  This program also
% calculates the rms value of the mag. current.

% Load the magnetization curve.  It is in two
% columns, with the first column being mmf and
% the second column being flux.
load p32.mag;
mmf_data = p32(:,1);
flux_data = p32(:,2);

% Initialize values
S = 1000;                     % Apparent power (VA)
Vrms = 240;                   % Rms voltage (V)
VM = Vrms * sqrt(2);          % Max voltage (V)
NP = 1000;                    % Primary turns

% Calculate angular velocity for 50 Hz
freq = 50;                    % Freq (Hz)
```

```
w = 2 * pi * freq;

% Calculate flux versus time
time = 0:1/2500:1/25;       % 0 to 1/25 sec
flux = -VM/(w*NP) * cos(w .* time);

% Calculate the mmf corresponding to a given flux
% using the MATLAB interpolation function.
mmf = interp1(flux_data,mmf_data,flux);

% Calculate the magnetization current
im = mmf / NP;

% Calculate the rms value of the current
irms = sqrt(sum(im.^2)/length(im));
disp(['The rms current at 50 Hz is ', num2str(irms)]);

% Calculate the full-load current
i_fl = S / Vrms;

% Calculate the percentage of full-load current
percnt = irms / i_fl * 100;
disp(['The magnetization current is ' num2str(percnt) ...
      '% of full-load current.']);

% Plot the magnetization current.
figure(1);
plot(time,im);
title ('\bfMagnetization Current at 240 V and 50 Hz');
xlabel ('\bfTime (s)');
ylabel ('\bf\itI_{m} \rm(A)');
axis([0 0.04 -0.5 0.5]);
grid on;
```

When this program is executed, the results are

```
» prob3_5b
The rms current at 50 Hz is 0.22973
The magnetization current is 5.5134% of full-load current.
```

The rms magnetization current is 0.318 A. Since the full-load current is 1000 VA / 240 V = 4.17 A, the magnetization current is 5.51% of the full-load current. The resulting plot is shown below.



(c) The magnetization current is a higher percentage of the full-load current for the 50 Hz case than for the 60 Hz case. This is true because the peak flux is higher for the 50 Hz waveform, driving the core further into saturation.

3-6. A 15-kVA 8000/230-V distribution transformer has an impedance referred to the primary of 80 + *j*300 Ω. The components of the excitation branch referred to the primary side are $R_C = 350\text{ k}\Omega$ and $X_M = 70\text{ k}\Omega$.

(a) If the primary voltage is 7967 V and the load impedance is Z_L = 3.2 + *j*1.5 Ω, what is the secondary voltage of the transformer? What is the voltage regulation of the transformer?

(b) If the load is disconnected and a capacitor of –*j*3.5 Ω is connected in its place, what is the secondary voltage of the transformer? What is its voltage regulation under these conditions?

SOLUTION

(a) The easiest way to solve this problem is to refer all components to the *primary* side of the transformer. The turns ratio is *a* = 8000/230 = 34.78. Thus the load impedance referred to the primary side is

$$Z_L' = (34.78)^2 (3.2 + j1.5\,\Omega) = 3871 + j1815\,\Omega$$

The referred secondary current is

$$\mathbf{I}_S' = \frac{7967\angle 0°\text{ V}}{(80 + j300\,\Omega) + (3871 + j1815\,\Omega)} = \frac{7967\angle 0°\text{ V}}{4481\angle 28.2°\,\Omega} = 1.78\angle -28.2°\text{ A}$$

and the referred secondary voltage is

$$\mathbf{V}_S' = \mathbf{I}_S' Z_L' = (1.78\angle -28.2°\text{ A})(3871 + j1815\,\Omega) = 7610\angle -3.1°\text{ V}$$

The actual secondary voltage is thus

$$\mathbf{V}_S = \frac{\mathbf{V}_S'}{a} = \frac{7610\angle -3.1°\text{ V}}{34.78} = 218.8\angle -3.1°\text{ V}$$

The voltage regulation is

$$\text{VR} = \frac{7967 - 7610}{7610} \times 100\% = 4.7\%$$

(b) As before, the easiest way to solve this problem is to refer all components to the *primary* side of the transformer. The turns ratio is again $a = 34.78$. Thus the load impedance referred to the primary side is

$$Z_L' = (34.78)^2(-j3.5\,\Omega) = -j4234\,\Omega$$

The referred secondary current is

$$\mathbf{I}_S' = \frac{7967\angle 0°\text{ V}}{(80 + j300\,\Omega) + (-j4234\,\Omega)} = \frac{7967\angle 0°\text{ V}}{3935\angle -88.8°\,\Omega} = 2.025\angle 88.8°\text{ A}$$

and the referred secondary voltage is

$$\mathbf{V}_S' = \mathbf{I}_S' Z_L' = (2.25\angle 88.8°\text{ A})(-j4234\ \Omega) = 8573\angle -1.2°\text{ V}$$

The actual secondary voltage is thus

$$\mathbf{V}_S = \frac{\mathbf{V}_S'}{a} = \frac{8573\angle -1.2°\text{ V}}{34.78} = 246.5\angle -1.2°\text{ V}$$

The voltage regulation is

$$\text{VR} = \frac{7967 - 8573}{8573} \times 100\% = -7.07\%$$

3-7. A 5000-kVA 230/13.8-kV single-phase power transformer has a per-unit resistance of 1 percent and a per-unit reactance of 5 percent (data taken from the transformer's nameplate). The open-circuit test performed on the low-voltage side of the transformer yielded the following data:

$$V_{\text{OC}} = 13.8\text{ kV} \qquad I_{\text{OC}} = 15.1\text{ A} \qquad P_{\text{OC}} = 44.9\text{ kW}$$

(a) Find the equivalent circuit referred to the low-voltage side of this transformer.

(b) If the voltage on the secondary side is 13.8 kV and the power supplied is 4000 kW at 0.8 PF lagging, find the voltage regulation of the transformer. Find its efficiency.

SOLUTION

(a) The open-circuit test was performed on the low-voltage side of the transformer, so it can be used to directly find the components of the excitation branch relative to the low-voltage side.

$$|Y_{\text{EX}}| = |G_C - jB_M| = \frac{15.1\text{ A}}{13.8\text{ kV}} = 0.0010942$$

$$\theta = \cos^{-1}\frac{P_{\text{OC}}}{V_{\text{OC}} I_{\text{OC}}} = \cos^{-1}\frac{44.9\text{ kW}}{(13.8\text{ kV})(15.1\text{ A})} = 77.56°$$

$$Y_{\text{EX}} = G_C - jB_M = 0.0010942\angle -77.56°\text{ S} = 0.0002358 - j0.0010685\text{ S}$$

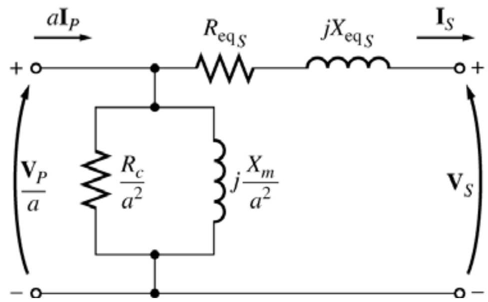
$$R_C = \frac{1}{G_C} = 4240\,\Omega$$

$$X_M = \frac{1}{B_M} = 936\,\Omega$$

The base impedance of this transformer referred to the secondary side is

$$Z_{\text{base}} = \frac{V_{\text{base}}^{\;2}}{S_{\text{base}}} = \frac{(13.8\text{ kV})^2}{5000\text{ kVA}} = 38.09\ \Omega$$

so $R_{\text{EQ}} = (0.01)(38.09\ \Omega) = 0.38\ \Omega$ and $X_{\text{EQ}} = (0.05)(38.09\ \Omega) = 1.9\ \Omega$. The resulting equivalent circuit is shown below:



$R_{\text{eq,s}} = 0.38\ \Omega$ $\quad X_{\text{eq,s}} = j1.9\ \Omega$

$R_{C,s} = 4240\ \Omega$ $\quad X_{M,s} = 936\ \Omega$

(b) If the load on the secondary side of the transformer is 4000 kW at 0.8 PF lagging and the secondary voltage is 13.8 kV, the secondary current is

$$I_S = \frac{P_{\text{LOAD}}}{V_S\ \text{PF}} = \frac{4000\text{ kW}}{(13.8\text{ kV})(0.8)} = 362.3\text{ A}$$

$$\mathbf{I}_S = 362.3\angle -36.87°\text{ A}$$

The voltage on the primary side of the transformer (referred to the secondary side) is

$$\mathbf{V}_P' = \mathbf{V}_S + \mathbf{I}_S Z_{\text{EQ}}$$

$$\mathbf{V}_P' = 13{,}800\angle 0°\text{ V} + (362.3\angle -36.87°\text{ A})(0.38 + j1.9\ \Omega) = 14{,}330\angle 1.9°\text{ V}$$

There is a voltage drop of 14 V under these load conditions. Therefore the voltage regulation of the transformer is

$$\text{VR} = \frac{14{,}330 - 13{,}800}{13{,}800} \times 100\% = 3.84\%$$

The transformer copper losses and core losses are

$$P_{\text{CU}} = I_S^{\;2} R_{\text{EQ},S} = (362.3\text{ A})^2 (0.38\ \Omega) = 49.9\text{ kW}$$

$$P_{\text{core}} = \frac{\left(V_P'\right)^2}{R_C} = \frac{(14{,}330\text{ V})^2}{4240\ \Omega} = 48.4\text{ W}$$

Therefore the efficiency of this transformer at these conditions is

$$\eta = \frac{P_{\text{OUT}}}{P_{\text{OUT}} + P_{\text{CU}} + P_{\text{core}}} \times 100\% = \frac{4000\text{ W}}{4000\text{ W} + 49.9\text{ W} + 48.4\text{ W}} = 97.6\%$$

3-8. A 150-MVA 15/200-kV single-phase power transformer has a per-unit resistance of 1.2 percent and a per-unit reactance of 5 percent (data taken from the transformer's nameplate). The magnetizing impedance is $j100$ per unit.

(a) Find the equivalent circuit referred to the low-voltage side of this transformer.

(b) Calculate the voltage regulation of this transformer for a full-load current at power factor of 0.8 lagging.

(c) Assume that the primary voltage of this transformer is a constant 15 kV, and plot the secondary voltage as a function of load current for currents from no-load to full-load. Repeat this process for power factors of 0.8 lagging, 1.0, and 0.8 leading.

SOLUTION

(a) The base impedance of this transformer referred to the primary (low-voltage) side is

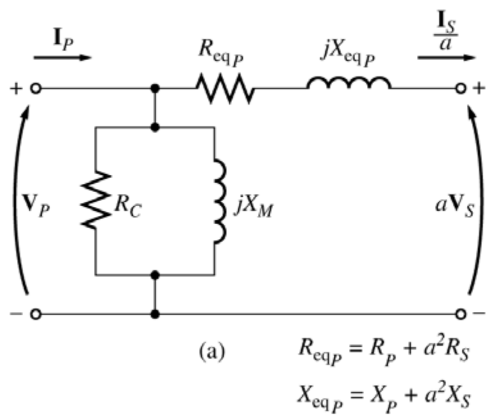
$$Z_{\text{base}} = \frac{V_{\text{base}}^{\,2}}{S_{\text{base}}} = \frac{(15\text{ kV})^2}{150\text{ MVA}} = 1.5\ \Omega$$

so $R_{\text{EQ}} = (0.012)(1.5\ \Omega) = 0.018\ \Omega$

$$X_{\text{EQ}} = (0.05)(1.5\ \Omega) = 0.075\ \Omega$$

$$X_M = (100)(1.5\ \Omega) = 150\ \Omega$$

The equivalent circuit is



$R_{\text{EQ},P} = 0.018\ \Omega$ $\qquad X_{\text{EQ},P} = j0.075\ \Omega$

$R_C = \text{not specified}$ $\qquad X_M = 150\ \Omega$

(b) If the load on the *secondary* (high voltage) side of the transformer is 150 MVA at 0.8 PF lagging, and the referred secondary voltage is 15 kV, then the referred secondary current is

$$I_S' = \frac{P_{\text{LOAD}}}{V_S\ \text{PF}} = \frac{150\text{ MVA}}{(15\text{ kV})(0.8)} = 12{,}500\text{ A}$$

$$\mathbf{I}_S' = 12{,}500\angle -36.87°\text{ A}$$

The voltage on the primary side of the transformer is

$$\mathbf{V}_P = \mathbf{V}_S' + \mathbf{I}_S' Z_{\text{EQ},P}$$

$$\mathbf{V}_P = 15{,}000\angle 0°\text{ V} + (12{,}500\angle -36.87°\text{ A})(0.018 + j0.075\ \Omega) = 15{,}755\angle 2.24°\text{ V}$$

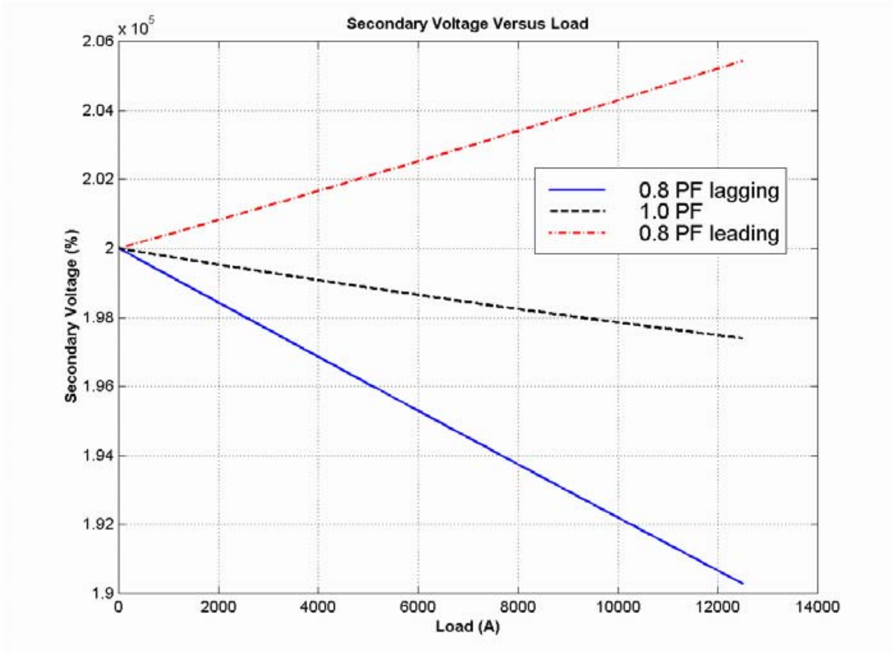
Therefore the voltage regulation of the transformer is

$$\text{VR} = \frac{15{,}755 - 15{,}000}{15{,}000} \times 100\% = 5.03\%$$

(c) This problem is repetitive in nature, and is ideally suited for MATLAB. A program to calculate the secondary voltage of the transformer as a function of load is shown below:

```
% M-file: prob3_8.m
% M-file to calculate and plot the secondary voltage
% of a transformer as a function of load for power
% factors of 0.8 lagging, 1.0, and 0.8 leading.
% These calculations are done using an equivalent
% circuit referred to the primary side.

% Define values for this transformer
VP = 15000;                 % Primary voltage (V)
amps = 0:125:12500;         % Current values (A)
Req = 0.018;                % Equivalent R (ohms)
Xeq = 0.075;                % Equivalent X (ohms)

% Calculate the current values for the three
% power factors.  The first row of I contains
% the lagging currents, the second row contains
% the unity currents, and the third row contains
% the leading currents.
I(1,:) = amps .* ( 0.8 - j*0.6);  % Lagging
I(2,:) = amps .* ( 1.0        );  % Unity
I(3,:) = amps .* ( 0.8 + j*0.6);  % Leading

% Calculate VS referred to the primary side
% for each current and power factor.
aVS = VP - (Req.*I + j.*Xeq.*I);

% Refer the secondary voltages back to the
% secondary side using the turns ratio.
VS = aVS * (200/15);

% Plot the secondary voltage versus load
plot(amps,abs(VS(1,:)),'b-','LineWidth',2.0);
hold on;
plot(amps,abs(VS(2,:)),'k--','LineWidth',2.0);
plot(amps,abs(VS(3,:)),'r-.','LineWidth',2.0);
title ('\bfSecondary Voltage Versus Load');
xlabel ('\bfLoad (A)');
ylabel ('\bfSecondary Voltage (%)');
legend('0.8 PF lagging','1.0 PF','0.8 PF leading');
grid on;
hold off;
```

The resulting plot of secondary voltage versus load is shown below:



3-9. A three-phase transformer bank is to handle 400 kVA and have a 34.5/13.8-kV voltage ratio. Find the rating of each individual transformer in the bank (high voltage, low voltage, turns ratio, and apparent power) if the transformer bank is connected to *(a)* Y-Y, *(b)* Y-Δ, *(c)* Δ-Y, *(d)* Δ-Δ.

SOLUTION For these four connections, the apparent power rating of each transformer is 1/3 of the total apparent power rating of the three-phase transformer.

The ratings for **each transformer** in the bank for each connection are given below:

Connection	Primary Voltage	Secondary Voltage	Apparent Power	Turns Ratio
Y-Y	19.9 kV	7.97 kV	133 kVA	2.50:1
Y-Δ	19.9 kV	13.8 kV	133 kVA	1.44:1
Δ-Y	34.5 kV	7.97 kV	133 kVA	4.33:1
Δ-Δ	34.5 kV	13.8 kV	133 kVA	2.50:1

3-10. A Y-connected of three identical 100-kVA 7967/**277**-V[2] transformers is supplied with power directly from a large constant-voltage bus. In the short-circuit test, the recorded values on the high-voltage side for one of these transformers are

$$V_{SC} = 560\text{ V} \qquad I_{SC} = 12.6\text{ A} \qquad P_{SC} = 3300\text{ W}$$

(a) If this bank delivers a rated load at 0.88 PF lagging and rated voltage, what is the line-to-line voltage on the primary of the transformer bank?

(b) What is the voltage regulation under these conditions?

(c) Assume that the primary line voltage of this transformer bank is a constant 13.8 kV, and plot the secondary line voltage as a function of load current for currents from no-load to full-load. Repeat this process for power factors of 0.85 lagging, 1.0, and 0.85 leading.

(d) Plot the voltage regulation of this transformer as a function of load current for currents from no-load to full-load. Repeat this process for power factors of 0.85 lagging, 1.0, and 0.85 leading.

[2] This voltage was misprinted as 7967/480-V in the first printing of the text. This error should be corrected in all subsequent printings.

SOLUTION From the short-circuit information, it is possible to determine the per-phase impedance of the transformer bank referred to the high-voltage side. The primary of this transformer is Y-connected, so the short-circuit phase voltage is

$$V_{\phi,\mathrm{SC}} = \frac{V_{\mathrm{SC}}}{\sqrt{3}} = 323.3\ \mathrm{V}$$

the short-circuit phase current is

$$I_{\phi,\mathrm{SC}} = 12.6\ \mathrm{A}$$

and the power per phase is

$$P_{\phi,\mathrm{SC}} = \frac{P_{\mathrm{SC}}}{3} = 1100\ \mathrm{W}$$

Thus the per-phase impedance is

$$\left|Z_{\mathrm{EQ}}\right| = \left|R_{\mathrm{EQ}} + jX_{\mathrm{EQ}}\right| = \frac{323.3\ \mathrm{V}}{12.6\ \mathrm{A}} = 25.66\ \Omega$$

$$\theta = \cos^{-1}\frac{P_{\mathrm{SC}}}{V_{\mathrm{SC}}I_{\mathrm{SC}}} = \cos^{-1}\frac{1100\ \mathrm{W}}{(323.3\ \mathrm{V})(12.6\ \mathrm{A})} = 74.3^\circ$$

$$Z_{\mathrm{EQ}} = R_{\mathrm{EQ}} + jX_{\mathrm{EQ}} = 25.66\angle 74.3^\circ\ \Omega = 6.94 + j24.7\ \Omega$$

$$R_{\mathrm{EQ}} = 6.94\ \Omega$$

$$X_{\mathrm{EQ}} = j24.7\ \Omega$$

(a) If this Y-Y transformer bank delivers rated kVA at 0.88 power factor lagging while the secondary voltage is a rated value, then each transformer delivers 33.3 kVA at a voltage of 277 V and 0.88 PF lagging. Referred to the *primary side* of one of the transformers, this load is equivalent to 33.3 kVA at 7967 V and 0.88 PF lagging. The equivalent current flowing in the secondary of one transformer referred to the primary side is

$$I_{\phi,S}' = \frac{33.3\ \mathrm{kVA}}{7967\ \mathrm{V}} = 4.184\ \mathrm{A}$$

$$\mathbf{I}_{\phi,S}' = 4.184\angle -28.36^\circ\ \mathrm{A}$$

The voltage on the primary side of a single transformer is thus

$$\mathbf{V}_{\phi,P} = \mathbf{V}_{\phi,S}' + \mathbf{I}_{\phi,S}' Z_{\mathrm{EQ},P}$$

$$\mathbf{V}_{\phi,P} = 7967\angle 0^\circ\ \mathrm{V} + \left(4.184\angle -28.36^\circ\ \mathrm{A}\right)\left(6.94 + j24.7\ \Omega\right) = 8042\angle 0.55^\circ\ \mathrm{V}$$

The line-to-line voltage on the primary of the transformer is

$$V_{\mathrm{LL},P} = \sqrt{3}\,V_{\phi,P} = \sqrt{3}(8042\ \mathrm{V}) = 13.93\ \mathrm{kV}$$

(b) The voltage regulation of each transformer in the bank, and therefore of the entire transformer bank, is

$$\mathrm{VR} = \frac{8042 - 7967}{7967}\times 100\% = 0.94\%$$

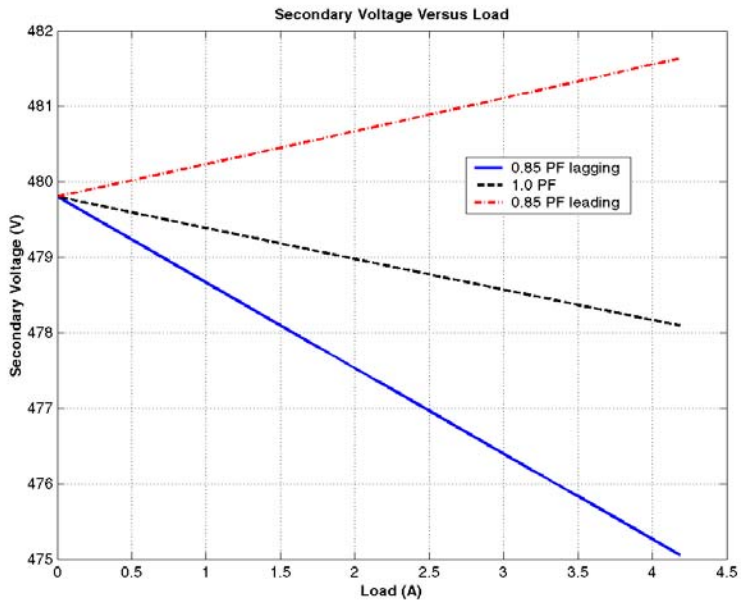
> **Note:** It is much easier to solve problems of this sort in the per-unit system, as we shall see in the next problem.

(c) This sort of repetitive operation is best performed with MATLAB. A suitable MATLAB program is shown below:

```
% M-file: prob3_10c.m
% M-file to calculate and plot the secondary voltage
% of a three-phase Y-Y transformer bank as a function
% of load for power factors of 0.85 lagging, 1.0,
% and 0.85 leading.  These calculations are done using
% an equivalent circuit referred to the primary side.

% Define values for this transformer
VL = 13800;                 % Primary line voltage (V)
VPP = VL / sqrt(3);         % Primary phase voltage (V)
amps = 0:0.04184:4.184;     % Phase current values (A)
Req = 6.94;                 % Equivalent R (ohms)
Xeq = 24.7;                 % Equivalent X (ohms)

% Calculate the current values for the three
% power factors.  The first row of I contains
% the lagging currents, the second row contains
% the unity currents, and the third row contains
% the leading currents.
re = 0.85;
im = sin(acos(re));
I(1,:) = amps .* ( re - j*im);  % Lagging
I(2,:) = amps .* ( 1.0      );  % Unity
I(3,:) = amps .* ( re + j*im);  % Leading

% Calculate secondary phase voltage referred
% to the primary side for each current and
% power factor.
aVSP = VPP - (Req.*I + j.*Xeq.*I);

% Refer the secondary phase voltages back to
% the secondary side using the turns ratio.
% Because this is a delta-connected secondary,
% this is also the line voltage.
VSP = aVSP * (277/7967);

% Convert secondary phase voltage to line
% voltage.
VSL = sqrt(3) * VSP;

% Plot the secondary voltage versus load
plot(amps,abs(VSL(1,:)),'b-','LineWidth',2.0);
hold on;
plot(amps,abs(VSL(2,:)),'k--','LineWidth',2.0);
plot(amps,abs(VSL(3,:)),'r-.','LineWidth',2.0);
title ('\bfSecondary Voltage Versus Load');
xlabel ('\bfLoad (A)');
ylabel ('\bfSecondary Voltage (V)');
legend('0.85 PF lagging','1.0 PF','0.85 PF leading');
```

```
grid on;
hold off;
```

The resulting plot is shown below:

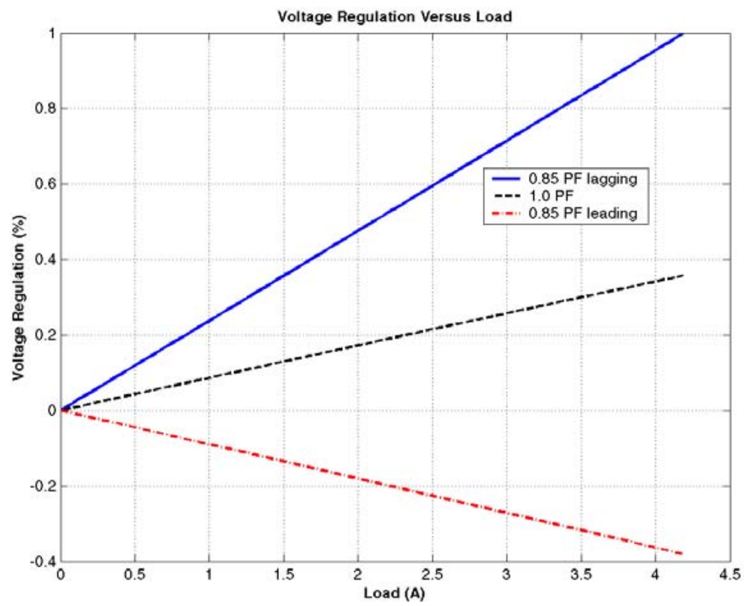


(d) This sort of repetitive operation is best performed with MATLAB. A suitable MATLAB program is shown below:

```
% M-file: prob3_10d.m
% M-file to calculate and plot the voltage regulation
% of a three-phase Y-Y transformer bank as a function
% of load for power factors of 0.85 lagging, 1.0,
% and 0.85 leading.  These calculations are done
% using an equivalent circuit referred to the primary side.

% Define values for this transformer
VL = 13800;                 % Primary line voltage (V)
VPP = VL / sqrt(3);         % Primary phase voltage (V)
amps = 0:0.04184:4.184;     % Phase current values (A)
Req = 6.94;                 % Equivalent R (ohms)
Xeq = 24.7;                 % Equivalent X (ohms)

% Calculate the current values for the three
% power factors.  The first row of I contains
% the lagging currents, the second row contains
% the unity currents, and the third row contains
% the leading currents.
re = 0.85;
im = sin(acos(re));
I(1,:) = amps .* ( re - j*im);  % Lagging
I(2,:) = amps .* ( 1.0      );  % Unity
I(3,:) = amps .* ( re + j*im);  % Leading

% Calculate secondary phase voltage referred
% to the primary side for each current and
% power factor.
```

```
aVSP = VPP - (Req.*I + j.*Xeq.*I);

% Calculate the voltage regulation.
VR = (VPP - abs(aVSP)) ./ abs(aVSP) .* 100;

% Plot the voltage regulation versus load
plot(amps,VR(1,:),'b-','LineWidth',2.0);
hold on;
plot(amps,VR(2,:),'k--','LineWidth',2.0);
plot(amps,VR(3,:),'r-.','LineWidth',2.0);
title ('\bfVoltage Regulation Versus Load');
xlabel ('\bfLoad (A)');
ylabel ('\bfVoltage Regulation (%)');
legend('0.85 PF lagging','1.0 PF','0.85 PF leading');
grid on;
hold off;
```

The resulting plot is shown below:



3-11. A 100,000-kVA 230/115-kV Δ-Δ three-phase power transformer has a per-unit resistance of 0.02 pu and a per-unit reactance of 0.055 pu. The excitation branch elements are $R_C = 110\text{ pu}$ and $X_M = 20\text{ pu}$.

(a) If this transformer supplies a load of 80 MVA at 0.85 PF lagging, draw the phasor diagram of one phase of the transformer.

(b) What is the voltage regulation of the transformer bank under these conditions?

(c) Sketch the equivalent circuit referred to the low-voltage side of one phase of this transformer. Calculate all the transformer impedances referred to the low-voltage side.

SOLUTION

(a) The transformer supplies a load of 80 MVA at 0.85 PF lagging. Therefore, the secondary line current of the transformer is

$$I_{LS} = \frac{S}{\sqrt{3}V_{LS}} = \frac{80,000,000\text{ VA}}{\sqrt{3}(115,000\text{ V})} = 402\text{ A}$$

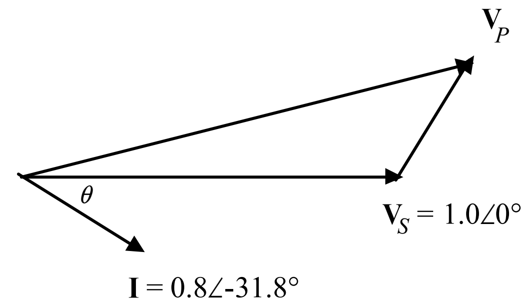
The base value of the secondary line current is

$$I_{LS,\text{base}} = \frac{S_\text{base}}{\sqrt{3}V_{LS,\text{base}}} = \frac{100{,}000{,}000\text{ VA}}{\sqrt{3}(115{,}000\text{ V})} = 502\text{ A}$$

so the per-unit secondary current is

$$\mathbf{I}_{LS,\text{pu}} = \frac{I_{LS}}{I_{LS,\text{pu}}} = \frac{402\text{ A}}{502\text{ A}}\angle\cos^{-1}(0.85) = 0.8\angle -31.8°$$

The per-unit phasor diagram is shown below:



(b) The per-unit primary voltage of this transformer is

$$\mathbf{V}_P = \mathbf{V}_S + \mathbf{I}\,Z_\text{EQ} = 1.0\angle 0° + (0.8\angle -31.8°)(0.02 + j0.055) = 1.037\angle 1.6°$$

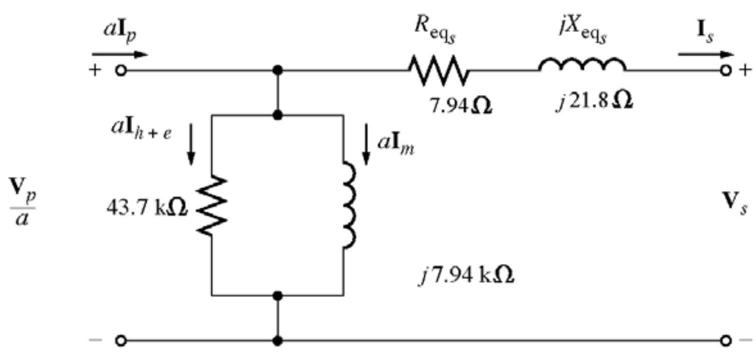
and the voltage regulation is

$$\text{VR} = \frac{1.037 - 1.0}{1.0} \times 100\% = 3.7\%$$

(c) The base impedance of the transformer referred to the low-voltage side is:

$$Z_\text{base} = \frac{3\,V_{\phi,\text{base}}{}^2}{S_\text{base}} = \frac{2(115\text{ kV})^2}{100\text{ MVA}} = 397\ \Omega$$

Each per-unit impedance is converted to actual ohms referred to the low-voltage side by multiplying it by this base impedance. The resulting equivalent circuit is shown below:



$$R_{\text{EQ},S} = (0.02)(397\ \Omega) = 7.94\ \Omega$$
$$X_{\text{EQ},S} = (0.055)(397\ \Omega) = 21.8\ \Omega$$
$$R_C = (110)(397\ \Omega) = 43.7\text{ k}\Omega$$
$$X_M = (20)(397\ \Omega) = 7.94\text{ k}\Omega$$

3-12. An autotransformer is used to connect a 12.6-kV distribution line to a 13.8-kV distribution line. It must be capable of handling 2000 kVA. There are three phases, connected Y-Y with their neutrals solidly grounded.

(a) What must the N_C / N_{SE} turns ratio be to accomplish this connection?

(b) How much apparent power must the windings of each autotransformer handle?

(c) If one of the autotransformers were reconnected as an ordinary transformer, what would its ratings be?

SOLUTION

(a) The transformer is connected Y-Y, so the primary and secondary phase voltages are the line voltages divided by $\sqrt{3}$. The turns ratio of each autotransformer is given by

$$\frac{V_H}{V_L} = \frac{N_C + N_{SE}}{N_C} = \frac{13.8\text{ kV}/\sqrt{3}}{12.6\text{ kV}/\sqrt{3}}$$
$$12.6\, N_C + 12.6\, N_{SE} = 13.8\, N_C$$
$$12.6\, N_{SE} = 1.2\, N_C$$

Therefore, N_C / N_{SE} = 10.5.

(b) The power advantage of this autotransformer is

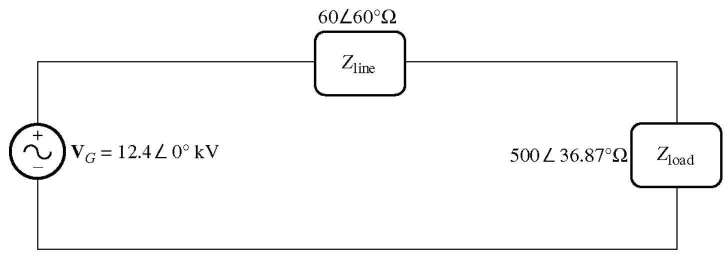
$$\frac{S_{IO}}{S_W} = \frac{N_C + N_{SE}}{N_C} = \frac{N_C + 10.5 N_C}{N_C} = 11.5$$

so 1/11.5 of the power in each transformer goes through the windings. Since 1/3 of the total power is associated with each phase, the windings in each autotransformer must handle

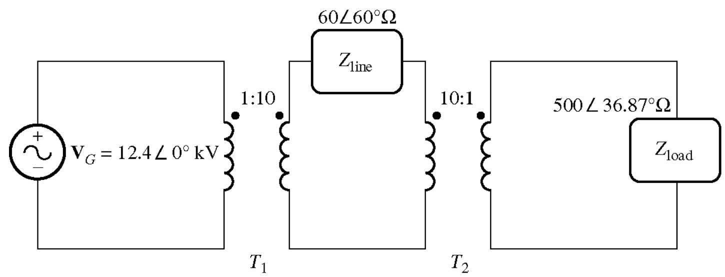
$$S_W = \frac{2000\text{ kVA}}{(3)(11.5)} = 58\text{ kVA}$$

(c) The voltages across each phase of the autotransformer are $13.8\text{ kV}/\sqrt{3}$ = 7967 V and $12.6\text{ kV}/\sqrt{3}$ = 7275 V. The voltage across the common winding (N_C) is 7275 kV, and the voltage across the series winding (N_{SE}) is 7967 kV – 7275 kV = 692 V. Therefore, a single phase of the autotransformer connected as an ordinary transformer would be rated at 7275/692 V and 58 kVA.

3-13. A 12.4-kV single-phase generator supplies power to a load through a transmission line. The load's impedance is $Z_{\text{load}} = 500\angle 36.87°\ \Omega$, and the transmission line's impedance is $Z_{\text{line}} = 60\angle 60°\ \Omega$.



(*a*)



(*b*)

(a) If the generator is directly connected to the load (Figure P3-3*a*), what is the ratio of the load voltage to the generated voltage? What are the transmission losses of the system?

(b) If a 1:10 step-up transformer is placed at the output of the generator and a 10:1 transformer is placed at the load end of the transmission line, what is the new ratio of the load voltage to the generated voltage (Figure P3-3*b*)? What are the transmission losses of the system now? (*Note*: The transformers may be assumed to be ideal.)

SOLUTION

(a) In the case of the directly-connected load, the line current is

$$\mathbf{I}_{\text{line}} = \mathbf{I}_{\text{load}} = \frac{12.4\angle 0° \text{ kV}}{60\angle 60° \ \Omega \ + 500\angle 36.87° \ \Omega} = 22.32\angle -39.3° \text{ A}$$

The load voltage is

$$\mathbf{V}_{\text{load}} = \mathbf{I}_{\text{load}} Z_{\text{load}} = (22.32\angle -39.3° \text{ A})(500\angle 36.87° \ \Omega) = 11.16\angle -2.43° \text{ kV}$$

The ratio of the load voltage to the generated voltage is 11.16/12.4 = 0.90. The transmission losses in the system are

$$P_{\text{loss}} = I_{\text{line}}^{\ 2} R_{\text{line}} = (22.32 \text{ A})^2 (30 \ \Omega) = 14.9 \text{ kW}$$

(b) In this case, a 1:10 step-up transformer precedes the transmission line and a 10:1 step-down transformer follows the transmission line. If the transformers are removed by referring the transmission line to the voltage levels found on either end, then the impedance of the transmission line becomes

$$Z_{\text{line}}' = \left(\frac{1}{10}\right)^2 Z_{\text{line}} = \left(\frac{1}{10}\right)^2 (60\angle 60° \ \Omega) = 0.60\angle 60° \ \Omega$$

The current in the referred transmission line and in the load becomes

$$\mathbf{I}_{\text{line}}' = \mathbf{I}_{\text{load}} = \frac{12.4\angle 0° \text{ kV}}{0.60\angle 60° \ \Omega \ + 500\angle 36.87° \ \Omega} = 24.773\angle -36.90° \text{ A}$$

The load voltage is

$$\mathbf{V}_{\text{load}} = \mathbf{I}_{\text{load}} Z_{\text{load}} = (24.773\angle -36.90^\circ \text{ A})(500\angle 36.87^\circ \ \Omega) = 12.386\angle -0.03^\circ \text{ kV}$$

The ratio of the load voltage to the generated voltage is 12.386/12.4 = 0.999. Also, the transmission losses in the system are reduced. The current in the transmission line is

$$I_{\text{line}} = \left(\frac{1}{10}\right) I_{\text{load}} = \left(\frac{1}{10}\right)(24.77 \text{ A}) = 2.477 \text{ A}$$

and the losses in the transmission line are

$$P_{\text{loss}} = I_{\text{line}}{}^2 R_{\text{line}} = (2.477 \text{ A})^2 (30 \ \Omega) = 184 \text{ W}$$

Transmission losses have decreased by a factor of more than 80!

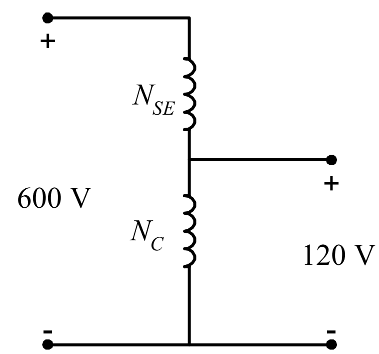
3-14. A 5000-VA 480/120-V conventional transformer is to be used to supply power from a 600-V source to a 120-V load. Consider the transformer to be ideal, and assume that all insulation can handle 600 V.

(a) Sketch the transformer connection that will do the required job.

(b) Find the kilovoltampere rating of the transformer in the configuration.

(c) Find the maximum primary and secondary currents under these conditions.

SOLUTION *(a)* For this configuration, the common winding must be the *smaller* of the two windings, and $N_{\text{SE}} = 4N_C$. The transformer connection is shown below:



(b) The kVA rating of the autotransformer can be found from the equation

$$S_{\text{IO}} = \frac{N_{\text{SE}} + N_C}{N_{\text{SE}}} S_W = \frac{4N_C + N_C}{4N_C}(5000 \text{ VA}) = 6250 \text{ VA}$$

(c) The maximum primary current for this configuration will be

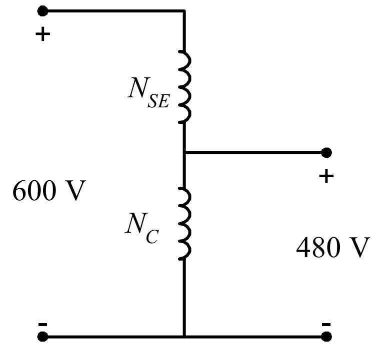
$$I_P = \frac{S}{V_P} = \frac{6250 \text{ VA}}{600 \text{ V}} = 10.4 \text{ A}$$

and the maximum secondary current is

$$I_S = \frac{S}{V_S} = \frac{6250 \text{ VA}}{120 \text{ V}} = 52.1 \text{ A}$$

3-15. A 5000-VA 480/120-V conventional transformer is to be used to supply power from a 600-V source to a 480-V load. Consider the transformer to be ideal, and assume that all insulation can handle 600 V. Answer the questions of Problem 3-14 for this transformer.

SOLUTION *(a)* For this configuration, the common winding must be the *larger* of the two windings, and $N_C = 4N_{\text{SE}}$. The transformer connection is shown below:



(b) The kVA rating of the autotransformer can be found from the equation

$$S_{IO} = \frac{N_{SE} + N_C}{N_{SE}} S_W = \frac{N_{SE} + 4N_{SE}}{N_{SE}} (5000 \text{ VA}) = 25{,}000 \text{ VA}$$

(c) The maximum primary current for this configuration will be

$$I_P = \frac{S}{V_P} = \frac{25{,}000 \text{ VA}}{600 \text{ V}} = 41.67 \text{ A}$$

and the maximum secondary current is

$$I_S = \frac{S}{V_S} = \frac{25{,}000 \text{ VA}}{480 \text{ V}} = 52.1 \text{ A}$$

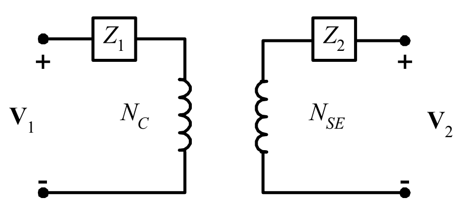
Note that the apparent power handling capability of the autotransformer is *much* higher when there is only a small difference between primary and secondary voltages. Autotransformers are normally used when there is a relatively small difference between the two voltage levels.

3-16. Prove the following statement: If a transformer having a series impedance Z_{eq} is connected as an autotransformer, its per-unit series impedance Z'_{eq} as an autotransformer will be

$$Z'_{eq} = \frac{N_{SE}}{N_{SE} + N_C} Z_{eq}$$

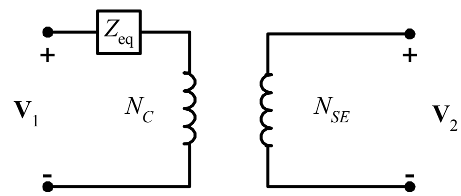
Note that this expression is the reciprocal of the autotransformer power advantage.

SOLUTION The impedance of a transformer can be found by shorting the secondary winding and determining the ratio of the voltage to the current of its primary winding. For the transformer connected as an ordinary transformer, the impedance referred to the primary (N_C) is:

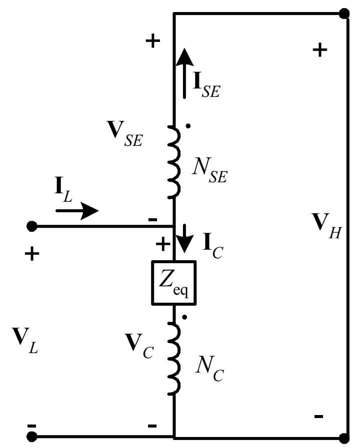


$$Z_{eq} = Z_1 + \left(\frac{N_C}{N_{SE}}\right)^2 Z_2$$

The corresponding equivalent circuit is:



When this transformer is connected as an autotransformer, the circuit is as shown below. If the output windings of the autotransformer are shorted out, the voltages $\mathbf{V}_H$ (and hence $\mathbf{V}_C$) will be zero, and the voltage $\mathbf{V}_L$ will be



$$\mathbf{V}_L = \mathbf{I}_C Z_{\text{eq}}$$

where Z_{eq} is the impedance of the ordinary transformer. However,

$$\mathbf{I}_L = \mathbf{I}_C + \mathbf{I}_{SE} = \mathbf{I}_C + \frac{N_C}{N_{SE}}\mathbf{I}_C = \frac{N_{SE}+N_C}{N_{SE}}\mathbf{I}_C$$

or $$\mathbf{I}_C = \frac{N_{\text{SE}}}{N_{\text{SE}}+N_C}\mathbf{I}_L$$

so the input voltage can be expressed in terms of the input current as:

$$\mathbf{V}_L = \mathbf{I}_C Z_{\text{eq}} = \frac{N_{SE}}{N_{SE}+N_C}\mathbf{I}_L Z_{\text{eq}}$$

The input impedance of the autotransformer is *defined* as $Z_{\text{eq}} = \mathbf{V}_L / \mathbf{I}_L$, so

$$\boxed{Z_{\text{eq}}{}' = \frac{\mathbf{V}_L}{\mathbf{I}_L} = \frac{N_{SE}}{N_{SE}+N_C} Z_{\text{eq}}}$$

This is the expression that we were trying to prove.

3-17. Three 25-kVA 24,000/277-V distribution transformers are connected in Δ-Y. The open-circuit test was performed on the low-voltage side of this transformer bank, and the following data were recorded:

$$V_{\text{line,OC}} = 480 \text{ V} \quad I_{\text{line,OC}} = 4.10 \text{ A} \qquad P_{3\phi,\text{OC}} = 945 \text{ W}$$

The short-circuit test was performed on the high-voltage side of this transformer bank, and the following data were recorded:

$$V_{\text{line,SC}} = 1400 \text{ V} \quad I_{\text{line,SC}} = 1.80 \text{ A} \qquad P_{3\phi,\text{SC}} = 912 \text{ W}$$

(a) Find the per-unit equivalent circuit of this transformer bank.

(b) Find the voltage regulation of this transformer bank at the rated load and 0.90 PF lagging.

(c) What is the transformer bank's efficiency under these conditions?

SOLUTION *(a)* The equivalent of this three-phase transformer bank can be found just like the equivalent circuit of a single-phase transformer if we work on a per-phase bases. The open-circuit test data on the low-voltage side can be used to find the excitation branch impedances referred to the secondary side of the transformer bank. Since the low-voltage side of the transformer is Y-connected, the per-phase open-circuit quantities are:

$$V_{\phi,\text{OC}} = 277 \text{ V} \qquad I_{\phi,\text{OC}} = 4.10 \text{ A} \qquad P_{\phi,\text{OC}} = 315 \text{ W}$$

The excitation admittance is given by

$$|Y_{EX}| = \frac{I_{\phi,OC}}{V_{\phi,OC}} = \frac{4.10 \text{ A}}{277 \text{ V}} = 0.01483 \text{ S}$$

The admittance angle is

$$\theta = -\cos^{-1}\left(\frac{P_{\phi,OC}}{V_{\phi,OC}\, I_{\phi,OC}}\right) = -\cos^{-1}\left(\frac{315 \text{ W}}{(277 \text{ V})\ (4.10 \text{ A})}\right) = -73.9°$$

Therefore,

$$Y_{EX} = G_C - jB_M = 0.01483\angle -73.9° = 0.00411 - j0.01425$$
$$R_C = 1/G_C = 243\ \Omega$$
$$X_M = 1/B_M = 70.2\ \Omega$$

The *base impedance* referred to the low-voltage side is

$$Z_{\text{base},S} = \frac{(V_{\phi,S})^2}{S_\phi} = \frac{(277 \text{ V})^2}{25 \text{ kVA}} = 3.069\ \Omega$$

so the excitation branch elements can be expressed in per-unit as

$$R_C = \frac{243\ \Omega}{3.069\ \Omega} = 79.2 \text{ pu} \qquad X_M = \frac{70.2\ \Omega}{3.069\ \Omega} = 22.9 \text{ pu}$$

The short-circuit test data can be used to find the series impedances referred to the high-voltage side, since the short-circuit test data was taken on the high-voltage side. Note that the high-voltage is Δ-connected, so $V_{\phi,\text{SC}} = V_{\text{SC}} = 1400 \text{ V}$, $I_{\phi,\text{SC}} = I_{\text{SC}}/\sqrt{3} = 1.039 \text{ A}$, and $P_{\phi,\text{SC}} = P_{\text{SC}}/3 = 304 \text{ W}$.

$$|Z_{EQ}| = \frac{V_{\phi,SC}}{I_{\phi,SC}} = \frac{1400 \text{ V}}{1.039 \text{ A}} = 1347\ \Omega$$

$$\theta = \cos^{-1}\left(\frac{P_{\phi,SC}}{V_{\phi,SC}\, I_{\phi,SC}}\right) = \cos^{-1}\left(\frac{304 \text{ W}}{(1400 \text{ V})\ (1.039 \text{ A})}\right) = 77.9°$$

$$Z_{EQ} = R_{EQ} + jX_{EQ} = 1347\angle 77.9° = 282 + j1371\ \Omega$$

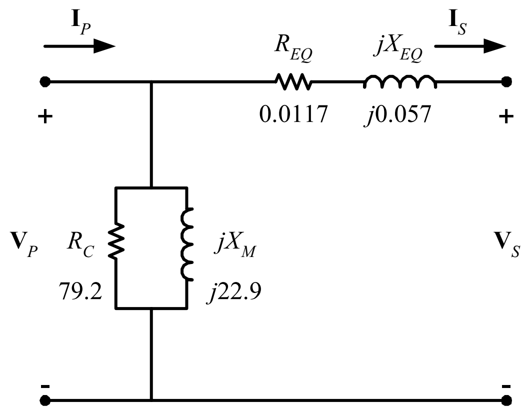
The *base impedance* referred to the high-voltage side is

$$Z_{\text{base},P} = \frac{\left(V_{\phi,P}\right)^2}{S_\phi} = \frac{\left(24{,}000\text{ V}\right)^2}{25\text{ kVA}} = 24{,}040\ \Omega$$

The resulting per-unit impedances are

$$R_{EQ} = \frac{282\ \Omega}{24{,}040\ \Omega} = 0.0117\text{ pu} \qquad X_{EQ} = \frac{1371\ \Omega}{24{,}040\ \Omega} = 0.057\text{ pu}$$

The per-unit, per-phase equivalent circuit of the transformer bank is shown below:



(b) If this transformer is operating at rated load and 0.90 PF lagging, then current flow will be at an angle of $-\cos^{-1}(0.9)$, or –25.8°. The voltage at the primary side of the transformer will be

$$\mathbf{V}_P = \mathbf{V}_S + \mathbf{I}_S Z_{\text{EQ}} = 1.0\angle 0° + \left(1.0\angle -25.8°\right)\left(0.0117 + j0.057\right) = 1.037\angle 2.65°$$

The voltage regulation of this transformer bank is

$$\text{VR} = \frac{1.037\text{-}1.0}{1.0} \times 100\% = 3.7\%$$

(c) The output power of this transformer bank is

$$P_{\text{OUT}} = V_S I_S \cos\theta = (1.0)(1.0)(0.9) = 0.9\text{ pu}$$

The copper losses are

$$P_{\text{CU}} = I_S^{\,2} R_{\text{EQ}} = (1.0)^2 (0.0117) = 0.0117\text{ pu}$$

The core losses are

$$P_{\text{core}} = \frac{V_P^{\,2}}{R_C} = \frac{(1.067)^2}{79.2} = 0.014\text{ pu}$$

Therefore, the total input power to the transformer bank is

$$P_{\text{IN}} = P_{\text{OUT}} + P_{\text{CU}} + P_{\text{core}} = 0.9 + 0.0117 + 0.014 = 0.9257$$

and the efficiency of the transformer bank is

$$\eta = \frac{P_{\text{OUT}}}{P_{\text{IN}}} \times 100\% = \frac{0.9}{0.9257} \times 100\% = 97.2\%$$

3-18. A 20-kVA 20,000/480-V 60-Hz distribution transformer is tested with the following results:

Open-circuit test (measured from secondary side)	**Short-circuit test (measured from primary side)**
$V_{OC} = 480$ V	$V_{SC} = 1130$ V
$I_{OC} = 1.51$ A	$I_{SC} = 1.00$ A
$V_{OC} = 271$ W	$P_{SC} = 260$ W

(a) Find the per-unit equivalent circuit for this transformer at 60 Hz.

(b) What would the rating of this transformer be if it were operated on a 50-Hz power system?

(c) Sketch the equivalent circuit of this transformer referred to the primary side *if it is operating at 50 Hz.*

SOLUTION

(a) The base impedance of this transformer referred to the primary side is

$$Z_{\text{base},P} = \frac{(V_P)^2}{S} = \frac{(20{,}000 \text{ V})^2}{20 \text{ kVA}} = 20 \text{ k}\Omega$$

The base impedance of this transformer referred to the secondary side is

$$Z_{\text{base},S} = \frac{(V_S)^2}{S} = \frac{(480 \text{ V})^2}{20 \text{ kVA}} = 11.52 \ \Omega$$

The open circuit test yields the values for the excitation branch (referred to the *secondary* side):

$$|Y_{EX}| = \frac{I_{\phi,OC}}{V_{\phi,OC}} = \frac{1.51 \text{ A}}{480 \text{ V}} = 0.00315 \ \text{ S}$$

$$\theta = -\cos^{-1}\left(\frac{P_{OC}}{V_{OC}\ I_{OC}}\right) = -\cos^{-1}\left(\frac{271 \text{ W}}{(480 \text{ V})\ (1.51 \text{ A})}\right) = -68°$$

$$Y_{EX} = G_C - jB_M = 0.00315\angle -68° = 0.00118 - j0.00292$$

$$R_C = 1/G_C = 847 \ \Omega$$

$$X_M = 1/B_M = 342 \ \Omega$$

The excitation branch elements can be expressed in per-unit as

$$R_C = \frac{847 \ \Omega}{11.52 \ \Omega} = 73.5 \text{ pu} \qquad X_M = \frac{342 \ \Omega}{11.52 \ \Omega} = 29.7 \text{ pu}$$

The short circuit test yields the values for the series impedances (referred to the *primary* side):

$$|Z_{EQ}| = \frac{V_{SC}}{I_{SC}} = \frac{1130 \text{ V}}{1.00 \text{ A}} = 1130 \ \ \Omega$$

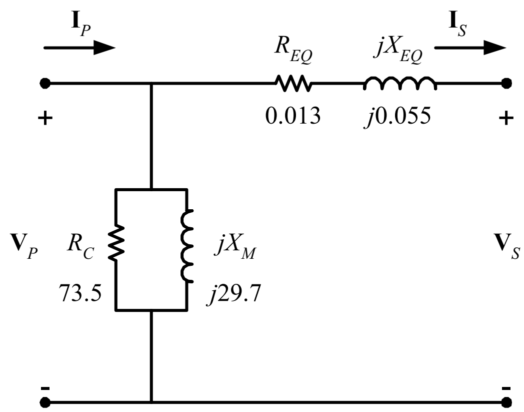
$$\theta = \cos^{-1}\left(\frac{P_{SC}}{V_{SC}\ I_{SC}}\right) = \cos^{-1}\left(\frac{260 \text{ W}}{(1130 \text{ V})\ (1.00 \text{ A})}\right) = 76.7°$$

$$Z_{EQ} = R_{EQ} + jX_{EQ} = 1130\angle 76.7° = 260 + j1100 \ \Omega$$

The resulting per-unit impedances are

$$R_{EQ} = \frac{260 \ \Omega}{20{,}000 \ \Omega} = 0.013 \text{ pu} \qquad X_{EQ} = \frac{1100 \ \Omega}{20{,}000 \ \Omega} = 0.055 \text{ pu}$$

The per-unit equivalent circuit is



(b) If this transformer were operated at 50 Hz, both the voltage and apparent power would have to be derated by a factor of 50/60, so its ratings would be 16.67 kVA, 16,667/400 V, and 50 Hz.

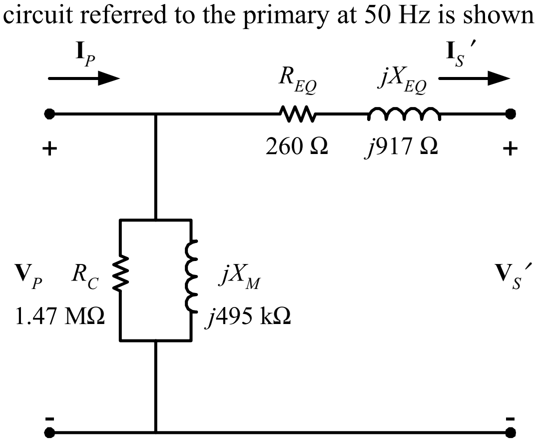
(c) The transformer parameters referred to the primary side *at 60 Hz* are:

$$R_C = Z_{\text{base}} R_{C,\text{pu}} = (20\text{ k}\Omega)(73.5) = 1.47\text{ M}\Omega$$
$$X_M = Z_{\text{base}} X_{M,\text{pu}} = (20\text{ k}\Omega)(29.7) = 594\text{ k}\Omega$$
$$R_{\text{EQ}} = Z_{\text{base}} R_{\text{EQ},\text{pu}} = (20\text{ k}\Omega)(0.013) = 260\ \Omega$$
$$X_{\text{EQ}} = Z_{\text{base}} X_{\text{EQ},\text{pu}} = (20\text{ k}\Omega)(0.055) = 1100\ \Omega$$

At 50 Hz, the resistance will be unaffected but the reactances are reduced in direct proportion to the decrease in frequency. At 50 Hz, the reactances are

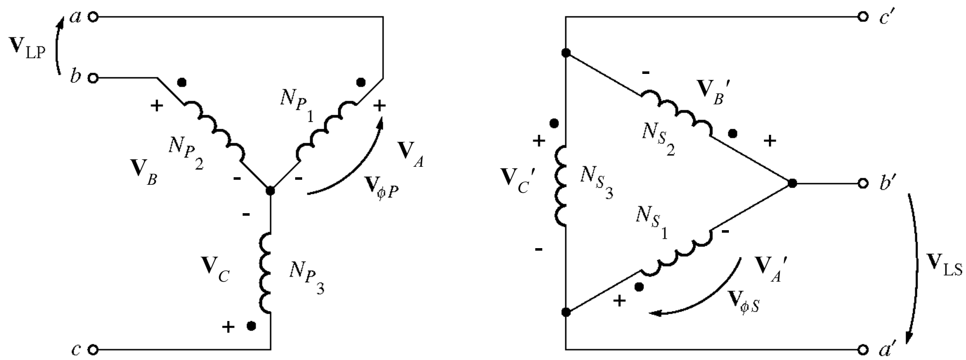
$$X_M = 495\text{ k}\Omega$$
$$X_{\text{EQ}} = 917\ \Omega$$

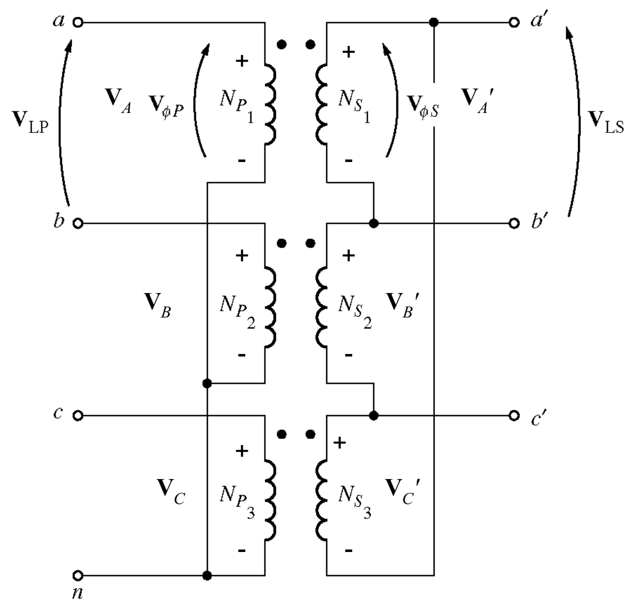
The resulting equivalent circuit referred to the primary at 50 Hz is shown below:



3-19. Prove that the three-phase system of voltages on the secondary of the Y-Δ transformer shown in Figure 3-37*b* lags the three-phase system of voltages on the primary of the transformer by 30°.

SOLUTION The figure is reproduced below:





(*b*)

Assume that the phase voltages on the primary side are given by

$$\mathbf{V}_A = V_{\phi P}\angle 0^\circ \qquad \mathbf{V}_B = V_{\phi P}\angle -120^\circ \qquad \mathbf{V}_C = V_{\phi P}\angle 120^\circ$$

Then the phase voltages on the secondary side are given by

$$\mathbf{V}_A' = V_{\phi S}\angle 0^\circ \qquad \mathbf{V}_B' = V_{\phi S}\angle -120^\circ \qquad \mathbf{V}_C' = V_{\phi S}\angle 120^\circ$$

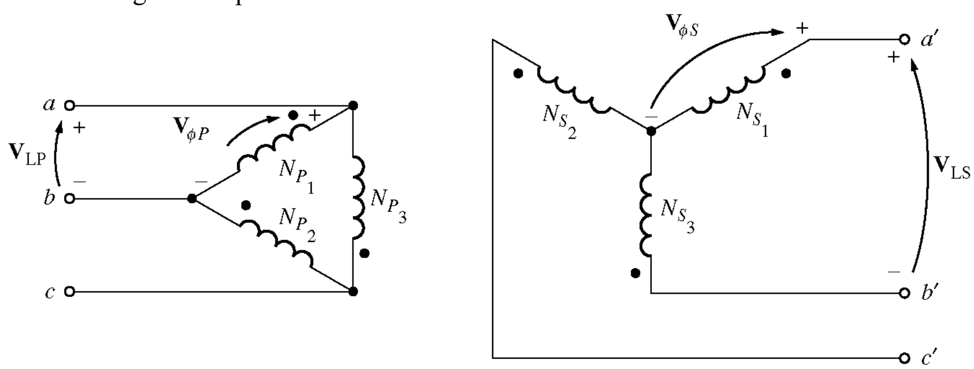
where $V_{\phi S} = V_{\phi P} / a$. Since this is a Y-Δ transformer bank, the line voltage $\mathbf{V}_{ab}$ on the primary side is

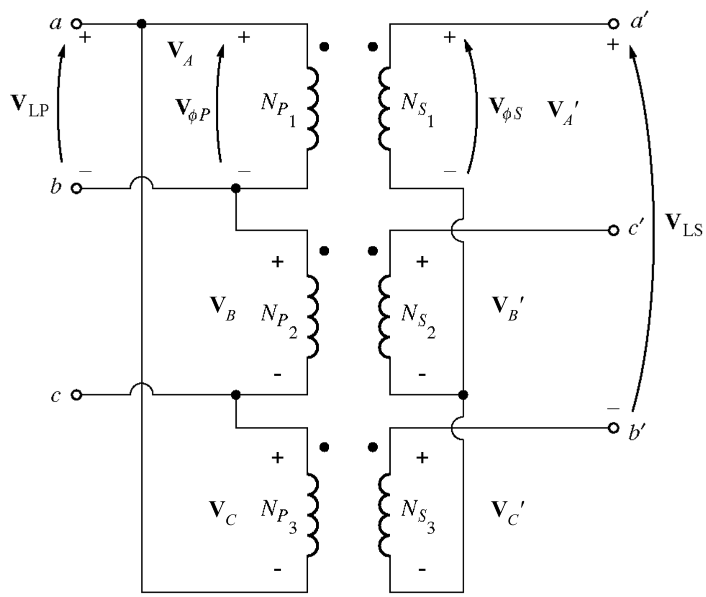
$$\mathbf{V}_{ab} = \mathbf{V}_A - \mathbf{V}_B = V_{\phi P}\angle 0^\circ - V_{\phi P}\angle -120^\circ = \sqrt{3}V_{\phi P}\angle 30^\circ$$

and the voltage $\mathbf{V}_{a'b'} = \mathbf{V}_A' = V_{\phi S}\angle 0^\circ$. *Note that the line voltage on the secondary side lags the line voltage on the primary side by 30°.*

3-20. Prove that the three-phase system of voltages on the secondary of the Δ-Y transformer shown in Figure 3-37*c* lags the three-phase system of voltages on the primary of the transformer by 30°.

SOLUTION The figure is reproduced below:





(c)

Assume that the phase voltages on the primary side are given by

$$\mathbf{V}_A = V_{\phi P}\angle 0° \qquad \mathbf{V}_B = V_{\phi P}\angle -120° \qquad \mathbf{V}_C = V_{\phi P}\angle 120°$$

Then the phase voltages on the secondary side are given by

$$\mathbf{V}_A' = V_{\phi S}\angle 0° \qquad \mathbf{V}_B' = V_{\phi S}\angle -120° \qquad \mathbf{V}_C' = V_{\phi S}\angle 120°$$

where $V_{\phi S} = V_{\phi P} / a$. Since this is a Δ-Y transformer bank, the line voltage $\mathbf{V}_{ab}$ on the primary side is just equal to $\mathbf{V}_A = V_{\phi P}\angle 0°$. The line voltage on the secondary side is given by

$$\mathbf{V}_{a'b'} = \mathbf{V}_A - \mathbf{V}_C = V_{\phi P}\angle 0° - V_{\phi P}\angle 120° = \sqrt{3}V_{\phi P}\angle -30°$$

Note that the line voltage on the secondary side lags the line voltage on the primary side by 30°.

3-21. A single-phase 10-kVA 480/120-V transformer is to be used as an autotransformer tying a 600-V distribution line to a 480-V load. When it is tested as a conventional transformer, the following values are measured on the primary (480-V) side of the transformer:

Open-circuit test	Short-circuit test
$V_{OC} = 480$ V	$V_{SC} = 10.0$ V
$I_{OC} = 0.41$ A	$I_{SC} = 10.6$ A
$V_{OC} = 38$ W	$P_{SC} = 26$ W

(a) Find the per-unit equivalent circuit of this transformer when it is connected in the conventional manner. What is the efficiency of the transformer at rated conditions and unity power factor? What is the voltage regulation at those conditions?

(b) Sketch the transformer connections when it is used as a 600/480-V step-down autotransformer.

(c) What is the kilovoltampere rating of this transformer when it is used in the autotransformer connection?

(d) Answer the questions in *(a)* for the autotransformer connection.

SOLUTION

(a) The base impedance of this transformer referred to the primary side is

$$Z_{\text{base},P} = \frac{(V_P)^2}{S} = \frac{(480\text{ V})^2}{10\text{ kVA}} = 23.04\ \Omega$$

The open circuit test yields the values for the excitation branch (referred to the *primary* side):

$$|Y_{EX}| = \frac{I_{\phi,OC}}{V_{\phi,OC}} = \frac{0.41\text{ A}}{480\text{ V}} = 0.000854\ \text{ S}$$

$$\theta = -\cos^{-1}\left(\frac{P_{OC}}{V_{OC}\ I_{OC}}\right) = -\cos^{-1}\left(\frac{38\text{ W}}{(480\text{ V})\ (0.41\text{ A})}\right) = -78.87°$$

$$Y_{EX} = G_C - jB_M = 0.000854\angle -78.87° = 0.000165 - j0.000838$$

$$R_C = 1/G_C = 6063\ \Omega$$

$$X_M = 1/B_M = 1193\ \Omega$$

The excitation branch elements can be expressed in per-unit as

$$R_C = \frac{6063\ \Omega}{23.04\ \Omega} = 263\text{ pu} \qquad X_M = \frac{1193\ \Omega}{23.04\ \Omega} = 51.8\text{ pu}$$

The short circuit test yields the values for the series impedances (referred to the *primary* side):

$$|Z_{EQ}| = \frac{V_{SC}}{I_{SC}} = \frac{10.0\text{ V}}{10.6\text{ A}} = 0.943\ \ \Omega$$

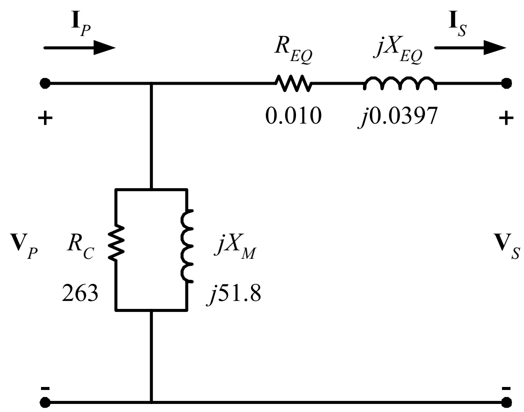
$$\theta = \cos^{-1}\left(\frac{P_{SC}}{V_{SC}\ I_{SC}}\right) = \cos^{-1}\left(\frac{26\text{ W}}{(10.0\text{ V})\ (10.6\text{ A})}\right) = 75.8°$$

$$Z_{EQ} = R_{EQ} + jX_{EQ} = 0.943\angle 75.8° = 0.231 + j0.915\ \Omega$$

The resulting per-unit impedances are

$$R_{EQ} = \frac{0.231\ \Omega}{23.04\ \Omega} = 0.010\text{ pu} \qquad X_{EQ} = \frac{0.915\ \Omega}{23.04\ \Omega} = 0.0397\text{ pu}$$

The per-unit equivalent circuit is



At rated conditions and unity power factor, the input power to this transformer would be P_{IN} = 1.0 pu. The core losses (in resistor R_C) would be

$$P_{\text{core}} = \frac{V^2}{R_C} = \frac{(1.0)^2}{263} = 0.00380 \text{ pu}$$

The copper losses (in resistor R_{EQ}) would be

$$P_{\text{CU}} = I^2 R_{\text{EQ}} = (1.0)^2 (0.010) = 0.010 \text{ pu}$$

The output power of the transformer would be

$$P_{\text{OUT}} = P_{\text{OUT}} - P_{\text{CU}} - P_{\text{core}} = 1.0 - 0.010 - 0.0038 = 0.986$$

and the transformer efficiency would be

$$\eta = \frac{P_{\text{OUT}}}{P_{\text{IN}}} \times 100\% = \frac{0.986}{1.0} \times 100\% = 98.6\%$$

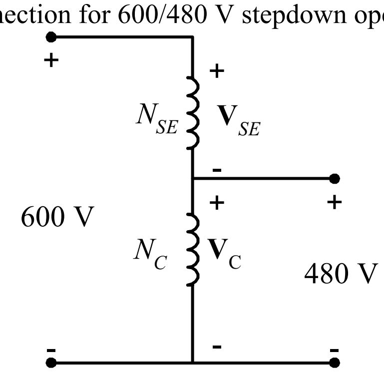
The output voltage of this transformer is

$$\mathbf{V}_{\text{OUT}} = \mathbf{V}_{\text{IN}} - \mathbf{I}Z_{\text{EQ}} = 1.0 - (1.0\angle 0°)(0.01 + j0.0397) = 0.991\angle -2.3°$$

The voltage regulation of the transformer is

$$\text{VR} = \frac{1.0\text{-}0.991}{0.991} \times 100\% = 0.9\%$$

(b) The autotransformer connection for 600/480 V stepdown operation is



(c) When used as an autotransformer, the kVA rating of this transformer becomes:

$$S_{IO} = \frac{N_C + N_{SE}}{N_{SE}} S_W = \frac{4+1}{1}(10 \text{ kVA}) = 50 \text{ kVA}$$

(d) As an autotransformer, the per-unit series impedance Z_{EQ} is decreased by the reciprocal of the power advantage, so the series impedance becomes

$$R_{EQ} = \frac{0.010}{5} = 0.002 \text{ pu}$$

$$X_{EQ} = \frac{0.0397}{5} = 0.00794 \text{ pu}$$

while the magnetization branch elements are basically unchanged. At rated conditions and unity power factor, the input power to this transformer would be P_{IN} = 1.0 pu. The core losses (in resistor R_C) would be

$$P_{core} = \frac{V^2}{R_C} = \frac{(1.0)^2}{263} = 0.00380 \text{ pu}$$

The copper losses (in resistor R_{EQ}) would be

$$P_{CU} = I^2 R_{EQ} = (1.0)^2 (0.002) = 0.002 \text{ pu}$$

The output power of the transformer would be

$$P_{OUT} = P_{OUT} - P_{CU} - P_{core} = 1.0 - 0.002 - 0.0038 = 0.994$$

and the transformer efficiency would be

$$\eta = \frac{P_{OUT}}{P_{IN}} \times 100\% = \frac{0.994}{1.0} \times 100\% = 99.4\%$$

The output voltage of this transformer is

$$\mathbf{V}_{OUT} = \mathbf{V}_{IN} - \mathbf{I} Z_{EQ} = 1.0 - (1.0\angle 0°)(0.002 + j0.00794) = 0.998\angle -0.5°$$

The voltage regulation of the transformer is

$$\text{VR} = \frac{1.0\text{-}0.998}{0.998} \times 100\% = 0.2\%$$

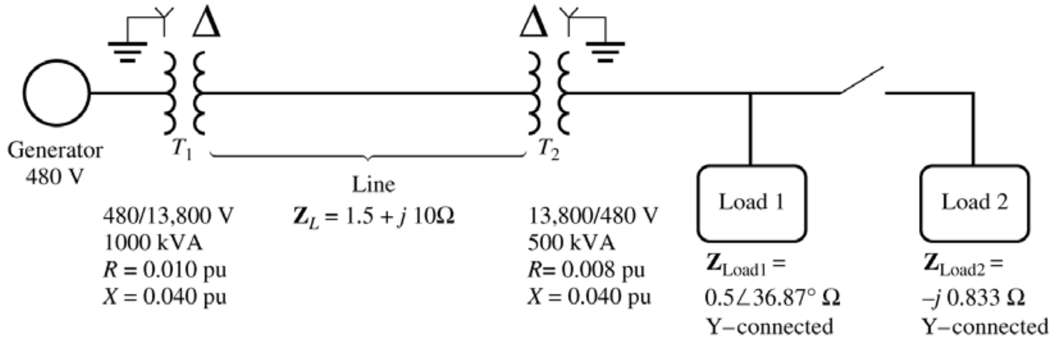
3-22. Figure P3-4 shows a power system consisting of a three-phase 480-V 60-Hz generator supplying two loads through a transmission line with a pair of transformers at either end.

(a) Sketch the per-phase equivalent circuit of this power system.

(b) With the switch opened, find the real power *P*, reactive power *Q*, and apparent power *S* supplied by the generator. What is the power factor of the generator?

(c) With the switch closed, find the real power *P*, reactive power *Q*, and apparent power *S* supplied by the generator. What is the power factor of the generator?

(d) What are the transmission losses (transformer plus transmission line losses) in this system with the switch open? With the switch closed? What is the effect of adding Load 2 to the system?



Region 1	**Region 2**	**Region 3**
$S_{\text{base1}} = 1000$ kVA	$S_{\text{base2}} = 1000$ kVA	$S_{\text{base3}} = 1000$ kVA
$V_{L,\text{base2}} = 480$ V	$V_{L,\text{base2}} = 13{,}800$ V	$V_{L,\text{base3}} = 480$ V

SOLUTION This problem can best be solved using the per-unit system of measurements. The power system can be divided into three regions by the two transformers. If the per-unit base quantities in Region 1 are chosen to be S_{base1} = 1000 kVA and $V_{L,\text{base1}}$ = 480 V, then the base quantities in Regions 2 and 3 will be as shown above. The base impedances of each region will be:

$$Z_{\text{base1}} = \frac{3V_{\phi 1}{}^2}{S_{\text{base1}}} = \frac{3(277\text{ V})^2}{1000\text{ kVA}} = 0.238\ \Omega$$

$$Z_{\text{base2}} = \frac{3V_{\phi 2}{}^2}{S_{\text{base2}}} = \frac{3(7967\text{ V})^2}{1000\text{ kVA}} = 190.4\ \Omega$$

$$Z_{\text{base3}} = \frac{3V_{\phi 3}{}^2}{S_{\text{base3}}} = \frac{3(277\text{ V})^2}{1000\text{ kVA}} = 0.238\ \Omega$$

(a) To get the per-unit, per-phase equivalent circuit, we must convert each impedance in the system to per-unit on the base of the region in which it is located. The impedance of transformer T_1 is already in per-unit to the proper base, so we don't have to do anything to it:

$$R_{1,\text{pu}} = 0.010$$

$$X_{1,\text{pu}} = 0.040$$

The impedance of transformer T_2 is already in per-unit, but it is per-unit to the base of transformer T_2, so it must be converted to the base of the power system.

$$(R,\ X,\ Z)_{\text{pu on base 2}} = (R,\ X,\ Z)_{\text{pu on base 1}} \frac{(V_{\text{base 1}})^2 (S_{\text{base 2}})}{(V_{\text{base 2}})^2 (S_{\text{base 1}})} \qquad \text{(3-66)}$$

$$R_{2,\text{pu}} = 0.020 \frac{(7967\text{ V})^2 (1000\text{ kVA})}{(7967\text{ V})^2 (1000\text{ kVA})} = 0.040$$

$$X_{2,\text{pu}} = 0.085 \frac{(7967\text{ V})^2 (1000\text{ kVA})}{(7967\text{ V})^2 (1000\text{ kVA})} = 0.170$$

The per-unit impedance of the transmission line is

$$Z_{\text{line,pu}} = \frac{Z_{\text{line}}}{Z_{\text{base2}}} = \frac{1.5 + j10\ \Omega}{190.4\ \Omega} = 0.00788 + j0.0525$$

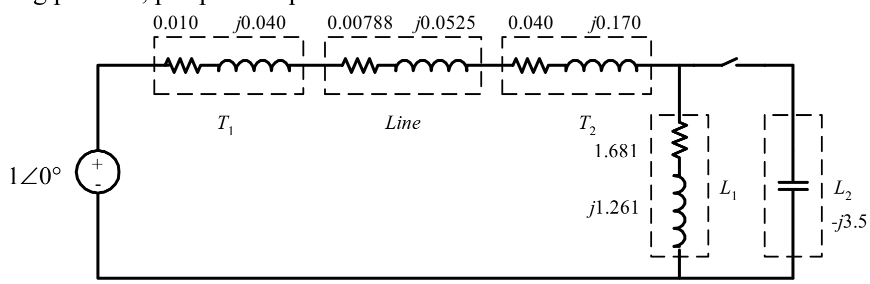
The per-unit impedance of Load 1 is

$$Z_{\text{load1,pu}} = \frac{Z_{\text{load1}}}{Z_{\text{base3}}} = \frac{0.5\angle 36.87^\circ\ \Omega}{0.238\ \Omega} = 1.681 + j1.261$$

The per-unit impedance of Load 2 is

$$Z_{\text{load2,pu}} = \frac{Z_{\text{load2}}}{Z_{\text{base3}}} = \frac{-j0.833\ \Omega}{0.238\ \Omega} = -j3.5$$

The resulting per-unit, per-phase equivalent circuit is shown below:



(b) With the switch opened, the equivalent impedance of this circuit is

$$Z_{\text{EQ}} = 0.010 + j0.040 + 0.00788 + j0.0525 + 0.040 + j0.170 + 1.681 + j1.261$$

$$Z_{\text{EQ}} = 1.7389 + j1.5235 = 2.312\angle 41.2^\circ$$

The resulting current is

$$\mathbf{I} = \frac{1\angle 0^\circ}{2.312\angle 41.2^\circ} = 0.4325\angle -41.2^\circ$$

The load voltage under these conditions would be

$$\mathbf{V}_{\text{Load,pu}} = \mathbf{I}\ Z_{\text{Load}} = (0.4325\angle -41.2^\circ)(1.681 + j1.261) = 0.909\angle -4.3^\circ$$

$$V_{\text{Load}} = V_{\text{Load,pu}} V_{\text{base3}} = (0.909)(480\text{ V}) = 436\text{ V}$$

The power supplied to the load is

$$P_{\text{Load,pu}} = I^2 R_{\text{Load}} = (0.4325)^2 (1.681) = 0.314$$

$$P_{\text{Load}} = P_{\text{Load,pu}} S_{\text{base}} = (0.314)(1000\text{ kVA}) = 314\text{ kW}$$

The power supplied by the generator is

$$P_{G,\text{pu}} = VI\cos\theta = (1)(0.4325)\cos 41.2^\circ = 0.325$$

$$Q_{G,\text{pu}} = VI\sin\theta = (1)(0.4325)\sin 41.2^\circ = 0.285$$

$$S_{G,\text{pu}} = VI = (1)(0.4325) = 0.4325$$

$$P_G = P_{G,\text{pu}} S_{\text{base}} = (0.325)(1000\text{ kVA}) = 325\text{ kW}$$

$$Q_G = Q_{G,\text{pu}} S_{\text{base}} = (0.285)(1000\text{ kVA}) = 285\text{ kVAR}$$

$$S_G = S_{G,\text{pu}} S_{\text{base}} = (0.4325)(1000\text{ kVA}) = 432.5\text{ kVA}$$

The power factor of the generator is

$$\text{PF} = \cos 41.2^\circ = 0.752 \text{ lagging}$$

(c) With the switch closed, the equivalent impedance of this circuit is

$$Z_{\text{EQ}} = 0.010 + j0.040 + 0.00788 + j0.0525 + 0.040 + j0.170 + \frac{(1.681 + j1.261)(-j3.5)}{1.681 + j1.261 - j3.5}$$

$$Z_{\text{EQ}} = 0.010 + j0.040 + 0.00788 + j0.0525 + 0.040 + j0.170 + (2.627 - j0.0011)$$

$$Z_{\text{EQ}} = 2.685 + j0.261 = 2.698\angle 5.6°$$

The resulting current is

$$\mathbf{I} = \frac{1\angle 0°}{2.698\angle 5.6°} = 0.371\angle -5.6°$$

The load voltage under these conditions would be

$$\mathbf{V}_{\text{Load,pu}} = \mathbf{I}\, Z_{\text{Load}} = (0.371\angle -5.6°)(2.627 - j0011) = 0.975\angle -5.6°$$

$$V_{\text{Load}} = V_{\text{Load,pu}} V_{\text{base3}} = (0.975)(480\text{ V}) = 468\text{ V}$$

The power supplied to the two loads is

$$P_{\text{Load,pu}} = I^2 R_{\text{Load}} = (0.371)^2 (2.627) = 0.361$$

$$P_{\text{Load}} = P_{\text{Load,pu}} S_{\text{base}} = (0.361)(1000\text{ kVA}) = 361\text{ kW}$$

The power supplied by the generator is

$$P_{G,\text{pu}} = VI\cos\theta = (1)(0.371)\cos 5.6° = 0.369$$

$$Q_{G,\text{pu}} = VI\sin\theta = (1)(0.371)\sin 5.6° = 0.036$$

$$S_{G,\text{pu}} = VI = (1)(0.371) = 0.371$$

$$P_G = P_{G,\text{pu}} S_{\text{base}} = (0.369)(1000\text{ kVA}) = 369\text{ kW}$$

$$Q_G = Q_{G,\text{pu}} S_{\text{base}} = (0.036)(1000\text{ kVA}) = 36\text{ kVAR}$$

$$S_G = S_{G,\text{pu}} S_{\text{base}} = (0.371)(1000\text{ kVA}) = 371\text{ kVA}$$

The power factor of the generator is

$$\text{PF} = \cos 5.6° = 0.995\text{ lagging}$$

(d) The transmission losses with the switch *open* are:

$$P_{\text{line,pu}} = I^2 R_{\text{line}} = (0.4325)^2 (0.00788) = 0.00147$$

$$P_{\text{line}} = P_{line,\text{pu}} S_{\text{base}} = (0.00147)(1000\text{ kVA}) = 1.47\text{ kW}$$

The transmission losses with the switch *closed* are:

$$P_{\text{line,pu}} = I^2 R_{\text{line}} = (0.371)^2 (0.00788) = 0.00108$$

$$P_{\text{line}} = P_{line,\text{pu}} S_{\text{base}} = (0.00108)(1000\text{ kVA}) = 1.08\text{ kW}$$

Load 2 improved the power factor of the system, increasing the load voltage and the total power supplied to the loads, while simultaneously decreasing the current in the transmission line and the transmission line losses. This problem is a good example of the advantages of power factor correction in power systems.