Chapter 9: Transmission Lines

9-1. Calculate the dc resistance in ohms per kilometer for an aluminum conductor with a 3 cm diameter.

SOLUTION The resistance per meter of aluminum conductor is given by Equation (9-2):

$$r_{DC} = \frac{\rho}{A} = \frac{\rho}{\pi r^2}$$

where $\rho = 2.83 \times 10^{-8}$ Ω-m. This value is

$$r_{DC} = \frac{\left(2.83\times10^{-8}\ \Omega\text{-m}\right)}{\pi\left(0.015\text{ m}\right)^2} = 4.004\times10^{-5}\ \Omega/\text{m}$$

Therefore the total DC resistance per kilometer would be

$$R_{DC} = \left(4.004\times10^{-5}\ \Omega/\text{m}\right)\left(\frac{1000\text{ m}}{1\text{ km}}\right) = 0.040\ \Omega/\text{km}$$

9-2. Calculate the dc resistance in ohms per *mile* for a hard-drawn copper conductor with a 1 inch diameter. (Note that 1 mile = 1.609 km).

SOLUTION The resistance per meter of hard-drawn copper conductor is given by Equation (9-2):

$$r_{DC} = \frac{\rho}{A} = \frac{\rho}{\pi r^2}$$

where $\rho = 1.77 \times 10^{-8}$ Ω-m. Note that the radius in this equation must be in units of meters. This value is

$$r_{DC} = \frac{\left(1.77\times10^{-8}\ \Omega\text{-m}\right)}{\pi\left[\left(0.5\text{ in}\right)\left(\frac{0.0254\text{ m}}{1\text{ in}}\right)\right]^2} = 3.493\times10^{-5}\ \Omega/\text{m}$$

Therefore the total DC resistance per mile would be $1000\ r_{DC} = 0.03367\ \Omega$.

$$R_{DC} = \left(3.493\times10^{-5}\ \Omega/\text{m}\right)\left(\frac{1609\text{ m}}{1\text{ mile}}\right) = 0.0562\ \Omega/\text{mile}$$

Problems 9-3 through 9-7 refer to a single phase, 8 kV, 50-Hz, 50 km-long transmission line consisting of two aluminum conductors with a 3 cm diameter separated by a spacing of 2 meters.

9-3. Calculate the inductive reactance of this line in ohms.

SOLUTION The series inductance per meter of this transmission line is given by Equation (9-22).

$$l = \frac{\mu}{\pi}\left(\frac{1}{4} + \ln\frac{D}{r}\right)\quad \text{H/m} \tag{9-22}$$

where $\mu = \mu_0 = 4\pi\times10^{-7}$ H/m.

$$l = \frac{\mu_0}{\pi}\left(\frac{1}{4} + \ln\frac{2.0\text{ m}}{0.015\text{ m}}\right) = \frac{4\pi\times10^{-7}\text{ H/m}}{\pi}\left(\frac{1}{4} + \ln\frac{2.0\text{ m}}{0.015\text{ m}}\right) = 2.057\times10^{-6}\text{ H/m}$$

Therefore the inductance of this transmission line will be

$$L = \left(2.057\times10^{-6}\text{ H/m}\right)\left(50{,}000\text{ m}\right) = 0.1029\text{ H}$$

The inductive reactance of this transmission line is

$$X = j\omega L = j2\pi f L = j2\pi\left(50\text{ Hz}\right)\left(0.1029\text{ H}\right) = j32.3\ \Omega$$

9-4. Assume that the 50 Hz ac resistance of the line is 5% greater than its dc resistance, and calculate the series impedance of the line in ohms per km.

SOLUTION The DC resistance per meter of this transmission line is given by Equation (9-22).

$$r_{\mathrm{DC}} = \frac{\rho}{A} = \frac{\rho}{\pi r^2}$$

where $\rho = 2.83 \times 10^{-8}$ Ω-m. This value is

$$r_{\mathrm{DC}} = \frac{\left(2.83 \times 10^{-8}\ \Omega\text{-m}\right)}{\pi\left(0.015\ \mathrm{m}\right)^2} = 4.004 \times 10^{-5}\ \Omega/\mathrm{m}$$

Therefore the total DC resistance of the line would be

$$R_{\mathrm{DC}} = \left(4.004 \times 10^{-5}\ \Omega/\mathrm{m}\right)\left(50{,}000\ \mathrm{m}\right) = 2.0\ \Omega$$

The AC resistance of the line would be

$$R_{\mathrm{AC}} = \left(2.0\ \Omega\right)\left(1.05\right) = 2.1\ \Omega$$

The total series impedance of this line would be $Z = 2.1 + j30.5\ \Omega$, so the impedance per kilometer would be

$$Z = \left(2.1 + j32.3\ \Omega\right)/\left(50\ \mathrm{km}\right) = 0.042 + j0.646\ \Omega/\mathrm{km}$$

9-5. Calculate the shunt admittance of the line in siemens per km.

SOLUTION The shunt capacitance per meter of this transmission line is given by Equation (9-41).

$$c = \frac{\pi\varepsilon}{\ln\left(\dfrac{D}{r}\right)} \qquad (9\text{-}41)$$

$$c = \frac{\pi\left(8.854 \times 10^{-12}\ \mathrm{F/m}\right)}{\ln\left(\dfrac{2.0}{0.015}\right)} = 5.69 \times 10^{-12}\ \mathrm{F/m}$$

Therefore the capacitance per kilometer will be

$$c = 5.69 \times 10^{-9}\ \mathrm{F/km}$$

The shunt admittance of this transmission line per kilometer will be

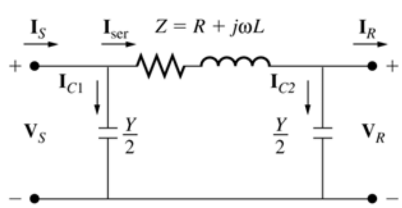
$$y_{sh} = j2\pi fc = j2\pi\left(50\ \mathrm{Hz}\right)\left(5.69 \times 10^{-9}\ \mathrm{F/km}\right) = j1.79 \times 10^{-6}\ \mathrm{S/km}$$

Therefore the total shunt admittance will be

$$Y_{sh} = \left(j1.79 \times 10^{-6}\ \mathrm{S/km}\right)\left(50\ \mathrm{km}\right) = j8.95 \times 10^{-5}\ \mathrm{S}$$

9-6. The single-phase transmission line is operating with the receiving side of the line open-circuited. The sending end voltage is 8 kV at 50 Hz. How much charging current is flowing in the line?

SOLUTION Although this line is in the "short" range of lengths, we will treat it as a medium-length line, because we must include the capacitances if we wish to calculate charging currents. The appropriate transmission line model is shown below.



The charging current can be calculated by open-circuiting the output of the transmission line and calculating $\mathbf{I}_S$:

$$\mathbf{I}_S = \frac{Y\mathbf{V}_S}{2} + \frac{\mathbf{V}_S}{Z + \frac{1}{Y/2}}$$

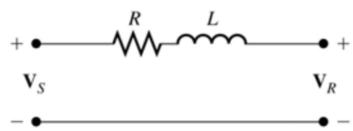
$$\mathbf{I}_S = \frac{\left(j8.95\times10^{-5}\ \mathrm{S}\right)\left(8000\angle0°\ \mathrm{V}\right)}{2} + \frac{8000\angle0°\ \mathrm{V}}{\left(2.1\ + j32.3\ \Omega\right) + \frac{1}{j8.95\times10^{-5}\ \mathrm{S}/2}}$$

$$\mathbf{I}_S = 0.358\angle90°\ \mathrm{A} + 0.358\angle90°\ \mathrm{A} = 0.716\angle90°\ \mathrm{A}$$

9-7. The single-phase transmission line is now supplying 8 kV to an 800 kVA, 0.9 PF lagging single-phase load.

(a) What is the sending end voltage and current of this transmission line?
(b) What is the efficiency of the transmission line under these conditions?
(c) What is the voltage regulation of the transmission line under these conditions?

SOLUTION At 50 km length, we can treat this transmission line as a "short" line and ignore the effects of the shunt admittance. The corresponding equivalent circuit is shown below.



The transmission line is supplying a voltage of 8 kV at the load, so the magnitude of the current flowing to the load is

$$I = \frac{S}{V} = \frac{800\ \mathrm{kVA}}{8\ \mathrm{kV}} = 100\ \mathrm{A}$$

(a) If we assume that the voltage at the load is arbitrarily assigned to be at 0° phase, and the power factor of the load is 0.9 lagging, the phasor current flowing to the load is $\mathbf{I} = 100\angle-25.8°\ \mathrm{A}$. The voltage at the sending end of the transmission line is then

$$\mathbf{V}_S = \mathbf{V}_R + Z\mathbf{I}$$

$$\mathbf{V}_S = 8000\angle0°\ \mathrm{V} + \left(2.1\ + j32.3\ \Omega\right)\left(100\angle-25.8°\ \mathrm{A}\right)$$

$$\mathbf{V}_S = 10000\angle16.4°\ \mathrm{V}$$

(b) The complex output power from the transmission line is

$$\mathbf{S}_{\mathrm{OUT}} = \mathbf{V}_R\mathbf{I}^* = \left(8000\angle0°\ \mathrm{V}\right)\left(100\angle-25.8°\ \mathrm{A}\right)^* = 800,000\angle25.8°\ \mathrm{VA}$$

Therefore the output power is

$$P_{\mathrm{OUT}} = 800\ \cos25.8°\ = 720\ \mathrm{kW}$$

The complex input power to the transmission line is

$$\mathbf{S}_{\mathrm{IN}} = \mathbf{V}_S\mathbf{I}^* = \left(10000\angle16.4°\ \mathrm{V}\right)\left(100\angle-25.8°\ \mathrm{A}\right)^* = 1,000,000\angle42.2°\ \mathrm{VA}$$

Therefore the input power is

$$P_{\mathrm{IN}} = 1000\ \cos42.2°\ = 741\ \mathrm{kW}$$

The resulting efficiency is

$$\eta = \frac{P_{\mathrm{OUT}}}{P_{\mathrm{IN}}}\times100\% = \frac{720\ \mathrm{kW}}{741\ \mathrm{kW}}\times100\% = 97.2\%$$

(c) The voltage regulation of the transmission line is

$$\text{VR} = \frac{V_S - V_R}{V_R} \times 100\% = \frac{10000 - 8000}{8000} \times 100\% = 25\%$$

Problems 9-8 through 9-10 refer to a single phase, 8 kV, 50-Hz, 50 km-long underground cable consisting of two aluminum conductors with a 3 cm diameter separated by a spacing of 15 cm.

9-8. The single-phase transmission line referred to in Problems 9-3 through 9-7 is to be replaced by an underground cable. The cable consists of two aluminum conductors with a 3 cm diameter, separated by a center-to-center spacing of 15 cm. As before, assume that the 50 Hz ac resistance of the line is 5% greater than its dc resistance, and calculate the series impedance and shunt admittance of the line in ohms per km and siemens per km. Also, calculate the total impedance and admittance for the entire line.

SOLUTION The series inductance per meter of this transmission line is given by Equation (9-22).

$$l = \frac{\mu}{\pi}\left(\frac{1}{4} + \ln\frac{D}{r}\right) \quad \text{H/m} \tag{9-22}$$

where $\mu = \mu_0 = 4\pi \times 10^{-7}$ H/m .

$$l = \frac{\mu_0}{\pi}\left(\frac{1}{4} + \ln\frac{0.15 \text{ m}}{0.015 \text{ m}}\right) = \frac{4\pi \times 10^{-7} \text{ H/m}}{\pi}\left(\frac{1}{4} + \ln\frac{0.15 \text{ m}}{0.015 \text{ m}}\right) = 1.021 \times 10^{-6} \text{ H/m}$$

Therefore the inductance of this transmission line will be

$$L = \left(1.021 \times 10^{-6} \text{ H/m}\right)\left(50{,}000 \text{ m}\right) = 0.0511 \text{ H}$$

The inductive reactance of this transmission line is

$$X = j\omega L = j2\pi fL = j2\pi(50 \text{ Hz})(0.0511 \text{ H}) = j16.05\ \Omega$$

The resistance of this transmission line is the same as for the overhead transmission line calculated previously: $R_{AC} = 2.1\ \Omega$. The total series impedance of this entire line would be $Z = 2.1\ + j16.05\ \Omega$, so the impedance per kilometer would be

$$Z = (2.1\ + j16.05\ \Omega)/(50 \text{ km}) = 0.042 + j0.321\ \Omega\text{/km}$$

The shunt capacitance per meter of this transmission line is given by Equation (9-41).

$$c = \frac{\pi\varepsilon}{\ln\left(\frac{D}{r}\right)} \tag{9-41}$$

$$c = \frac{\pi\left(8.854 \times 10^{-12} \text{ F/m}\right)}{\ln\left(\frac{0.15}{0.015}\right)} = 1.21 \times 10^{-11} \text{ F/m}$$

Therefore the capacitance per kilometer will be

$$c = 1.21 \times 10^{-8} \text{ F/km}$$

The shunt admittance of this transmission line per kilometer will be

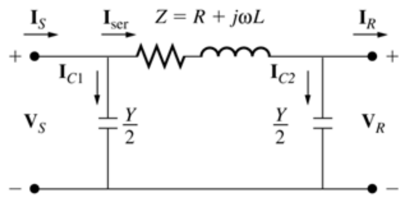
$$y_{sh} = j2\pi fc = j2\pi(50 \text{ Hz})\left(1.21 \times 10^{-8} \text{ F/km}\right) = j3.80 \times 10^{-6} \text{ S/km}$$

Therefore the total shunt admittance will be

$$Y_{sh} = \left(j3.80 \times 10^{-6} \text{ S/km}\right)(50 \text{ km}) = j1.90 \times 10^{-4} \text{ S}$$

9-9. The underground cable is operating with the receiving side of the line open-circuited. The sending end voltage is 8 kV at 50 Hz. How much charging current is flowing in the line? How does this charging current in the cable compare to the charging current of the overhead transmission line?

SOLUTION Although this line is in the "short" range of lengths, we will treat it as a medium-length line, because we must include the capacitances if we wish to calculate charging currents. The appropriate transmission line model is shown below.



The charging current can be calculated by open-circuiting the output of the transmission line and calculating $\mathbf{I}_S$:

$$\mathbf{I}_S = \frac{Y\mathbf{V}_S}{2} + \frac{\mathbf{V}_S}{Z + \frac{1}{Y/2}}$$

$$\mathbf{I}_S = \frac{\left(j1.90\times10^{-4}\text{ S}\right)\left(8000\angle0^\circ\text{ V}\right)}{2} + \frac{8000\angle0^\circ\text{ V}}{\left(2.1\ + j32.3\ \Omega\right) + \frac{1}{j1.90\times10^{-4}\text{ S}/2}}$$

$$\mathbf{I}_S = 0.760\angle90^\circ\text{ A} + 0.762\angle90^\circ\text{ A} = 1.522\angle90^\circ\text{ A}$$

Since the shunt admittance of the underground cable is more than twice as large as shunt admittance of the overhead transmission line, the charging current of the underground cable is more than twice as large.

9-10. The underground cable is now supplying 8 kV to an 800 kVA, 0.9 PF lagging single-phase load.

(a) What is the sending end voltage and current of this transmission line?
(b) What is the efficiency of the transmission line under these conditions?
(c) What is the voltage regulation of the transmission line under these conditions?

SOLUTION At 50 km length, we can treat this transmission line as a "short" line and ignore the effects of the shunt admittance. (Note, however, that this assumption is not as good as it was for the overhead transmission line. The higher shunt admittance makes its effects harder to ignore.) The transmission line is supplying a voltage of 8 kV at the load, so the magnitude of the current flowing to the load is

$$I = \frac{S}{V} = \frac{800\text{ kVA}}{8\text{ kV}} = 100\text{ A}$$

(a) If we assume that the voltage at the load is arbitrarily assigned to be at 0° phase, and the power factor of the load is 0.9 lagging, the phasor current flowing to the load is $\mathbf{I} = 100\angle-25.8^\circ\text{ A}$. The voltage at the sending end of the transmission line is then

$$\mathbf{V}_S = \mathbf{V}_R + Z\mathbf{I}$$

$$\mathbf{V}_S = 8000\angle0^\circ\text{ V} + \left(2.1\ + j16.05\ \Omega\right)\left(100\angle-25.8^\circ\text{ A}\right)$$

$$\mathbf{V}_S = 8990\angle8.7^\circ\text{ V}$$

(b) The complex output power from the transmission line is

$$\mathbf{S}_{\text{OUT}} = \mathbf{V}_R\mathbf{I}^* = \left(8000\angle0^\circ\text{ V}\right)\left(100\angle-25.8^\circ\text{ A}\right)^* = 800{,}000\angle25.8^\circ\text{ VA}$$

Therefore the output power is

$$P_{\text{OUT}} = 800\ \cos25.8^\circ\ = 720\text{ kW}$$

The complex input power to the transmission line is

$$\mathbf{S}_{\text{IN}} = \mathbf{V}_S \mathbf{I}^* = (8990\angle 8.7°\text{ V})(100\angle -25.8°\text{ A})^* = 899{,}000\angle 34.5°\text{ VA}$$

Therefore the input power is

$$P_{\text{IN}} = 899 \ \cos 34.5° \ = 740.9\text{ kW}$$

The resulting efficiency is

$$\eta = \frac{P_{\text{OUT}}}{P_{\text{IN}}} \times 100\% = \frac{720\text{ kW}}{740.9\text{ kW}} \times 100\% = 97.2\%$$

(c) The voltage regulation of the transmission line is

$$\text{VR} = \frac{V_S - V_R}{V_R} \times 100\% = \frac{8990 - 8000}{8000} \times 100\% = 12.4\%$$

9-11. A 138 kV, 200 MVA, 60 Hz, three-phase, power transmission line is 100 km long, and has the following characteristics:

$r = 0.103\ \Omega/\text{km}$

$x = 0.525\ \Omega/\text{km}$

$y = 3.3 \times 10^{-6}\ \text{S/km}$

(a) What is per phase series impedance and shunt admittance of this transmission line?
(b) Should it be modeled as a short, medium, or long transmission line?
(c) Calculate the *ABCD* constants of this transmission line.
(d) Sketch the phasor diagram of this transmission line when the line is supplying rated voltage and apparent power at a 0.90 power factor lagging.
(e) Calculate the sending end voltage if the line is supplying rated voltage and apparent power at 0.90 PF lagging.
(f) What is the voltage regulation of the transmission line for the conditions in *(e)*?
(g) What is the efficiency of the transmission line for the conditions in *(e)*?

SOLUTION

(a) The per-phase series impedance of this transmission line is

$$Z = (0.103 + j0.525\ \Omega/\text{km})(100\text{ km}) = 10.3 + j52.5\ \Omega$$

The per-phase shunt admittance of this transmission line is

$$Y = (j3.3\times 10^{-6}\text{ S/km})(100\text{ km}) = j0.00033\text{ S}$$

(b) This transmission line should be modeled as a medium length transmission line.

(c) The ABCD constants for a medium length line are given by the following equations:

$$A = \frac{ZY}{2} + 1 \qquad B = Z$$
$$C = Y\left(\frac{ZY}{4} + 1\right) \quad D = \frac{ZY}{2} + 1 \tag{9-73}$$

$$A = \frac{ZY}{2} + 1 = \frac{(10.3 + j52.5\ \Omega)(j0.00033\text{ S})}{2} + 1 = 0.9913\angle 0.1°$$

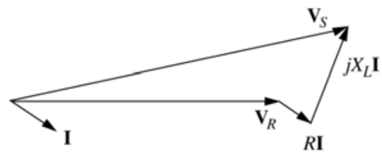
$$B = Z = 10.3 + j52.5\ \Omega = 53.5\angle 78.9°\ \Omega$$

$$C = Y\left(\frac{ZY}{4} + 1\right) = (j0.00033\text{ S})\left[\frac{(10.3 + j52.5\ \Omega)(j0.00033\text{ S})}{4} + 1\right]$$

$$C = 3.286 \times 10^{-4} \angle 90° \text{ S}$$

$$D = \frac{ZY}{2} + 1 = \frac{(10.3 + j52.5\ \Omega)(j0.00033\text{ S})}{2} + 1 = 0.9913\angle 0.1°$$

(d) The phasor diagram is shown below:



(e) The rated line voltage is 138 kV, so the rated phase voltage is 138 kV / $\sqrt{3}$ = 79.67 kV, and the rated current is

$$I_L = \frac{S_{\text{out}}}{\sqrt{3}V_{LL}} = \frac{200{,}000{,}000\text{ VA}}{\sqrt{3}(138{,}000\text{ V})} = 837\text{ A}$$

If the phase voltage at the receiving end is assumed to be at a phase angle of 0°, then the phase voltage at the receiving end will be $\mathbf{V}_R = 79.67\angle 0°\text{ kV}$, and the phase current at the receiving end will be $\mathbf{I}_R = 837\angle -25.8°\text{ A}$. The current and voltage at the sending end of the transmission line are given by the following equations:

$$\mathbf{V}_S = A\mathbf{V}_R + B\mathbf{I}_R$$

$$\mathbf{V}_S = (0.9913\angle 0.1°)(79.67\angle 0°\text{ kV}) + (53.5\angle 78.9°\ \Omega)(837\angle -25.8°\text{ A})$$

$$\mathbf{V}_S = 111.8\angle 18.75°\text{ kV}$$

$$\mathbf{I}_S = C\mathbf{V}_R + D\mathbf{I}_R$$

$$\mathbf{I}_S = (3.286 \times 10^{-4}\angle 90°\text{ S})(79.67\angle 0°\text{ kV}) + (0.9913\angle 0.1°)(837\angle -25.8°\text{ A})$$

$$\mathbf{I}_S = 818.7\angle -24.05°\text{ A}$$

(f) The voltage regulation of the transmission line is

$$\text{VR} = \frac{V_S - V_R}{V_R} \times 100\% = \frac{111.8\text{ kV} - 79.67\text{ kV}}{79.67\text{ kV}} \times 100\% = 40.3\%$$

(g) The output power from the transmission line is

$$P_{\text{OUT}} = S\text{ PF} = (200\text{ MVA})(0.9) = 180\text{ MW}$$

The input power to the transmission line is

$$P_{\text{IN}} = 3V_{\phi,S}I_{\phi,S}\cos\theta = 3(111.8\text{ kV})(818.7\text{ A})\cos(42.8°) = 201.5\text{ MW}$$

The resulting efficiency is

$$\eta = \frac{P_{\text{IN}}}{P_{\text{OUT}}} \times 100\% = \frac{180\text{ kW}}{201.5\text{ kW}} \times 100\% = 89.3\%$$

9-12. If the series resistance and shunt admittance of the transmission line in Problem 9-11 are ignored, what would the value of the angle δ be at rated conditions and 0.90 PF lagging?

SOLUTION If the series resistance and shunt admittance are ignored, then the sending end voltage of the transmission line would be

$$\mathbf{V}_S = \mathbf{V}_R + jX\mathbf{I}_R$$

$$\mathbf{V}_S = 79.67\angle 0^\circ \text{ kV} + (j52.5\ \Omega)(837\angle -25.8^\circ \text{ A})$$

$$\mathbf{V}_S = 106.4\angle 21.8^\circ \text{ kV}$$

Since δ is the angle between $\mathbf{V}_S$ and $\mathbf{V}_R$, $\delta = 21.8^\circ$.

9-13. If the series resistance and shunt admittance of the transmission line in Problem 9-11 are *not* ignored, what would the value of the angle δ be at rated conditions and 0.90 PF lagging?

SOLUTION From Problem 9-11, $\mathbf{V}_S = 111.8\angle 18.75^\circ$ kV and $\mathbf{V}_R = 79.67\angle 0^\circ$ kV, so the angle δ is 18.75° when the series resistance and shunt admittance are also considered.

9-14. Assume that the transmission line of Problem 9-11 is to supply a load at 0.90 PF lagging with no more than a 5% voltage drop and a torque angle $\delta \le 30^\circ$. Treat the line as a medium-length transmission line. What is the maximum power that this transmission line can supply without violating one of these constraints? Which constraint is violated first?

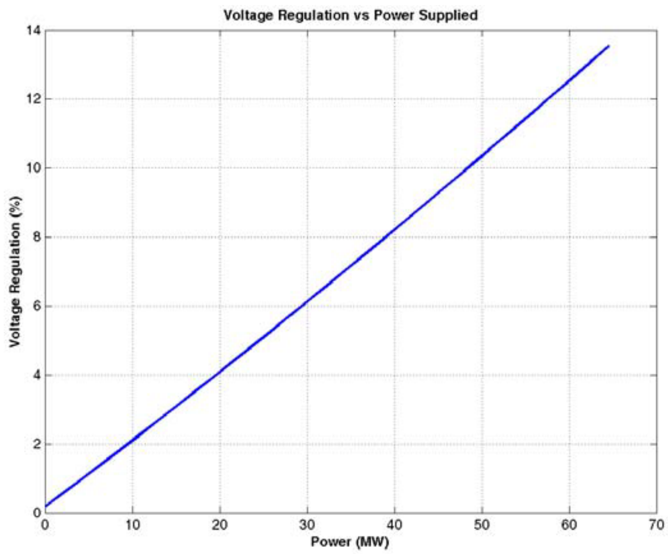
SOLUTION A MATLAB program to calculate the voltage regulation and angle δ as a function of the power supplied to a load an a power factor of 0.9 lagging is shown below.

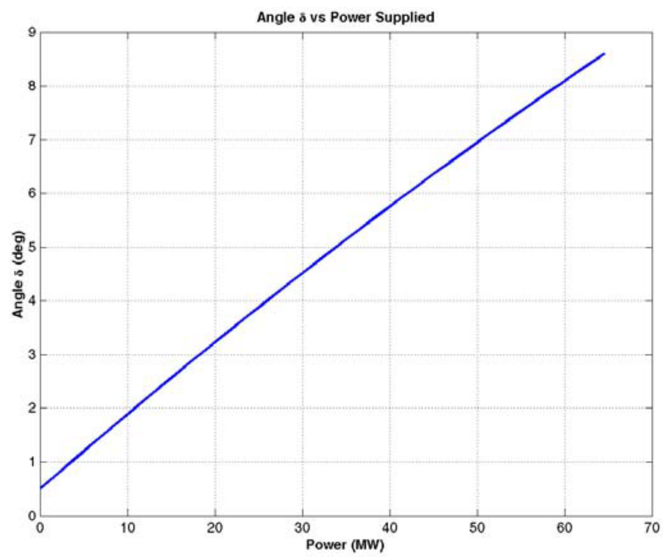
```
% M-file: prob9_13.m
% M-file to calculate the voltage drop and angle delta
%   for a transmission line as load is increased.

% First, initialize the values needed in this program.
v_r = p2r(79670,0);      % Receiving end voltage
v_s = 0;                 % Sending end voltage (will calculate)
r = 0.103;               % Resistance in ohms/km
x = 0.525;               % Reactance in ohms/km
y = 3.3e-6;              % Shunt admittance in S/km
l = 100;                 % Line length (k)

% Calculate series impedance and shunt admittance
Z = (r + j*x) * l;
Y = y * l;

% Calculate ABCD constants
A = Y*Z/2 + 1;
B = Z;
C = Y*(Y*Z/4 + 1);
D = Y*Z/2 + 1;

% Calculate the transmitted power for various currents
% assuming a power factor of 0.9 lagging (-25.8 degrees).
% This calculation uses the equation for complex power
% P = 3 * v_r * conj(i_r).
i_r = (0:10:300) * p2r(1,-25.8);
power = real( 3 * v_r * conj(i_r));

% Calculate sending end voltage and current for the
% various loads
v_s = A * v_r + B * i_r;
i_s = C * v_r + D * i_r;
```

```
% Calculate VR for each current step
VR = ( abs(v_s) - abs(v_r) ) ./ abs(v_r);

% Calculate angle delta for each current step
[v_r_mag, v_r_angle] = r2p(v_r);
for ii = 1:length(i_r)
   [v_s_mag, v_s_angle] = r2p(v_s(ii));
   delta(ii) = v_s_angle - v_r_angle;
end

% Which limit is reached first?
for ii = 1:length(i_r)
   if VR(ii) > 0.05

      % VR exceeded first--tell user
      str = ['VR exceeds 5% at ' num2str(power(ii)/1e6) ' MW'];
      disp(str);
      break;

   elseif abs(delta) > 30

      % Delta exceeded first--tell user
      str = ['delta exceeds 30 deg at ' num2str(power(ii)/1e6) ' MW'];
      disp(str);
      break;

   end
end

% Plot the VR and delta vs power
figure(1);
plot(power/1e6,VR*100,'b-','LineWidth',2.0);
xlabel('\bfPower (MW)');
ylabel('\bfVoltage Regulation (%)');
title ('\bfVoltage Regulation vs Power Supplied');
grid on;

figure(2);
plot(power/1e6,delta,'b-','LineWidth',2.0);
xlabel('\bfPower (MW)');
ylabel('\bfAngle \delta (deg)');
title ('\bfAngle \delta vs Power Supplied');
grid on;
```

When this program is executed, the results are as shown below. Note that the voltage regulation is the first limit to be exceeded by this transmission line of the load is at 0.9 PF lagging. Try the same calculation with other power factors. How does the maximum power supplied vary?

```
>> prob9_13
VR exceeds 5% at 25.8222 MW
```





9-15. The transmission line of Problem 9-11 is connected between two infinite busses, as shown in Figure P9-1. Answer the following questions about this transmission line.

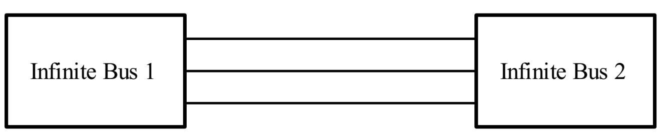


Figure P9-1 A three-phase transmission line connecting two infinite busses together.

(a) If the per-phase (line-to-neutral) voltage on the sending infinite bus is $80\angle 10°$ kV and the per-phase voltage on the receiving infinite bus is $76\angle 0°$ kV, how much real and reactive power are being supplied by the transmission line to the receiving bus?

(b) If the per-phase voltage on the sending infinite bus is changed to $82\angle 10°$ kV, how much real and reactive power are being supplied by the transmission line to the receiving bus? Which changed more, the real or the reactive power supplied to the load?

(c) If the per-phase voltage on the sending infinite bus is changed to $80\angle 15°$ kV, how much real and reactive power are being supplied by the transmission line to the receiving bus? Compared to the conditions in part *(a)*, which changed more, the real or the reactive power supplied to the load?

(d) From the above results, how could real power flow be controlled in a transmission line? How could reactive power flow be controlled in a transmission line?

SOLUTION

(a) If the shunt admittance of the transmission line is ignored, the relationship between the voltages and currents on this transmission line is

$$\mathbf{V}_S = \mathbf{V}_R + R\mathbf{I} + jX\mathbf{I}$$

where $\mathbf{I}_S = \mathbf{I}_R = \mathbf{I}$. Therefore we can calculate the current in the transmission line as

$$\mathbf{I} = \frac{\mathbf{V}_S - \mathbf{V}_R}{R + jX}$$

$$\mathbf{I} = \frac{80,000\angle 10° - 76,000\angle 0°}{10.3 + j52.5\ \Omega} = 265\angle -0.5°\ \text{A}$$

The real and reactive power supplied by this transmission line is

$$P = 3V_{\phi,R} I_\phi \cos\theta = 3(76\ \text{kV})(265\ \text{A})\cos(0.5°) = 60.4\ \text{MW}$$

$$Q = 3V_{\phi,R} I_\phi \sin\theta = 3(76\ \text{kV})(265\ \text{A})\sin(0.5°) = 0.53\ \text{MVAR}$$

(b) If the sending end voltage is changed to $82\angle 10°$ kV, the current is

$$\mathbf{I} = \frac{82,000\angle 10° - 76,000\angle 0°}{10.3 + j52.5\ \Omega} = 280\angle -7.7°\ \text{A}$$

The real and reactive power supplied by this transmission line is

$$P = 3V_{\phi,R} I_\phi \cos\theta = 3(76\ \text{kV})(280\ \text{A})\cos(7.7°) = 63.3\ \text{MW}$$

$$Q = 3V_{\phi,R} I_\phi \sin\theta = 3(76\ \text{kV})(280\ \text{A})\sin(7.7°) = 8.56\ \text{MVAR}$$

In this case, there was a relatively small change in *P* (3 MW) and a relatively large change in *Q* (8 MVAR) supplied to the receiving bus.

(c) If the sending end voltage is changed to $82\angle 10°$ kV, the current is

$$\mathbf{I} = \frac{80,000\angle 15° - 76,000\angle 0°}{10.3 + j52.5\ \Omega} = 388\angle 7.2°\ \text{A}$$

The real and reactive power supplied by this transmission line is

$$P = 3V_{\phi,R} I_\phi \cos\theta = 3(76\ \text{kV})(388\ \text{A})\cos(-7.2°) = 87.8\ \text{MW}$$

$$Q = 3V_{\phi,R} I_\phi \sin\theta = 3(76\ \text{kV})(388\ \text{A})\sin(-7.2°) = -11\ \text{MVAR}$$

In this case, there was a relatively large change in *P* (27.4 MW) and a relatively small change in *Q* (11.5 MVAR) supplied to the receiving bus.

(d) From the above results, we can see that real power flow can be adjusted by changing the phase angle between the two voltages at the two ends of the transmission line, while reactive power flow can be changed by changing the relative magnitude of the two voltages on either side of the transmission line.

9-16. A 50 Hz three phase transmission line is 300 km long. It has a total series impedance of $23 + j75\ \Omega$ and a shunt admittance of $j500\ \mu\text{S}$. It delivers 150 MW at 220 kV, with a power factor of 0.88 lagging. Find the voltage at the sending end using *(a)* the short line approximation. *(b)* The medium-length line approximation. *(c)* The long line equation. How accurate are the short and medium-length line approximations for this case?

SOLUTION

(a) In the short line approximation, the shunt admittance is ignored. The *ABCD* constants for this line are:

$$\begin{aligned} A &= 1 \quad B = Z \\ C &= 0 \quad D = 1 \end{aligned} \tag{9-67}$$

$$A = 1$$

$$B = Z = 23 + j75\ \Omega = 78.4\angle 73°\ \Omega$$

$$C = 0\ \text{S}$$

$$D = 1$$

The receiving end line voltage is 220 kV, so the rated phase voltage is 220 kV / $\sqrt{3}$ = 127 kV, and the current is

$$I_L = \frac{S_{\text{out}}}{\sqrt{3}V_{LL}} = \frac{150{,}000{,}000\ \text{W}}{\sqrt{3}\left(220{,}000\ \text{V}\right)} = 394\ \text{A}$$

If the phase voltage at the receiving end is assumed to be at a phase angle of 0°, then the phase voltage at the receiving end will be $\mathbf{V}_R = 127\angle 0°\ \text{kV}$, and the phase current at the receiving end will be $\mathbf{I}_R = 394\angle -28.4°\ \text{A}$. The current and voltage at the sending end of the transmission line are given by the following equations:

$$\mathbf{V}_S = A\mathbf{V}_R + B\mathbf{I}_R$$

$$\mathbf{V}_S = (1)(127\angle 0°\ \text{kV}) + (78.4\angle 73°\ \Omega)(394\angle -28.4°\ \text{A})$$

$$\mathbf{V}_S = 151\angle 8.2°\ \text{kV}$$

$$\mathbf{I}_S = C\mathbf{V}_R + D\mathbf{I}_R$$

$$\mathbf{I}_S = (0\ \text{S})(133\angle 0°\ \text{kV}) + (1)(394\angle -28.4°\ \text{A})$$

$$\mathbf{I}_S = 394\angle -28.4°\ \text{A}$$

(b) In the medium length line approximation, the shunt admittance divided into two equal pieces at either end of the line. The *ABCD* constants for this line are:

$$\begin{aligned} A &= \frac{ZY}{2} + 1 \qquad B = Z \\ C &= Y\left(\frac{ZY}{4} + 1\right) \quad D = \frac{ZY}{2} + 1 \end{aligned} \tag{9-73}$$

$$A = \frac{ZY}{2} + 1 = \frac{(23 + j75\ \Omega)(j0.0005\ \text{S})}{2} + 1 = 0.9813\angle 0.34°$$

$$B = Z = 23 + j75\ \Omega = 78.4\angle 73°\ \Omega$$

$$C = Y\left(\frac{ZY}{4} + 1\right) = (j0.0005\ \text{S})\left[\frac{(23 + j75\ \Omega)(j0.0005\ \text{S})}{4} + 1\right]$$

$$C = 4.953 \times 10^{-4} \angle 90.2° \text{ S}$$

$$D = \frac{ZY}{2} + 1 = \frac{(23 + j75\ \Omega)(j0.0005\text{ S})}{2} + 1 = 0.9813 \angle 0.34°$$

The receiving end line voltage is 220 kV, so the rated phase voltage is 220 kV / $\sqrt{3}$ = 127 kV, and the current is

$$I_L = \frac{S_{\text{out}}}{\sqrt{3} V_{LL}} = \frac{150{,}000{,}000 \text{ W}}{\sqrt{3}(220{,}000 \text{ V})} = 394 \text{ A}$$

If the phase voltage at the receiving end is assumed to be at a phase angle of 0°, then the phase voltage at the receiving end will be $\mathbf{V}_R = 127 \angle 0° \text{ kV}$, and the phase current at the receiving end will be $\mathbf{I}_R = 394 \angle -28.4° \text{ A}$. The current and voltage at the sending end of the transmission line are given by the following equations:

$$\mathbf{V}_S = A\mathbf{V}_R + B\mathbf{I}_R$$

$$\mathbf{V}_S = 148.4 \angle 8.7° \text{ kV}$$

$$\mathbf{I}_S = C\mathbf{V}_R + D\mathbf{I}_R$$

$$\mathbf{I}_S = 361 \angle -19.2° \text{ A}$$

(c) In the long transmission line, the *ABCD* constants are based on modified impedances and admittances:

$$Z' = Z \frac{\sinh \gamma d}{\gamma d} \tag{9-74}$$

$$Y' = Y \frac{\tanh(\gamma d / 2)}{\gamma d / 2} \tag{9-75}$$

and the corresponding *ABCD* constants are

$$\begin{aligned} A &= \frac{Z'Y'}{2} + 1 \qquad B = Z' \\ C &= Y'\left(\frac{Z'Y'}{4} + 1\right) \qquad D = \frac{Z'Y'}{2} + 1 \end{aligned} \tag{9-76}$$

The propagation constant of this transmission line is $\gamma = \sqrt{yz}$

$$\gamma = \sqrt{yz} = \sqrt{\left(\frac{j500 \times 10^{-6}\text{ S}}{300\text{ km}}\right)\left(\frac{23 + j75\ \Omega}{300\text{ km}}\right)} = 0.00066 \angle 81.5°$$

$$\gamma d = (0.00066 \angle 81.5°)(300\text{ km}) = 0.198 \angle 81.5°$$

The modified parameters are

$$Z' = Z \frac{\sinh \gamma d}{\gamma d} = (23 + j75\ \Omega) \frac{\sinh(0.198 \angle 81.5°)}{0.198 \angle 81.5°} = 77.9 \angle 73°\ \Omega$$

$$Y' = Y \frac{\tanh(\gamma d / 2)}{\gamma d / 2} = 5.01 \times 10^{-4} \angle 89.9° \text{ S}$$

and the *ABCD* constants are

$$A = \frac{Z'Y'}{2} + 1 = 0.983\angle 0.33°$$

$$B = Z' = 77.9\angle 73°\ \Omega$$

$$C = Y'\left(\frac{Z'Y'}{4} + 1\right) = 4.97\angle 90.1°\ \text{S}$$

$$D = \frac{Z'Y'}{2} + 1 = 0.983\angle 0.33°$$

The receiving end line voltage is 220 kV, so the rated phase voltage is 220 kV / $\sqrt{3}$ = 127 kV, and the current is

$$I_L = \frac{S_{\text{out}}}{\sqrt{3}V_{LL}} = \frac{150{,}000{,}000\ \text{W}}{\sqrt{3}\left(220{,}000\ \text{V}\right)} = 394\ \text{A}$$

If the phase voltage at the receiving end is assumed to be at a phase angle of 0°, then the phase voltage at the receiving end will be $\mathbf{V}_R = 127\angle 0°\ \text{kV}$, and the phase current at the receiving end will be $\mathbf{I}_R = 394\angle -28.4°\ \text{A}$. The current and voltage at the sending end of the transmission line are given by the following equations:

$$\mathbf{V}_S = A\mathbf{V}_R + B\mathbf{I}_R$$

$$\mathbf{V}_S = 148.2\angle 8.7°\ \text{kV}$$

$$\mathbf{I}_S = C\mathbf{V}_R + D\mathbf{I}_R$$

$$\mathbf{I}_S = 361.2\angle -19.2°\ \text{A}$$

The short transmission line approximate was rather inaccurate, but the medium and long line models were both in good agreement with each other.

9-17. A 60 Hz, three phase, 110 kV transmission line has a length of 100 miles and a series impedance of $0.20 + j0.85\ \Omega$/mile and a shunt admittance of 6×10^{-6} S/mile. The transmission line is supplying 60 MW at a power factor of 0.85 lagging, and the receiving end voltage is 110 kV.

(a) What are the voltage, current, and power factor at the receiving end of this line?
(b) What are the voltage, current, and power factor at the sending end of this line?
(c) How much power is being lost in this transmission line?
(d) What is the current angle δ of this transmission line? How close is the transmission line to its steady-state stability limit?

SOLUTION This transmission line may be considered to be a medium-line line. The impedance *Z* and admittance *Y* of this line are:

$$Z = \left(0.20 + j0.85\ \Omega/\text{mile}\right)\left(100\ \text{miles}\right) = 20 + j85\ \Omega$$

$$Y = \left(j6\times 10^{-6}/\text{mile}\right)\left(100\ \text{miles}\right) = j0.0006\ \text{S}$$

The *ABCD* constants for this line are:

$$A = \frac{ZY}{2} + 1 = \frac{\left(20 + j85\ \Omega\right)\left(j0.0006\ \text{S}\right)}{2} + 1 = 0.9745\angle 0.35°$$

$$B = Z = 20 + j85\ \Omega = 87.3\angle 76.8°\ \Omega$$

$$C = Y\left(\frac{ZY}{4} + 1\right) = \left(j0.0006\ \text{S}\right)\left[\frac{\left(20 + j85\ \Omega\right)\left(j0.0006\ \text{S}\right)}{4} + 1\right] = 0.00059\angle 90.2°\ \text{S}$$

$$D = \frac{ZY}{2} + 1 = \frac{(20 + j85\ \Omega)(j0.0006\ \text{S})}{2} + 1 = 0.9745\angle 0.35°$$

(a) Assuming that the receiving end voltage is at 0°, the receiving end phase voltage and current are.

$$\mathbf{V}_R = 110\angle 0°\ \text{kV}/\sqrt{3} = 63.5\angle 0°\ \text{kV}$$

$$I_R = \frac{P}{\sqrt{3}V\cos\theta} = \frac{60\ \text{MW}}{\sqrt{3}(110\ \text{kV})(0.85)} = 370\ \text{A}$$

$$\mathbf{I}_R = 370\angle -31.7°\ \text{A}$$

The receiving end power factor is 0.85 lagging. The receiving end line voltage and current are 110 kV and 370 A, respectively.

(b) The sending end voltage and current are given by

$$\mathbf{V}_S = A\mathbf{V}_R + B\mathbf{I}_R = A(63.5\angle 0°\ \text{kV}) + B(370\angle -31.7°\ \text{A})$$

$$\mathbf{V}_S = 87.8\angle 15.35°\ \text{kV}$$

$$\mathbf{I}_S = C\mathbf{V}_R + D\mathbf{I}_R = C(63.5\angle 0°\ \text{kV}) + D(370\angle -31.7°\ \text{A})$$

$$\mathbf{I}_S = 342.4\angle -26°\ \text{A}$$

The sending end power factor is $\cos\left[15.35° - (-26°)\right] = \cos(41.4°) = 0.751\ \text{lagging}$. The sending end line voltage and current are $\sqrt{3}(87.8\ \text{kV}) = 152\ \text{kV}$ and 342 A, respectively.

(c) The power at the sending end of the transmission line is

$$P_S = 3V_{\phi,S}I_{\phi,S}\cos\theta = 3(87,800)(342)(0.751) = 67.7\ \text{MW}$$

The power at the receiving end of the transmission line is

$$P_R = 3V_{\phi,R}I_{\phi,R}\cos\theta = 3(63,500)(370)(0.85) = 59.9\ \text{MW}$$

Therefore the losses in the transmission line are approximately 7.8 MW.

(d) The angle δ is 15.35°. It is about 1/4 of the way to the line's static stability limit.