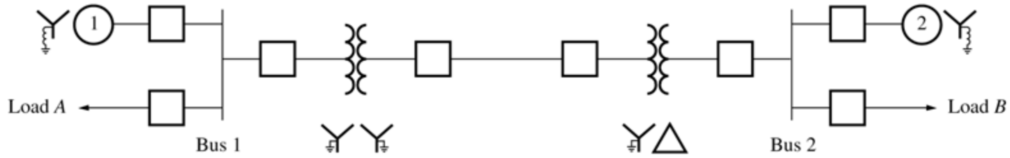
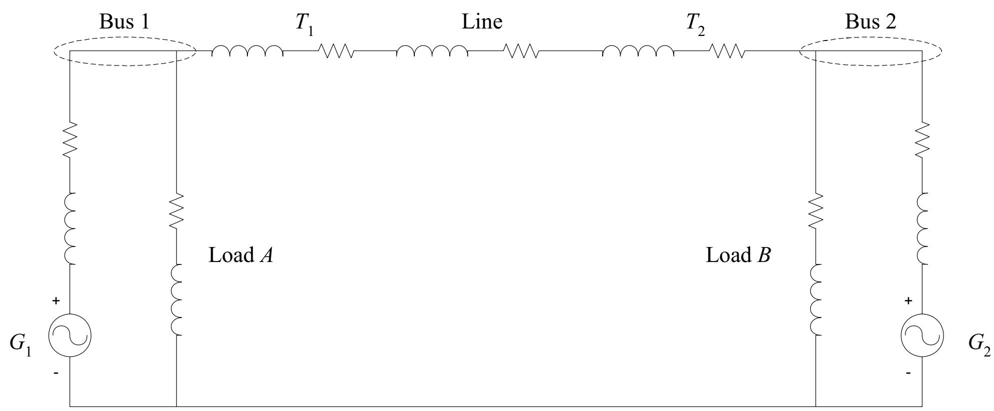
Chapter 10: *Power System Representation and Equations*

10-1. Sketch the per-phase, per-unit equivalent circuit of the power system in Figure 10-2. (Treat each load on the systems as a resistance in series with a reactance.) Note that you do not have enough information to actually calculate the values of components in the equivalent circuit.



SOLUTION The per-phase, per-unit equivalent circuit would be:



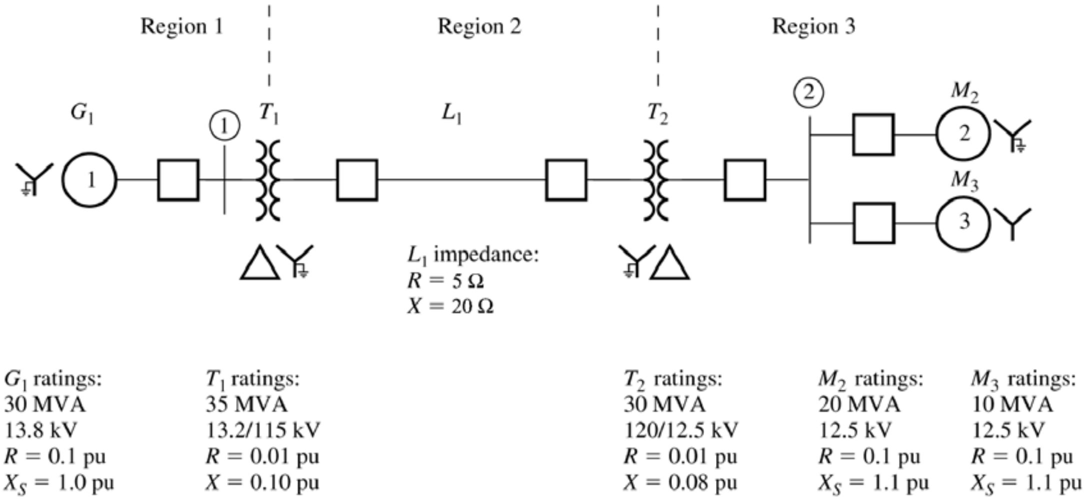
10-2. A 20,000 kVA, 110/13.8 kV, Y-Δ three phase transformer has a series impedance of 0.02 + j0.08 pu. Find the per-unit impedance of this transformer in a power system with a base apparent power of 500 MVA and a base voltage on the high side of 120 kV.

SOLUTION The per-unit impedance to the new base would be:

$$\text{per-unit } Z_{\text{new}} = \text{per-unit } Z_{\text{given}} \left(\frac{V_{\text{given}}}{V_{\text{new}}}\right)^2 \left(\frac{S_{\text{new}}}{S_{\text{given}}}\right) \tag{10-8}$$

$$\text{per-unit } Z_{\text{new}} = (0.02 + j0.08)\left(\frac{110 \text{ kV}}{120 \text{ kV}}\right)^2 \left(\frac{500{,}000 \text{ kVA}}{20{,}000 \text{ kVA}}\right) = 0.42 + j1.68 \text{ pu}$$

10-3. Find the per-phase equivalent circuit of the power system shown in Figure P10-1.



SOLUTION The per-phase equivalent circuit must be created on some system base voltage and apparent power. Since this problem has not specified the system base values, we will use the ratings of generator G_1 as the system base values at that point. Therefore, the system base apparent power is $S_{\text{base}} = 30\text{ MVA}$, and the system base voltages in each region are:

$$V_{\text{base},1} = 13.8\text{ kV}$$

$$V_{\text{base},2} = \left(\frac{115\text{ kV}}{13.2\text{ kV}}\right) V_{\text{base},1} = \left(\frac{115\text{ kV}}{13.2\text{ kV}}\right)(13.8\text{ kV}) = 120\text{ kV}$$

$$V_{\text{base},3} = \left(\frac{12.5\text{ kV}}{120\text{ kV}}\right) V_{\text{base},2} = \left(\frac{12.5\text{ kV}}{120\text{ kV}}\right)(120\text{ kV}) = 12.5\text{ kV}$$

The base impedance of Region 2 is:

$$Z_{\text{base},2} = \frac{\left(V_{LL,\text{ base},2}\right)^2}{S_{3\phi,\text{ base}}} = \frac{(120{,}000\text{ V})^2}{30{,}000{,}000\text{ VA}} = 480\ \Omega$$

The per unit resistance and reactance of G_1 are already on the proper base:

$$Z_{G1} = 0.1 + j1.0\text{ pu}$$

The per unit resistance and reactance of T_1 are:

$$\text{per-unit } Z_{\text{new}} = \text{per-unit } Z_{\text{given}} \left(\frac{V_{\text{given}}}{V_{\text{new}}}\right)^2 \left(\frac{S_{\text{new}}}{S_{\text{given}}}\right)$$

$$Z_{T1} = (0.01 + j0.10)\left(\frac{13.2\text{ kV}}{13.8\text{ kV}}\right)^2 \left(\frac{30{,}000\text{ kVA}}{35{,}000\text{ kVA}}\right) = 0.00784 + j0.0784\text{ pu}$$

The per unit resistance and reactance of the transmission line are:

$$Z_{\text{line}} = \frac{Z}{Z_{\text{base}}} = \frac{5 + j20\ \Omega}{480\ \Omega} = 0.0104\ + j0.0417\text{ pu}$$

The per unit resistance and reactance of T_2 are already on the right base:

$$Z_{T2} = 0.01 + j0.08\text{ pu}$$

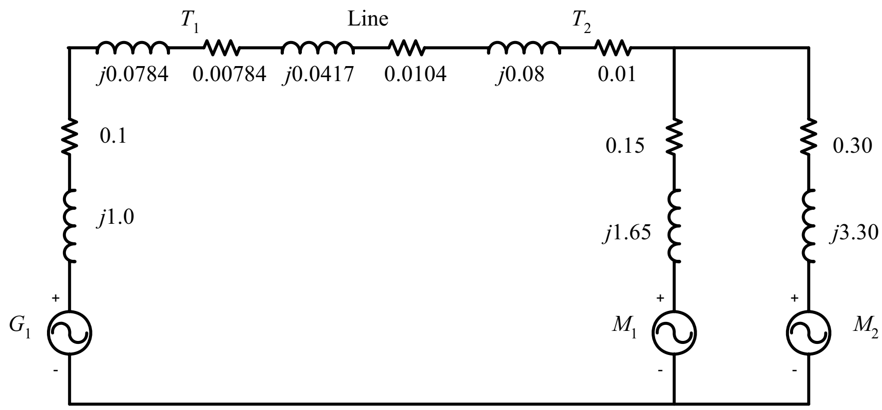
The per unit resistance and reactance of M_1 are:

$$Z_{M1} = (0.1 + j1.1)\left(\frac{12.5\text{ kV}}{12.5\text{ kV}}\right)^2 \left(\frac{30{,}000\text{ kVA}}{20{,}000\text{ kVA}}\right) = 0.15 + j1.65\text{ pu}$$

The per unit resistance and reactance of M_1 are:

$$Z_{M2} = (0.1 + j1.1)\left(\frac{12.5\text{ kV}}{12.5\text{ kV}}\right)^2 \left(\frac{30{,}000\text{ kVA}}{10{,}000\text{ kVA}}\right) = 0.30 + j3.30\text{ pu}$$

The resulting per-phase equivalent circuit is:



10-4. Two 4.16 kV three-phase synchronous motors are connected to the same bus. The motor ratings are:

Motor 1: 5,000 hp, 0.8 PF lagging, 95% efficiency, R = 3%, X_S = 90%

Motor 2: 3,000 hp, 1.0 PF, 95% efficiency, R = 3%, X_S = 90%

Calculate the per-unit impedances of these motors to a base of 20 MVA, 4.16 kV. (Note: To calculate these values, you will first have to determine the rated apparent power of each motor considering its rated output power, efficiency, and power factor.)

SOLUTION The rated input power of Motor 1 is

$$P_1 = \frac{P_{\text{out}}}{\eta} = \frac{(5{,}000\text{ hp})\left(\frac{746\text{ W}}{1\text{ hp}}\right)}{0.95} = 393\text{ kW}$$

The apparent power rating is

$$S_1 = \frac{P_1}{\text{PF}} = \frac{393\text{ kW}}{0.8} = 491\text{ kVA}$$

The per-unit impedances of Motor 1 are specified to the motor's own base. The impedances converted to the specified base are:

$$\text{per-unit } Z_{\text{new}} = \text{per-unit } Z_{\text{given}} \left(\frac{V_{\text{given}}}{V_{\text{new}}}\right)^2 \left(\frac{S_{\text{new}}}{S_{\text{given}}}\right) \tag{10-8}$$

$$Z_1 = (0.03 + j0.90)\left(\frac{4.16\text{ kV}}{4.16\text{ kV}}\right)^2 \left(\frac{20{,}000\text{ kVA}}{491\text{ kVA}}\right) = 1.22 + j36.7\text{ pu}$$

The rated input power of Motor 2 is

$$P_2 = \frac{P_{\text{out}}}{\eta} = \frac{(3{,}000 \text{ hp})\left(\dfrac{746 \text{ W}}{1 \text{ hp}}\right)}{0.95} = 236 \text{ kW}$$

The apparent power rating is

$$S_2 = \frac{P_2}{\text{PF}} = \frac{236 \text{ kW}}{1.0} = 236 \text{ kVA}$$

The per-unit impedances of Motor 2 are specified to the motor's own base. The impedances converted to the specified base are:

$$\text{per-unit } Z_{\text{new}} = \text{per-unit } Z_{\text{given}} \left(\frac{V_{\text{given}}}{V_{\text{new}}}\right)^2 \left(\frac{S_{\text{new}}}{S_{\text{given}}}\right) \qquad (10\text{-}8)$$

$$Z_2 = (0.03 + j0.90)\left(\frac{4.16 \text{ kV}}{4.16 \text{ kV}}\right)^2 \left(\frac{20{,}000 \text{ kVA}}{236 \text{ kVA}}\right) = 2.54 + j76.3 \text{ pu}$$

10-5. A Y-connected synchronous generator rated 100 MVA, 13.2 kV has a rated impedance of R = 5% and X_S = 80% per-unit. It is connected to a $j10\ \Omega$ transmission line through a 13.8/120 kV, 100 MVA, Δ-Y transformer with a rated impedance of R = 2% and X = 8% per unit. The base for the power system is 200 MVA, 120 kV at the transmission line.

(a) Sketch the one-line diagram of this power system, with symbols labeled appropriately.
(b) Find per-unit impedance of the generator on the system base.
(c) Find per-unit impedance of the transformer on the system base.
(d) Find per-unit impedance of the transmission line on the system base.
(e) Find the per-phase, per-unit equivalent circuit of this power system.

SOLUTION The base quantities for this power system in Region 2 are:

$$S_{\text{base}} = 200 \text{ MVA}$$

$$V_{\text{base},2} = 120 \text{ kV}$$

$$Z_{\text{base},2} = \frac{(V_{LL,\text{ base},2})^2}{S_{3\phi,\text{ base}}} = \frac{(120{,}000 \text{ V})^2}{200{,}000{,}000 \text{ VA}} = 72\ \Omega$$

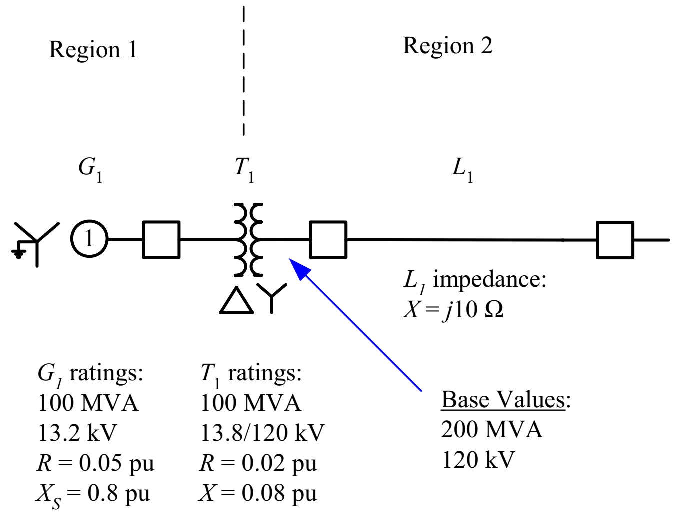
The base quantities for this power system in Region 1 are:

$$S_{\text{base}} = 200 \text{ MVA}$$

$$V_{\text{base},1} = \left(\frac{13.8 \text{ kV}}{120 \text{ kV}}\right) V_{\text{base},2} = \left(\frac{13.8 \text{ kV}}{120 \text{ kV}}\right)(120 \text{ kV}) = 13.8 \text{ kV}$$

$$Z_{\text{base},1} = \frac{(V_{LL,\text{ base},1})^2}{S_{3\phi,\text{ base}}} = \frac{(13{,}800 \text{ V})^2}{200{,}000{,}000 \text{ VA}} = 0.952\ \Omega$$

(a) The one-line diagram for this power system is shown below:



(b) The per-unit impedance of the generator on the system base is:

$$\text{per-unit } Z_{\text{new}} = \text{per-unit } Z_{\text{given}} \left(\frac{V_{\text{given}}}{V_{\text{new}}}\right)^2 \left(\frac{S_{\text{new}}}{S_{\text{given}}}\right) \tag{10-8}$$

$$Z_{G1} = (0.05 + j0.80)\left(\frac{13.2 \text{ kV}}{13.8 \text{ kV}}\right)^2 \left(\frac{200 \text{ MVA}}{100 \text{ MVA}}\right) = 0.0915 + j1.464 \text{ pu}$$

(c) The per-unit impedance of the transformer on the system base is:

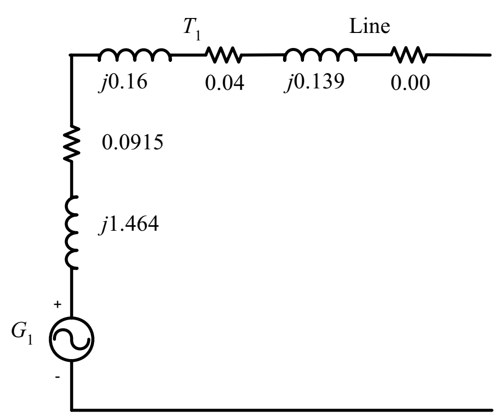
$$\text{per-unit } Z_{\text{new}} = \text{per-unit } Z_{\text{given}} \left(\frac{V_{\text{given}}}{V_{\text{new}}}\right)^2 \left(\frac{S_{\text{new}}}{S_{\text{given}}}\right) \tag{10-8}$$

$$Z_{T1} = (0.02 + j0.08)\left(\frac{13.8 \text{ kV}}{13.8 \text{ kV}}\right)^2 \left(\frac{200 \text{ MVA}}{100 \text{ MVA}}\right) = 0.04 + j0.16 \text{ pu}$$

(d) The per unit resistance and reactance of the transmission line are:

$$Z_{\text{line}} = \frac{Z}{Z_{\text{base}}} = \frac{j10\ \Omega}{72\ \Omega} = j0.139 \text{ pu}$$

(e) The resulting per-phase equivalent circuit is:



10-6. Assume that the power system of the previous problem is connected to a resistive Y-connected load of 200 Ω per phase. If the internal generated voltage of the generator is $\mathbf{E}_A = 1.10\angle 20°$ per unit, what is the terminal voltage of the generator? How much power is being supplied to the load?

SOLUTION The Y-connected load is connected to the end of the transmission line in Region 2, so $Z_{\text{base},2} = 72\ \Omega$, and the per-unit impedance of the load is

$$Z_{\text{load}} = \frac{Z}{Z_{\text{base}}} = \frac{200\ \Omega}{72\ \Omega} = 2.78 \text{ pu}$$

The resulting current flow is

$$\mathbf{I}_{\text{line}} = \frac{\mathbf{E}_A}{Z_{G1} + Z_{T1} + Z_{\text{line}} + Z_{\text{load}}}$$

$$\mathbf{I}_{\text{line}} = \frac{1.10\angle 20°}{(0.0915 + j1.464) + (0.04 + j0.16) + (j0.139) + (2.78)}$$

$$\mathbf{I}_{\text{line}} = 0.323\angle -11.2°$$

The per-phase terminal voltage of the generator will be

$$\mathbf{V}_\phi = \mathbf{E}_A - \mathbf{I}_A R_A - j\mathbf{I}_A X_S$$

$$\mathbf{V}_\phi = 1.10\angle 20° - (0.323\angle -11.2°)(0.0915) - j(0.323\angle -11.2°)(1.464)$$

$$\mathbf{V}_\phi = 0.917\angle -5.14°$$

Therefore, the terminal voltage will be $(0.917)(13.8 \text{ kV}) = 12.7 \text{ kV}$. The per-unit power supplied to the load is

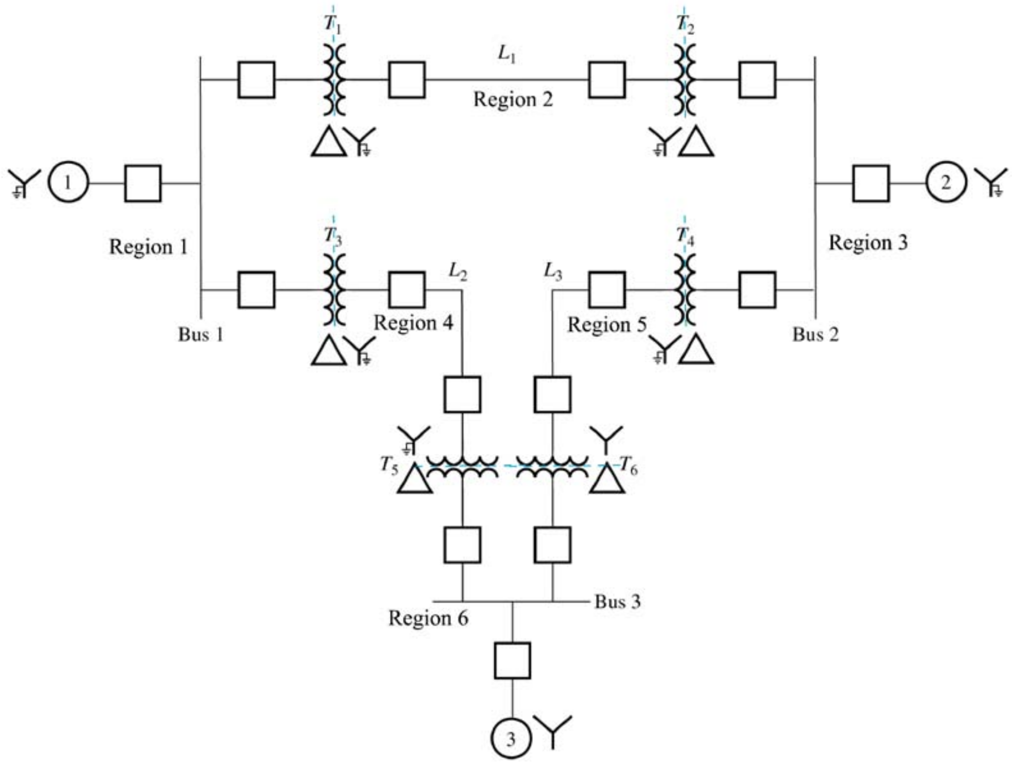
$$P_{\text{pu}} = I_{\text{pu}}^{\ 2} R = (0.323)^2 (2.78) = 0.290$$

Therefore, the total power supplied to the load is $(0.290)(200 \text{ MVA}) = 58 \text{ MW}$.

10-7. Figure P10-2 shows a one-line diagram of a three-phase power system. The ratings of the various components in the system are:

Synchronous Generator 1:	40 MVA, 13.8 kV, R = 3%, X_S = 80%
Synchronous Motor 2:	20 MVA, 13.8 kV, R = 3%, X_S = 80%
Synchronous Motor 3:	10 MVA, 13.2 kV, R = 3%, X_S = 100%
Y-Δ Transformers:	20 MVA, 13.8/138 kV, R = 2%, X = 10%
Line 1:	R = 10 Ω, X = 50 Ω
Line 2:	R = 5 Ω, X = 30 Ω
Line 3:	R = 5 Ω, X = 30 Ω

The per-unit system base for this power system is 40 MVA, 128 kV in transmission line 1. Create the per-phase, per-unit equivalent circuit for this power system.



SOLUTION This power system has been divided into regions at the transformers, with the base voltage and apparent power specified in Region 2 to be 128 kV and 40 MVA. The base quantities for this power system in all regions are:

$$S_{\text{base}} = 40 \text{ MVA}$$

$$V_{\text{base},1} = \left(\frac{13.8 \text{ kV}}{138 \text{ kV}}\right) V_{\text{base},2} = \left(\frac{13.8 \text{ kV}}{138 \text{ kV}}\right)(128 \text{ kV}) = 12.8 \text{ kV}$$

$$Z_{\text{base},1} = \frac{\left(V_{LL,\text{ base},1}\right)^2}{S_{3\phi,\text{ base}}} = \frac{(12{,}800 \text{ V})^2}{40{,}000{,}000 \text{ VA}} = 4.096\ \Omega$$

$$V_{\text{base},2} = 128 \text{ kV}$$

$$Z_{\text{base},2} = \frac{\left(V_{LL,\text{ base},2}\right)^2}{S_{3\phi,\text{ base}}} = \frac{(128{,}000 \text{ V})^2}{40{,}000{,}000 \text{ VA}} = 409.6\ \Omega$$

$$V_{\text{base},3} = \left(\frac{13.8 \text{ kV}}{138 \text{ kV}}\right) V_{\text{base},2} = \left(\frac{13.8 \text{ kV}}{138 \text{ kV}}\right)(128 \text{ kV}) = 12.8 \text{ kV}$$

$$Z_{\text{base},3} = \frac{\left(V_{LL,\text{ base},3}\right)^2}{S_{3\phi,\text{ base}}} = \frac{(12{,}800 \text{ V})^2}{40{,}000{,}000 \text{ VA}} = 4.096\ \Omega$$

$$V_{\text{base},4} = \left(\frac{138 \text{ kV}}{13.8 \text{ kV}}\right) V_{\text{base},1} = \left(\frac{138 \text{ kV}}{13.8 \text{ kV}}\right)(12.8 \text{ kV}) = 128 \text{ kV}$$

$$Z_{\text{base},4} = \frac{\left(V_{LL,\text{ base},3}\right)^2}{S_{3\phi,\text{ base}}} = \frac{(128{,}000 \text{ V})^2}{40{,}000{,}000 \text{ VA}} = 409.6\ \Omega$$

$$V_{\text{base},5} = \left(\frac{138 \text{ kV}}{13.8 \text{ kV}}\right) V_{\text{base},3} = \left(\frac{138 \text{ kV}}{13.8 \text{ kV}}\right)(12.8 \text{ kV}) = 128 \text{ kV}$$

$$Z_{\text{base},5} = \frac{\left(V_{LL,\text{ base},3}\right)^2}{S_{3\phi,\text{ base}}} = \frac{(128{,}000 \text{ V})^2}{40{,}000{,}000 \text{ VA}} = 409.6\ \Omega$$

$$V_{\text{base},6} = \left(\frac{13.8 \text{ kV}}{138 \text{ kV}}\right) V_{\text{base},4} = \left(\frac{13.8 \text{ kV}}{138 \text{ kV}}\right)(128 \text{ kV}) = 12.8 \text{ kV}$$

$$Z_{\text{base},6} = \frac{\left(V_{LL,\text{ base},6}\right)^2}{S_{3\phi,\text{ base}}} = \frac{(12{,}800 \text{ V})^2}{40{,}000{,}000 \text{ VA}} = 4.096\ \Omega$$

The base quantities for this power system in Region 1 are:

$$S_{\text{base}} = 200 \text{ MVA}$$

$$V_{\text{base},1} = \left(\frac{13.8 \text{ kV}}{138 \text{ kV}}\right) V_{\text{base},2} = \left(\frac{13.8 \text{ kV}}{138 \text{ kV}}\right)(128 \text{ kV}) = 12.8 \text{ kV}$$

$$Z_{\text{base},1} = \frac{\left(V_{LL,\text{ base},1}\right)^2}{S_{3\phi,\text{ base}}} = \frac{(128{,}000 \text{ V})^2}{40{,}000{,}000 \text{ VA}} = 409.6\ \Omega$$

The per-unit impedances of the various components to the system base are:

$$\text{per-unit } Z_{\text{new}} = \text{per-unit } Z_{\text{given}} \left(\frac{V_{\text{given}}}{V_{\text{new}}}\right)^2 \left(\frac{S_{\text{new}}}{S_{\text{given}}}\right) \tag{10-8}$$

$$Z_{G1} = (0.03 + j0.80)\left(\frac{13.8 \text{ kV}}{12.8 \text{ kV}}\right)^2 \left(\frac{40 \text{ MVA}}{40 \text{ MVA}}\right) = 0.0349 + j0.930 \text{ pu}$$

$$Z_{M2} = (0.03 + j0.80)\left(\frac{13.8 \text{ kV}}{12.8 \text{ kV}}\right)^2 \left(\frac{40 \text{ MVA}}{20 \text{ MVA}}\right) = 0.0697 + j1.860 \text{ pu}$$

$$Z_{M3} = (0.03 + j1.00)\left(\frac{13.8 \text{ kV}}{12.8 \text{ kV}}\right)^2 \left(\frac{40 \text{ MVA}}{10 \text{ MVA}}\right) = 0.140 + j4.65 \text{ pu}$$

$$Z_{T1} = (0.02 + j0.10)\left(\frac{13.8 \text{ kV}}{12.8 \text{ kV}}\right)^2 \left(\frac{40 \text{ MVA}}{20 \text{ MVA}}\right) = 0.0465 + j0.233 \text{ pu}$$

$$Z_{L1} = \frac{Z}{Z_{\text{base}}} = \frac{10 + j50\ \Omega}{409.6\ \Omega} = 0.0244 + j0.1221 \text{ pu}$$

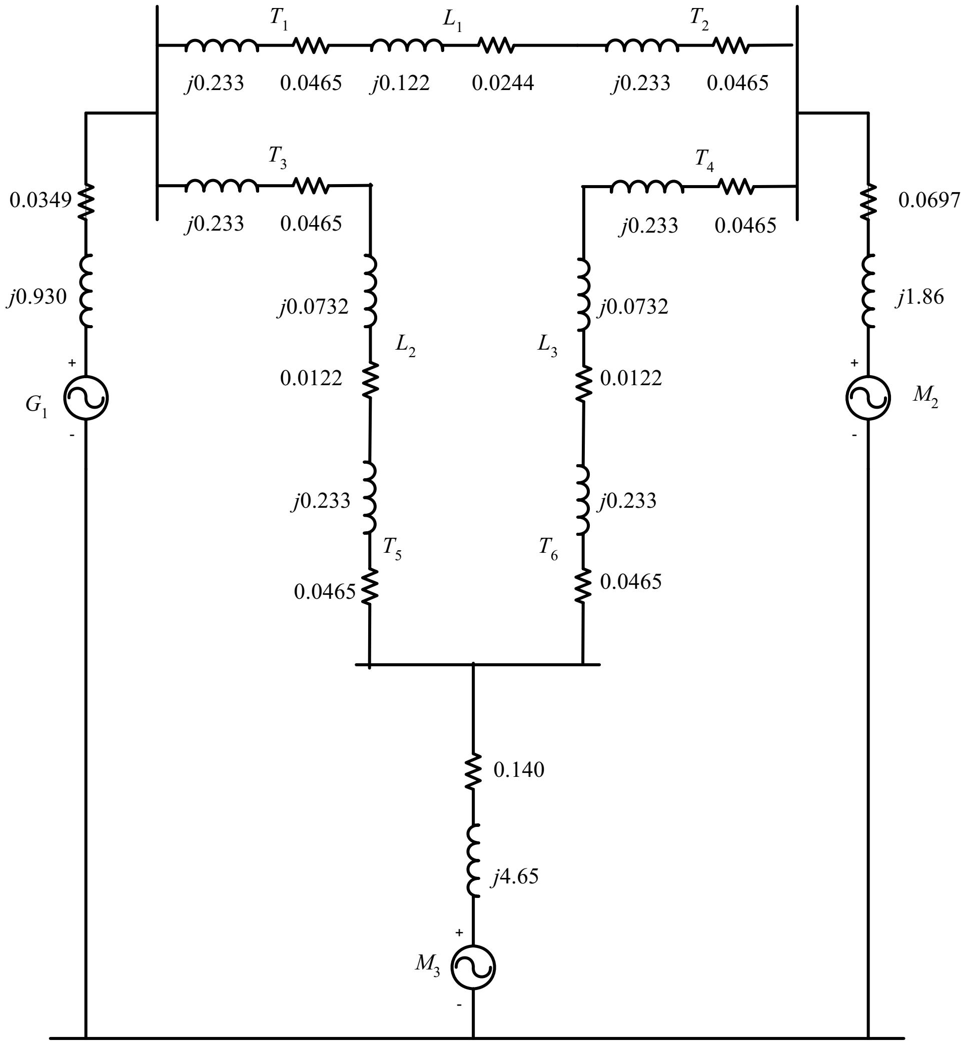
$$Z_{L2} = \frac{Z}{Z_{\text{base}}} = \frac{5 + j30\ \Omega}{409.6\ \Omega} = 0.0122 + j0.0732 \text{ pu}$$

$$Z_{L3} = \frac{Z}{Z_{\text{base}}} = \frac{5 + j30\ \Omega}{409.6\ \Omega} = 0.0122 + j0.0732 \text{ pu}$$

All transformers in this figure are Y-Δ, and the base quantities are the same for all of them, so the per-unit transformer impedances are:

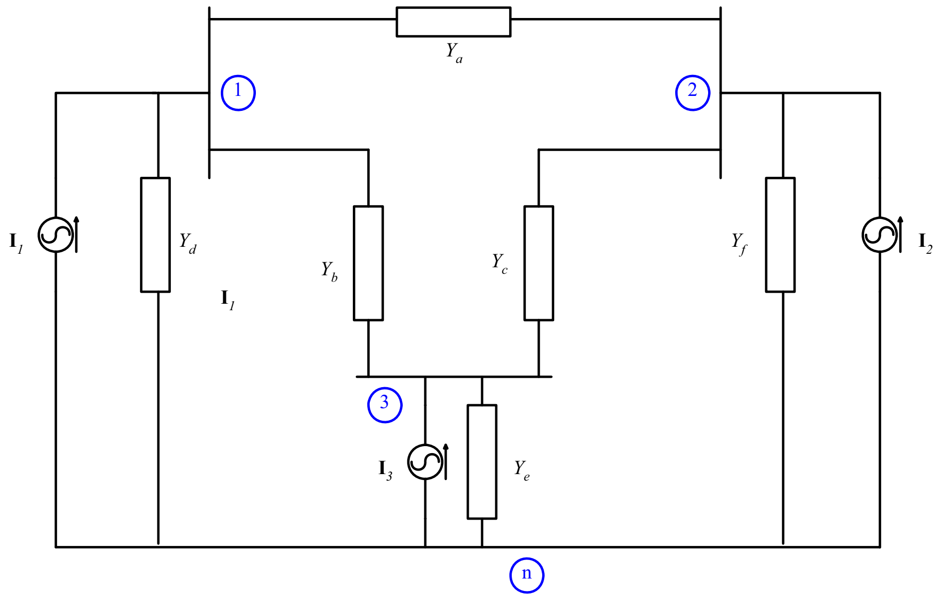
$$Z_{T1} = Z_{T2} = Z_{T3} = Z_{T4} = Z_{T5} = Z_{T6} = 0.0465 + j0.233 \text{ pu}$$

The resulting per-phase, per-unit equivalent circuit is shown below:



10-8. Calculate the bus admittance matrix $\mathbf{Y}_{\text{bus}}$ and the bus impedance matrix $\mathbf{Z}_{\text{bus}}$ for the power system shown in Figure P10-2.

SOLUTION The voltage sources can be converted to current sources, and the series impedances between each bus can be replaced by a single admittance, resulting in the system shown below. Note that we have labeled each bus with a number.



The admittances in this circuit are:

$$Y_a = \frac{1}{Z_{T1} + Z_{L1} + Z_{T2}} = \frac{1}{(0.0465 + j0.233 \text{ pu}) + (0.0244 + j0.122) + (0.0465 + j0.233 \text{ pu})}$$

$$Y_a = 0.3265 - j1.6355 \text{ pu}$$

$$Y_b = \frac{1}{Z_{T3} + Z_{L2} + Z_{T5}} = \frac{1}{(0.0465 + j0.233 \text{ pu}) + (0.0122 + j0.0732) + (0.0465 + j0.233 \text{ pu})}$$

$$Y_b = 0.3486 - j1.7866 \text{ pu}$$

$$Y_c = \frac{1}{Z_{T4} + Z_{L3} + Z_{T6}} = \frac{1}{(0.0465 + j0.233 \text{ pu}) + (0.0122 + j0.0732) + (0.0465 + j0.233 \text{ pu})}$$

$$Y_c = 0.3486 - j1.7866 \text{ pu}$$

$$Y_d = \frac{1}{Z_{G1}} = \frac{1}{0.0349 + j0.930} = 0.3537 \; - \; j0.9425 \text{ pu}$$

$$Y_e = \frac{1}{Z_{M3}} = \frac{1}{0.140 + j4.65} = \; 0.0064 \; - \; j0.2149 \text{ pu}$$

$$Y_f = \frac{1}{Z_{M2}} = \frac{1}{0.0697 + j1.860} = 0.0201 \; - \; j0.5369 \text{ pu}$$

The bus admittance matrix $\mathbf{Y}_{\text{bus}}$ is:

$$\mathbf{Y}_{\text{bus}} = \begin{bmatrix} Y_a + Y_b + Y_d & -Y_a & -Y_b \\ -Y_a & Y_a + Y_c + Y_f & -Y_c \\ -Y_b & -Y_c & Y_b + Y_c + Y_e \end{bmatrix}$$

$$\mathbf{Y}_{\text{bus}} = \begin{bmatrix} 1.0288 - j4.3646 & -0.3265 + j1.6355 & -0.3486 + j1.7866 \\ -0.3265 + j1.6355 & 0.6952 - j3.9590 & -0.3486 + j1.7866 \\ -0.3486 + j1.7866 & -0.3486 + j1.7866 & 0.7036 - j3.7881 \end{bmatrix}$$

The bus impedance matrix $\mathbf{Z}_{\text{bus}}$ is:

$$\mathbf{Z}_{\text{bus}} = \mathbf{Y}_{\text{bus}}^{-1}$$

$$\mathbf{Z}_{\text{bus}} = \begin{bmatrix} 0.1532 + j0.6048 & 0.1068 + j0.4875 & 0.1181 + j0.5172 \\ 0.1068 + j0.4875 & 0.1251 + j0.7046 & 0.1045 + j0.5642 \\ 0.1181 + j0.5172 & 0.1045 + j0.5642 & 0.1479 + j0.7670 \end{bmatrix}$$

10-9. Assume that internal generated voltages of the generators and motors in the per-unit equivalent circuit of the previous problem have the following values:

$$\mathbf{E}_{A1} = 1.15\angle 22°$$

$$\mathbf{E}_{A2} = 1.00\angle -20°$$

$$\mathbf{E}_{A3} = 0.95\angle -15°$$

(a) Find the per-unit voltages on each bus in the power system.
(b) Find the actual voltages on each bus in the power system.
(c) Find the current flowing in each transmission line in the power system.
(d) Determine the magnitude and direction of the real and reactive power flowing in each transmission line.
(e) Are any of the components in the power system overloaded?

SOLUTION The Norton equivalent currents associated with each voltage source are:

$$\mathbf{I}_1 = \frac{\mathbf{E}_{A1}}{Z_{G1}} = \frac{1.15\angle 22°}{0.0349 + j0.930} = 0.7832 - j0.8526 = 1.158\angle -47.4°$$

$$\mathbf{I}_2 = \frac{\mathbf{E}_{A2}}{Z_{M2}} = \frac{1.00\angle -20°}{0.0697 + j1.86} = -0.1647 - j0.5114 = 0.5373\angle -107.9°$$

$$\mathbf{I}_3 = \frac{\mathbf{E}_{A3}}{Z_{M3}} = \frac{0.95\angle -15°}{0.14 + j4.65} = -0.0469 - j0.1988 = 0.204\angle -103.2°$$

The nodal equations for this power system are:

$$\mathbf{Y}_{\text{bus}}\mathbf{V} = \mathbf{I}$$

$$\mathbf{V} = \begin{bmatrix} 1.0288 - j4.3646 & -0.3265 + j1.6355 & -0.3486 + j1.7866 \\ -0.3265 + j1.6355 & 0.6952 - j3.9590 & -0.3486 + j1.7866 \\ -0.3486 + j1.7866 & -0.3486 + j1.7866 & 0.7036 - j3.7881 \end{bmatrix}^{-1} \begin{bmatrix} 0.7832 - j0.8526 \\ -0.1647 - j0.5114 \\ -0.0469 - j0.1988 \end{bmatrix}$$

$$\mathbf{V} = \begin{bmatrix} 0.9646 + j0.1604 \\ 0.9463 + j0.0635 \\ 0.9503 + j0.0926 \end{bmatrix} = \begin{bmatrix} 0.978\angle 9.4° \\ 0.948\angle 3.8° \\ 0.955\angle 5.6° \end{bmatrix}$$

(a) The per-unit voltages at each bus in the power system are:

$$\mathbf{V}_1 = 0.978\angle 9.4°$$

$$\mathbf{V}_2 = 0.948\angle 3.8°$$

$$\mathbf{V}_3 = 0.955\angle 5.6°$$

(b) The actual voltages at each bus in the power system are:

$$V_1 = V_{1,\text{pu}}V_{\text{base}} = (0.978)(12.8\text{ kV}) = 12.5\text{ kV}$$

$$V_2 = V_{2,\text{pu}}V_{\text{base}} = (0.948)(12.8\text{ kV}) = 12.1\text{ kV}$$

$$V_3 = V_{3,\text{pu}}V_{\text{base}} = (0.955)(12.8\text{ kV}) = 12.2\text{ kV}$$

(c) The current flowing in Line 1 is: (start here)

$$\mathbf{I}_{L1} = Y_a\left(\mathbf{V}_1 - \mathbf{V}_2\right) = \left(0.3265 - j1.6355\right)\left(0.978\angle 9.4° - 0.948\angle 3.8°\right)$$

$$\mathbf{I}_{L1} = 0.1645 + j0.0017 = 0.165\angle 0.6°$$

The current flowing in Line 2 is:

$$\mathbf{I}_{L2} = Y_b\left(\mathbf{V}_1 - \mathbf{V}_3\right) = \left(0.3486 - j1.7866\right)\left(0.978\angle 9.4° - 0.955\angle 5.6°\right)$$

$$\mathbf{I}_{L2} = 0.1263 - j0.0029 = 0.1269\angle -0.9°$$

The current flowing in Line 3 is:

$$\mathbf{I}_{L3} = Y_c\left(\mathbf{V}_2 - \mathbf{V}_3\right) = \left(0.3486 - j1.7866\right)\left(0.948\angle 3.8° - 0.955\angle 5.6°\right)$$

$$\mathbf{I}_{L3} = -0.0533 + j0.0029 = 0.0534\angle -177°$$

(d) The real and reactive power flowing from Bus 1 to Bus 2 in Line 1 is:

$$\mathbf{S}_{L1} = \mathbf{V}_1\mathbf{I}_{L1}^{*} = \left(0.978\angle 9.4°\right)\left(0.165\angle 0.6°\right)^{*}$$

$$\mathbf{S}_{L1} = 0.159 + j0.0248 = 0.161\angle 8.9°$$

$$P_{L1} = S_{\text{base}}P_{L1,\text{pu}} = \left(40\text{ MVA}\right)\left(0.159\right) = 6.4\text{ MW}$$

$$Q_{L1} = S_{\text{base}}Q_{L1,\text{pu}} = \left(40\text{ MVA}\right)\left(0.0248\right) = 1.0\text{ MVAR}$$

The real and reactive power flowing from Bus 1 to Bus 3 in Line 2 is:

$$\mathbf{S}_{L2} = \mathbf{V}_1\mathbf{I}_{L2}^{*} = \left(0.978\angle 9.4°\right)\left(0.1269\angle -0.9°\right)^{*}$$

$$\mathbf{S}_{L2} = 0.1215 + j0.0221 = 0.124\angle 10.3°$$

$$P_{L2} = S_{\text{base}}P_{L2,\text{pu}} = \left(40\text{ MVA}\right)\left(0.1215\right) = 4.86\text{ MW}$$

$$Q_{L2} = S_{\text{base}}Q_{L2,\text{pu}} = \left(40\text{ MVA}\right)\left(0.0221\right) = 0.88\text{ MVAR}$$

The real and reactive power flowing from Bus 2 to Bus 3 in Line 3 is:

$$\mathbf{S}_{L3} = \mathbf{V}_3\mathbf{I}_{L3}^{*} = \left(0.955\angle 5.6°\right)\left(0.0534\angle -177°\right)^{*}$$

$$\mathbf{S}_{L3} = -0.0509 - j0.0021 = 0.051\angle -178°$$

$$P_{L3} = S_{\text{base}}P_{L3,\text{pu}} = \left(40\text{ MVA}\right)\left(-0.0509\right) = -2.04\text{ MW}$$

$$Q_{L3} = S_{\text{base}}Q_{L3,\text{pu}} = \left(40\text{ MVA}\right)\left(-0.0021\right) = -0.09\text{ MVAR}$$

The negative power here means that the power is really flowing from Bus 3 to Bus 2.

(e) None of the components in the power system are even close to being overloaded.