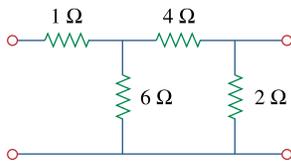


**Chapter 19, Problem 1.**

Obtain the  $z$  parameters for the network in Fig. 19.65.

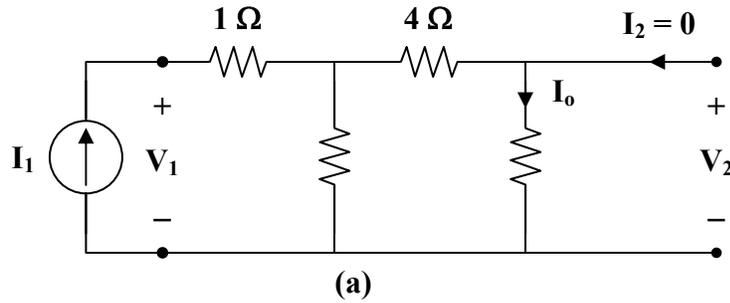


**Figure 19.65**

For Prob. 19.1 and 19.28.

**Chapter 19, Solution 1.**

To get  $z_{11}$  and  $z_{21}$ , consider the circuit in Fig. (a).

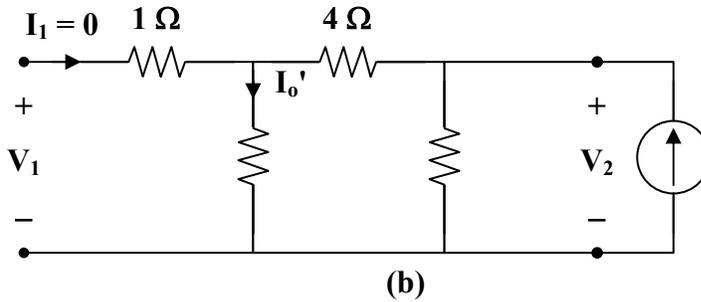


$$z_{11} = \frac{V_1}{I_1} = 1 + 6 \parallel (4 + 2) = 4 \Omega$$

$$I_o = \frac{1}{2} I_1, \quad V_2 = 2 I_o = I_1$$

$$z_{21} = \frac{V_2}{I_1} = 1 \Omega$$

To get  $z_{22}$  and  $z_{12}$ , consider the circuit in Fig. (b).



$$z_{22} = \frac{V_2}{I_2} = 2 \parallel (4 + 6) = 1.667 \Omega$$

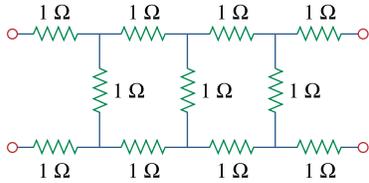
$$I_o' = \frac{2}{2 + 10} I_2 = \frac{1}{6} I_2, \quad V_1 = 6 I_o' = I_2$$

$$z_{12} = \frac{V_1}{I_2} = 1 \Omega$$

Hence, 
$$[z] = \begin{bmatrix} 4 & 1 \\ 1 & 1.667 \end{bmatrix} \Omega$$

**Chapter 19, Problem 2.**

\* Find the impedance parameter equivalent of the network in Fig. 19.66.



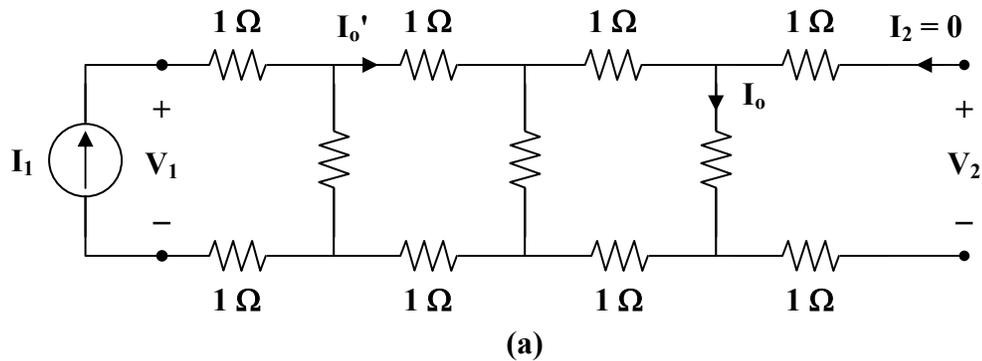
**Figure 19.66**

For Prob. 19.2.

\* An asterisk indicates a challenging problem.

**Chapter 19, Solution 2.**

Consider the circuit in Fig. (a) to get  $z_{11}$  and  $z_{21}$ .



$$z_{11} = \frac{V_1}{I_1} = 2 + 1 \parallel [2 + 1 \parallel (2 + 1)]$$

$$z_{11} = 2 + 1 \parallel \left(2 + \frac{3}{4}\right) = 2 + \frac{(1)(11/4)}{1 + 11/4} = 2 + \frac{11}{15} = 2.733$$

$$I_o = \frac{1}{1+3} I_o' = \frac{1}{4} I_o'$$

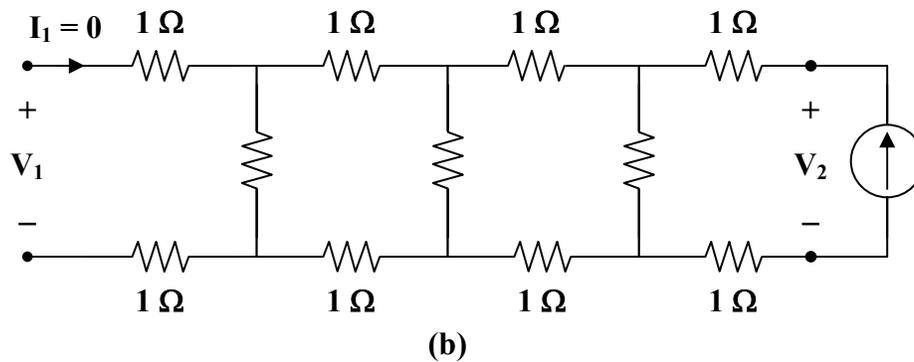
$$I_o' = \frac{1}{1+11/4} I_1 = \frac{4}{15} I_1$$

$$I_o = \frac{1}{4} \cdot \frac{4}{15} I_1 = \frac{1}{15} I_1$$

$$V_2 = I_o = \frac{1}{15} I_1$$

$$z_{21} = \frac{V_2}{I_1} = \frac{1}{15} = z_{12} = 0.06667$$

To get  $z_{22}$ , consider the circuit in Fig. (b).



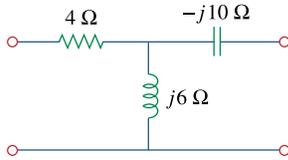
$$z_{22} = \frac{V_2}{I_2} = 2 + 1 \parallel (2 + 1 \parallel 3) = z_{11} = 2.733$$

Thus,

$$[z] = \begin{bmatrix} 2.733 & 0.06667 \\ 0.06667 & 2.733 \end{bmatrix} \Omega$$

### Chapter 19, Problem 3.

Find the  $z$  parameters of the circuit in Fig. 19.67.



**Figure 19.67**

For Prob. 19.3.

### Chapter 19, Solution 3.

$$z_{12} = j6 = z_{21}$$

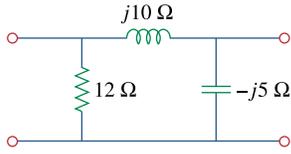
$$z_{11} - z_{12} = 4 \quad \longrightarrow \quad z_{11} = z_{12} + 4 = 4 + j6 \Omega$$

$$z_{22} - z_{12} = -j10 \quad \longrightarrow \quad z_{22} = z_{12} - j10 = -j4 \Omega$$

$$[z] = \begin{bmatrix} 4 + j6 & j6 \\ j6 & -j4 \end{bmatrix} \Omega = \begin{bmatrix} 4 + j6 & j6 \\ j6 & -j4 \end{bmatrix} \Omega$$

### Chapter 19, Problem 4.

Calculate the  $z$  parameters for the circuit in Fig. 19.68.

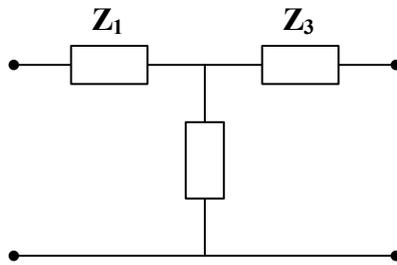


**Figure 19.68**

For Prob. 19.4.

### Chapter 19, Solution 4.

Transform the  $\Pi$  network to a T network.



$$\mathbf{Z}_1 = \frac{(12)(j10)}{12 + j10 - j5} = \frac{j120}{12 + j5}$$
$$\mathbf{Z}_2 = \frac{-j60}{12 + j5}$$
$$\mathbf{Z}_3 = \frac{50}{12 + j5}$$

The  $z$  parameters are

$$\mathbf{z}_{12} = \mathbf{z}_{21} = \mathbf{Z}_2 = \frac{(-j60)(12 - j5)}{144 + 25} = -1.775 - j4.26$$

$$\mathbf{z}_{11} = \mathbf{Z}_1 + \mathbf{z}_{12} = \frac{(j120)(12 - j5)}{169} + \mathbf{z}_{12} = 1.775 + j4.26$$

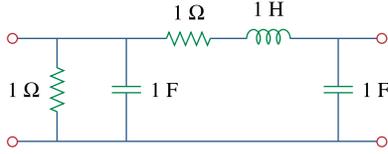
$$\mathbf{z}_{22} = \mathbf{Z}_3 + \mathbf{z}_{21} = \frac{(50)(12 - j5)}{169} + \mathbf{z}_{21} = 1.7758 - j5.739$$

Thus,

$$[\mathbf{z}] = \begin{bmatrix} 1.775 + j4.26 & -1.775 - j4.26 \\ -1.775 - j4.26 & 1.7758 - j5.739 \end{bmatrix} \Omega$$

**Chapter 19, Problem 5.**

Obtain the  $z$  parameters for the network in Fig. 19.69 as functions of  $s$ .

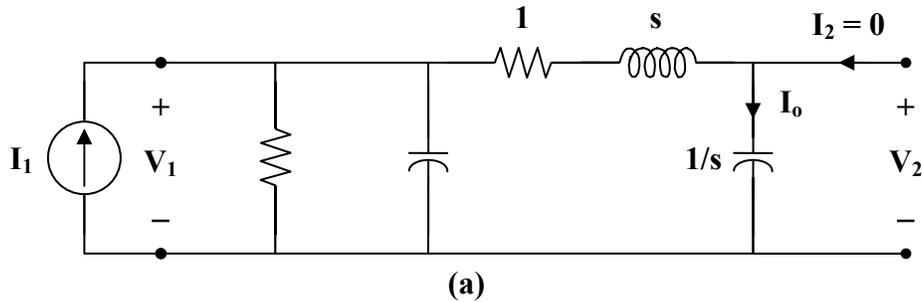


**Figure 19.69**

For Prob. 19.5.

**Chapter 19, Solution 5.**

Consider the circuit in Fig. (a).



$$z_{11} = 1 \parallel \frac{1}{s} \parallel \left(1 + s + \frac{1}{s}\right) = \frac{\frac{1}{s}}{1 + \frac{1}{s}} \parallel \left(1 + s + \frac{1}{s}\right) = \frac{\left(\frac{1}{s+1}\right)\left(1 + s + \frac{1}{s}\right)}{\left(\frac{1}{s+1}\right) + 1 + s + \frac{1}{s}}$$

$$z_{11} = \frac{s^2 + s + 1}{s^3 + 2s^2 + 3s + 1}$$

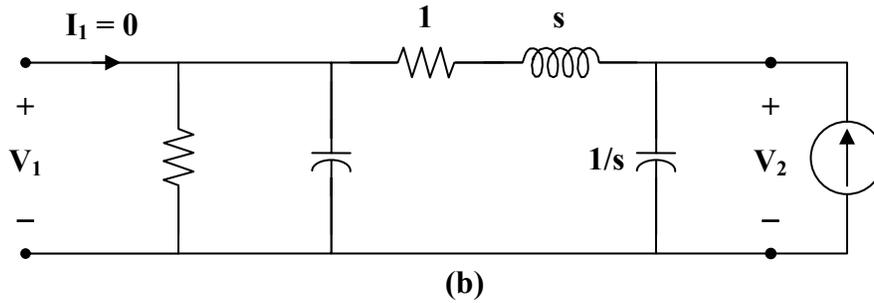
$$I_o = \frac{1 \parallel \frac{1}{s}}{1 \parallel \frac{1}{s} + 1 + s + \frac{1}{s}} I_1 = \frac{\frac{1}{s+1}}{\frac{1}{s+1} + 1 + s + \frac{1}{s}} I_1 = \frac{\frac{s}{s+1}}{\frac{s}{s+1} + s^2 + s + 1} I_1$$

$$I_o = \frac{s}{s^3 + 2s^2 + 3s + 1} I_1$$

$$V_2 = \frac{1}{s} I_o = \frac{I_1}{s^3 + 2s^2 + 3s + 1}$$

$$z_{21} = \frac{V_2}{I_1} = \frac{1}{s^3 + 2s^2 + 3s + 1}$$

Consider the circuit in Fig. (b).



$$\begin{aligned} z_{22} &= \frac{V_2}{I_2} = \frac{1}{s} \parallel \left( 1 + s + 1 \parallel \frac{1}{s} \right) = \frac{1}{s} \parallel \left( 1 + s + \frac{1}{s+1} \right) \\ z_{22} &= \frac{\left( \frac{1}{s} \right) \left( 1 + s + \frac{1}{s+1} \right)}{\frac{1}{s} + 1 + s + \frac{1}{s+1}} = \frac{1 + s + \frac{1}{s+1}}{1 + s + s^2 + \frac{s}{s+1}} \\ z_{22} &= \frac{s^2 + 2s + 2}{s^3 + 2s^2 + 3s + 1} \end{aligned}$$

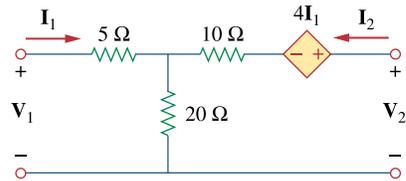
$$z_{12} = z_{21}$$

Hence,

$$[z] = \begin{bmatrix} \frac{s^2 + s + 1}{s^3 + 2s^2 + 3s + 1} & \frac{1}{s^3 + 2s^2 + 3s + 1} \\ \frac{1}{s^3 + 2s^2 + 3s + 1} & \frac{s^2 + 2s + 2}{s^3 + 2s^2 + 3s + 1} \end{bmatrix}$$

**Chapter 19, Problem 6.**

Compute the  $z$  parameters of the circuit in Fig. 19.70.

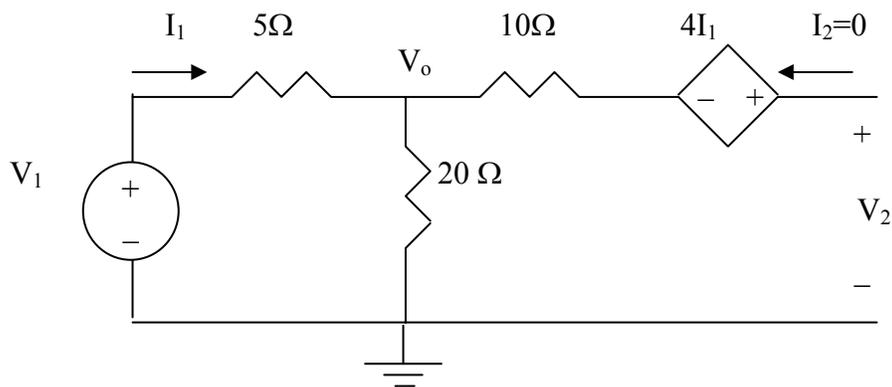


**Figure 19.70**

For Prob. 19.6 and 19.73.

**Chapter 19, Solution 6.**

To find  $z_{11}$  and  $z_{21}$ , consider the circuit below.



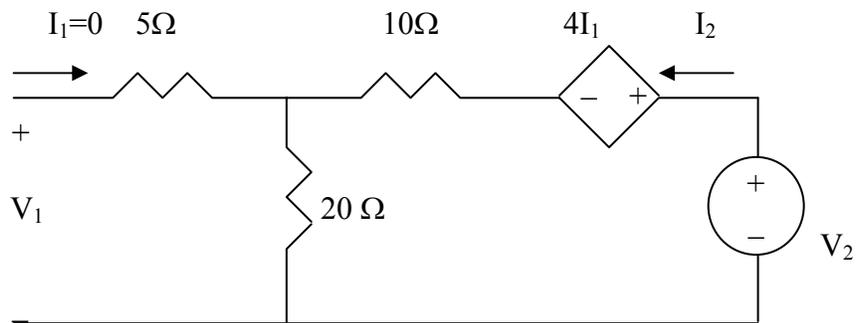
$$z_{11} = \frac{V_1}{I_1} = \frac{(20+5)I_1}{I_1} = 25 \Omega$$

$$V_o = \frac{20}{25}V_1 = 20I_1$$

$$-V_o - 4I_2 + V_2 = 0 \quad \longrightarrow \quad V_2 = V_o + 4I_2 = 20I_1 + 4I_2 = 24I_1$$

$$z_{21} = \frac{V_2}{I_1} = 24 \Omega$$

To find  $z_{12}$  and  $z_{22}$ , consider the circuit below.



$$V_2 = (10 + 20)I_2 = 30I_2$$

$$z_{22} = \frac{V_2}{I_2} = 30 \Omega$$

$$V_1 = 20I_2$$

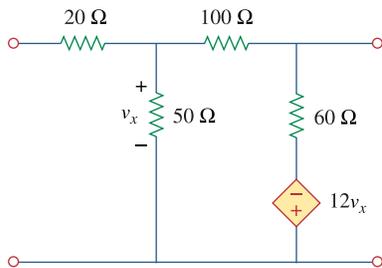
$$z_{12} = \frac{V_1}{I_2} = 20 \Omega$$

Thus,

$$[z] = \begin{bmatrix} 25 & 20 \\ 24 & 30 \end{bmatrix} \Omega$$

### Chapter 19, Problem 7.

Calculate the impedance-parameter equivalent of the circuit in Fig. 19.71.

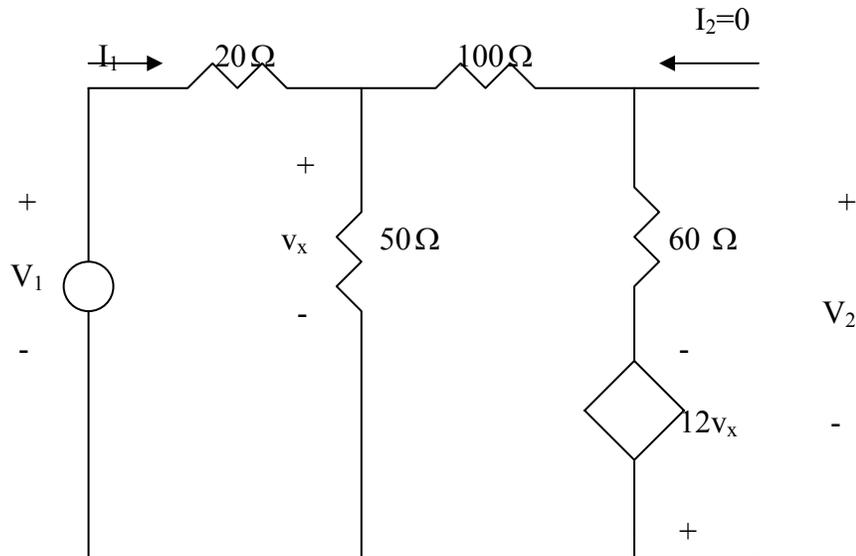


**Figure 19.71**

For Prob. 19.7 and 19.80.

### Chapter 19, Solution 7.

To get  $z_{11}$  and  $z_{21}$ , we consider the circuit below.

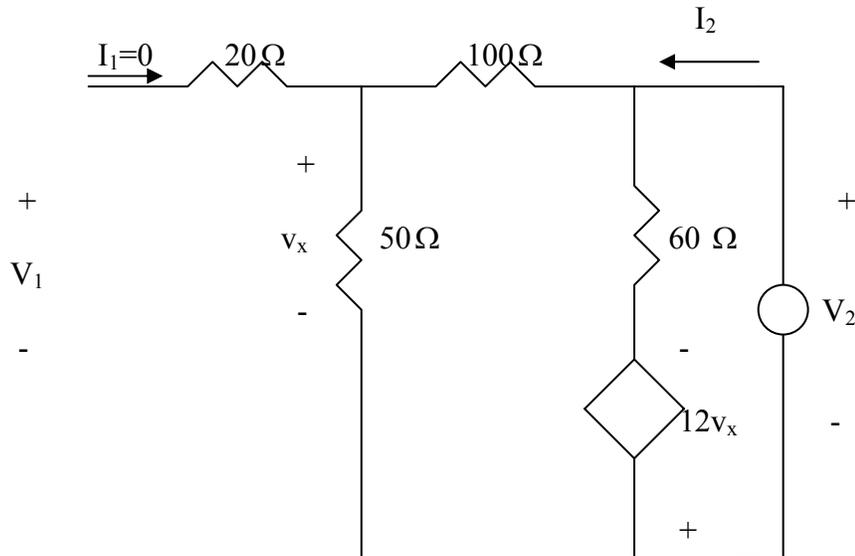


$$\frac{V_1 - V_x}{20} = \frac{V_x}{50} + \frac{V_x + 12V_x}{160} \longrightarrow V_x = \frac{40}{121} V_1$$

$$I_1 = \frac{V_1 - V_x}{20} = \frac{81}{121} \left( \frac{V_1}{20} \right) \longrightarrow z_{11} = \frac{V_1}{I_1} = 29.88$$

$$\begin{aligned} V_2 &= 60 \left( \frac{13V_x}{160} \right) - 12V_x = -\frac{57}{8} V_x = -\frac{57}{8} \left( \frac{40}{121} \right) V_1 = -\frac{57}{8} \left( \frac{40}{121} \right) \frac{20 \times 121}{81} I_1 \\ &= -70.37 I_1 \longrightarrow z_{21} = \frac{V_2}{I_1} = -70.37 \end{aligned}$$

To get  $z_{12}$  and  $z_{22}$ , we consider the circuit below.



$$V_x = \frac{50}{100 + 50} V_2 = \frac{1}{3} V_2, \quad I_2 = \frac{V_2}{150} + \frac{V_2 + 12V_x}{60} = 0.09 V_2$$

$$z_{22} = \frac{V_2}{I_2} = 1/0.09 = 11.11$$

$$V_1 = V_x = \frac{1}{3} V_2 = \frac{11.11}{3} I_2 = 3.704 I_2 \longrightarrow z_{12} = \frac{V_1}{I_2} = 3.704$$

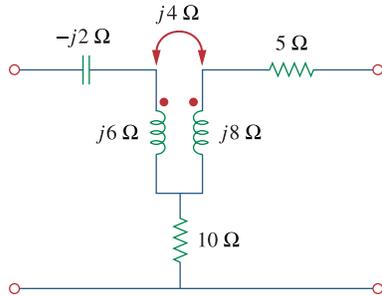
Thus,

$$[z] = \begin{bmatrix} 29.88 & 3.704 \\ -70.37 & 11.11 \end{bmatrix} \Omega$$

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**Chapter 19, Problem 8.**

Find the  $z$  parameters of the two-port in Fig. 19.72.

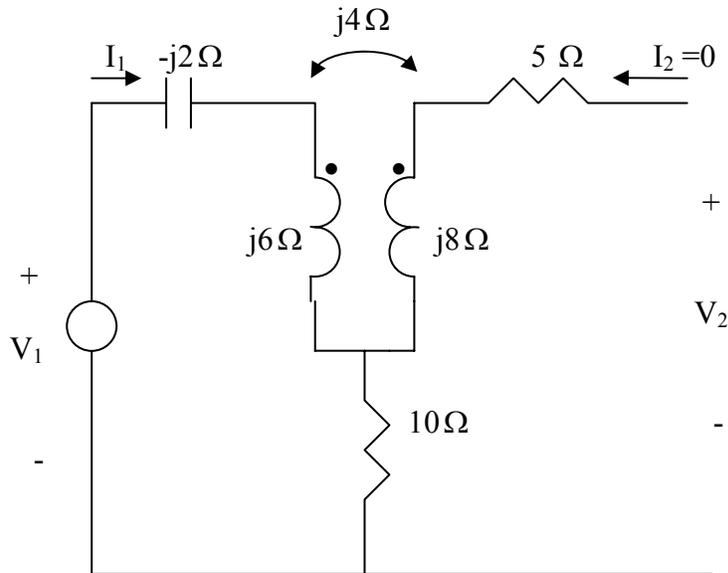


**Figure 19.72**

For Prob. 19.8.

**Chapter 19, Solution 8.**

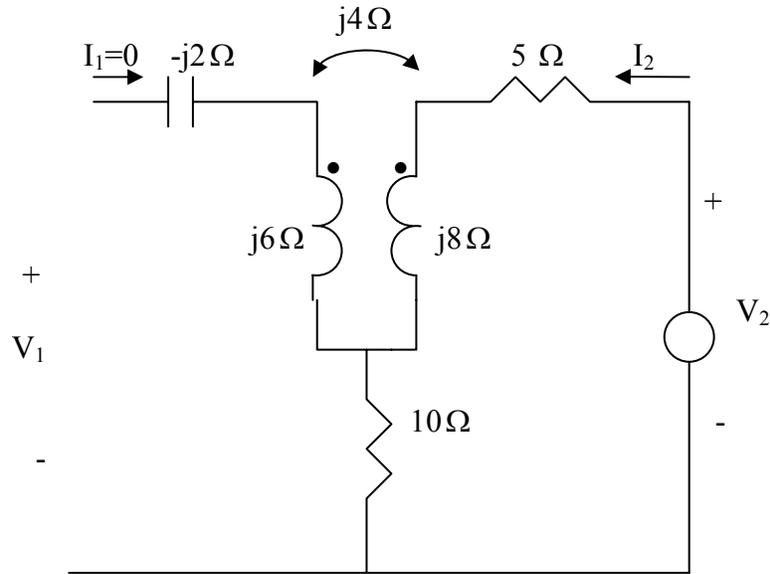
To get  $z_{11}$  and  $z_{21}$ , consider the circuit below.



$$V_1 = (10 - j2 + j6)I_1 \quad \longrightarrow \quad z_{11} = \frac{V_1}{I_1} = 10 + j4$$

$$V_2 = -10I_1 - j4I_1 \quad \longrightarrow \quad z_{21} = \frac{V_2}{I_1} = -(10 + j4)$$

To get  $z_{22}$  and  $z_{12}$ , consider the circuit below.



$$V_2 = (5 + 10 + j8)I_2 \quad \longrightarrow \quad z_{22} = \frac{V_2}{I_2} = 15 + j8$$

$$V_1 = -(10 + j4)I_2 \quad \longrightarrow \quad z_{12} = \frac{V_1}{I_2} = -(10 + j4)$$

Thus,

$$[z] = \begin{bmatrix} (10 + j4) & -(10 + j4) \\ -(10 + j4) & (15 + j8) \end{bmatrix} \Omega$$

### Chapter 19, Problem 9.

The  $y$  parameters of a network are:

$$[\mathbf{y}] = \begin{bmatrix} 0.5 & -0.2 \\ -0.2 & 0.4 \end{bmatrix}$$

Determine the  $z$  parameters for the network.

### Chapter 19, Solution 9.

$$z_{11} = \frac{y_{22}}{\Delta y} = \frac{0.4}{0.16} = 2.5, \quad \Delta y = y_{11}y_{22} - y_{21}y_{12} = 0.5 \times 0.4 - 0.2 \times 0.2 = 0.16$$

$$z_{12} = \frac{-y_{12}}{\Delta y} = \frac{0.2}{0.16} = 1.25 = z_{21}$$

$$z_{22} = \frac{y_{11}}{\Delta y} = \frac{0.5}{0.16} = 3.125$$

Thus,

$$[\mathbf{z}] = \begin{bmatrix} 2.5 & 1.25 \\ 1.25 & 3.125 \end{bmatrix} \Omega$$

**Chapter 19, Problem 10.**

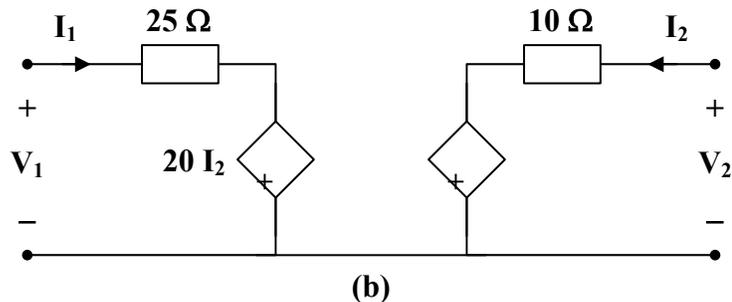
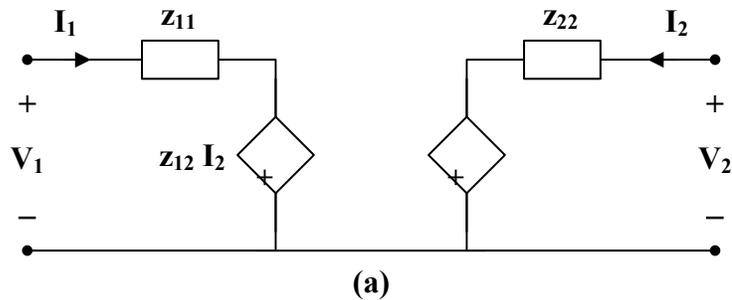
Construct a two-port that realizes each of the following  $z$  parameters.

(a)  $[z] = \begin{bmatrix} 25 & 20 \\ 5 & 10 \end{bmatrix} \Omega$

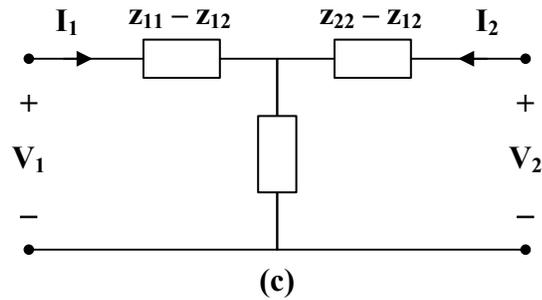
(b)  $[z] = \begin{bmatrix} 1 + \frac{3}{s} & \frac{1}{s} \\ \frac{1}{s} & 2s + \frac{1}{s} \end{bmatrix} \Omega$

**Chapter 19, Solution 10.**

- (a) This is a non-reciprocal circuit so that **the two-port looks like the one shown in Figs. (a) and (b).**



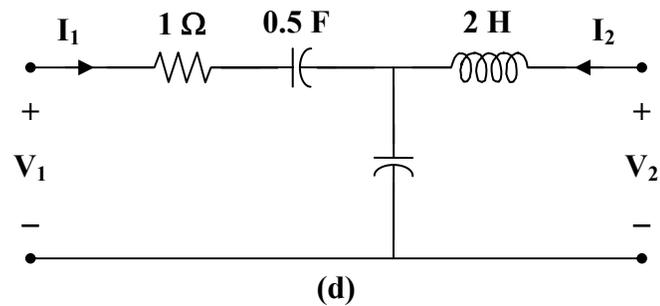
- (b) This is a reciprocal network and the two-port look like the one shown in Figs. (c) and (d).



$$z_{11} - z_{12} = 1 + \frac{2}{s} = 1 + \frac{1}{0.5s}$$

$$z_{22} - z_{12} = 2s$$

$$z_{12} = \frac{1}{s}$$



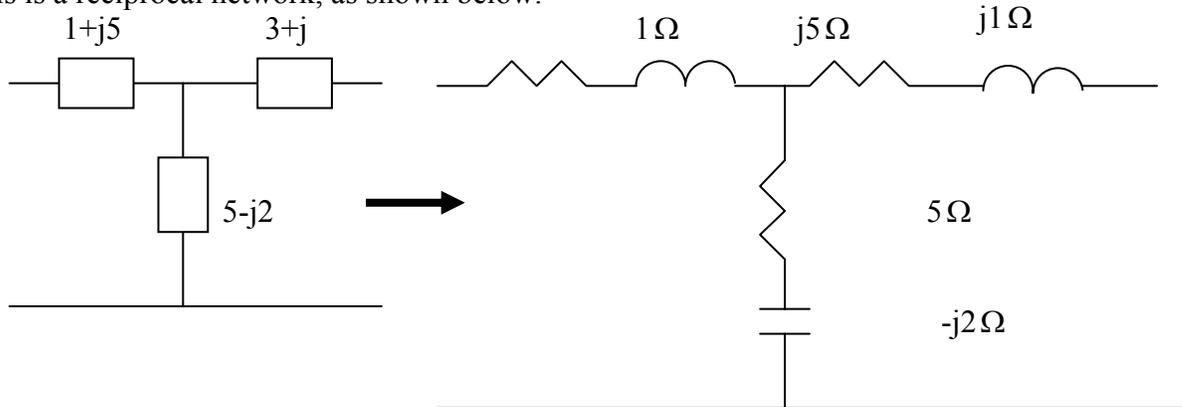
### Chapter 19, Problem 11.

Determine a two-port network that is represented by the following  $z$  parameters:

$$[\mathbf{z}] = \begin{bmatrix} 6 + j3 & 5 - j2 \\ 5 - j2 & 8 - j \end{bmatrix} \Omega$$

### Chapter 19, Solution 11.

This is a reciprocal network, as shown below.

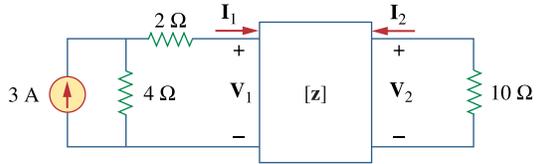


**Chapter 19, Problem 12.**

For the circuit shown in Fig. 19.73, let

$$[\mathbf{z}] = \begin{bmatrix} 10 & -6 \\ -4 & 12 \end{bmatrix}$$

Find  $I_1$ ,  $I_2$ ,  $V_1$ , and  $V_2$ .



**Figure 19.73**

For Prob. 19.12.

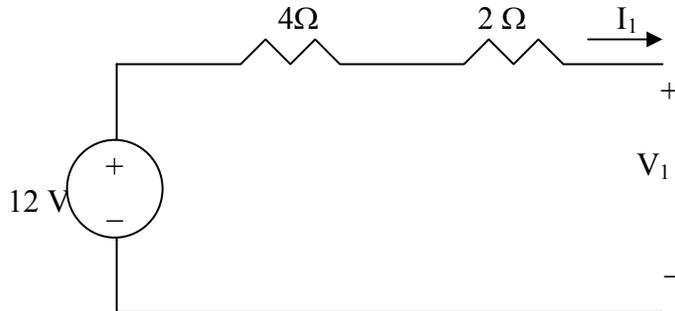
**Chapter 19, Solution 12.**

$$V_1 = 10I_1 - 6I_2 \quad (1)$$

$$V_2 = -4I_2 + 12I_2 \quad (2)$$

$$V_2 = -10I_2 \quad (3)$$

If we convert the current source to a voltage source, that portion of the circuit becomes what is shown below.



$$-12 + 6I_1 + V_1 = 0 \quad \longrightarrow \quad V_1 = 12 - 6I_1 \quad (4)$$

Substituting (3) and (4) into (1) and (2), we get

$$12 - 6I_1 = 10I_1 - 6I_2 \quad \longrightarrow \quad 12 = 16I_1 - 6I_2 \quad (5)$$

$$-10I_2 = -4I_1 + 12I_2 \quad \longrightarrow \quad 0 = -4I_1 + 22I_2 \quad \longrightarrow \quad I_1 = 5.5I_2 \quad (6)$$

From (5) and (6),

$$12 = 88I_2 - 6I_2 = 82I_2 \quad \longrightarrow \quad I_2 = \underline{0.1463 \text{ A}}$$

$$I_1 = 5.5I_2 = \underline{0.8049 \text{ A}}$$

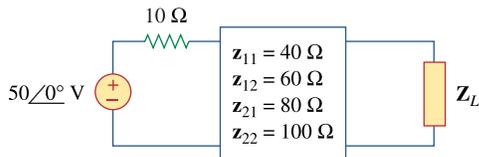
$$V_2 = -10I_2 = \underline{-1.463 \text{ V}}$$

$$V_1 = 12 - 6I_1 = \underline{7.1706 \text{ V}}$$

### Chapter 19, Problem 13.

Determine the average power delivered to  $Z_L = 5 + j4$  in the network of Fig. 19.74.

*Note:* The voltage is rms.

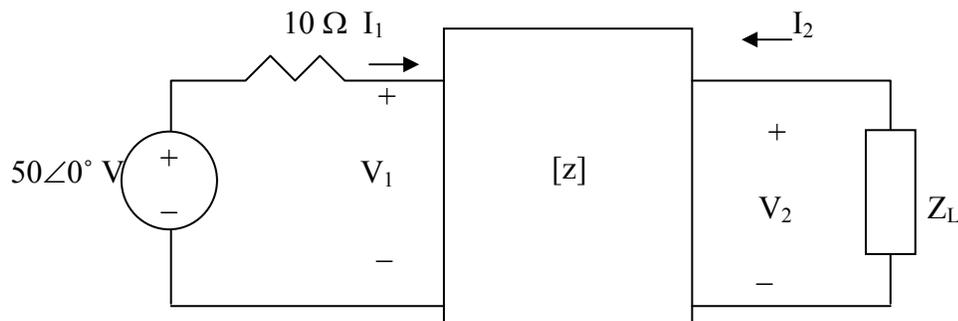


**Figure 19.74**

For Prob. 19.13.

### Chapter 19, Solution 13.

Consider the circuit as shown below.



$$V_1 = 40I_1 + 60I_2 \quad (1)$$

$$V_2 = 80I_1 + 100I_2 \quad (2)$$

$$V_2 = -I_2 Z_L = -I_2(5 + j4) \quad (3)$$

$$50 = V_1 + 10I_1 \quad \longrightarrow \quad V_1 = 50 - 10I_1 \quad (4)$$

Substituting (4) in (1)

$$50 - 10I_1 = 40I_1 + 60I_2 \quad \longrightarrow \quad 5 = 5I_1 + 6I_2 \quad (5)$$

Substituting (3) into (2),

$$-I_2(5 + j4) = 80I_1 + 100I_2 \quad \longrightarrow \quad 0 = 80I_1 + (105 + j4)I_2 \quad (6)$$

Solving (5) and (6) gives

$$I_2 = -7.423 + j3.299 \text{ A}$$

We can check the answer using MATLAB.

First we need to rewrite equations 1-4 as follows,

$$\begin{bmatrix} 1 & 0 & -40 & -60 \\ 0 & 1 & -80 & -100 \\ 0 & 1 & 0 & 5+j4 \\ 1 & 0 & 10 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ I_1 \\ I_2 \end{bmatrix} = A * X = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 50 \end{bmatrix} = U$$

```
>> A=[1,0,-40,-60;0,1,-80,-100;0,1,0,(5+4i);1,0,10,0]
```

```
A =
```

```
1.0e+002 *
```

```
0.0100      0      -0.4000      -0.6000
      0      0.0100     -0.8000     -1.0000
      0      0.0100      0      0.0500 + 0.0400i
0.0100      0      0.1000      0
```

```
>> U=[0;0;0;50]
```

```
U =
```

```
0
0
0
50
```

```
>> X=inv(A)*U
```

```
X =
```

```
-49.0722 +39.5876i
50.3093 +13.1959i
9.9072 - 3.9588i
-7.4227 + 3.2990i
```

$$P = |I_2|^2 5 = \underline{\underline{329.9 \text{ W}}}.$$

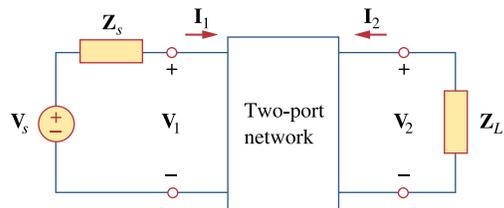
### Chapter 19, Problem 14.

For the two-port network shown in Fig. 19.75, show that at the output terminals,

$$\mathbf{Z}_{\text{Th}} = \mathbf{z}_{22} - \frac{\mathbf{z}_{12}\mathbf{z}_{21}}{\mathbf{z}_{11} + \mathbf{Z}_s}$$

and

$$\mathbf{V}_{\text{Th}} = \frac{\mathbf{z}_{21}}{\mathbf{z}_{11} + \mathbf{Z}_s} \mathbf{V}_s$$

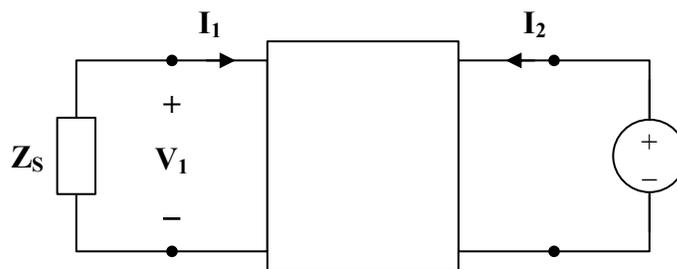


**Figure 19.75**

For Prob. 19.14 and 19.41.

### Chapter 19, Solution 14.

To find  $\mathbf{Z}_{\text{Th}}$ , consider the circuit in Fig. (a).



(a)

$$\mathbf{V}_1 = \mathbf{z}_{11} \mathbf{I}_1 + \mathbf{z}_{12} \mathbf{I}_2 \quad (1)$$

$$\mathbf{V}_2 = \mathbf{z}_{21} \mathbf{I}_1 + \mathbf{z}_{22} \mathbf{I}_2 \quad (2)$$

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But

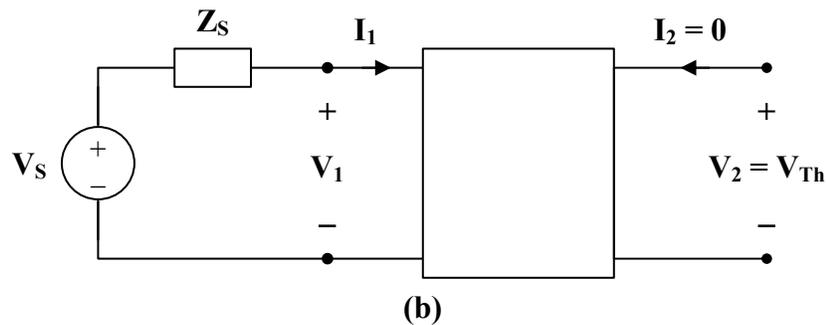
$$V_2 = 1, \quad V_1 = -Z_s I_1$$

Hence,  $0 = (z_{11} + Z_s) I_1 + z_{12} I_2 \longrightarrow I_1 = \frac{-z_{12}}{z_{11} + Z_s} I_2$

$$1 = \left( \frac{-z_{21} z_{12}}{z_{11} + Z_s} + z_{22} \right) I_2$$

$$Z_{Th} = \frac{V_2}{I_2} = \frac{1}{I_2} = \underline{z_{22} - \frac{z_{21} z_{12}}{z_{11} + Z_s}}$$

To find  $V_{Th}$ , consider the circuit in Fig. (b).



$$I_2 = 0, \quad V_1 = V_s - I_1 Z_s$$

Substituting these into (1) and (2),

$$V_s - I_1 Z_s = z_{11} I_1 \longrightarrow I_1 = \frac{V_s}{z_{11} + Z_s}$$

$$V_2 = z_{21} I_1 = \frac{z_{21} V_s}{z_{11} + Z_s}$$

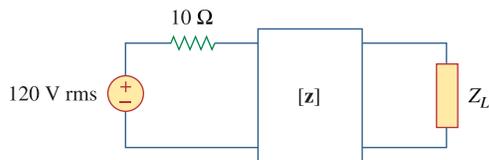
$$V_{Th} = V_2 = \underline{\frac{z_{21} V_s}{z_{11} + Z_s}}$$

### Chapter 19, Problem 15.

For the two-port circuit in Fig. 19.76,

$$[\mathbf{z}] = \begin{bmatrix} 40 & 60 \\ 80 & 120 \end{bmatrix} \Omega$$

- (a) Find  $Z_L$  for maximum power transfer to the load.  
(b) Calculate the maximum power delivered to the load.



**Figure 19.76**

For Prob. 19.15.

### Chapter 19, Solution 15.

- (a) From Prob. 18.12,

$$Z_{Th} = z_{22} - \frac{z_{12}z_{21}}{z_{11} + Z_s} = 120 - \frac{80 \times 60}{40 + 10} = 24$$

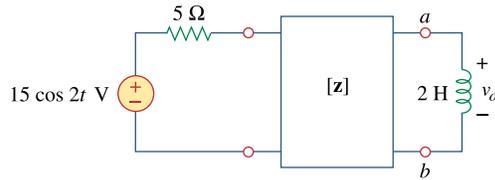
$$\underline{Z_L = Z_{Th} = 24\Omega}$$

(b)  $V_{Th} = \frac{z_{21}}{z_{11} + Z_s} V_s = \frac{80}{40 + 10} (120) = 192$

$$P_{max} = \left( \frac{V_{Th}}{2R_{Th}} \right)^2 R_{Th} = 4^2 \times 24 = \underline{\underline{384W}}$$

### Chapter 19, Problem 16.

For the circuit in Fig. 19.77, at  $\omega = 2 \text{ rad/s}$ ,  $\mathbf{z}_{11} = 10\Omega$ ,  $\mathbf{z}_{12} = \mathbf{z}_{21} = j6\Omega$ ,  $\mathbf{z}_{22} = 4\Omega$ . Obtain the Thevenin equivalent circuit at terminals  $a$ - $b$  and calculate  $v_o$ .

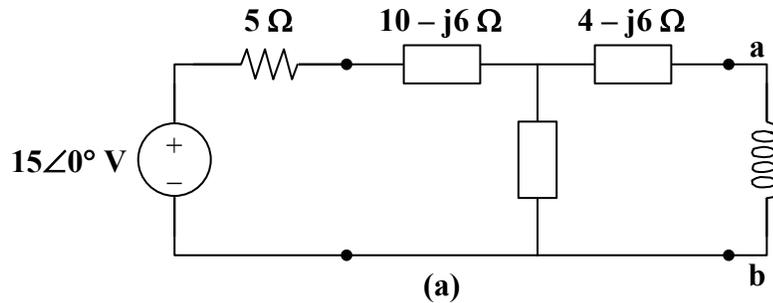


**Figure 19.77**

For Prob. 19.16.

**Chapter 19, Solution 16.**

As a reciprocal two-port, the given circuit can be represented as shown in Fig. (a).



At terminals a-b,

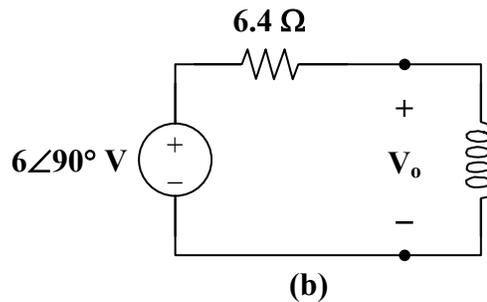
$$Z_{Th} = (4 - j6) + j6 \parallel (5 + 10 - j6)$$

$$Z_{Th} = 4 - j6 + \frac{j6(15 - j6)}{15} = 4 - j6 + 2.4 + j6$$

$$Z_{Th} = \underline{\underline{6.4 \Omega}}$$

$$V_{Th} = \frac{j6}{j6 + 5 + 10 - j6} (15 \angle 0^\circ) = j6 = \underline{\underline{6 \angle 90^\circ \text{ V}}}$$

The Thevenin equivalent circuit is shown in Fig. (b).



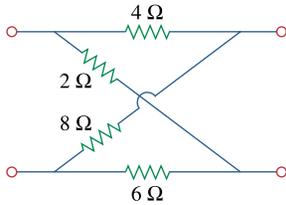
From this,

$$V_o = \frac{j4}{6.4 + j4} (j6) = 3.18 \angle 148^\circ$$

$$v_o(t) = \underline{\underline{3.18 \cos(2t + 148^\circ) \text{ V}}}$$

**Chapter 19, Problem 17.**

\* Determine the  $z$  and  $y$  parameters for the circuit in Fig. 19.78.



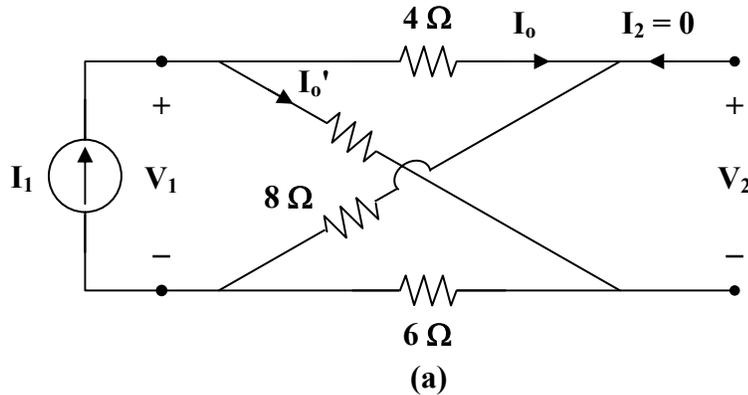
**Figure 19.78**

For Prob. 19.17.

\* An asterisk indicates a challenging problem.

**Chapter 19, Solution 17.**

To obtain  $z_{11}$  and  $z_{21}$ , consider the circuit in Fig. (a).



In this case, the 4- $\Omega$  and 8- $\Omega$  resistors are in series, since the same current,  $I_o$ , passes through them. Similarly, the 2- $\Omega$  and 6- $\Omega$  resistors are in series, since the same current,  $I_o'$ , passes through them.

$$z_{11} = \frac{V_1}{I_1} = (4 + 8) \parallel (2 + 6) = 12 \parallel 8 = \frac{(12)(8)}{20} = 4.8 \Omega$$

$$I_o = \frac{8}{8 + 12} I_1 = \frac{2}{5} I_1 \quad I_o' = \frac{3}{5} I_1$$

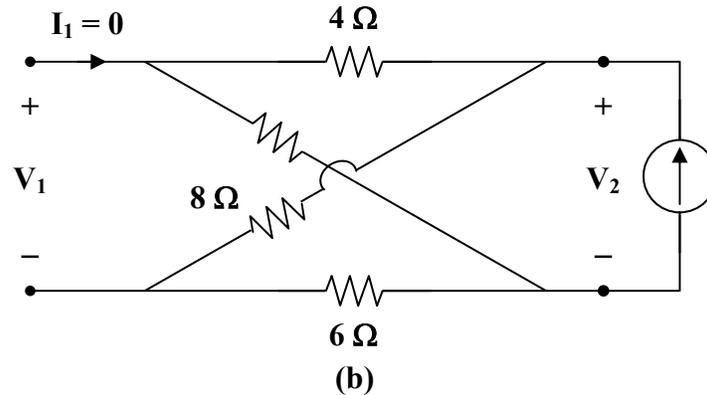
But

$$-V_2 - 4I_o + 2I_o' = 0$$

$$V_2 = -4I_o + 2I_o' = \frac{-8}{5}I_1 + \frac{6}{5}I_1 = \frac{-2}{5}I_1$$

$$z_{21} = \frac{V_2}{I_1} = \frac{-2}{5} = -0.4 \Omega$$

To get  $z_{22}$  and  $z_{12}$ , consider the circuit in Fig. (b).



$$z_{22} = \frac{V_2}{I_2} = (4 + 2) \parallel (8 + 6) = 6 \parallel 14 = \frac{(6)(14)}{20} = 4.2 \Omega$$

$$z_{12} = z_{21} = -0.4 \Omega$$

Thus,

$$[z] = \begin{bmatrix} 4.8 & -0.4 \\ -0.4 & 4.2 \end{bmatrix} \Omega$$

We may take advantage of Table 18.1 to get  $[y]$  from  $[z]$ .

$$\Delta_z = (4.8)(4.2) - (0.4)^2 = 20$$

$$y_{11} = \frac{z_{22}}{\Delta_z} = \frac{4.2}{20} = 0.21$$

$$y_{12} = \frac{-z_{12}}{\Delta_z} = \frac{0.4}{20} = 0.02$$

$$y_{21} = \frac{-z_{21}}{\Delta_z} = \frac{0.4}{20} = 0.02$$

$$y_{22} = \frac{z_{11}}{\Delta_z} = \frac{4.8}{20} = 0.24$$

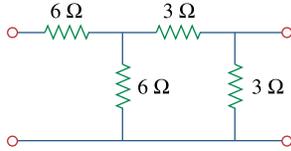
Thus,

$$[y] = \begin{bmatrix} 0.21 & 0.02 \\ 0.02 & 0.24 \end{bmatrix} S$$

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**Chapter 19, Problem 18.**

Calculate the y parameters for the two-port in Fig. 19.79.

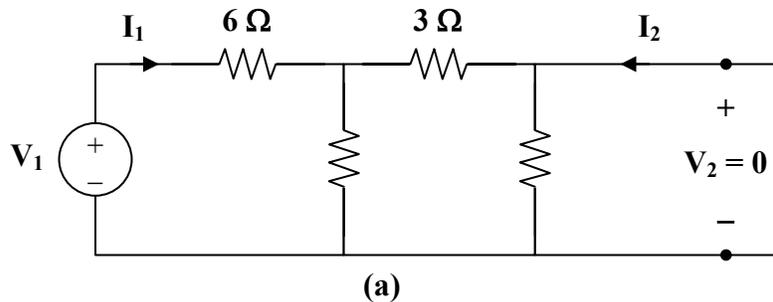


**Figure 19.79**

For Prob. 19.18 and 19.37.

**Chapter 19, Solution 18.**

To get  $y_{11}$  and  $y_{21}$ , consider the circuit in Fig.(a).



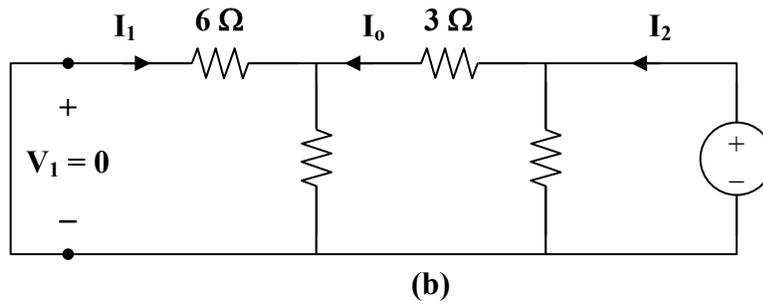
$$V_1 = (6 + 6 \parallel 3)I_1 = 8I_1$$

$$y_{11} = \frac{I_1}{V_1} = \frac{1}{8}$$

$$I_2 = \frac{-6}{6+3}I_1 = \frac{-2}{3} \frac{V_1}{8} = \frac{-V_1}{12}$$

$$y_{21} = \frac{I_2}{V_1} = \frac{-1}{12}$$

To get  $y_{22}$  and  $y_{12}$ , consider the circuit in Fig.(b).



$$y_{22} = \frac{I_2}{V_2} = \frac{1}{3 \parallel (3 + 6 \parallel 6)} = \frac{1}{3 \parallel 6} = \frac{1}{2}$$

$$I_1 = \frac{-I_0}{2}, \quad I_0 = \frac{3}{3+6} I_2 = \frac{1}{3} I_2$$

$$I_1 = \frac{-I_2}{6} = \left( \frac{-1}{6} \right) \left( \frac{1}{2} V_2 \right) = \frac{-V_2}{12}$$

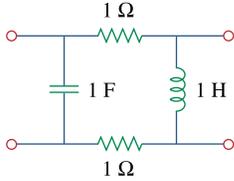
$$y_{12} = \frac{I_1}{V_2} = \frac{-1}{12} = y_{21}$$

Thus,

$$[y] = \underline{\underline{\begin{bmatrix} \frac{1}{8} & \frac{-1}{12} \\ \frac{-1}{12} & \frac{1}{2} \end{bmatrix} \text{ S}}}$$

**Chapter 19, Problem 19.**

Find the  $y$  parameters of the two-port in Fig. 19.80 in terms of  $s$ .

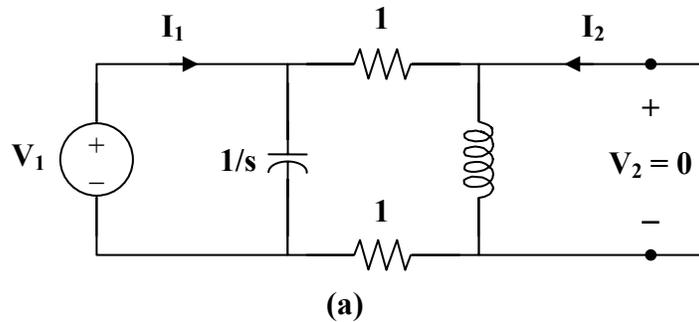


**Figure 19.80**

For Prob. 19.19.

**Chapter 19, Solution 19.**

Consider the circuit in Fig.(a) for calculating  $y_{11}$  and  $y_{21}$ .



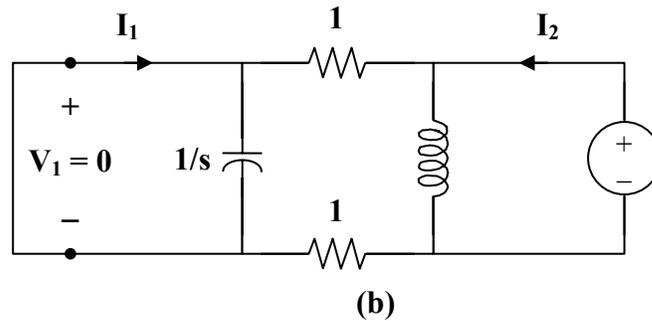
$$V_1 = \left( \frac{1}{s} \parallel 2 \right) I_1 = \frac{2/s}{2 + (1/s)} I_1 = \frac{2}{2s + 1} I_1$$

$$y_{11} = \frac{I_1}{V_1} = \frac{2s + 1}{2} = s + 0.5$$

$$I_2 = \frac{(-1/s)}{(1/s) + 2} I_1 = \frac{-I_1}{2s + 1} = \frac{-V_1}{2}$$

$$y_{21} = \frac{I_2}{V_1} = -0.5$$

To get  $y_{22}$  and  $y_{12}$ , refer to the circuit in Fig.(b).



$$V_2 = (s \parallel 2) I_2 = \frac{2s}{s+2} I_2$$

$$y_{22} = \frac{I_2}{V_2} = \frac{s+2}{2s} = 0.5 + \frac{1}{s}$$

$$I_1 = \frac{-s}{s+2} I_2 = \frac{-s}{s+2} \cdot \frac{s+2}{2s} V_2 = \frac{-V_2}{2}$$

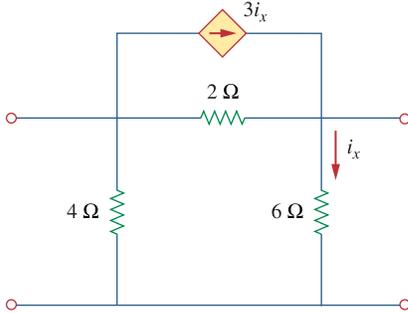
$$y_{12} = \frac{I_1}{V_2} = -0.5$$

Thus,

$$[y] = \underline{\underline{\begin{bmatrix} s+0.5 & -0.5 \\ -0.5 & 0.5+1/s \end{bmatrix} S}}$$

**Chapter 19, Problem 20.**

Find the y parameters for the circuit in Fig. 19.81.

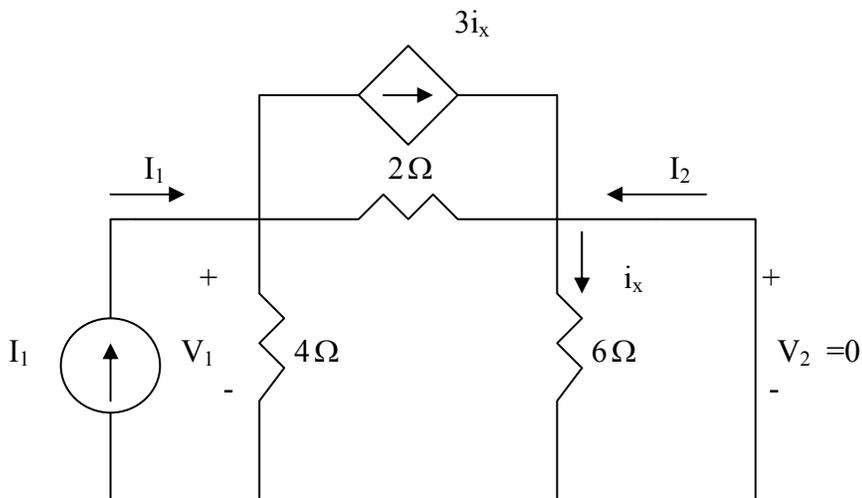


**Figure 19.81**

For Prob. 19.20.

**Chapter 19, Solution 20.**

To get  $y_{11}$  and  $y_{21}$ , consider the circuit below.

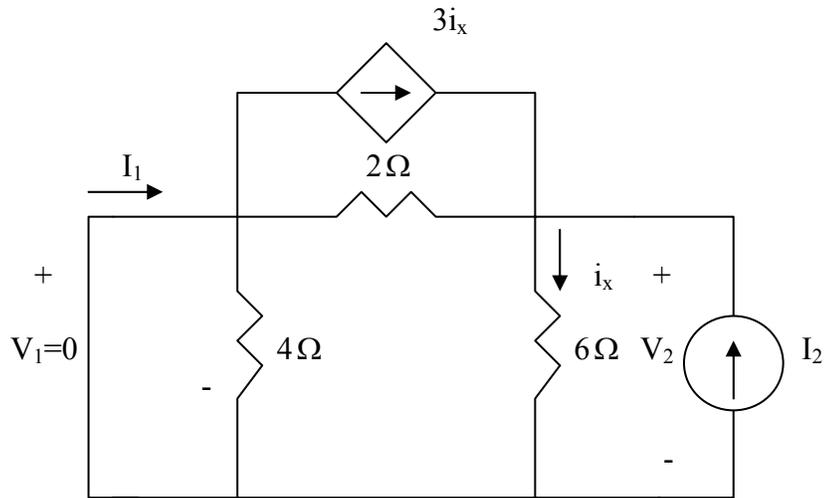


Since 6-ohm resistor is short-circuited,  $i_x = 0$

$$V_1 = I_1(4 // 2) = \frac{8}{6} I_1 \quad \longrightarrow \quad y_{11} = \frac{I_1}{V_1} = 0.75$$

$$I_2 = -\frac{4}{4+2} I_1 = -\frac{2}{3} \left( \frac{6}{8} V_1 \right) = -\frac{1}{2} V_1 \quad \longrightarrow \quad y_{21} = \frac{I_2}{V_1} = -0.5$$

To get  $y_{22}$  and  $y_{12}$ , consider the circuit below.



$$i_x = \frac{V_2}{6}, \quad I_2 = i_x - 3i_x + \frac{V_2}{2} = \frac{V_2}{6} \quad \longrightarrow \quad y_{22} = \frac{I_2}{V_2} = \frac{1}{6} = 0.1667$$

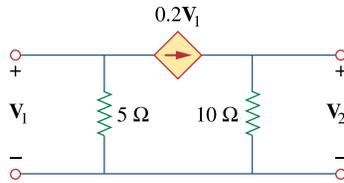
$$I_1 = 3i_x - \frac{V_2}{2} = 0 \quad \longrightarrow \quad y_{12} = \frac{I_1}{V_2} = 0$$

Thus,

$$[y] = \begin{bmatrix} 0.75 & 0 \\ -0.5 & 0.1667 \end{bmatrix} \text{S}$$

**Chapter 19, Problem 21.**

Obtain the admittance parameter equivalent circuit of the two-port in Fig. 19.82.

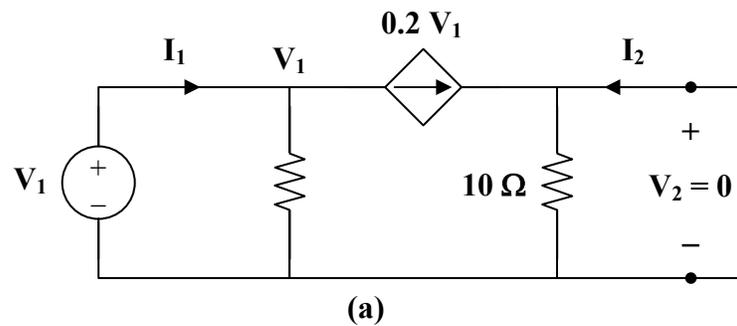


**Figure 19.82**

For Prob. 19.21.

**Chapter 19, Solution 21.**

To get  $y_{11}$  and  $y_{21}$ , refer to Fig. (a).

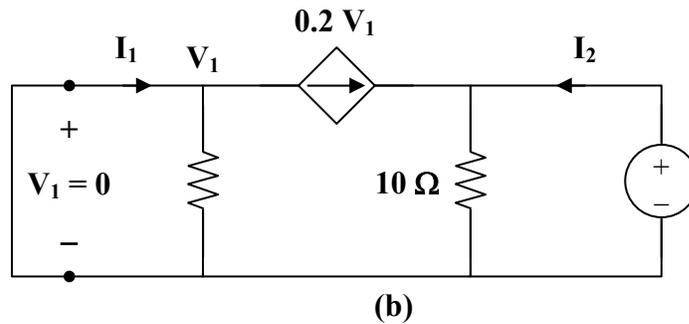


At node 1,

$$I_1 = \frac{V_1}{5} + 0.2V_1 = 0.4V_1 \longrightarrow y_{11} = \frac{I_1}{V_1} = 0.4$$

$$I_2 = -0.2V_1 \longrightarrow y_{21} = \frac{I_2}{V_1} = -0.2$$

To get  $y_{22}$  and  $y_{12}$ , refer to the circuit in Fig. (b).



Since  $V_1 = 0$ , the dependent current source can be replaced with an open circuit.

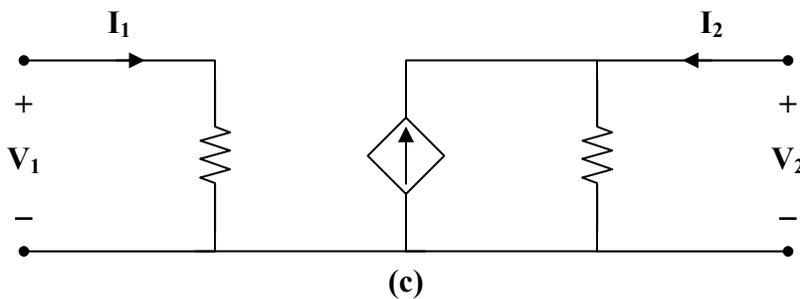
$$V_2 = 10I_2 \longrightarrow y_{22} = \frac{I_2}{V_2} = \frac{1}{10} = 0.1$$

$$y_{12} = \frac{I_1}{V_2} = 0$$

Thus,

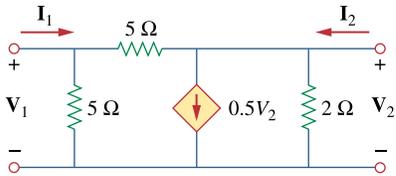
$$[y] = \begin{bmatrix} 0.4 & 0 \\ -0.2 & 0.1 \end{bmatrix} \text{S}$$

Consequently, **the y parameter equivalent circuit is shown in Fig. (c).**



**Chapter 19, Problem 22.**

Obtain the  $y$  parameters of the two-port network in Fig. 19.83.

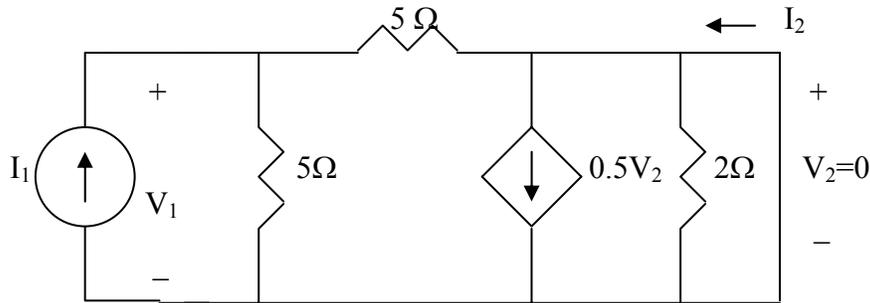


**Figure 19.83**

For Prob. 19.22.

**Chapter 19, Solution 22.**

To obtain  $y_{11}$  and  $y_{21}$ , consider the circuit below.

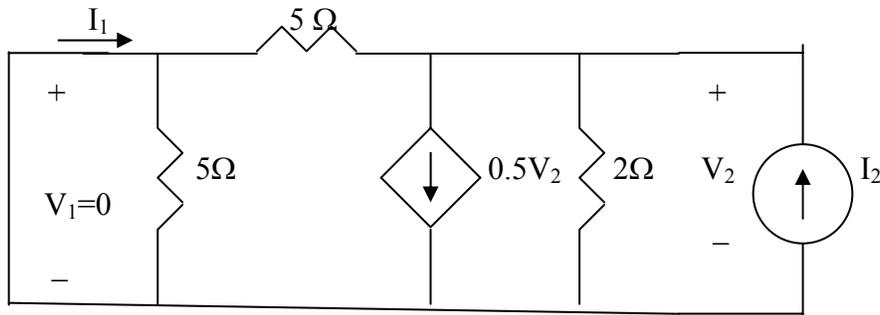


The 2-Ω resistor is short-circuited.

$$V_1 = 5 \frac{I_1}{2} \longrightarrow y_{11} = \frac{I_1}{V_1} = \frac{2}{5} = 0.4$$

$$I_2 = \frac{1}{2} I_1 \longrightarrow y_{21} = \frac{I_2}{V_1} = \frac{\frac{1}{2} I_1}{2.5 I_1} = 0.2$$

To obtain  $y_{12}$  and  $y_{22}$ , consider the circuit below.



At the top node, KCL gives

$$I_2 = 0.5V_2 + \frac{V_2}{2} + \frac{V_2}{5} = 1.2V_2 \longrightarrow y_{22} = \frac{I_2}{V_2} = 1.2$$

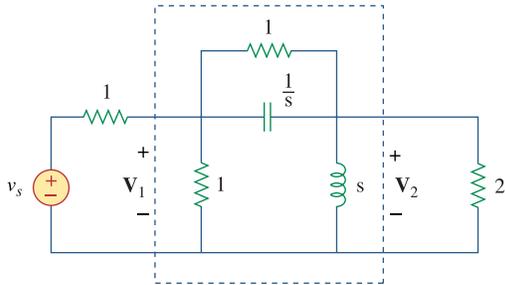
$$I_1 = -\frac{V_2}{5} = -0.2V_2 \longrightarrow y_{12} = \frac{I_1}{V_2} = -0.2$$

Hence,

$$[y] = \begin{bmatrix} 0.4 & -0.2 \\ 0.2 & 1.2 \end{bmatrix} \text{ S}$$

**Chapter 19, Problem 23.**

- (a) Find the  $y$  parameters of the two-port in Fig. 19.84.  
 (b) Determine  $\mathbf{V}_2(s)$  for  $v_s = 2u(t)\text{V}$ .



**Figure 19.84**

For Prob. 19.23.

**Chapter 19, Solution 23.**

(a)

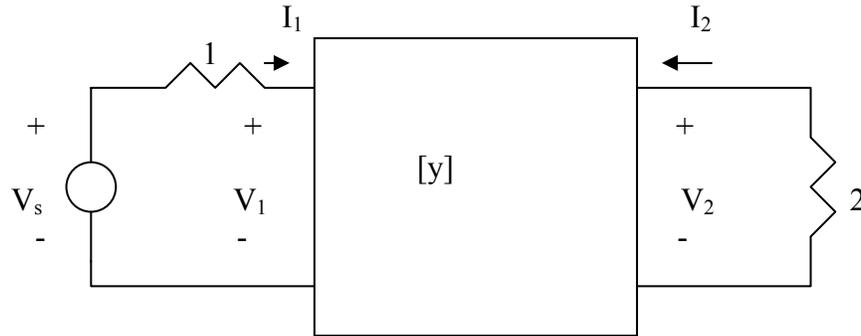
$$-y_{12} = 1 / \left( 1 // \frac{1}{s} \right) = 1 + s \quad \longrightarrow \quad y_{12} = -(s + 1)$$

$$y_{11} + y_{12} = 1 \quad \longrightarrow \quad y_{11} = 1 - y_{12} = 1 + s + 1 = s + 2$$

$$y_{22} + y_{12} = s \quad \longrightarrow \quad y_{22} = \frac{1}{s} - y_{12} = \frac{1}{s} + s + 1 = \frac{s^2 + s + 1}{s}$$

$$[y] = \begin{bmatrix} s + 2 & -(s + 1) \\ -(s + 1) & \frac{s^2 + s + 1}{s} \end{bmatrix}$$

(b) Consider the network below.



$$V_s = I_1 + V_1 \text{ or } V_s - V_1 = I_1 \quad (1)$$

$$V_2 = -2I_2 \quad (2)$$

$$I_1 = y_{11}V_1 + y_{12}V_2 \quad (3)$$

$$I_2 = y_{21}V_1 + y_{22}V_2 \quad (4)$$

From (1) and (3)

$$V_s - V_1 = y_{11}V_1 + y_{12}V_2 \quad \longrightarrow \quad V_s = (1 + y_{11})V_1 + y_{12}V_2 \quad (5)$$

From (2) and (4),

$$-0.5V_2 = y_{21}V_1 + y_{22}V_2 \quad \longrightarrow \quad V_1 = -\frac{1}{y_{21}}(0.5 + y_{22})V_2 \quad (6)$$

Substituting (6) into (5),

$$\begin{aligned} V_s &= -\frac{(1 + y_{11})(0.5 + y_{22})}{y_{21}}V_2 + y_{12}V_2 \\ &= \frac{2}{s} \quad \longrightarrow \quad V_2 = \frac{2/s}{\left[ y_{12} - \frac{1}{y_{21}}(1 + y_{11})(0.5 + y_{22}) \right]} \\ V_2 &= \frac{2/s}{-(s+1) + \frac{1}{s+1}(1+s+2)\left(\frac{1}{2} + \frac{s^2+s+1}{s}\right)} = \frac{0.8(s+1)}{(s^2 + 1.8s + 1.2)} \end{aligned}$$

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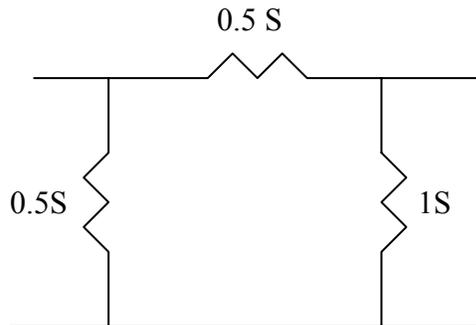
**Chapter 19, Problem 25.**

Draw the two-port network that has the following  $y$  parameters:

$$[y] = \begin{bmatrix} 1 & -0.5 \\ -0.5 & 1.5 \end{bmatrix} \text{S}$$

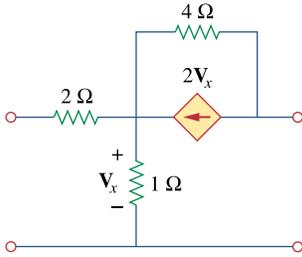
**Chapter 19, Solution 25.**

This is a reciprocal network and is shown below.



**Chapter 19, Problem 26.**

Calculate  $[y]$  for the two-port in Fig. 19.85.

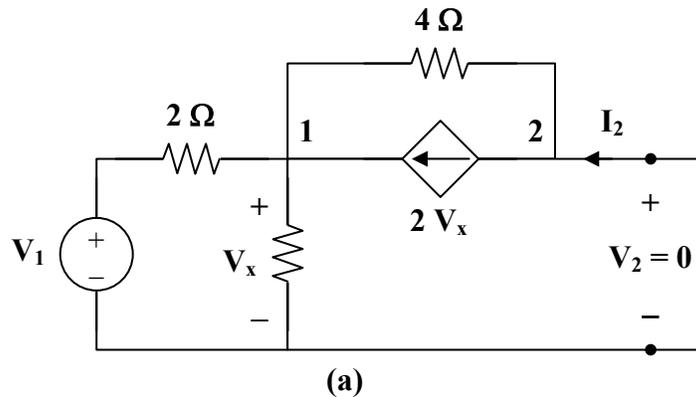


**Figure 19.85**

For Prob. 19.26.

**Chapter 19, Solution 26.**

To get  $y_{11}$  and  $y_{21}$ , consider the circuit in Fig. (a).



At node 1,

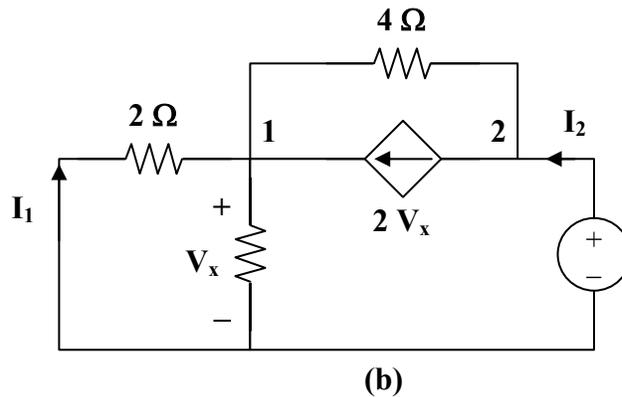
$$\frac{V_1 - V_x}{2} + 2V_x = \frac{V_x}{1} + \frac{V_x}{4} \longrightarrow 2V_1 = -V_x \quad (1)$$

But 
$$I_1 = \frac{V_1 - V_x}{2} = \frac{V_1 + 2V_1}{2} = 1.5V_1 \longrightarrow y_{11} = \frac{I_1}{V_1} = 1.5$$

Also, 
$$I_2 + \frac{V_x}{4} = 2V_x \longrightarrow I_2 = 1.75V_x = -3.5V_1$$

$$y_{21} = \frac{I_2}{V_1} = -3.5$$

To get  $y_{22}$  and  $y_{12}$ , consider the circuit in Fig.(b).



At node 2,

$$I_2 = 2V_x + \frac{V_2 - V_x}{4} \quad (2)$$

At node 1,

$$2V_x + \frac{V_2 - V_x}{4} = \frac{V_x}{2} + \frac{V_x}{1} = \frac{3}{2}V_x \quad \longrightarrow \quad V_2 = -V_x \quad (3)$$

Substituting (3) into (2) gives

$$I_2 = 2V_x - \frac{1}{2}V_x = 1.5V_x = -1.5V_2$$

$$y_{22} = \frac{I_2}{V_2} = -1.5$$

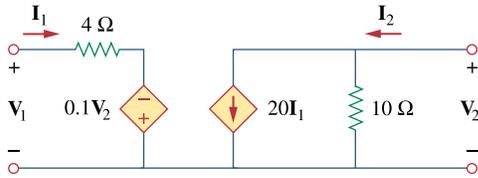
$$I_1 = \frac{-V_x}{2} = \frac{V_2}{2} \quad \longrightarrow \quad y_{12} = \frac{I_1}{V_2} = 0.5$$

Thus,

$$[y] = \begin{bmatrix} 1.5 & 0.5 \\ -3.5 & -1.5 \end{bmatrix} \text{S}$$

**Chapter 19, Problem 27.**

Find the y parameters for the circuit in Fig. 19.86.

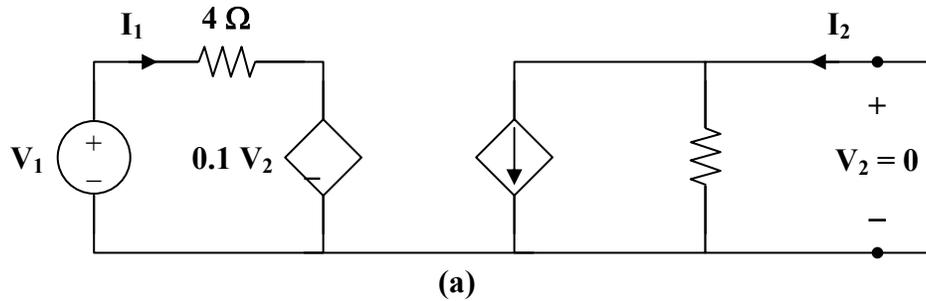


**Figure 19.86**

For Prob. 19.27.

**Chapter 19, Solution 27.**

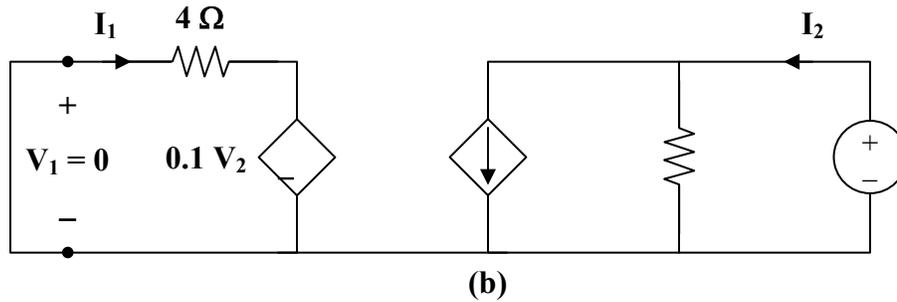
Consider the circuit in Fig. (a).



$$V_1 = 4I_1 \longrightarrow y_{11} = \frac{I_1}{V_1} = \frac{I_1}{4I_1} = 0.25$$

$$I_2 = 20I_1 = 5V_1 \longrightarrow y_{21} = \frac{I_2}{V_1} = 5$$

Consider the circuit in Fig. (b).



$$4\mathbf{I}_1 = 0.1\mathbf{V}_2 \longrightarrow \mathbf{y}_{12} = \frac{\mathbf{I}_1}{\mathbf{V}_2} = \frac{0.1}{4} = 0.025$$

$$\mathbf{I}_2 = 20\mathbf{I}_1 + \frac{\mathbf{V}_2}{10} = 0.5\mathbf{V}_2 + 0.1\mathbf{V}_2 = 0.6\mathbf{V}_2 \longrightarrow \mathbf{y}_{22} = \frac{\mathbf{I}_2}{\mathbf{V}_2} = 0.6$$

Thus,

$$[\mathbf{y}] = \underline{\underline{\begin{bmatrix} 0.25 & 0.025 \\ 5 & 0.6 \end{bmatrix} \text{S}}}$$

Alternatively, from the given circuit,

$$\mathbf{V}_1 = 4\mathbf{I}_1 - 0.1\mathbf{V}_2$$

$$\mathbf{I}_2 = 20\mathbf{I}_1 + 0.1\mathbf{V}_2$$

Comparing these with the equations for the h parameters show that

$$\mathbf{h}_{11} = 4, \quad \mathbf{h}_{12} = -0.1, \quad \mathbf{h}_{21} = 20, \quad \mathbf{h}_{22} = 0.1$$

Using Table 18.1,

$$\mathbf{y}_{11} = \frac{1}{\mathbf{h}_{11}} = \frac{1}{4} = 0.25, \quad \mathbf{y}_{12} = \frac{-\mathbf{h}_{12}}{\mathbf{h}_{11}} = \frac{0.1}{4} = 0.025$$

$$\mathbf{y}_{21} = \frac{\mathbf{h}_{21}}{\mathbf{h}_{11}} = \frac{20}{4} = 5, \quad \mathbf{y}_{22} = \frac{\Delta_{\mathbf{h}}}{\mathbf{h}_{11}} = \frac{0.4 + 2}{4} = 0.6$$

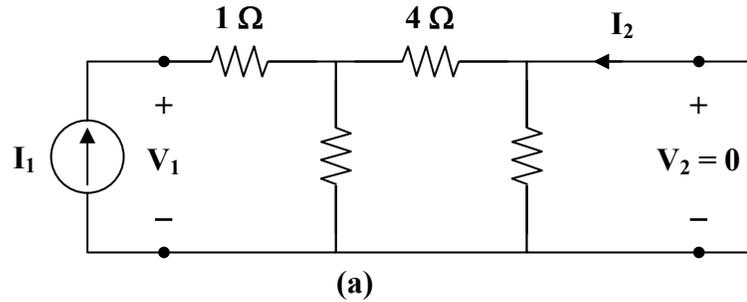
as above.

**Chapter 19, Problem 28.**

In the circuit of Fig. 19.65, the input port is connected to a 1-A dc current source. Calculate the power dissipated by the 2- $\Omega$  resistor by using the y parameters. Confirm your result by direct circuit analysis.

**Chapter 19, Solution 28.**

We obtain  $y_{11}$  and  $y_{21}$  by considering the circuit in Fig.(a).



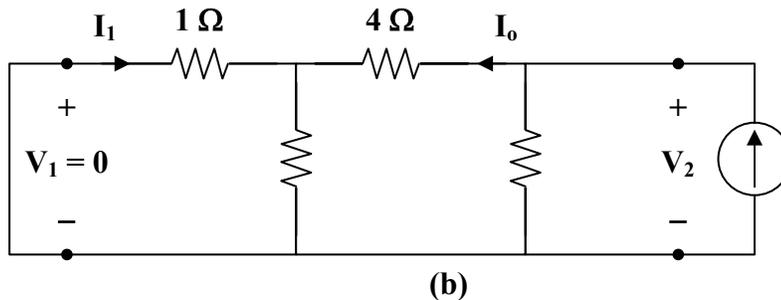
$$Z_{in} = 1 + 6 \parallel 4 = 3.4$$

$$y_{11} = \frac{I_1}{V_1} = \frac{1}{Z_{in}} = 0.2941$$

$$I_2 = \frac{-6}{10} I_1 = \left( \frac{-6}{10} \right) \left( \frac{V_1}{3.4} \right) = \frac{-6}{34} V_1$$

$$y_{21} = \frac{I_2}{V_1} = \frac{-6}{34} = -0.1765$$

To get  $y_{22}$  and  $y_{12}$ , consider the circuit in Fig. (b).



$$\frac{1}{y_{22}} = 2 \parallel (4 + 6 \parallel 1) = 2 \parallel \left(4 + \frac{6}{7}\right) = \frac{(2)(34/7)}{2 + (34/7)} = \frac{34}{24} = \frac{V_2}{I_2}$$

$$y_{22} = \frac{24}{34} = 0.7059$$

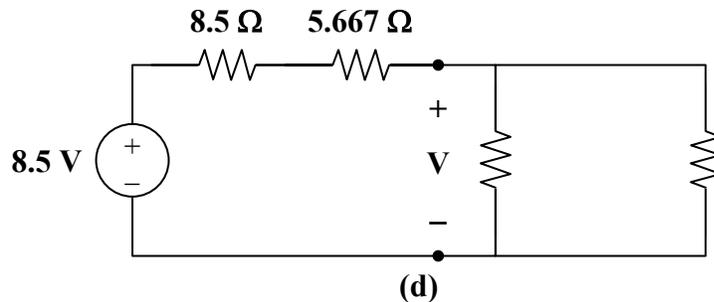
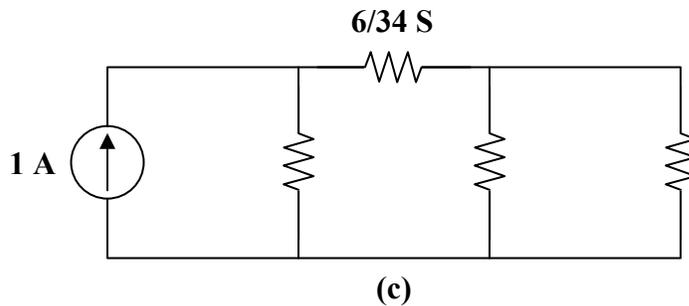
$$I_1 = \frac{-6}{7} I_o \quad I_o = \frac{2}{2 + (34/7)} I_2 = \frac{14}{48} I_2 = \frac{7}{34} V_2$$

$$I_1 = \frac{-6}{34} V_2 \longrightarrow y_{12} = \frac{I_1}{V_2} = \frac{-6}{34} = -0.1765$$

Thus,

$$[y] = \begin{bmatrix} 0.2941 & -0.1765 \\ -0.1765 & 0.7059 \end{bmatrix} \text{S}$$

The equivalent circuit is shown in Fig. (c). After transforming the current source to a voltage source, we have the circuit in Fig. (d).



$$V = \frac{(2 \parallel 1.889)(8.5)}{2 \parallel 1.889 + 8.5 + 5.667} = \frac{(0.9714)(8.5)}{0.9714 + 14.167} = 0.5454$$

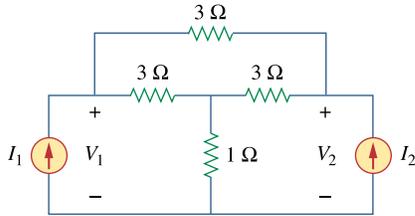
$$P = \frac{V^2}{R} = \frac{(0.5454)^2}{2} = \underline{\underline{0.1487 \text{ W}}}$$

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**Chapter 19, Problem 29.**

In the bridge circuit of Fig. 19.87,  $I_1 = 10 \text{ A}$  and  $I_2 = -4 \text{ A}$

- (a) Find  $V_1$  and  $V_2$  using  $y$  parameters.  
 (b) -Confirm the results in part (a) by direct circuit analysis.

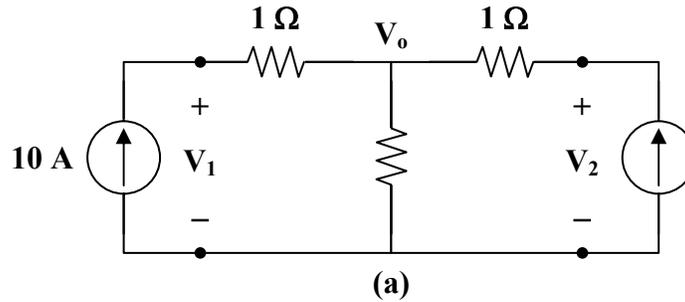


**Figure 19.87**

For Prob. 19.29.

**Chapter 19, Solution 29.**

- (a) Transforming the  $\Delta$  subnetwork to  $Y$  gives the circuit in Fig. (a).



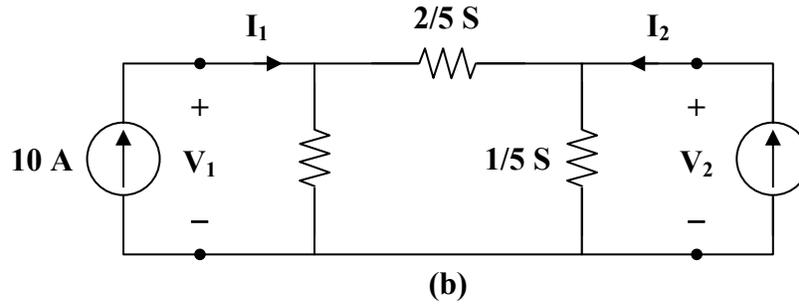
It is easy to get the  $z$  parameters

$$\mathbf{z}_{12} = \mathbf{z}_{21} = 2, \quad \mathbf{z}_{11} = 1 + 2 = 3, \quad \mathbf{z}_{22} = 3$$

$$\Delta_z = \mathbf{z}_{11} \mathbf{z}_{22} - \mathbf{z}_{12} \mathbf{z}_{21} = 9 - 4 = 5$$

$$\mathbf{y}_{11} = \frac{\mathbf{z}_{22}}{\Delta_z} = \frac{3}{5} = \mathbf{y}_{22}, \quad \mathbf{y}_{12} = \mathbf{y}_{21} = \frac{-\mathbf{z}_{12}}{\Delta_z} = \frac{-2}{5}$$

Thus, the equivalent circuit is as shown in Fig. (b).



$$I_1 = 10 = \frac{3}{5}V_1 - \frac{2}{5}V_2 \longrightarrow 50 = 3V_1 - 2V_2 \quad (1)$$

$$I_2 = -4 = \frac{-2}{5}V_1 + \frac{3}{5}V_2 \longrightarrow -20 = -2V_1 + 3V_2$$

$$10 = V_1 - 1.5V_2 \longrightarrow V_1 = 10 + 1.5V_2 \quad (2)$$

Substituting (2) into (1),

$$50 = 30 + 4.5V_2 - 2V_2 \longrightarrow V_2 = \underline{\underline{8 \text{ V}}}$$

$$V_1 = 10 + 1.5V_2 = \underline{\underline{22 \text{ V}}}$$

(b) For direct circuit analysis, consider the circuit in Fig. (a).

For the main non-reference node,

$$10 - 4 = \frac{V_o}{2} \longrightarrow V_o = 12$$

$$10 = \frac{V_1 - V_o}{1} \longrightarrow V_1 = 10 + V_o = \underline{\underline{22 \text{ V}}}$$

$$-4 = \frac{V_2 - V_o}{1} \longrightarrow V_2 = V_o - 4 = \underline{\underline{8 \text{ V}}}$$

### Chapter 19, Problem 30.

Find the  $h$  parameters for the networks in Fig. 19.88.

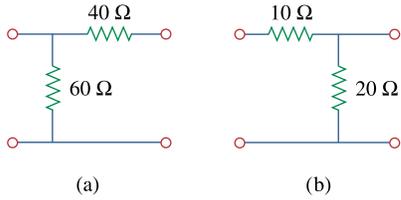


Figure 19.88

For Prob. 19.30.

### Chapter 19, Solution 30.

- (a) Convert to  $z$  parameters; then, convert to  $h$  parameters using Table 18.1.

$$\mathbf{z}_{11} = \mathbf{z}_{12} = \mathbf{z}_{21} = 60 \Omega, \quad \mathbf{z}_{22} = 100 \Omega$$

$$\Delta_z = \mathbf{z}_{11} \mathbf{z}_{22} - \mathbf{z}_{12} \mathbf{z}_{21} = 6000 - 3600 = 2400$$

$$\mathbf{h}_{11} = \frac{\Delta_z}{\mathbf{z}_{22}} = \frac{2400}{100} = 24, \quad \mathbf{h}_{12} = \frac{\mathbf{z}_{12}}{\mathbf{z}_{22}} = \frac{60}{100} = 0.6$$

$$\mathbf{h}_{21} = \frac{-\mathbf{z}_{21}}{\mathbf{z}_{22}} = -0.6, \quad \mathbf{h}_{22} = \frac{1}{\mathbf{z}_{22}} = 0.01$$

Thus,

$$\mathbf{[h]} = \underline{\underline{\begin{bmatrix} 24 \Omega & 0.6 \\ -0.6 & 0.01 \text{ S} \end{bmatrix}}}$$

- (b) Similarly,

$$\mathbf{z}_{11} = 30 \Omega \quad \mathbf{z}_{12} = \mathbf{z}_{21} = \mathbf{z}_{22} = 20 \Omega$$

$$\Delta_z = 600 - 400 = 200$$

$$\mathbf{h}_{11} = \frac{200}{20} = 10 \quad \mathbf{h}_{12} = \frac{20}{20} = 1$$

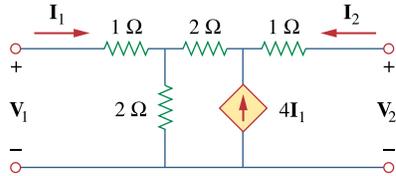
$$\mathbf{h}_{21} = -1 \quad \mathbf{h}_{22} = \frac{1}{20} = 0.05$$

Thus,

$$\mathbf{[h]} = \underline{\underline{\begin{bmatrix} 10 \Omega & 1 \\ -1 & 0.05 \text{ S} \end{bmatrix}}}$$

### Chapter 19, Problem 31.

Determine the hybrid parameters for the network in Fig. 19.89.

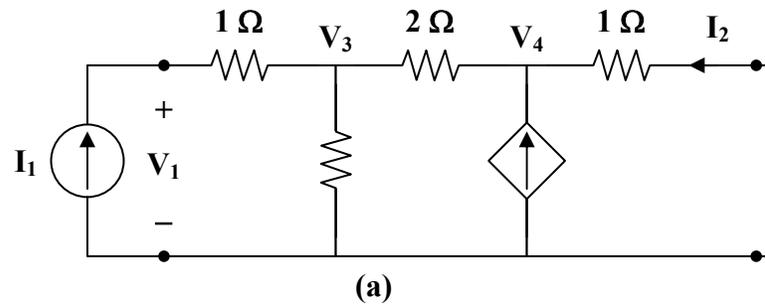


**Figure 19.89**

For Prob. 19.31.

### Chapter 19, Solution 31.

We get  $h_{11}$  and  $h_{21}$  by considering the circuit in Fig. (a).



At node 1,

$$I_1 = \frac{V_3}{2} + \frac{V_3 - V_4}{2} \longrightarrow 2I_1 = 2V_3 - V_4 \quad (1)$$

At node 2,

$$\frac{V_3 - V_4}{2} + 4I_1 = \frac{V_4}{1}$$

$$8I_1 = -V_3 + 3V_4 \longrightarrow 16I_1 = -2V_3 + 6V_4 \quad (2)$$

Adding (1) and (2),

$$18\mathbf{I}_1 = 5\mathbf{V}_4 \longrightarrow \mathbf{V}_4 = 3.6\mathbf{I}_1$$

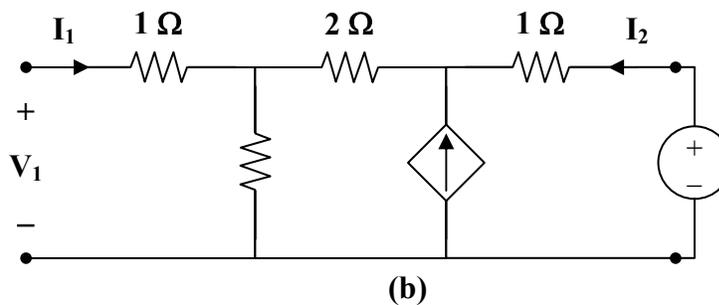
$$\mathbf{V}_3 = 3\mathbf{V}_4 - 8\mathbf{I}_1 = 2.8\mathbf{I}_1$$

$$\mathbf{V}_1 = \mathbf{V}_3 + \mathbf{I}_1 = 3.8\mathbf{I}_1$$

$$\mathbf{h}_{11} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = 3.8 \Omega$$

$$\mathbf{I}_2 = \frac{-\mathbf{V}_4}{1} = -3.6\mathbf{I}_1 \longrightarrow \mathbf{h}_{21} = \frac{\mathbf{I}_2}{\mathbf{I}_1} = -3.6$$

To get  $\mathbf{h}_{22}$  and  $\mathbf{h}_{12}$ , refer to the circuit in Fig. (b). The dependent current source can be replaced by an open circuit since  $4\mathbf{I}_1 = 0$ .



$$\mathbf{V}_1 = \frac{2}{2+2+1}\mathbf{V}_2 = \frac{2}{5}\mathbf{V}_2 \longrightarrow \mathbf{h}_{12} = \frac{\mathbf{V}_1}{\mathbf{V}_2} = 0.4$$

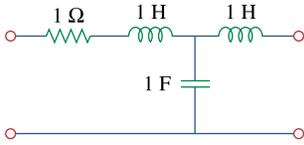
$$\mathbf{I}_2 = \frac{\mathbf{V}_2}{2+2+1} = \frac{\mathbf{V}_2}{5} \longrightarrow \mathbf{h}_{22} = \frac{\mathbf{I}_2}{\mathbf{V}_2} = \frac{1}{5} = 0.2 \text{ S}$$

Thus,

$$\underline{\underline{[\mathbf{h}] = \begin{bmatrix} 3.8 \Omega & 0.4 \\ -3.6 & 0.2 \text{ S} \end{bmatrix}}}$$

**Chapter 19, Problem 32.**

Find the  $h$  and  $g$  parameters of the two-port network in Fig. 19.90 as functions of  $s$ .

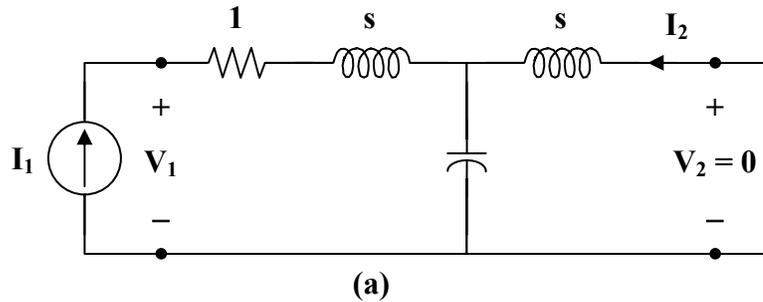


**Figure 19.90**

For Prob. 19.32.

**Chapter 19, Solution 32.**

(a) We obtain  $h_{11}$  and  $h_{21}$  by referring to the circuit in Fig. (a).



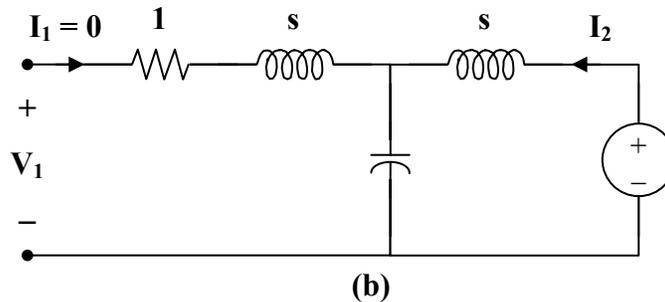
$$V_1 = \left(1 + s + s \parallel \frac{1}{s}\right) I_1 = \left(1 + s + \frac{s}{s^2 + 1}\right) I_1$$

$$h_{11} = \frac{V_1}{I_1} = s + 1 + \frac{s}{s^2 + 1}$$

By current division,

$$I_2 = \frac{-1/s}{s + 1/s} I_1 = \frac{-I_1}{s + 1} \longrightarrow h_{21} = \frac{I_2}{I_1} = \frac{-1}{s^2 + 1}$$

To get  $h_{22}$  and  $h_{12}$ , refer to Fig. (b).



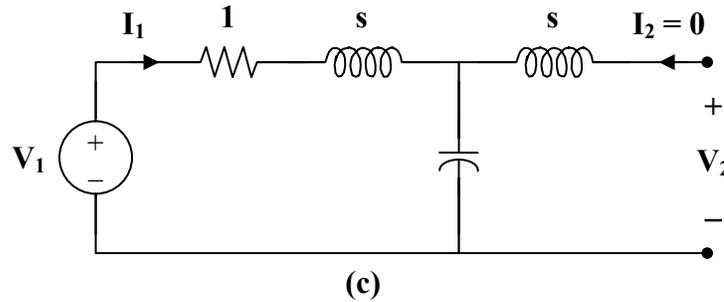
$$V_1 = \frac{1/s}{s + 1/s} V_2 = \frac{V_2}{s^2 + 1} \longrightarrow h_{12} = \frac{V_1}{V_2} = \frac{1}{s^2 + 1}$$

$$V_2 = \left(s + \frac{1}{s}\right) I_2 \longrightarrow h_{22} = \frac{I_2}{V_2} = \frac{1}{s + 1/s} = \frac{s}{s^2 + 1}$$

Thus,

$$[\mathbf{h}] = \begin{bmatrix} s + 1 + \frac{s}{s^2 + 1} & \frac{1}{s^2 + 1} \\ \frac{-1}{s^2 + 1} & \frac{s}{s^2 + 1} \end{bmatrix}$$

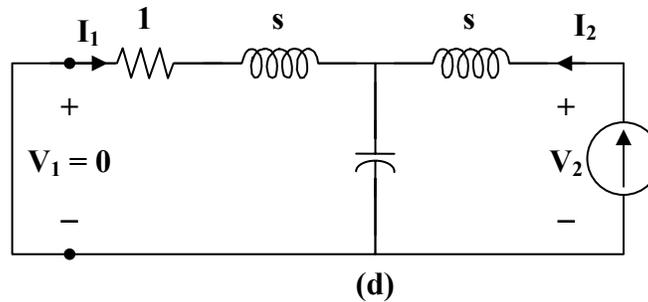
(b) To get  $g_{11}$  and  $g_{21}$ , refer to Fig. (c).



$$V_1 = \left(1 + s + \frac{1}{s}\right) I_1 \longrightarrow g_{11} = \frac{I_1}{V_1} = \frac{1}{1 + s + 1/s} = \frac{s}{s^2 + s + 1}$$

$$V_2 = \frac{1/s}{1 + s + 1/s} V_1 = \frac{V_1}{s^2 + s + 1} \longrightarrow g_{21} = \frac{V_2}{V_1} = \frac{1}{s^2 + s + 1}$$

To get  $g_{22}$  and  $g_{12}$ , refer to Fig. (d).



$$V_2 = \left(s + \frac{1}{s} \parallel (s+1)\right) I_2 = \left(s + \frac{(s+1)/s}{1 + s + 1/s}\right) I_2$$

$$g_{22} = \frac{V_2}{I_2} = s + \frac{s+1}{s^2 + s + 1}$$

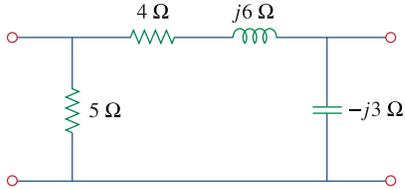
$$I_1 = \frac{-1/s}{1 + s + 1/s} I_2 = \frac{-I_2}{s^2 + s + 1} \longrightarrow g_{12} = \frac{I_1}{I_2} = \frac{-1}{s^2 + s + 1}$$

Thus,

$$[g] = \begin{bmatrix} \frac{s}{s^2 + s + 1} & \frac{-1}{s^2 + s + 1} \\ \frac{1}{s^2 + s + 1} & s + \frac{s+1}{s^2 + s + 1} \end{bmatrix}$$

**Chapter 19, Problem 33.**

Obtain the  $h$  parameters for the two-port of Fig. 19.91.

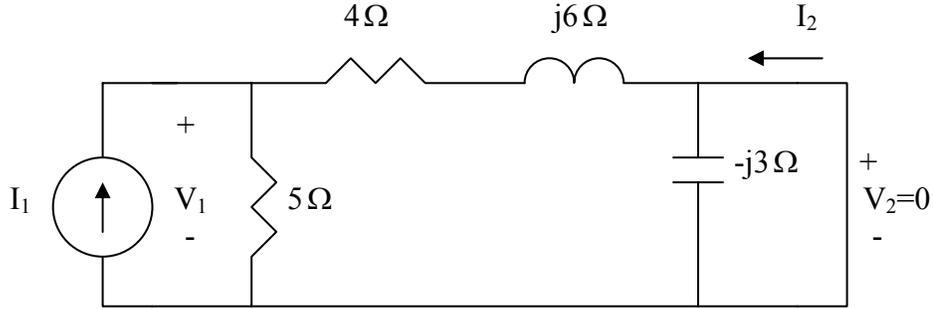


**Figure 19.91**

For Prob. 19.33.

**Chapter 19, Solution 33.**

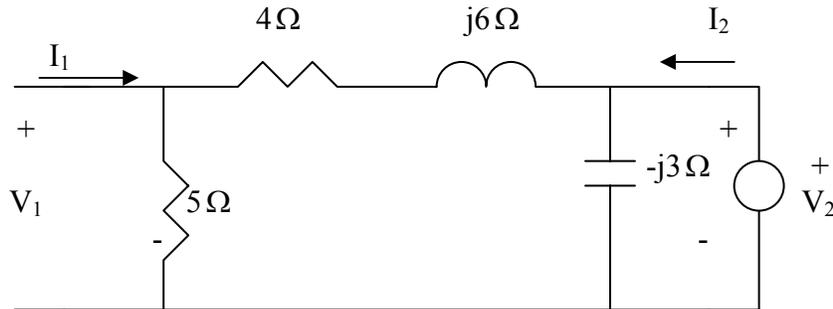
To get  $h_{11}$  and  $h_{21}$ , consider the circuit below.



$$V_1 = 5 // (4 + j6) I_1 = \frac{5(4 + j6)I_1}{9 + j6} \quad h_{11} = \frac{V_1}{I_1} = 3.0769 + j1.2821$$

$$\text{Also, } I_2 = -\frac{5}{9 + j6} I_1 \quad \longrightarrow \quad h_{21} = \frac{I_2}{I_1} = -0.3846 + j0.2564$$

To get  $h_{22}$  and  $h_{12}$ , consider the circuit below.



$$V_1 = \frac{5}{9 + j6} V_2 \quad \longrightarrow \quad h_{12} = \frac{V_1}{V_2} = \frac{5}{9 + j6} = 0.3846 - j0.2564$$

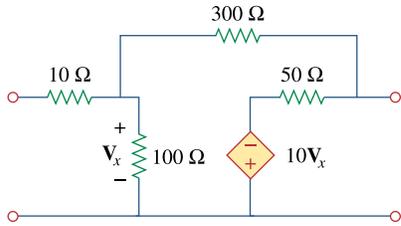
$$V_2 = -j3 // (9 + j6) I_2 \quad \longrightarrow \quad h_{22} = \frac{I_2}{V_2} = \frac{1}{-j3 // (9 + j6)} = \frac{9 + j3}{-j3(9 + j6)} = 0.0769 + j0.2821$$

Thus,

$$[h] = \begin{bmatrix} 3.077 + j1.2821 & 0.3846 - j0.2564 \\ -0.3846 + j0.2564 & 0.0769 + j0.2821 \end{bmatrix}$$

**Chapter 19, Problem 34.**

Obtain the  $h$  and  $g$  parameters of the two-port in Fig. 19.92.

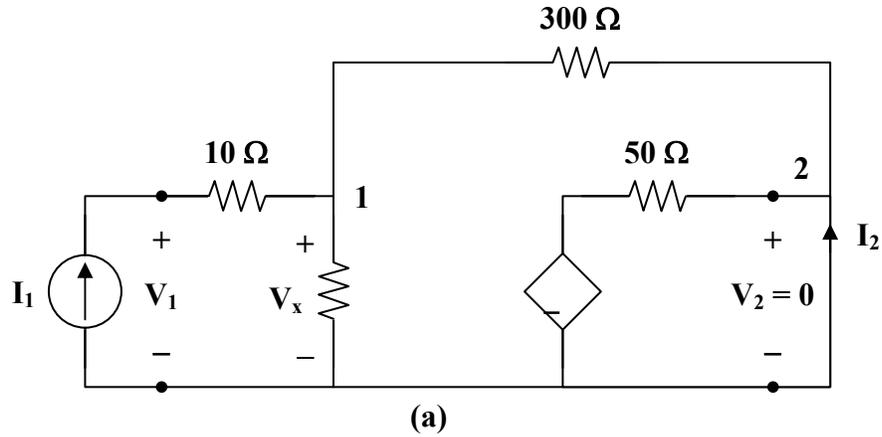


**Figure 19.92**

For Prob. 19.34.

**Chapter 19, Solution 34.**

Refer to Fig. (a) to get  $h_{11}$  and  $h_{21}$ .



At node 1,

$$I_1 = \frac{V_x}{100} + \frac{V_x - 0}{300} \longrightarrow 300I_1 = 4V_x \quad (1)$$

$$V_x = \frac{300}{4}I_1 = 75I_1$$

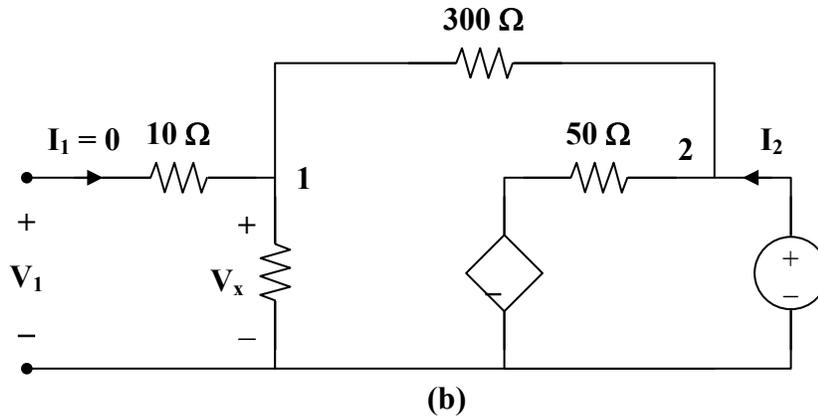
But  $V_1 = 10I_1 + V_x = 85I_1 \longrightarrow h_{11} = \frac{V_1}{I_1} = 85 \Omega$

At node 2,

$$\mathbf{I}_2 = \frac{0 + 10\mathbf{V}_x}{50} - \frac{\mathbf{V}_x}{300} = \frac{\mathbf{V}_x}{5} - \frac{\mathbf{V}_x}{300} = \frac{75}{5}\mathbf{I}_1 - \frac{75}{300}\mathbf{I}_1 = 14.75\mathbf{I}_1$$

$$\mathbf{h}_{21} = \frac{\mathbf{I}_2}{\mathbf{I}_1} = 14.75$$

To get  $\mathbf{h}_{22}$  and  $\mathbf{h}_{12}$ , refer to Fig. (b).



At node 2,

$$\mathbf{I}_2 = \frac{\mathbf{V}_2}{400} + \frac{\mathbf{V}_2 + 10\mathbf{V}_x}{50} \longrightarrow 400\mathbf{I}_2 = 9\mathbf{V}_2 + 80\mathbf{V}_x$$

But

$$\mathbf{V}_x = \frac{100}{400}\mathbf{V}_2 = \frac{\mathbf{V}_2}{4}$$

Hence,

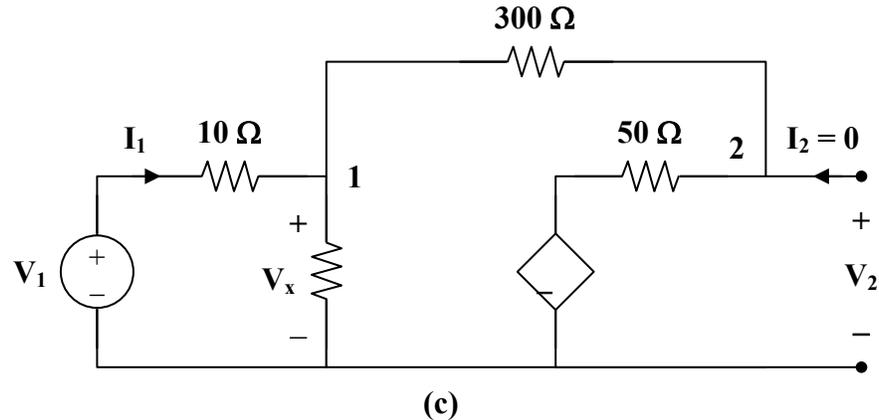
$$400\mathbf{I}_2 = 9\mathbf{V}_2 + 20\mathbf{V}_2 = 29\mathbf{V}_2$$

$$\mathbf{h}_{22} = \frac{\mathbf{I}_2}{\mathbf{V}_2} = \frac{29}{400} = 0.0725 \text{ S}$$

$$\mathbf{V}_1 = \mathbf{V}_x = \frac{\mathbf{V}_2}{4} \longrightarrow \mathbf{h}_{12} = \frac{\mathbf{V}_1}{\mathbf{V}_2} = \frac{1}{4} = 0.25$$

$$\mathbf{[h]} = \underline{\underline{\begin{bmatrix} 85 \Omega & 0.25 \\ 14.75 & 0.0725 \text{ S} \end{bmatrix}}}$$

To get  $g_{11}$  and  $g_{21}$ , refer to Fig. (c).



At node 1,

$$I_1 = \frac{V_x}{100} + \frac{V_x + 10V_x}{350} \longrightarrow 350I_1 = 14.5V_x \quad (2)$$

But  $I_1 = \frac{V_1 - V_x}{10} \longrightarrow 10I_1 = V_1 - V_x$

or  $V_x = V_1 - 10I_1 \quad (3)$

Substituting (3) into (2) gives

$$350I_1 = 14.5V_1 - 145I_1 \longrightarrow 495I_1 = 14.5V_1$$

$$g_{11} = \frac{I_1}{V_1} = \frac{14.5}{495} = 0.02929 \text{ S}$$

At node 2,

$$V_2 = (50) \left( \frac{11}{350} V_x \right) - 10V_x = -8.4286V_x$$

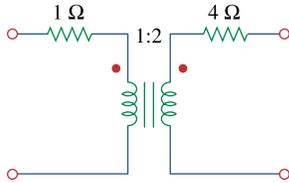
$$= -8.4286V_1 + 84.286I_1 = -8.4286V_1 + (84.286) \left( \frac{14.5}{495} \right) V_1$$

$$V_2 = -5.96V_1 \longrightarrow g_{21} = \frac{V_2}{V_1} = -5.96$$



**Chapter 19, Problem 35.**

Determine the  $h$  parameters for the network in Fig. 19.93.

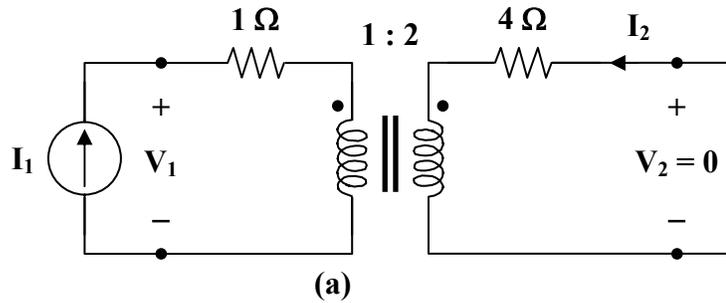


**Figure 19.93**

For Prob. 19.35.

**Chapter 19, Solution 35.**

To get  $h_{11}$  and  $h_{21}$  consider the circuit in Fig. (a).

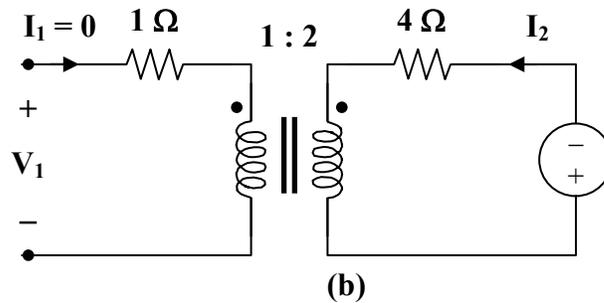


$$Z_R = \frac{4}{n^2} = \frac{4}{4} = 1$$

$$V_1 = (1+1)I_1 = 2I_1 \longrightarrow h_{11} = \frac{V_1}{I_1} = 2 \Omega$$

$$\frac{I_1}{I_2} = \frac{-N_2}{N_1} = -2 \longrightarrow h_{21} = \frac{I_2}{I_1} = \frac{-1}{2} = -0.5$$

To get  $h_{22}$  and  $h_{12}$ , refer to Fig. (b).



Since  $I_1 = 0$ ,  $I_2 = 0$ .

Hence,  $h_{22} = 0$ .

At the terminals of the transformer, we have  $V_1$  and  $V_2$  which are related as

$$\frac{V_2}{V_1} = \frac{N_2}{N_1} = n = 2 \longrightarrow h_{12} = \frac{V_1}{V_2} = \frac{1}{2} = 0.5$$

Thus,

$$[\mathbf{h}] = \begin{bmatrix} 2 \Omega & 0.5 \\ -0.5 & 0 \end{bmatrix}$$

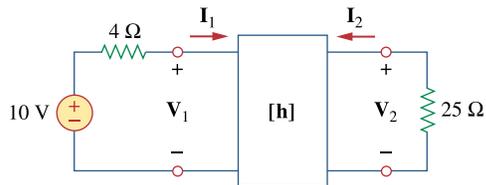
### Chapter 19, Problem 36.

For the two-port in Fig. 19.94,

$$[\mathbf{h}] \begin{bmatrix} 16\Omega & 3 \\ -2 & 0.01\text{S} \end{bmatrix}$$

Find:

- (a)  $V_2 / V_1$       (b)  $I_2 / I_1$   
(c)  $I_1 / V_1$       (d)  $V_2 / I_1$

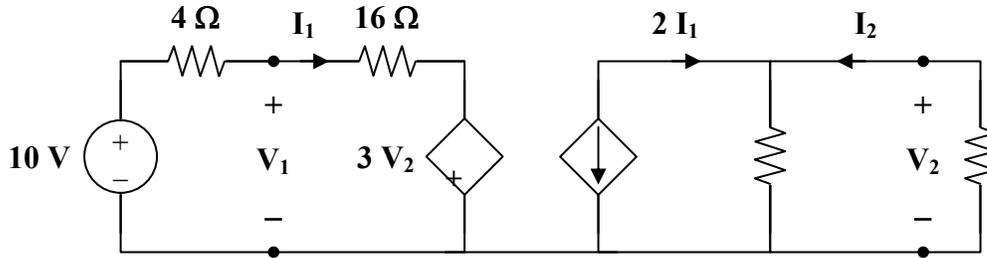


**Figure 19.94**

For Prob. 19.36.

**Chapter 19, Solution 36.**

We replace the two-port by its equivalent circuit as shown below.



$$100 \parallel 25 = 20 \Omega$$

$$V_2 = (20)(2I_1) = 40I_1 \quad (1)$$

$$-10 + 20I_1 + 3V_2 = 0$$

$$10 = 20I_1 + (3)(40I_1) = 140I_1$$

$$I_1 = \frac{1}{14}, \quad V_2 = \frac{40}{14}$$

$$V_1 = 16I_1 + 3V_2 = \frac{136}{14}$$

$$I_2 = \left(\frac{100}{125}\right)(2I_1) = \frac{-8}{70}$$

$$(a) \quad \frac{V_2}{V_1} = \frac{40}{136} = \underline{\underline{0.2941}}$$

$$(b) \quad \frac{I_2}{I_1} = \underline{\underline{-1.6}}$$

$$(c) \quad \frac{I_1}{V_1} = \frac{1}{136} = \underline{\underline{7.353 \times 10^{-3} \text{ S}}}$$

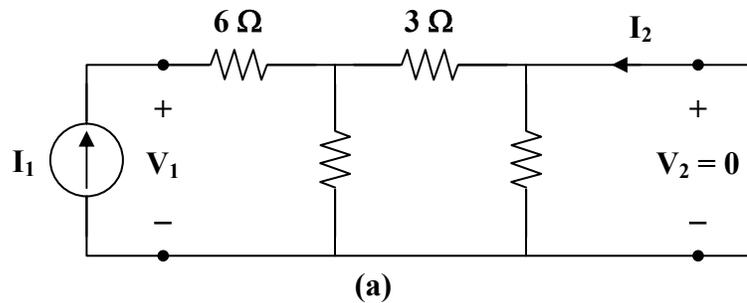
$$(d) \quad \frac{V_2}{I_1} = \frac{40}{1} = \underline{\underline{40 \Omega}}$$

**Chapter 19, Problem 37.**

The input port of the circuit in Fig. 19.79 is connected to a 10-V dc voltage source while the output port is terminated by a 5- $\Omega$  resistor. Find the voltage across the 5- $\Omega$  resistor by using  $h$  parameters of the circuit. Confirm your result by using direct circuit analysis.

**Chapter 19, Solution 37.**

- (a) We first obtain the  $h$  parameters. To get  $h_{11}$  and  $h_{21}$  refer to Fig. (a).

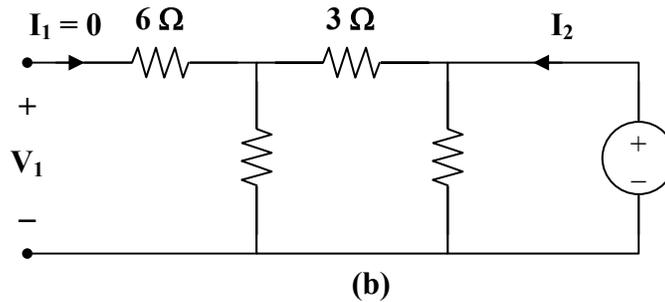


$$3 \parallel 6 = 2$$

$$V_1 = (6 + 2)I_1 = 8I_1 \quad \longrightarrow \quad h_{11} = \frac{V_1}{I_1} = 8 \Omega$$

$$I_2 = \frac{-6}{3+6}I_1 = \frac{-2}{3}I_1 \quad \longrightarrow \quad h_{21} = \frac{I_2}{I_1} = \frac{-2}{3}$$

To get  $h_{22}$  and  $h_{12}$ , refer to the circuit in Fig. (b).



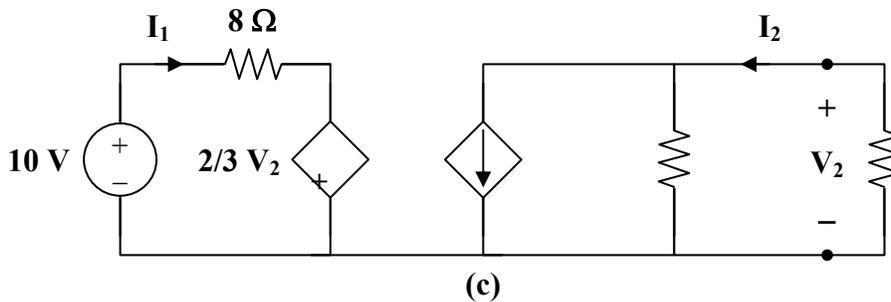
$$3 \parallel 9 = \frac{9}{4}$$

$$V_2 = \frac{9}{4} I_2 \longrightarrow h_{22} = \frac{I_2}{V_2} = \frac{4}{9}$$

$$V_1 = \frac{6}{6+3} V_2 = \frac{2}{3} V_2 \longrightarrow h_{12} = \frac{V_1}{V_2} = \frac{2}{3}$$

$$[h] = \begin{bmatrix} 8 \Omega & \frac{2}{3} \\ -\frac{2}{3} & \frac{4}{9} \text{ S} \end{bmatrix}$$

The equivalent circuit of the given circuit is shown in Fig. (c).



$$8I_1 + \frac{2}{3}V_2 = 10 \quad (1)$$

$$V_2 = \frac{2}{3}I_1 \left( 5 \parallel \frac{9}{4} \right) = \frac{2}{3}I_1 \left( \frac{45}{29} \right) = \frac{30}{29}I_1$$

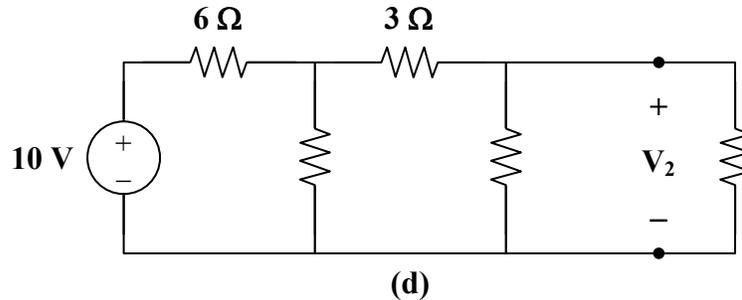
$$I_1 = \frac{29}{30}V_2 \quad (2)$$

Substituting (2) into (1),

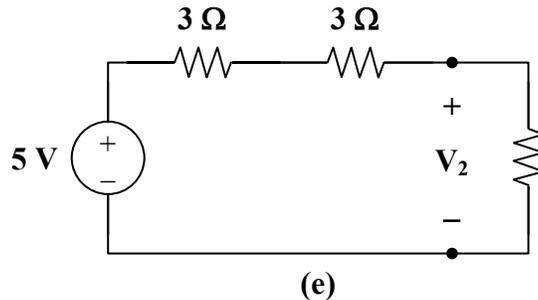
$$(8)\left(\frac{29}{30}\right)V_2 + \frac{2}{3}V_2 = 10$$

$$V_2 = \frac{300}{252} = \underline{\underline{1.19 \text{ V}}}$$

(b) By direct analysis, refer to Fig.(d).



Transform the 10-V voltage source to a  $\frac{10}{6}$ -A current source. Since  $6 \parallel 6 = 3 \Omega$ , we combine the two 6- $\Omega$  resistors in parallel and transform the current source back to  $\frac{10}{6} \times 3 = 5 \text{ V}$  voltage source shown in Fig. (e).



$$3 \parallel 5 = \frac{(3)(5)}{8} = \frac{15}{8}$$

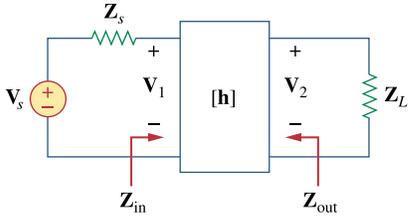
$$V_2 = \frac{15/8}{6 + 15/8} (5) = \frac{75}{63} = \underline{\underline{1.1905 \text{ V}}}$$

**Chapter 19, Problem 38.**

The  $h$  parameters of the two-port of Fig. 19.95 are:

$$[\mathbf{h}] = \begin{bmatrix} 600\Omega & 0.04 \\ 30 & 2\text{mS} \end{bmatrix}$$

Given the  $Z_s = 2\text{k}\Omega$  and  $Z_L = 400\Omega$ , find  $Z_{in}$  and  $Z_{out}$ .



**Figure 19.95**

For Prob. 19.38.

**Chapter 19, Solution 38.**

From eq. (19.75),

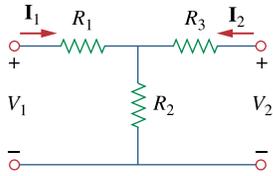
$$Z_{in} = h_{ie} - \frac{h_{re}h_{fe}R_L}{1 + h_{oe}R_L} = h_{i1} - \frac{h_{12}h_{21}R_L}{1 + h_{22}R_L} = 600 - \frac{0.04 \times 30 \times 400}{1 + 2 \times 10^{-3} \times 400} = \underline{333.33 \Omega}$$

From eq. (19.79),

$$Z_{out} = \frac{R_s + h_{ie}}{(R_s + h_{ie})h_{oe} - h_{re}h_{fe}} = \frac{R_s + h_{i1}}{(R_s + h_{i1})h_{22} - h_{21}h_{12}} = \frac{2,000 + 600}{2600 \times 2 \times 10^{-3} - 30 \times 0.04} = \underline{650 \Omega}$$

**Chapter 19, Problem 39.**

Obtain the  $g$  parameters for the wye circuit of Fig. 19.96.

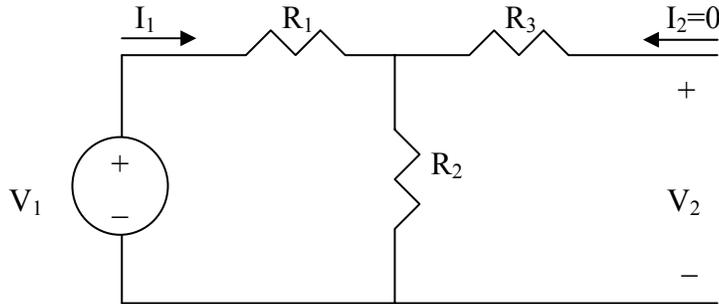


**Figure 19.96**

For Prob. 19.39.

**Chapter 19, Solution 39.**

We obtain  $g_{11}$  and  $g_{21}$  using the circuit below.

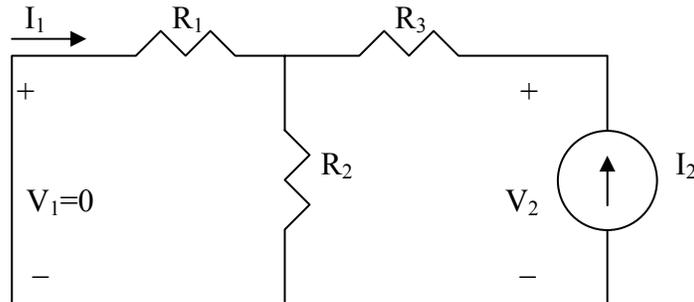


$$I_1 = \frac{V_1}{R_1 + R_2} \longrightarrow g_{11} = \frac{I_1}{V_1} = \frac{1}{R_1 + R_2}$$

By voltage division,

$$V_2 = \frac{R_2}{R_1 + R_2} V_1 \longrightarrow g_{21} = \frac{V_2}{V_1} = \frac{R_2}{R_1 + R_2}$$

We obtain  $g_{12}$  and  $g_{22}$  using the circuit below.



By current division,

$$I_1 = -\frac{R_2}{R_1 + R_2} I_2 \longrightarrow g_{12} = \frac{I_1}{I_2} = -\frac{R_2}{R_1 + R_2}$$

Also,

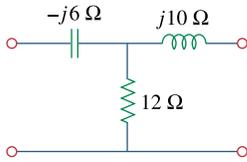
$$V_2 = I_2 (R_3 + R_1 // R_2) = I_2 \left( R_3 + \frac{R_1 R_2}{R_1 + R_2} \right) \quad g_{22} = \frac{V_2}{I_2} = R_3 + \frac{R_1 R_2}{R_1 + R_2}$$

$$g_{11} = \frac{1}{R_1 + R_2}, g_{12} = -\frac{R_2}{R_1 + R_2}$$

$$g_{21} = \frac{R_2}{R_1 + R_2}, g_{22} = R_3 + \frac{R_1 R_2}{R_1 + R_2}$$

**Chapter 19, Problem 40.**

Find the  $g$  parameters for the circuit in Fig. 19.97.

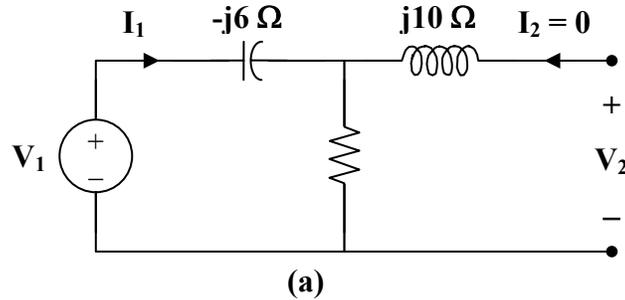


**Figure 19.97**

For Prob. 19.40.

**Chapter 19, Solution 40.**

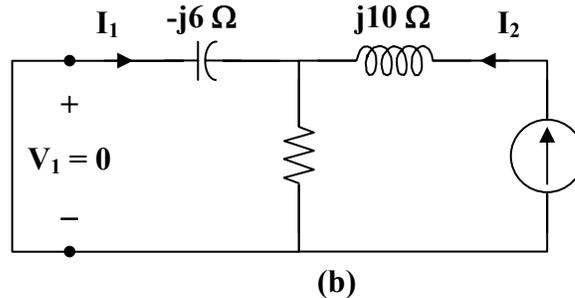
To get  $g_{11}$  and  $g_{21}$ , consider the circuit in Fig. (a).



$$V_1 = (12 - j6)I_1 \longrightarrow g_{11} = \frac{I_1}{V_1} = \frac{1}{12 - j6} = 0.0667 + j0.0333 \text{ S}$$

$$g_{21} = \frac{V_2}{V_1} = \frac{12I_1}{(12 - j6)I_1} = \frac{2}{2 - j} = 0.8 + j0.4$$

To get  $g_{12}$  and  $g_{22}$ , consider the circuit in Fig. (b).



$$I_1 = \frac{-12}{12 - j6} I_2 \longrightarrow g_{12} = \frac{I_1}{I_2} = \frac{-12}{12 - j6} = -g_{21} = -0.8 - j0.4$$

$$V_2 = (j10 + 12 \parallel -j6)I_2$$

$$g_{22} = \frac{V_2}{I_2} = j10 + \frac{(12)(-j6)}{12 - j6} = 2.4 + j5.2 \text{ } \Omega$$

$$[\mathbf{g}] = \begin{bmatrix} 0.0667 + j0.0333 \text{ S} & -0.8 - j0.4 \\ 0.8 + j0.4 & 2.4 + j5.2 \text{ } \Omega \end{bmatrix}$$

### Chapter 19, Problem 41.

For the two-port in Fig. 19.75, show that

$$\frac{\mathbf{I}_2}{\mathbf{I}_1} = \frac{-\mathbf{g}_{21}}{\mathbf{g}_{11}\mathbf{Z}_L + \Delta_g}$$

$$\frac{\mathbf{V}_2}{\mathbf{V}_s} = \frac{\mathbf{g}_{21}\mathbf{Z}_L}{(1 + \mathbf{g}_{11}\mathbf{Z}_s)(\mathbf{g}_{22} + \mathbf{Z}_L) - \mathbf{g}_{21}\mathbf{g}_{12}\mathbf{Z}_s}$$

where  $\Delta_g$  is the determinant of  $[\mathbf{g}]$  matrix.

## Chapter 19, Solution 41.

For the g parameters

$$\mathbf{I}_1 = \mathbf{g}_{11} \mathbf{V}_1 + \mathbf{g}_{12} \mathbf{I}_2 \quad (1)$$

$$\mathbf{V}_2 = \mathbf{g}_{21} \mathbf{V}_1 + \mathbf{g}_{22} \mathbf{I}_2 \quad (2)$$

But  $\mathbf{V}_1 = \mathbf{V}_s - \mathbf{I}_1 \mathbf{Z}_s$  and

$$\mathbf{V}_2 = -\mathbf{I}_2 \mathbf{Z}_L = \mathbf{g}_{21} \mathbf{V}_1 + \mathbf{g}_{22} \mathbf{I}_2$$

$$0 = \mathbf{g}_{21} \mathbf{V}_1 + (\mathbf{g}_{22} + \mathbf{Z}_L) \mathbf{I}_2$$

or 
$$\mathbf{V}_1 = \frac{-(\mathbf{g}_{22} + \mathbf{Z}_L)}{\mathbf{g}_{21}} \mathbf{I}_2$$

Substituting this into (1),

$$\mathbf{I}_1 = \frac{(\mathbf{g}_{22} \mathbf{g}_{11} + \mathbf{Z}_L \mathbf{g}_{11} - \mathbf{g}_{21} \mathbf{g}_{12})}{-\mathbf{g}_{21}} \mathbf{I}_2$$

or 
$$\frac{\mathbf{I}_2}{\mathbf{I}_1} = \frac{-\mathbf{g}_{21}}{\mathbf{g}_{11} \mathbf{Z}_L + \Delta_g}$$

Also, 
$$\begin{aligned} \mathbf{V}_2 &= \mathbf{g}_{21} (\mathbf{V}_s - \mathbf{I}_1 \mathbf{Z}_s) + \mathbf{g}_{22} \mathbf{I}_2 \\ &= \mathbf{g}_{21} \mathbf{V}_s - \mathbf{g}_{21} \mathbf{Z}_s \mathbf{I}_1 + \mathbf{g}_{22} \mathbf{I}_2 \\ &= \mathbf{g}_{21} \mathbf{V}_s + \mathbf{Z}_s (\mathbf{g}_{11} \mathbf{Z}_L + \Delta_g) \mathbf{I}_2 + \mathbf{g}_{22} \mathbf{I}_2 \end{aligned}$$

But 
$$\mathbf{I}_2 = \frac{-\mathbf{V}_2}{\mathbf{Z}_L}$$

$$\mathbf{V}_2 = \mathbf{g}_{21} \mathbf{V}_s - [\mathbf{g}_{11} \mathbf{Z}_s \mathbf{Z}_L + \Delta_g \mathbf{Z}_s + \mathbf{g}_{22}] \left[ \frac{\mathbf{V}_2}{\mathbf{Z}_L} \right]$$

$$\frac{\mathbf{V}_2 [\mathbf{Z}_L + \mathbf{g}_{11} \mathbf{Z}_s \mathbf{Z}_L + \Delta_g \mathbf{Z}_s + \mathbf{g}_{22}]}{\mathbf{Z}_L} = \mathbf{g}_{21} \mathbf{V}_s$$

$$\frac{\mathbf{V}_2}{\mathbf{V}_s} = \frac{\mathbf{g}_{21} \mathbf{Z}_L}{\mathbf{Z}_L + \mathbf{g}_{11} \mathbf{Z}_s \mathbf{Z}_L + \Delta_g \mathbf{Z}_s + \mathbf{g}_{22}}$$

$$\frac{\mathbf{V}_2}{\mathbf{V}_s} = \frac{\mathbf{g}_{21} \mathbf{Z}_L}{\mathbf{Z}_L + \mathbf{g}_{11} \mathbf{Z}_s \mathbf{Z}_L + \mathbf{g}_{11} \mathbf{g}_{22} \mathbf{Z}_s - \mathbf{g}_{21} \mathbf{g}_{12} \mathbf{Z}_s + \mathbf{g}_{22}}$$

$$\frac{\mathbf{V}_2}{\mathbf{V}_s} = \frac{\mathbf{g}_{21} \mathbf{Z}_L}{(1 + \mathbf{g}_{11} \mathbf{Z}_s)(\mathbf{g}_{22} + \mathbf{Z}_L) - \mathbf{g}_{12} \mathbf{g}_{21} \mathbf{Z}_s}$$

### Chapter 19, Problem 42.

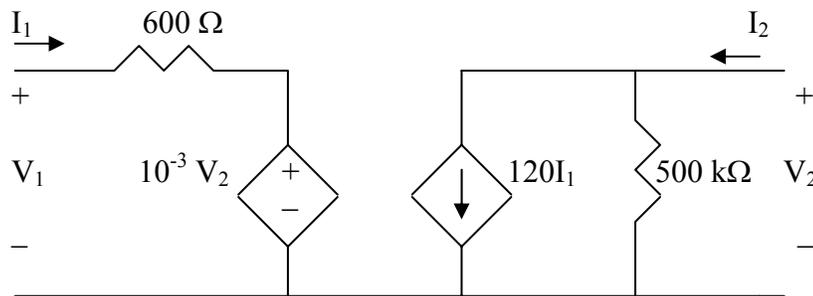
The  $h$  parameters of a two-port device are given by

$$\mathbf{h}_{11} = 600\Omega, \quad \mathbf{h}_{12} = 10^{-3}, \quad \mathbf{h}_{21} = 120, \quad \mathbf{h}_{22} = 2 \times 10^{-6} \text{ S}$$

Draw a circuit model of the device including the value of each element.

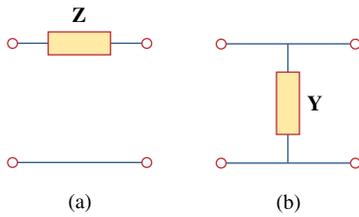
### Chapter 19, Solution 42.

With the help of Fig. 19.20, we obtain the circuit model below.



**Chapter 19, Problem 43.**

Find the transmission parameters for the single-element two-port networks in Fig. 19.98.

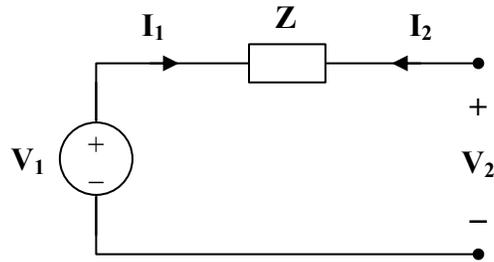


**Figure 19.98**

For Prob. 19.43.

**Chapter 19, Solution 43.**

(a) To find **A** and **C**, consider the network in Fig. (a).

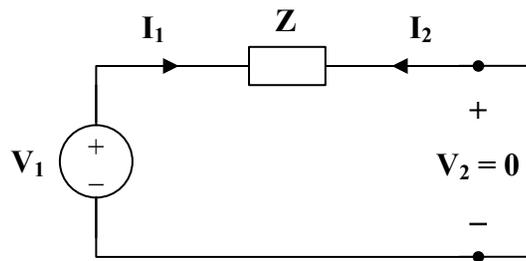


(a)

$$V_1 = V_2 \longrightarrow A = \frac{V_1}{V_2} = 1$$

$$I_1 = 0 \longrightarrow C = \frac{I_1}{V_2} = 0$$

To get **B** and **D**, consider the circuit in Fig. (b).



(b)

$$V_1 = ZI_1, \quad I_2 = -I_1$$

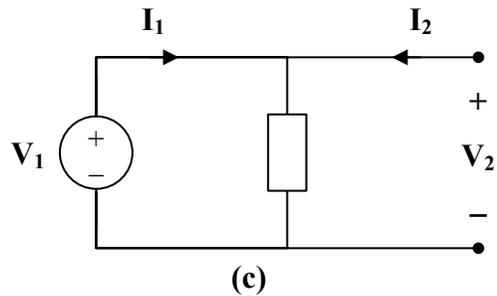
$$B = \frac{-V_1}{I_2} = \frac{-ZI_1}{-I_1} = Z$$

$$D = \frac{-I_1}{I_2} = 1$$

Hence,

$$[T] = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix}$$

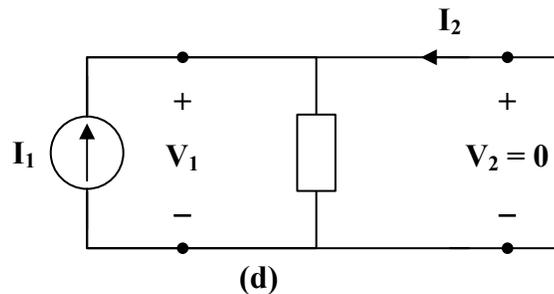
(b) To find **A** and **C**, consider the circuit in Fig. (c).



$$V_1 = V_2 \longrightarrow A = \frac{V_1}{V_2} = 1$$

$$V_1 = ZI_1 = V_2 \longrightarrow C = \frac{I_1}{V_2} = \frac{1}{Z} = Y$$

To get **B** and **D**, refer to the circuit in Fig.(d).



$$V_1 = V_2 = 0 \qquad I_2 = -I_1$$

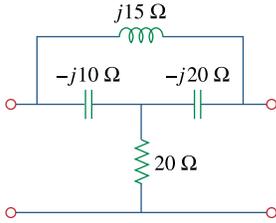
$$B = \frac{-V_1}{I_2} = 0, \qquad D = \frac{-I_1}{I_2} = 1$$

Thus,

$$\underline{[T]} = \underline{\begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix}}$$

**Chapter 19, Problem 44.**

Determine the transmission parameters of the circuit in Fig. 19.99.

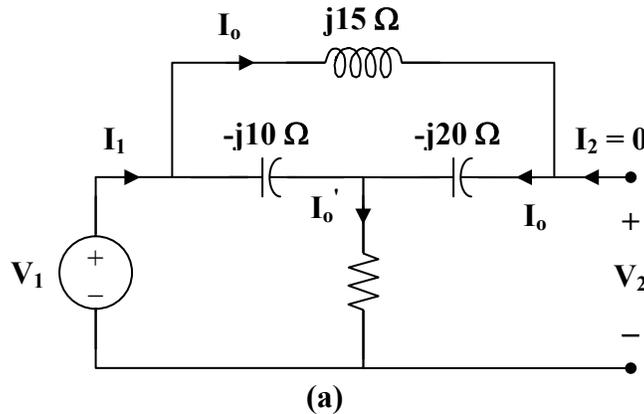


**Figure 19.99**

For Prob. 19.44.

**Chapter 19, Solution 44.**

To determine **A** and **C**, consider the circuit in Fig.(a).



$$V_1 = [20 + (-j10) \parallel (j15 - j20)] I_1$$

$$V_1 = \left[ 20 + \frac{(-j10)(-j5)}{-j15} \right] I_1 = \left[ 20 - j\frac{10}{3} \right] I_1$$

$$I_0' = I_1$$

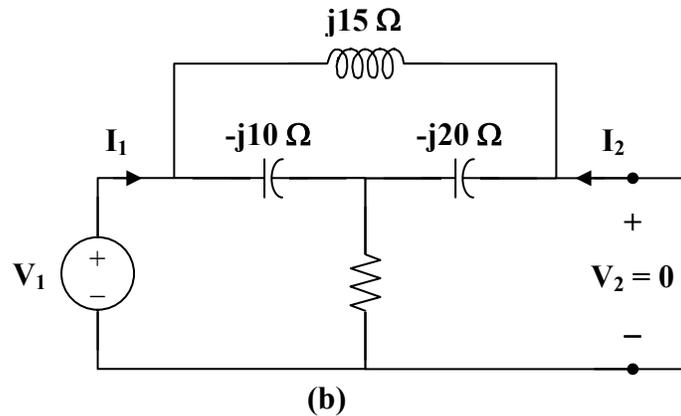
$$I_0 = \left( \frac{-j10}{-j10 - j5} \right) I_1 = \left( \frac{2}{3} \right) I_1$$

$$V_2 = (-j20) I_0 + 20 I_0' = -j\frac{40}{3} I_1 + 20 I_1 = \left( 20 - j\frac{40}{3} \right) I_1$$

$$\mathbf{A} = \frac{\mathbf{V}_1}{\mathbf{V}_2} = \frac{(20 - j10/3)\mathbf{I}_1}{\left(20 - j\frac{40}{3}\right)\mathbf{I}_1} = 0.7692 + j0.3461$$

$$\mathbf{C} = \frac{\mathbf{I}_1}{\mathbf{V}_2} = \frac{1}{20 - j\frac{40}{3}} = 0.03461 + j0.023$$

To find **B** and **D**, consider the circuit in Fig. (b).

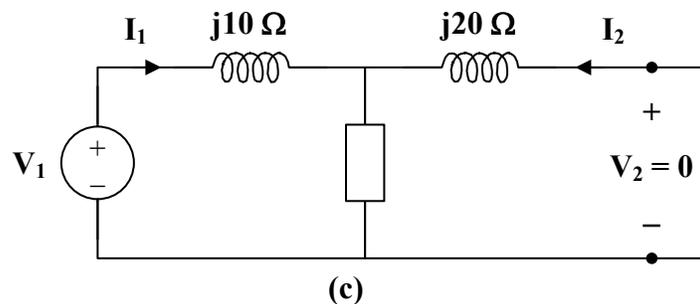


We may transform the  $\Delta$  subnetwork to a T as shown in Fig. (c).

$$\mathbf{Z}_1 = \frac{(j15)(-j10)}{j15 - j10 - j20} = j10$$

$$\mathbf{Z}_2 = \frac{(-j10)(-j20)}{-j15} = -j\frac{40}{3}$$

$$\mathbf{Z}_3 = \frac{(j15)(-j20)}{-j15} = j20$$



$$-\mathbf{I}_2 = \frac{20 - j40/3}{20 - j40/3 + j20} \mathbf{I}_1 = \frac{3 - j2}{3 + j} \mathbf{I}_1$$

$$\mathbf{D} = \frac{-\mathbf{I}_1}{\mathbf{I}_2} = \frac{3 + j}{3 - j2} = 0.5385 + j0.6923$$

$$\mathbf{V}_1 = \left[ j10 + \frac{(j20)(20 - j40/3)}{20 - j40/3 + j20} \right] \mathbf{I}_1$$

$$\mathbf{V}_1 = [j10 + 2(9 + j7)] \mathbf{I}_1 = j \mathbf{I}_1 (24 - j18)$$

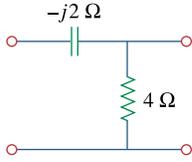
$$\mathbf{B} = \frac{-\mathbf{V}_1}{\mathbf{I}_2} = \frac{-j \mathbf{I}_1 (24 - j18)}{\frac{-(3 - j2)}{3 + j} \mathbf{I}_1} = \frac{6}{13} (-15 + j55)$$

$$\mathbf{B} = -6.923 + j25.385 \Omega$$

$$\underline{\underline{[\mathbf{T}] = \begin{bmatrix} \mathbf{0.7692 + j0.3461} & -6.923 + j25.385 \Omega \\ \mathbf{0.03461 + j0.023 S} & \mathbf{0.5385 + j0.6923} \end{bmatrix}}}$$

**Chapter 19, Problem 45.**

Find the **ABCD** parameters for the circuit in Fig. 19.100.

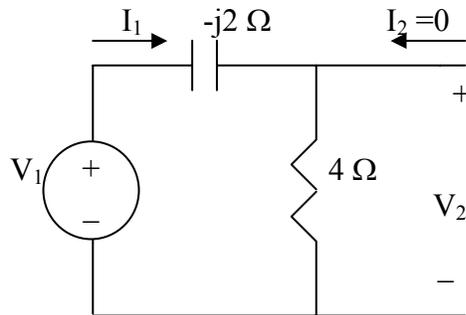


**Figure 19.100**

For Prob. 19.45.

### Chapter 19, Solution 45.

To determine A and C, consider the circuit below.

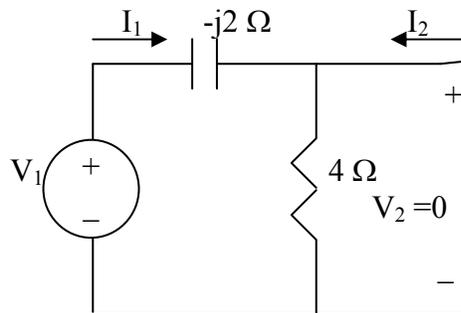


$$V_1 = (4 - j2)I_1, \quad V_2 = 4I_1$$

$$A = \frac{V_1}{V_2} = \frac{4 - j2}{4} = 1 - j0.5$$

$$C = \frac{I_1}{V_2} = \frac{I_1}{4I_1} = 0.25$$

To determine B and D, consider the circuit below.



The 4- $\Omega$  resistor is short-circuited. Hence,

$$I_2 = -I_1, \quad D = -\frac{I_1}{I_2} = 1$$

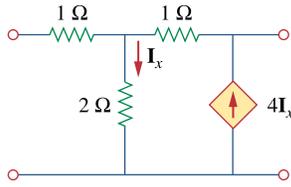
$$V_1 = -j2I_1 = j2I_2 \quad B = -\frac{V_1}{I_2} = -\frac{j2I_2}{I_2} = -j2\Omega$$

Hence,

$$\underline{\underline{\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 - j0.5 & -j2\Omega \\ 0.25\text{S} & 1 \end{bmatrix} = \begin{bmatrix} 1 - j0.5 & -j2\Omega \\ 0.25\text{S} & 1 \end{bmatrix}}}$$

**Chapter 19, Problem 46.**

Find the transmission parameters for the circuit in Fig. 19.101.

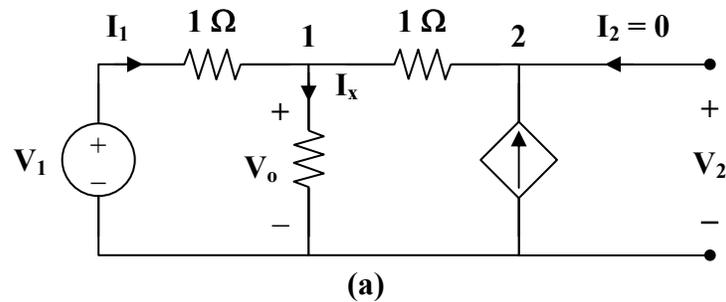


**Figure 19.101**

For Prob. 19.46.

**Chapter 19, Solution 46.**

To get **A** and **C**, refer to the circuit in Fig.(a).



At node 1,

$$I_1 = \frac{V_o}{2} + \frac{V_o - V_2}{1} \longrightarrow 2I_1 = 3V_o - 2V_2 \quad (1)$$

At node 2,

$$\frac{V_o - V_2}{1} = 4I_x = \frac{4V_o}{2} = 2V_o \longrightarrow V_o = -V_2 \quad (2)$$

From (1) and (2),

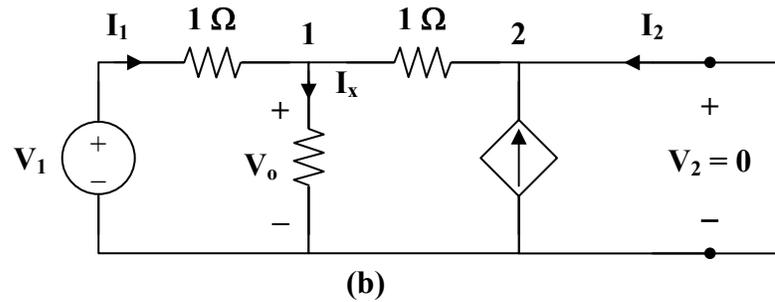
$$2I_1 = -5V_2 \longrightarrow C = \frac{I_1}{V_2} = \frac{-5}{2} = -2.5 \text{ S}$$

But 
$$I_1 = \frac{V_1 - V_o}{1} = V_1 + V_2$$

$$-2.5V_2 = V_1 + V_2 \longrightarrow V_1 = -3.5V_2$$

$$A = \frac{V_1}{V_2} = -3.5$$

To get **B** and **D**, consider the circuit in Fig. (b).



At node 1,

$$I_1 = \frac{V_o}{2} + \frac{V_o}{1} \longrightarrow 2I_1 = 3V_o \quad (3)$$

At node 2,

$$I_2 + \frac{V_o}{1} + 4I_x = 0$$

$$-I_2 = V_o + 2V_o = 0 \longrightarrow I_2 = -3V_o \quad (4)$$

Adding (3) and (4),

$$2I_1 + I_2 = 0 \longrightarrow I_1 = -0.5I_2 \quad (5)$$

$$D = \frac{-I_1}{I_2} = 0.5$$

But

$$I_1 = \frac{V_1 - V_o}{1} \longrightarrow V_1 = I_1 + V_o \quad (6)$$

Substituting (5) and (4) into (6),

$$V_1 = \frac{-1}{2}I_2 + \frac{-1}{3}I_2 = \frac{-5}{6}I_2$$

$$B = \frac{-V_1}{I_2} = \frac{5}{6} = 0.8333 \Omega$$

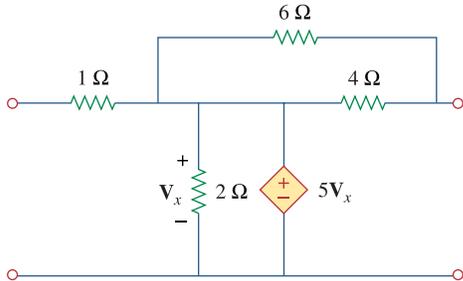
Thus,

$$[T] = \begin{bmatrix} -3.5 & 0.8333 \Omega \\ -2.5 \text{ S} & -0.5 \end{bmatrix}$$

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**Chapter 19, Problem 47.**

Obtain the **ABCD** parameters for the network in Fig. 19.102.

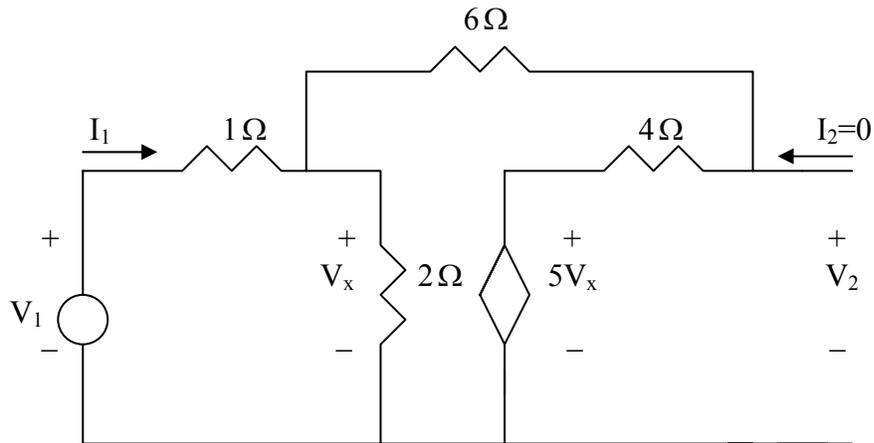


**Figure 19.102**

For Prob. 19.47.

**Chapter 19, Solution 47.**

To get A and C, consider the circuit below.

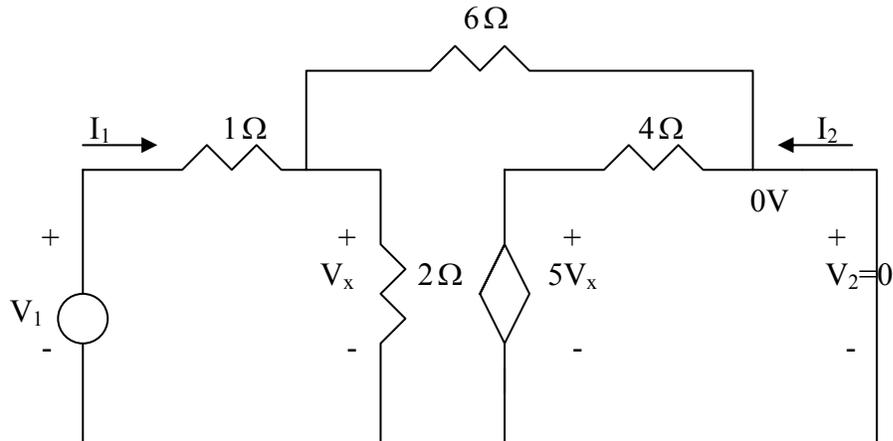


$$\frac{V_1 - V_x}{1} = \frac{V_x}{2} + \frac{V_x - 5V_x}{10} \quad \longrightarrow \quad V_1 = 1.1V_x$$

$$V_2 = 4(-0.4V_x) + 5V_x = 3.4V_x \quad \longrightarrow \quad A = \frac{V_1}{V_2} = 1.1/3.4 = 0.3235$$

$$I_1 = \frac{V_1 - V_x}{1} = 1.1V_x - V_x = 0.1V_x \quad \longrightarrow \quad C = \frac{I_1}{V_2} = 0.1/3.4 = 0.02941$$

To get B and D, consider the circuit below.



$$\frac{V_1 - V_x}{1} = \frac{V_x}{6} + \frac{V_x}{2} \quad \longrightarrow \quad V_1 = \frac{10}{6} V_x \quad (1)$$

$$I_2 = -\frac{5V_x}{4} - \frac{V_x}{6} = -\frac{17}{12} V_x \quad (2)$$

$$V_1 = I_1 + V_x \quad (3)$$

From (1) and (3)

$$I_1 = V_1 - V_x = \frac{4}{6} V_x \quad \longrightarrow \quad D = -\frac{I_1}{I_2} = \frac{4}{6} \left( \frac{12}{17} \right) = 0.4706$$

$$B = -\frac{V_1}{I_2} = \frac{10}{6} \left( \frac{12}{17} \right) = 1.176$$

$$[T] = \begin{bmatrix} 0.3235 & 1.176 \\ 0.02941 & 0.4706 \end{bmatrix}$$

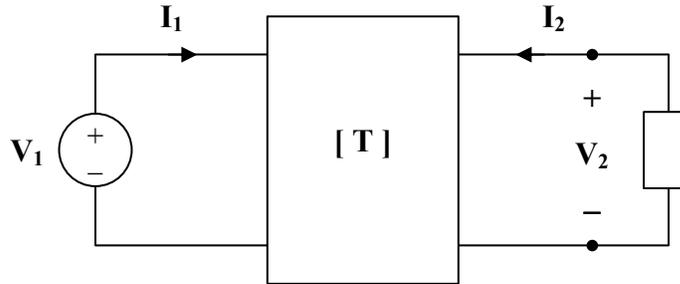
### Chapter 19, Problem 48.

For a two-port, let  $\mathbf{A} = 4$ ,  $\mathbf{B} = 30\ \Omega$ ,  $\mathbf{C} = 0.1\ \text{S}$ , and  $\mathbf{D} = 1.5$ . Calculate the input impedance,  $\mathbf{Z}_{\text{in}} = \mathbf{V}_1 / \mathbf{I}_1$  when:

- (a) the output terminals are short-circuited,
- (b) the output port is open-circuited,
- (c) the output port is terminated by a  $10\text{-}\Omega$  load.

**Chapter 19, Solution 48.**

(a) Refer to the circuit below.



$$V_1 = 4V_2 - 30I_2 \quad (1)$$

$$I_1 = 0.1V_2 - I_2 \quad (2)$$

When the output terminals are shorted,  $V_2 = 0$ .

So, (1) and (2) become

$$V_1 = -30I_2 \quad \text{and} \quad I_1 = -I_2$$

Hence,

$$Z_{in} = \frac{V_1}{I_1} = \underline{\underline{30 \Omega}}$$

(b) When the output terminals are open-circuited,  $I_2 = 0$ .

So, (1) and (2) become

$$V_1 = 4V_2$$

$$I_1 = 0.1V_2 \quad \text{or} \quad V_2 = 10I_1$$

$$V_1 = 40I_1$$

$$Z_{in} = \frac{V_1}{I_1} = \underline{\underline{40 \Omega}}$$

(c) When the output port is terminated by a  $10\text{-}\Omega$  load,  $V_2 = -10I_2$ .

So, (1) and (2) become

$$V_1 = -40I_2 - 30I_2 = -70I_2$$

$$I_1 = -I_2 - I_2 = -2I_2$$

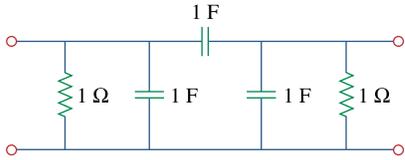
$$V_1 = 35I_1$$

$$Z_{in} = \frac{V_1}{I_1} = \underline{\underline{35 \Omega}}$$

Alternatively, we may use  $Z_{in} = \frac{AZ_L + B}{CZ_L + D}$

**Chapter 19, Problem 49.**

Using impedances in the  $s$  domain, obtain the transmission parameters for the circuit in Fig. 19.103.

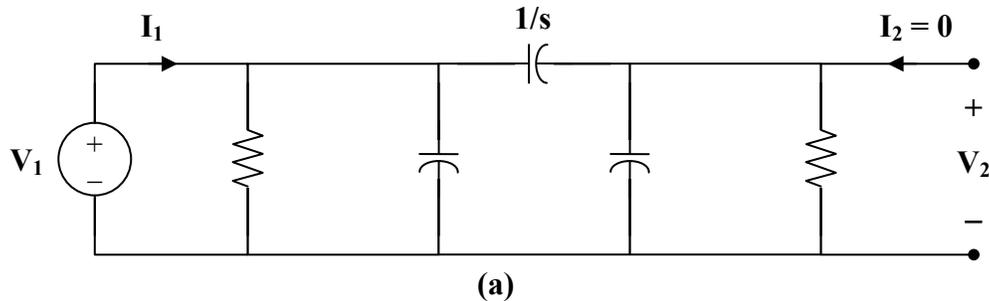


**Figure 19.103**

For Prob. 19.49.

**Chapter 19, Solution 49.**

To get **A** and **C**, refer to the circuit in Fig.(a).



$$1 \parallel \frac{1}{s} = \frac{1/s}{1 + 1/s} = \frac{1}{s+1}$$

$$V_2 = \frac{1 \parallel 1/s}{1/s + 1 \parallel 1/s} V_1$$

$$A = \frac{V_1}{V_2} = \frac{\frac{1}{s} + \frac{1}{s+1}}{\frac{1}{s+1}} = \frac{2s+1}{s}$$

$$V_1 = I_1 \left( \frac{1}{s+1} \right) \parallel \left( \frac{1}{s} + \frac{1}{s+1} \right) = I_1 \left( \frac{1}{s+1} \right) \parallel \left( \frac{2s+1}{s(s+1)} \right)$$

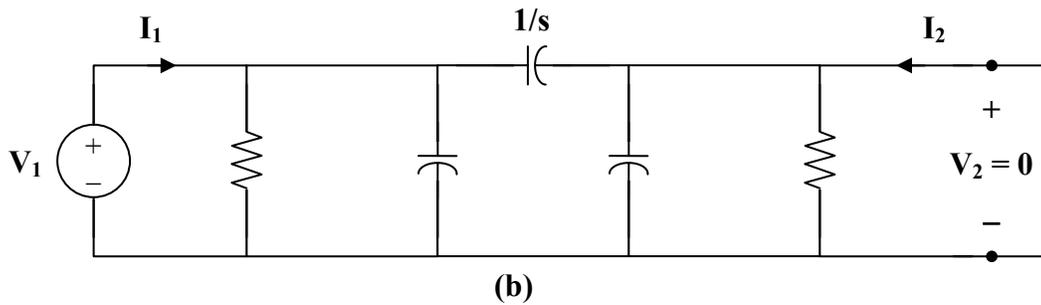
$$\frac{V_1}{I_1} = \frac{\left( \frac{1}{s+1} \right) \cdot \left( \frac{2s+1}{s(s+1)} \right)}{\frac{1}{s+1} + \frac{2s+1}{s(s+1)}} = \frac{2s+1}{(s+1)(3s+1)}$$

But 
$$V_1 = V_2 \cdot \frac{2s+1}{s}$$

Hence, 
$$\frac{V_2}{I_1} \cdot \frac{2s+1}{s} = \frac{2s+1}{(s+1)(3s+1)}$$

$$C = \frac{V_2}{I_1} = \frac{(s+1)(3s+1)}{s}$$

To get **B** and **D**, consider the circuit in Fig. (b).



$$V_1 = I_1 \left( 1 \parallel \frac{1}{s} \parallel \frac{1}{s} \right) = I_1 \left( 1 \parallel \frac{1}{2s} \right) = \frac{I_1}{2s+1}$$

$$I_2 = \frac{\frac{-1}{s+1} I_1}{\frac{1}{s+1} + \frac{1}{s}} = \frac{-s}{2s+1} I_1$$

$$D = \frac{-I_1}{I_2} = \frac{2s+1}{s} = 2 + \frac{1}{s}$$

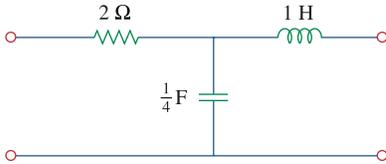
$$V_1 = \left( \frac{1}{2s+1} \right) \begin{pmatrix} 2s+1 \\ -s \end{pmatrix} I_2 = \frac{I_2}{-s} \longrightarrow B = \frac{-V_1}{I_2} = \frac{1}{s}$$

Thus,

$$[T] = \begin{bmatrix} \frac{2s+1}{s} & \frac{1}{s} \\ \frac{(s+1)(3s+1)}{s} & 2 + \frac{1}{s} \end{bmatrix}$$

**Chapter 19, Problem 50.**

Derive the  $s$ -domain expression for the  $t$  parameters of the circuit in Fig. 19.104.

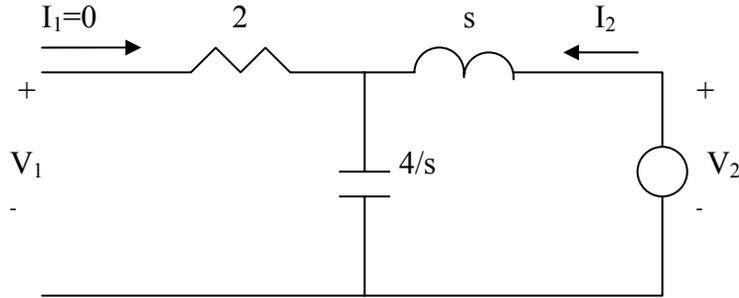


**Figure 19.104**

For Prob. 19.50.

**Chapter 19, Solution 50.**

To get a and c, consider the circuit below.

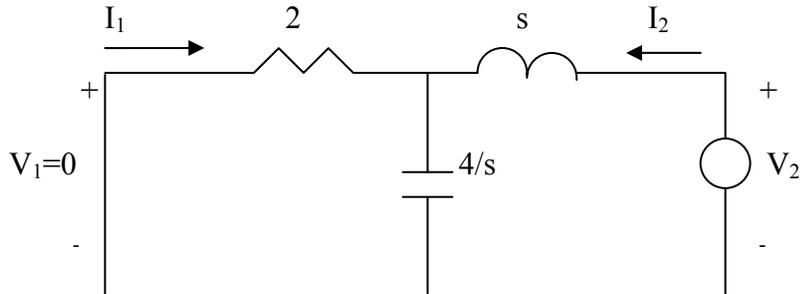


$$V_1 = \frac{4/s}{s+4/s} V_2 = \frac{4}{s^2+4} V_2 \quad \longrightarrow \quad a = \frac{V_2}{V_1} = 1 + 0.25s^2$$

$$V_2 = (s + 4/s)I_2 \text{ or}$$

$$I_2 = \frac{V_2}{s+4/s} = \frac{(1+0.25s^2)V_1}{s+4/s} \quad \longrightarrow \quad c = \frac{I_2}{V_1} = \frac{s+0.25s^3}{s^2+4}$$

To get b and d, consider the circuit below.



$$I_1 = \frac{-4/s}{2+4/s} I_2 = -\frac{2I_2}{s+2} \quad \longrightarrow \quad d = -\frac{I_2}{I_1} = 1 + 0.5s$$

$$V_2 = (s + 2 // \frac{4}{s}) I_2 = \frac{(s^2 + 2s + 4)}{s+2} I_2$$

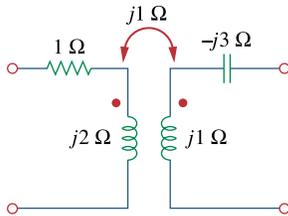
$$= -\frac{(s^2 + 2s + 4)(s+2)}{s+2} I_1 \quad \longrightarrow \quad b = -\frac{V_2}{I_1} = 0.5s^2 + s + 2$$

$$[t] = \begin{bmatrix} 0.25s^2 + 1 & 0.5s^2 + s + 2 \\ \frac{0.25s^2 + s}{s^2 + 4} & 0.5s + 1 \end{bmatrix}$$

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**Chapter 19, Problem 51.**

Obtain the  $t$  parameters for the network in Fig. 19.105.

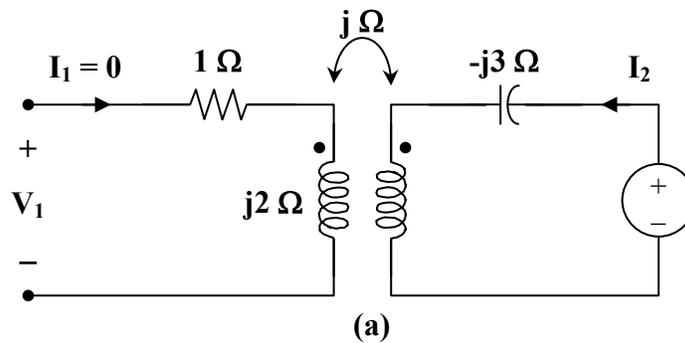


**Figure 19.105**

For Prob. 19.51.

**Chapter 19, Solution 51.**

To get **a** and **c**, consider the circuit in Fig. (a).



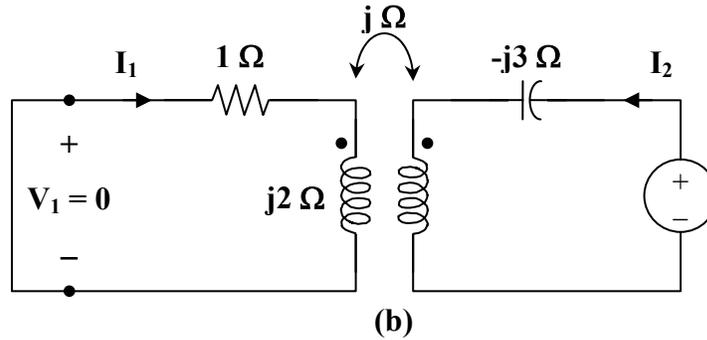
$$\mathbf{V}_2 = \mathbf{I}_2 (j - j3) = -j2 \mathbf{I}_2$$

$$\mathbf{V}_1 = -j \mathbf{I}_2$$

$$\mathbf{a} = \frac{\mathbf{V}_2}{\mathbf{V}_1} = \frac{-j2 \mathbf{I}_2}{-j \mathbf{I}_2} = 2$$

$$\mathbf{c} = \frac{\mathbf{I}_2}{\mathbf{V}_1} = \frac{1}{-j} = j$$

To get **b** and **d**, consider the circuit in Fig. (b).



For mesh 1,

$$0 = (1 + j2)I_1 - jI_2$$

or 
$$\frac{I_2}{I_1} = \frac{1 + j2}{j} = 2 - j$$

$$\mathbf{d} = \frac{-I_2}{I_1} = -2 + j$$

For mesh 2,

$$V_2 = I_2(j - j3) - jI_1$$

$$V_2 = I_1(2 - j)(-j2) - jI_1 = (-2 - j5)I_1$$

$$\mathbf{b} = \frac{-V_2}{I_1} = 2 + j5$$

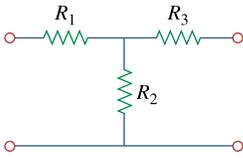
Thus,

$$[\mathbf{t}] = \underline{\underline{\begin{bmatrix} 2 & 2 + j5 \\ j & -2 + j \end{bmatrix}}}$$

**Chapter 19, Problem 52.**

(a) For the  $T$  network in Fig. 19.106, show that the  $h$  parameters are:

$$\mathbf{h}_{11} = R_1 + \frac{R_2 R_3}{R_1 + R_3}, \quad \mathbf{h}_{12} = \frac{R_2}{R_2 + R_3}$$
$$\mathbf{h}_{21} = -\frac{R_2}{R_2 + R_3}, \quad \mathbf{h}_{22} = \frac{1}{R_2 + R_3}$$



**Figure 19.106**

For Prob. 19.52.

(b) For the same network, show that the transmission parameters are:

$$\mathbf{A} = 1 + \frac{R_1}{R_2}, \quad \mathbf{B} = R_3 + \frac{R_1}{R_2}(R_2 + R_3)$$
$$\mathbf{C} = \frac{1}{R_2}, \quad \mathbf{D} = 1 + \frac{R_3}{R_2}$$

**Chapter 19, Solution 52.**

It is easy to find the z parameters and then transform these to h parameters and T parameters.

$$[\mathbf{z}] = \begin{bmatrix} R_1 + R_2 & R_2 \\ R_2 & R_2 + R_3 \end{bmatrix}$$

$$\begin{aligned} \Delta_z &= (R_1 + R_2)(R_2 + R_3) - R_2^2 \\ &= R_1R_2 + R_2R_3 + R_3R_1 \end{aligned}$$

$$(a) \quad [\mathbf{h}] = \begin{bmatrix} \frac{\Delta_z}{z_{22}} & \frac{z_{12}}{z_{22}} \\ \frac{z_{22}}{-z_{21}} & \frac{1}{z_{22}} \\ \frac{-z_{21}}{z_{22}} & \frac{1}{z_{22}} \end{bmatrix} = \begin{bmatrix} \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2 + R_3} & \frac{R_2}{R_2 + R_3} \\ \frac{-R_2}{R_2 + R_3} & \frac{1}{R_2 + R_3} \end{bmatrix}$$

Thus,

$$\underline{\underline{h_{11} = R_1 + \frac{R_2R_3}{R_2 + R_3}}}, \quad \underline{\underline{h_{12} = \frac{R_2}{R_2 + R_3} = -h_{21}}}, \quad \underline{\underline{h_{22} = \frac{1}{R_2 + R_3}}}$$

as required.

$$(b) \quad [\mathbf{T}] = \begin{bmatrix} \frac{z_{11}}{z_{21}} & \frac{\Delta_z}{z_{21}} \\ \frac{z_{21}}{1} & \frac{z_{22}}{z_{21}} \\ \frac{z_{21}}{z_{21}} & \frac{z_{21}}{z_{21}} \end{bmatrix} = \begin{bmatrix} \frac{R_1 + R_2}{R_2} & \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2} \\ \frac{1}{R_2} & \frac{R_2}{R_2 + R_3} \\ \frac{1}{R_2} & \frac{R_2}{R_2} \end{bmatrix}$$

Hence,

$$\underline{\underline{A = 1 + \frac{R_1}{R_2}}}, \quad \underline{\underline{B = R_3 + \frac{R_1}{R_2}(R_2 + R_3)}}, \quad \underline{\underline{C = \frac{1}{R_2}}}, \quad \underline{\underline{D = 1 + \frac{R_3}{R_2}}}$$

as required.

### Chapter 19, Problem 53.

Through derivation, express the  $z$  parameters in terms of the **ABCD** parameters.

### Chapter 19, Solution 53.

For the  $z$  parameters,

$$\mathbf{V}_1 = \mathbf{z}_{11} \mathbf{I}_1 + \mathbf{z}_{12} \mathbf{I}_2 \quad (1)$$

$$\mathbf{V}_2 = \mathbf{z}_{12} \mathbf{I}_1 + \mathbf{z}_{22} \mathbf{I}_2 \quad (2)$$

For **ABCD** parameters,

$$\mathbf{V}_1 = \mathbf{A} \mathbf{V}_2 - \mathbf{B} \mathbf{I}_2 \quad (3)$$

$$\mathbf{I}_1 = \mathbf{C} \mathbf{V}_2 - \mathbf{D} \mathbf{I}_2 \quad (4)$$

From (4),

$$\mathbf{V}_2 = \frac{\mathbf{I}_1}{\mathbf{C}} + \frac{\mathbf{D}}{\mathbf{C}} \mathbf{I}_2 \quad (5)$$

Comparing (2) and (5),

$$\mathbf{z}_{21} = \frac{1}{\mathbf{C}}, \quad \mathbf{z}_{22} = \frac{\mathbf{D}}{\mathbf{C}}$$

Substituting (5) into (3),

$$\begin{aligned} \mathbf{V}_1 &= \frac{\mathbf{A}}{\mathbf{C}} \mathbf{I}_1 + \left( \frac{\mathbf{AD}}{\mathbf{C}} - \mathbf{B} \right) \mathbf{I}_2 \\ &= \frac{\mathbf{A}}{\mathbf{C}} \mathbf{I}_1 + \frac{\mathbf{AD} - \mathbf{BC}}{\mathbf{C}} \mathbf{I}_2 \end{aligned} \quad (6)$$

Comparing (6) and (1),

$$\mathbf{z}_{11} = \frac{\mathbf{A}}{\mathbf{C}}, \quad \mathbf{z}_{12} = \frac{\mathbf{AD} - \mathbf{BC}}{\mathbf{C}} = \frac{\Delta_T}{\mathbf{C}}$$

Thus,

$$[\mathbf{Z}] = \begin{bmatrix} \frac{\mathbf{A}}{\mathbf{C}} & \frac{\Delta_T}{\mathbf{C}} \\ \frac{1}{\mathbf{C}} & \frac{\mathbf{D}}{\mathbf{C}} \end{bmatrix}$$

### Chapter 19, Problem 54.

Show that the transmission parameters of a two-port may be obtained from the  $y$  parameters as:

$$A = -\frac{y_{22}}{y_{21}}, \quad B = -\frac{1}{y_{21}}$$
$$C = -\frac{\Delta_y}{y_{21}}, \quad D = -\frac{y_{11}}{y_{21}}$$

### Chapter 19, Solution 54.

For the  $y$  parameters

$$\mathbf{I}_1 = y_{11} \mathbf{V}_1 + y_{12} \mathbf{V}_2 \quad (1)$$

$$\mathbf{I}_2 = y_{21} \mathbf{V}_1 + y_{22} \mathbf{V}_2 \quad (2)$$

From (2),

$$\mathbf{V}_1 = \frac{\mathbf{I}_2}{y_{21}} - \frac{y_{22}}{y_{21}} \mathbf{V}_2$$

or

$$\mathbf{V}_1 = \frac{-y_{22}}{y_{21}} \mathbf{V}_2 + \frac{1}{y_{21}} \mathbf{I}_2 \quad (3)$$

Substituting (3) into (1) gives

$$\mathbf{I}_1 = \frac{-y_{11} y_{22}}{y_{21}} \mathbf{V}_2 + y_{12} \mathbf{V}_2 + \frac{y_{11}}{y_{21}} \mathbf{I}_2$$

or

$$\mathbf{I}_1 = \frac{-\Delta_y}{y_{21}} \mathbf{V}_2 + \frac{y_{11}}{y_{21}} \mathbf{I}_2 \quad (4)$$

Comparing (3) and (4) with the following equations

$$\mathbf{V}_1 = \mathbf{A} \mathbf{V}_2 - \mathbf{B} \mathbf{I}_2$$

$$\mathbf{I}_1 = \mathbf{C} \mathbf{V}_2 - \mathbf{D} \mathbf{I}_2$$

clearly shows that

$$\underline{\underline{\mathbf{A} = \frac{-y_{22}}{y_{21}}}}, \quad \underline{\underline{\mathbf{B} = \frac{-1}{y_{21}}}}, \quad \underline{\underline{\mathbf{C} = \frac{-\Delta_y}{y_{21}}}}, \quad \underline{\underline{\mathbf{D} = \frac{-y_{11}}{y_{21}}}}$$

as required.

### Chapter 19, Problem 55.

Prove that the  $g$  parameters can be obtained from the  $z$  parameters as

$$\mathbf{g}_{11} = \frac{1}{\mathbf{z}_{11}}, \quad \mathbf{g}_{12} = -\frac{\mathbf{z}_{12}}{\mathbf{z}_{11}}$$
$$\mathbf{g}_{21} = \frac{\mathbf{z}_{21}}{\mathbf{z}_{11}}, \quad \mathbf{g}_{22} = \frac{\Delta_z}{\mathbf{z}_{11}}$$

### Chapter 19, Solution 55.

For the  $z$  parameters

$$\mathbf{V}_1 = \mathbf{z}_{11} \mathbf{I}_1 + \mathbf{z}_{12} \mathbf{I}_2 \quad (1)$$

$$\mathbf{V}_2 = \mathbf{z}_{21} \mathbf{I}_1 + \mathbf{z}_{22} \mathbf{I}_2 \quad (2)$$

From (1),

$$\mathbf{I}_1 = \frac{1}{\mathbf{z}_{11}} \mathbf{V}_1 - \frac{\mathbf{z}_{12}}{\mathbf{z}_{11}} \mathbf{I}_2 \quad (3)$$

Substituting (3) into (2) gives

$$\mathbf{V}_2 = \frac{\mathbf{z}_{21}}{\mathbf{z}_{11}} \mathbf{V}_1 + \left( \mathbf{z}_{22} - \frac{\mathbf{z}_{21} \mathbf{z}_{12}}{\mathbf{z}_{11}} \right) \mathbf{I}_2$$

or

$$\mathbf{V}_2 = \frac{\mathbf{z}_{21}}{\mathbf{z}_{11}} \mathbf{V}_1 + \frac{\Delta_z}{\mathbf{z}_{11}} \mathbf{I}_2 \quad (4)$$

Comparing (3) and (4) with the following equations

$$\mathbf{I}_1 = \mathbf{g}_{11} \mathbf{V}_1 + \mathbf{g}_{12} \mathbf{I}_2$$

$$\mathbf{V}_2 = \mathbf{g}_{21} \mathbf{V}_1 + \mathbf{g}_{22} \mathbf{I}_2$$

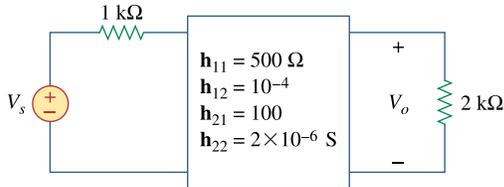
indicates that

$$\underline{\mathbf{g}_{11} = \frac{1}{\mathbf{z}_{11}}}, \quad \underline{\mathbf{g}_{12} = -\frac{\mathbf{z}_{12}}{\mathbf{z}_{11}}}, \quad \underline{\mathbf{g}_{21} = \frac{\mathbf{z}_{21}}{\mathbf{z}_{11}}}, \quad \underline{\mathbf{g}_{22} = \frac{\Delta_z}{\mathbf{z}_{11}}}$$

as required.

**Chapter 19, Problem 56.**

For the network of Fig. 19.107, obtain  $V_o/V_s$ .

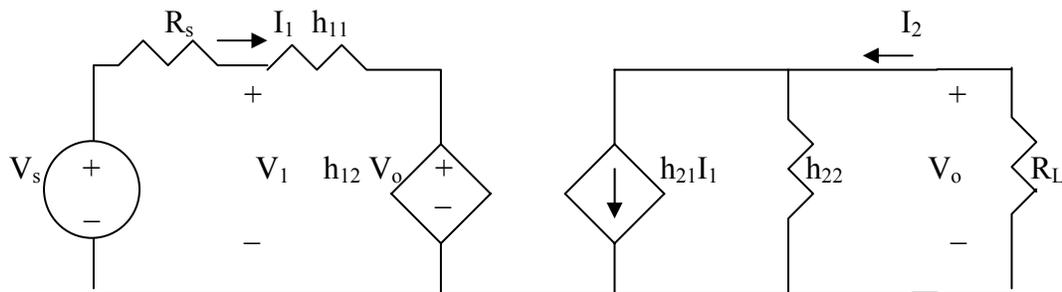


**Figure 19.107**

For Prob. 19.56.

**Chapter 19, Solution 56.**

Using Fig. 19.20, we obtain the equivalent circuit as shown below.



We can solve this using MATLAB. First, we generate 4 equations from the given circuit. It may help to let  $V_s = 10$  V.

$$\begin{aligned}
 -10 + R_s I_1 + V_1 &= 0 \text{ or } V_1 + 1000I_1 = 10 \\
 -10 + R_s I_1 + h_{11}I_1 + h_{12}V_o &= 0 \text{ or } 0.0001V_s + 1500 = 10 \\
 I_2 &= -V_o/R_L \text{ or } V_o + 2000I_2 = 0 \\
 h_{21}I_1 + h_{22}V_o - I_2 &= 0 \text{ or } 2 \times 10^{-6}V_o + 100I_1 - I_2 = 0
 \end{aligned}$$

```
>> A=[1,0,1000,0;0,0.0001,1500,0;0,1,0,2000;0,(2*10^-6),100,-1]
```

```
A =
1.0e+003 *
0.0010    0    1.0000    0
0    0.0000    1.5000    0
0    0.0010    0    2.0000
0    0.0000    0.1000   -0.0010
```

```
>> U=[10;10;0;0]
```

```
U =
10
10
0
0
```

```
>> X=inv(A)*U
```

```
X =
1.0e+003 *
0.0032
-1.3459
0.0000
0.0007
```

$$\text{Gain} = V_o / V_s = -1,345.9/10 = \underline{\underline{-134.59}}$$

There is a second approach we can take to check this problem. First, the resistive value of  $h_{22}$  is quite large, 500 k $\Omega$  versus  $R_L$  so can be ignored. Working on the right side of the circuit we obtain the following,

$$I_2 = 100I_1 \text{ which leads to } V_o = -I_2 \times 2k = -2 \times 10^5 I_1.$$

Now the left hand loop equation becomes,

$$-V_s + (1000 + 500 + 10^{-4}(-2 \times 10^5))I_1 = 1480I_1.$$

Solving for  $V_o/V_s$  we get,

$$V_o/V_s = -200,000/1480 = \underline{\underline{-134.14}}$$

Our answer checks!

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### Chapter 19, Problem 57.

Given the transmission parameters

$$[\mathbf{T}] = \begin{bmatrix} 3 & 20 \\ 1 & 7 \end{bmatrix}$$

obtain the other five two-port parameters.

### Chapter 19, Solution 57.

$$\Delta_T = (3)(7) - (20)(1) = 1$$

$$[\mathbf{z}] = \begin{bmatrix} \frac{\mathbf{A}}{\mathbf{C}} & \frac{\Delta_T}{\mathbf{C}} \\ \frac{1}{\mathbf{C}} & \frac{\mathbf{D}}{\mathbf{C}} \end{bmatrix} = \underline{\underline{\begin{bmatrix} 3 & 1 \\ 1 & 7 \end{bmatrix} \Omega}}$$

$$[\mathbf{y}] = \begin{bmatrix} \frac{\mathbf{D}}{\mathbf{B}} & \frac{-\Delta_T}{\mathbf{B}} \\ \frac{-1}{\mathbf{B}} & \frac{\mathbf{A}}{\mathbf{B}} \end{bmatrix} = \underline{\underline{\begin{bmatrix} 7 & -1 \\ 20 & 20 \\ -1 & 3 \\ 20 & 20 \end{bmatrix} \text{S}}}$$

$$[\mathbf{h}] = \begin{bmatrix} \frac{\mathbf{B}}{\mathbf{D}} & \frac{\Delta_T}{\mathbf{D}} \\ \frac{-1}{\mathbf{D}} & \frac{\mathbf{C}}{\mathbf{D}} \end{bmatrix} = \underline{\underline{\begin{bmatrix} \frac{20}{7} \Omega & \frac{1}{7} \\ -1 & \frac{1}{7} \text{S} \end{bmatrix}}}$$

$$[\mathbf{g}] = \begin{bmatrix} \frac{\mathbf{C}}{\mathbf{A}} & \frac{-\Delta_T}{\mathbf{A}} \\ \frac{1}{\mathbf{A}} & \frac{\mathbf{B}}{\mathbf{A}} \end{bmatrix} = \underline{\underline{\begin{bmatrix} \frac{1}{3} \text{S} & \frac{-1}{3} \\ 1 & \frac{20}{3} \Omega \end{bmatrix}}}$$

$$[\mathbf{t}] = \begin{bmatrix} \frac{\mathbf{D}}{\Delta_T} & \frac{\mathbf{B}}{\Delta_T} \\ \frac{\mathbf{C}}{\Delta_T} & \frac{\mathbf{A}}{\Delta_T} \end{bmatrix} = \underline{\underline{\begin{bmatrix} 7 & 20 \Omega \\ 1 \text{S} & 3 \end{bmatrix}}}$$

### Chapter 19, Problem 58.

A two-port is described by

$$\mathbf{V}_1 = \mathbf{I}_1 + 2\mathbf{V}_2, \quad \mathbf{I}_2 = -2\mathbf{I}_1 + 0.4\mathbf{V}_2$$

Find: (a) the  $y$  parameters, (b) the transmission parameters.

### Chapter 19, Solution 58.

The given set of equations is for the  $h$  parameters.

$$[\mathbf{h}] = \begin{bmatrix} 1 \Omega & 2 \\ -2 & 0.4 \text{ S} \end{bmatrix} \quad \Delta_h = (1)(0.4) - (2)(-2) = 4.4$$

$$(a) \quad [\mathbf{y}] = \begin{bmatrix} \frac{1}{\mathbf{h}_{11}} & \frac{-\mathbf{h}_{12}}{\mathbf{h}_{11}} \\ \frac{\mathbf{h}_{21}}{\mathbf{h}_{11}} & \frac{\Delta_h}{\mathbf{h}_{11}} \end{bmatrix} = \underline{\underline{\begin{bmatrix} 1 & -2 \\ -2 & 4.4 \end{bmatrix} \text{ S}}}$$

$$(b) \quad [\mathbf{T}] = \begin{bmatrix} \frac{-\Delta_h}{\mathbf{h}_{21}} & \frac{-\mathbf{h}_{11}}{\mathbf{h}_{21}} \\ \frac{-\mathbf{h}_{22}}{\mathbf{h}_{21}} & \frac{-1}{\mathbf{h}_{21}} \end{bmatrix} = \underline{\underline{\begin{bmatrix} 2.2 & 0.5 \Omega \\ 0.2 \text{ S} & 0.5 \end{bmatrix}}}$$

### Chapter 19, Problem 59.

Given that

$$[\mathbf{g}] = \begin{bmatrix} 0.06 \text{ S} & -0.4 \\ 0.2 & 2\Omega \end{bmatrix}$$

determine:

- (a)  $[\mathbf{z}]$       (b)  $[\mathbf{y}]$       (c)  $[\mathbf{h}]$       (d)  $[\mathbf{T}]$

### Chapter 19, Solution 59.

$$\Delta_g = (0.06)(2) - (-0.4)(0.2) = 0.12 + 0.08 = 0.2$$

$$(a) \quad [\mathbf{z}] = \begin{bmatrix} \frac{1}{\mathbf{g}_{11}} & \frac{-\mathbf{g}_{12}}{\mathbf{g}_{11}} \\ \frac{\mathbf{g}_{21}}{\Delta_g} & \frac{\Delta_g}{\mathbf{g}_{11}} \end{bmatrix} = \underline{\underline{\begin{bmatrix} 16.667 & 6.667 \\ 3.333 & 3.333 \end{bmatrix} \Omega}}$$

$$(b) \quad [\mathbf{y}] = \begin{bmatrix} \frac{\Delta_g}{\mathbf{g}_{22}} & \frac{\mathbf{g}_{12}}{\mathbf{g}_{22}} \\ \frac{-\mathbf{g}_{21}}{\mathbf{g}_{22}} & \frac{1}{\mathbf{g}_{22}} \end{bmatrix} = \underline{\underline{\begin{bmatrix} 0.1 & -0.2 \\ -0.1 & 0.5 \end{bmatrix} \text{ S}}}$$

$$(c) \quad [\mathbf{h}] = \begin{bmatrix} \frac{\mathbf{g}_{22}}{\Delta_g} & \frac{-\mathbf{g}_{12}}{\Delta_g} \\ \frac{-\mathbf{g}_{21}}{\Delta_g} & \frac{\mathbf{g}_{11}}{\Delta_g} \end{bmatrix} = \underline{\underline{\begin{bmatrix} 10 \Omega & 2 \\ -1 & 0.3 \text{ S} \end{bmatrix}}}$$

$$(d) \quad [\mathbf{T}] = \begin{bmatrix} \frac{1}{\mathbf{g}_{21}} & \frac{\mathbf{g}_{22}}{\mathbf{g}_{21}} \\ \frac{\mathbf{g}_{11}}{\Delta_g} & \frac{\Delta_g}{\mathbf{g}_{21}} \end{bmatrix} = \underline{\underline{\begin{bmatrix} 5 & 10 \Omega \\ 0.3 \text{ S} & 1 \end{bmatrix}}}$$

### Chapter 19, Problem 60.

Design a T network necessary to realize the following  $z$  parameters at  $\omega = 10^6$  rad/s

$$[z] = \begin{bmatrix} 4 + j3 & 2 \\ 2 & 5 - j \end{bmatrix} \text{ k}\Omega$$

### Chapter 19, Solution 60.

Comparing this with Fig. 19.5,

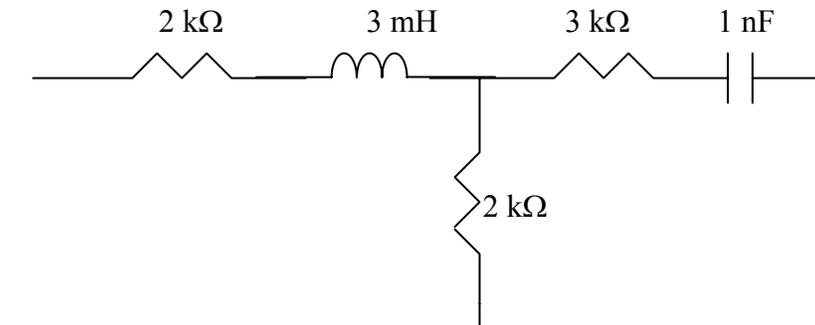
$$z_{11} - z_{12} = 4 + j3 - 2 = 2 + j3 \text{ k}\Omega$$

$$z_{22} - z_{12} = 5 - j - 2 = 3 - j \text{ k}\Omega$$

$$X_L = 3 \times 10^3 = \omega L \quad \longrightarrow \quad L = \frac{3 \times 10^3}{10^6} = 3 \text{ mH}$$

$$X_C = 1 \times 10^3 = 1/(\omega C) \text{ or } C = 1/(10^3 \times 10^6) = 1 \text{ nF}$$

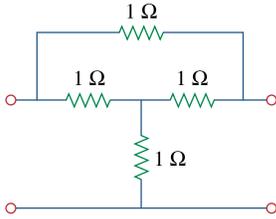
Hence, the resulting T network is shown below.



### Chapter 19, Problem 61.

For the bridge circuit in Fig. 19.108, obtain:

- (a) the  $z$  parameters
- (b) the  $h$  parameters
- (c) the transmission parameters

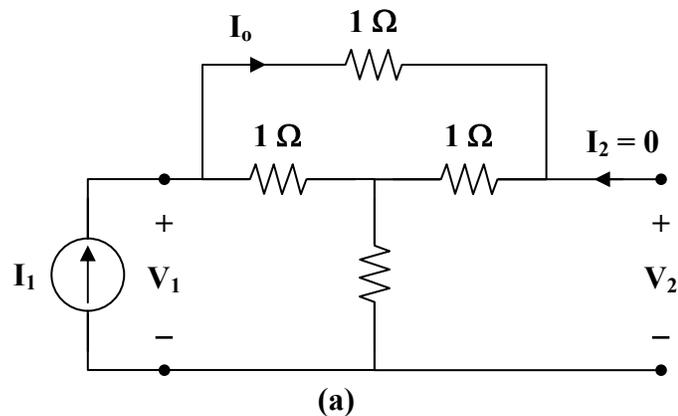


**Figure 19.108**

For Prob. 19.61.

### Chapter 19, Solution 61.

- (a) To obtain  $z_{11}$  and  $z_{21}$ , consider the circuit in Fig. (a).



$$V_1 = I_1 [1 + 1 \parallel (1 + 1)] = I_1 \left( 1 + \frac{2}{3} \right) = \frac{5}{3} I_1$$

$$z_{11} = \frac{V_1}{I_1} = \frac{5}{3}$$

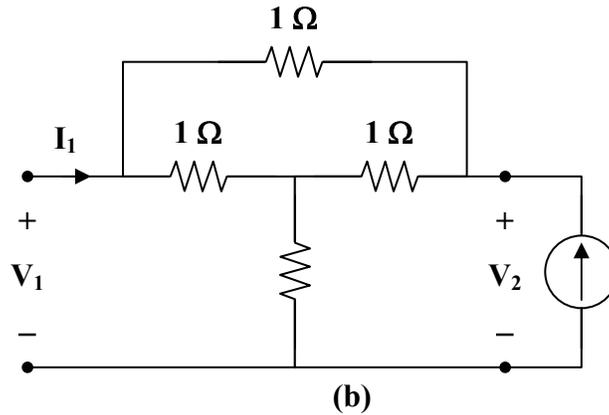
$$I_0 = \frac{1}{1+2} I_1 = \frac{1}{3} I_1$$

$$-V_2 + I_0 + I_1 = 0$$

$$V_2 = \frac{1}{3} I_1 + I_1 = \frac{4}{3} I_1$$

$$z_{21} = \frac{V_2}{I_1} = \frac{4}{3}$$

To obtain  $z_{22}$  and  $z_{12}$ , consider the circuit in Fig. (b).



Due to symmetry, this is similar to the circuit in Fig. (a).

$$z_{22} = z_{11} = \frac{5}{3}, \quad z_{21} = z_{12} = \frac{4}{3}$$

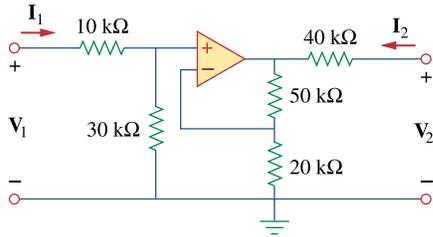
$$[\mathbf{z}] = \begin{bmatrix} \frac{5}{3} & \frac{4}{3} \\ \frac{4}{3} & \frac{5}{3} \end{bmatrix} \Omega$$

$$(b) \quad [\mathbf{h}] = \begin{bmatrix} \frac{\Delta_z}{z_{22}} & \frac{z_{12}}{z_{22}} \\ -\frac{z_{21}}{z_{22}} & \frac{1}{z_{22}} \end{bmatrix} = \begin{bmatrix} \frac{3}{5} \Omega & \frac{4}{5} \\ -\frac{4}{5} & \frac{3}{5} S \end{bmatrix}$$

$$(c) \quad [\mathbf{T}] = \begin{bmatrix} \frac{z_{11}}{z_{21}} & \frac{\Delta_z}{z_{21}} \\ \frac{1}{z_{21}} & \frac{z_{22}}{z_{21}} \end{bmatrix} = \begin{bmatrix} \frac{5}{4} & \frac{3}{4} \Omega \\ \frac{3}{4} S & \frac{5}{4} \end{bmatrix}$$

### Chapter 19, Problem 62.

Find the  $z$  parameters of the op amp circuit in Fig. 19.109. Obtain the transmission parameters.

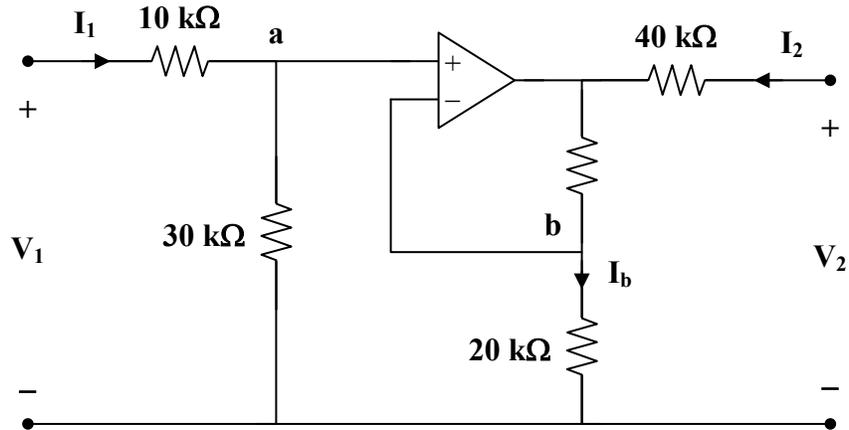


**Figure 19.109**

For Prob. 19.62.

**Chapter 19, Solution 62.**

Consider the circuit shown below.



Since no current enters the input terminals of the op amp,

$$V_1 = (10 + 30) \times 10^3 I_1 \quad (1)$$

But 
$$V_a = V_b = \frac{30}{40} V_1 = \frac{3}{4} V_1$$

$$I_b = \frac{V_b}{20 \times 10^3} = \frac{3}{80 \times 10^3} V_1$$

which is the same current that flows through the 50-kΩ resistor.

Thus, 
$$V_2 = 40 \times 10^3 I_2 + (50 + 20) \times 10^3 I_b$$

$$V_2 = 40 \times 10^3 I_2 + 70 \times 10^3 \cdot \frac{3}{80 \times 10^3} V_1$$

$$V_2 = \frac{21}{8} V_1 + 40 \times 10^3 I_2$$

$$V_2 = 105 \times 10^3 I_1 + 40 \times 10^3 I_2 \quad (2)$$

From (1) and (2),

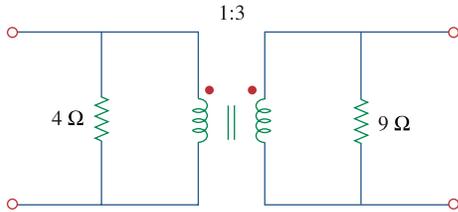
$$[z] = \begin{bmatrix} 40 & 0 \\ 105 & 40 \end{bmatrix} \text{ k}\Omega$$

$$\Delta_z = z_{11} z_{22} - z_{12} z_{21} = 16 \times 10^8$$

$$[T] = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \frac{z_{11}}{z_{21}} & \frac{\Delta_z}{z_{21}} \\ 1 & \frac{z_{22}}{z_{21}} \end{bmatrix} = \begin{bmatrix} 0.381 & 15.24 \text{ k}\Omega \\ 9.52 \mu\text{S} & 0.381 \end{bmatrix}$$

**Chapter 19, Problem 63.**

Determine the  $z$  parameters of the two-port in Fig. 19.110.

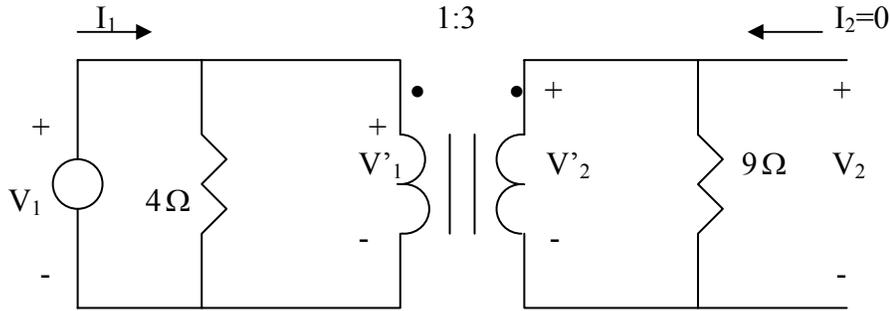


**Figure 19.110**

For Prob. 19.63.

**Chapter 19, Solution 63.**

To get  $z_{11}$  and  $z_{21}$ , consider the circuit below.

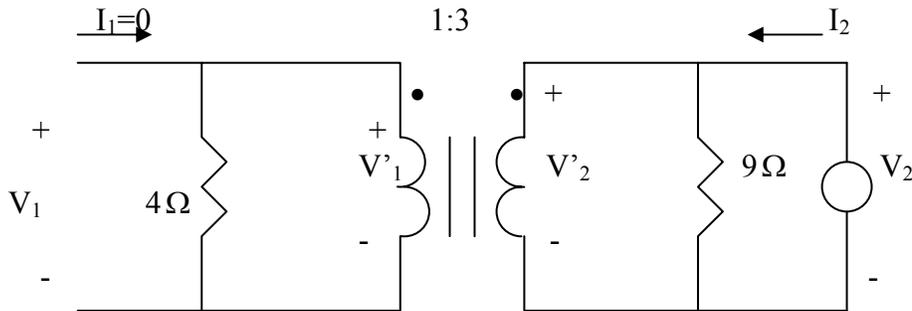


$$Z_R = \frac{9}{n^2} = 1, \quad n = 3$$

$$V_1 = (4 // Z_R) I_1 = \frac{4}{5} I_1 \quad \longrightarrow \quad z_{11} = \frac{V_1}{I_1} = 0.8$$

$$V_2 = V_2' = nV_1' = nV_1 = 3\left(\frac{4}{5}\right) I_1 \quad \longrightarrow \quad z_{21} = \frac{V_2}{I_1} = 2.4$$

To get  $z_{21}$  and  $z_{22}$ , consider the circuit below.



$$Z_R' = n^2 (4) = 36, \quad n = 3$$

$$V_2 = (9 // Z_R') I_2 = \frac{9 \times 36}{45} I_2 \quad \longrightarrow \quad z_{22} = \frac{V_2}{I_2} = 7.2$$

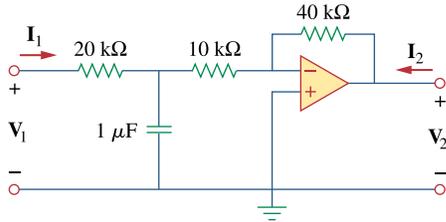
$$V_1 = \frac{V_2}{n} = \frac{V_2}{3} = 2.4 I_2 \quad \longrightarrow \quad z_{21} = \frac{V_1}{I_2} = 2.4$$

Thus,

$$[z] = \begin{bmatrix} 0.8 & 2.4 \\ 2.4 & 7.2 \end{bmatrix} \Omega$$

**Chapter 19, Problem 64.**

Determine the  $y$  parameters at  $\omega = 1,000 \text{ rad/s}$  for the op amp circuit in Fig. 19.111. Find the corresponding  $h$  parameters.



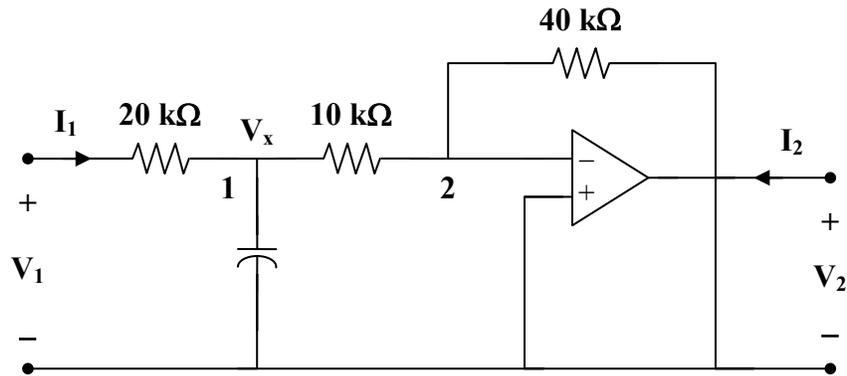
**Figure 19.111**

For Prob. 19.64.

**Chapter 19, Solution 64.**

$$1 \mu\text{F} \longrightarrow \frac{1}{j\omega C} = \frac{-j}{(10^3)(10^{-6})} = -j \text{ k}\Omega$$

Consider the op amp circuit below.



At node 1,

$$\frac{V_1 - V_x}{20} = \frac{V_x}{-j} + \frac{V_x - 0}{10}$$

$$V_1 = (3 + j20) V_x \quad (1)$$

At node 2,

$$\frac{V_x - 0}{10} = \frac{0 - V_2}{40} \longrightarrow V_x = \frac{-1}{4} V_2 \quad (2)$$

But 
$$I_1 = \frac{V_1 - V_x}{20 \times 10^3} \quad (3)$$

Substituting (2) into (3) gives

$$I_1 = \frac{V_1 + 0.25 V_2}{20 \times 10^3} = 50 \times 10^{-6} V_1 + 12.5 \times 10^{-6} V_2 \quad (4)$$

Substituting (2) into (1) yields

$$V_1 = \frac{-1}{4} (3 + j20) V_2$$

or 
$$0 = V_1 + (0.75 + j5) V_2 \quad (5)$$

Comparing (4) and (5) with the following equations

$$I_1 = y_{11} V_1 + y_{12} V_2$$

$$I_2 = y_{21} V_1 + y_{22} V_2$$

indicates that  $I_2 = 0$  and that

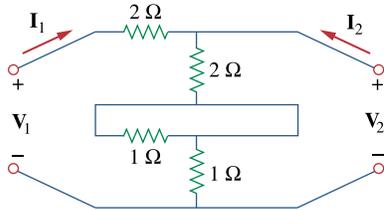
$$[y] = \underline{\underline{\begin{bmatrix} 50 \times 10^{-6} & 12.5 \times 10^{-6} \\ 1 & 0.75 + j5 \end{bmatrix} \text{ S}}}$$

$$\Delta_y = (77.5 + j25) \times 10^{-6} - 12.5 \times 10^{-6} = (65 + j250) \times 10^{-6}$$

$$[h] = \begin{bmatrix} \frac{1}{y_{11}} & \frac{-y_{12}}{y_{11}} \\ \frac{y_{21}}{y_{11}} & \frac{\Delta_y}{y_{11}} \end{bmatrix} = \underline{\underline{\begin{bmatrix} 2 \times 10^4 \Omega & -0.25 \\ 2 \times 10^4 & 1.3 + j5 \text{ S} \end{bmatrix}}}$$

### Chapter 19, Problem 65.

What is the y parameter presentation of the circuit in Fig. 19.112?



**Figure 19.112**

For Prob. 19.65.

### Chapter 19, Solution 65.

The network consists of two two-ports in series. It is better to work with z parameters and then convert to y parameters.

$$\text{For } N_a, \quad [z_a] = \begin{bmatrix} 4 & 2 \\ 2 & 2 \end{bmatrix}$$

$$\text{For } N_b, \quad [z_b] = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$$

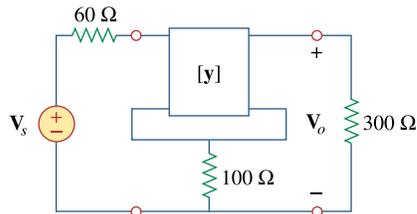
$$[z] = [z_a] + [z_b] = \begin{bmatrix} 6 & 3 \\ 3 & 3 \end{bmatrix}$$

$$\Delta_z = 18 - 9 = 9$$

$$[y] = \begin{bmatrix} \frac{z_{22}}{\Delta_z} & \frac{-z_{12}}{\Delta_z} \\ \frac{-z_{21}}{\Delta_z} & \frac{z_{11}}{\Delta_z} \end{bmatrix} = \underline{\underline{\begin{bmatrix} \frac{1}{3} & \frac{-1}{3} \\ \frac{-1}{3} & \frac{2}{3} \end{bmatrix} \text{ S}}}$$

**Chapter 19, Problem 66.**

In the two-port of Fig. 19.113, let  $y_{12} = y_{21} = 0$ ,  $y_{11} = 2 \text{ mS}$ , and  $y_{22} = 10 \text{ mS}$ . Find  $V_o/V_s$ .



**Figure 19.113**

For Prob. 19.66.

## Chapter 19, Solution 66.

Since we have two two-ports in series, it is better to convert the given y parameters to z parameters.

$$\Delta_y = y_{11} y_{22} - y_{12} y_{21} = (2 \times 10^{-3})(10 \times 10^{-3}) - 0 = 20 \times 10^{-6}$$

$$[\mathbf{z}_a] = \begin{bmatrix} \frac{y_{22}}{\Delta_y} & \frac{-y_{12}}{\Delta_y} \\ \frac{-y_{21}}{\Delta_y} & \frac{y_{11}}{\Delta_y} \end{bmatrix} = \begin{bmatrix} 500 \Omega & 0 \\ 0 & 100 \Omega \end{bmatrix}$$

$$[\mathbf{z}] = \begin{bmatrix} 500 & 0 \\ 0 & 100 \end{bmatrix} + \begin{bmatrix} 100 & 100 \\ 100 & 100 \end{bmatrix} = \begin{bmatrix} 600 & 100 \\ 100 & 200 \end{bmatrix}$$

i.e. 
$$\begin{aligned} \mathbf{V}_1 &= \mathbf{z}_{11} \mathbf{I}_1 + \mathbf{z}_{12} \mathbf{I}_2 \\ \mathbf{V}_2 &= \mathbf{z}_{21} \mathbf{I}_1 + \mathbf{z}_{22} \mathbf{I}_2 \end{aligned}$$

or 
$$\mathbf{V}_1 = 600 \mathbf{I}_1 + 100 \mathbf{I}_2 \quad (1)$$

$$\mathbf{V}_2 = 100 \mathbf{I}_1 + 200 \mathbf{I}_2 \quad (2)$$

But, at the input port,

$$\mathbf{V}_s = \mathbf{V}_1 + 60 \mathbf{I}_1 \quad (3)$$

and at the output port,

$$\mathbf{V}_2 = \mathbf{V}_o = -300 \mathbf{I}_2 \quad (4)$$

From (2) and (4),

$$\begin{aligned} 100 \mathbf{I}_1 + 200 \mathbf{I}_2 &= -300 \mathbf{I}_2 \\ \mathbf{I}_1 &= -5 \mathbf{I}_2 \end{aligned} \quad (5)$$

Substituting (1) and (5) into (3),

$$\begin{aligned} \mathbf{V}_s &= 600 \mathbf{I}_1 + 100 \mathbf{I}_2 + 60 \mathbf{I}_1 \\ &= (660)(-5) \mathbf{I}_2 + 100 \mathbf{I}_2 \\ &= -3200 \mathbf{I}_2 \end{aligned} \quad (6)$$

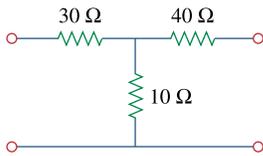
From (4) and (6),

$$\frac{\mathbf{V}_o}{\mathbf{V}_2} = \frac{-300 \mathbf{I}_2}{-3200 \mathbf{I}_2} = \underline{\underline{0.09375}}$$

**Chapter 19, Problem 67.**



If three copies of the circuit in Fig. 19.114 are connected in parallel, find the overall transmission parameters.

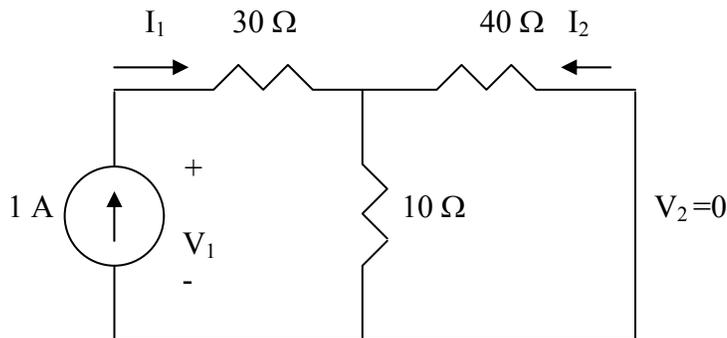


**Figure 19.114**

For Prob. 19.67.

**Chapter 19, Solution 67.**

We first find the  $y$  parameters, to find  $y_{11}$  and  $y_{21}$  consider the circuit below.

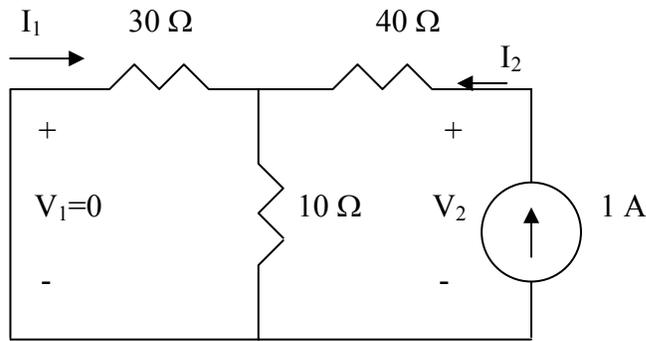


$$V_1 = I_1(30 + 10 // 40) = 38I_1 \quad \longrightarrow \quad y_{11} = \frac{I_1}{V_1} = \frac{1}{38}$$

By current division,

$$I_2 = \frac{-10}{50} I_1 = -0.2I_1 \quad \longrightarrow \quad y_{21} = \frac{I_2}{V_1} = \frac{-0.2I_1}{38I_1} = \frac{-1}{190}$$

To find  $y_{22}$  and  $y_{12}$  consider the circuit below.



$$V_2 = (40 + 10 // 30)I_2 = 47.5I_2 \quad \longrightarrow \quad y_{22} = \frac{I_2}{V_2} = \frac{2}{93} \quad y_{22} = 2/95$$

By current division,

$$I_1 = -\frac{10}{30+10}I_2 = -\frac{I_2}{4} \quad \longrightarrow \quad y_{12} = \frac{I_1}{V_2} = \frac{-\frac{1}{4}I_2}{47.5I_2} = -\frac{1}{190}$$

$$[Y] = \begin{bmatrix} 1/38 & -1/190 \\ -1/190 & 2/95 \end{bmatrix}$$

For three copies cascaded in parallel, we can use MATLAB.

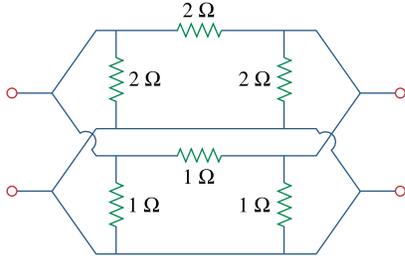
```
>> Y=[1/38,-1/190;-1/190,2/95]
Y =
    0.0263 -0.0053
   -0.0053  0.0211
>> Y3=3*Y
Y3 =
    0.0789 -0.0158
   -0.0158  0.0632
>> DY=0.0789*0.0632-0.0158*0.158
DY =
    0.0025
>> T=[0.0632/0.0158,1/0.0158;DY/0.0158,0.0789/0.0158]
T =
    4.0000  63.2911
    0.1576  4.9937
```

$$T = \begin{bmatrix} 4 & 63.29 \\ 0.1576 & 4.994 \end{bmatrix}$$

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### Chapter 19, Problem 68.

Obtain the  $h$  parameters for the network in Fig. 19.115.



**Figure 19.115**

For Prob. 19.68.

### Chapter 19, Solution 68.

For the upper network  $N_a$ ,  $[\mathbf{y}_a] = \begin{bmatrix} 4 & -2 \\ -2 & 4 \end{bmatrix}$

and for the lower network  $N_b$ ,  $[\mathbf{y}_b] = \begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix}$

For the overall network,

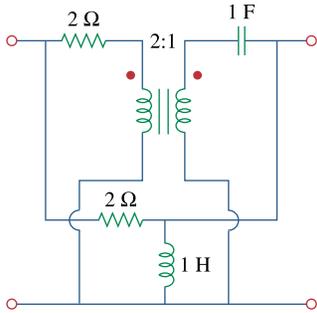
$$[\mathbf{y}] = [\mathbf{y}_a] + [\mathbf{y}_b] = \begin{bmatrix} 6 & -3 \\ -3 & 6 \end{bmatrix}$$

$$\Delta_y = 36 - 9 = 27$$

$$[\mathbf{h}] = \begin{bmatrix} \frac{1}{\Delta_y} & \frac{-\mathbf{y}_{12}}{\Delta_y} \\ \frac{\mathbf{y}_{11}}{\Delta_y} & \frac{\mathbf{y}_{11}}{\Delta_y} \\ \frac{\mathbf{y}_{21}}{\Delta_y} & \frac{\Delta_y}{\Delta_y} \\ \frac{\mathbf{y}_{11}}{\Delta_y} & \frac{\mathbf{y}_{11}}{\Delta_y} \end{bmatrix} = \begin{bmatrix} \frac{1}{6} \Omega & \frac{1}{2} \\ \frac{1}{2} & \frac{9}{2} \text{ S} \end{bmatrix}$$

**Chapter 19, Problem 69.**

\* The circuit in Fig. 19.116 may be regarded as two two-ports connected in parallel. Obtain the y parameters as functions of  $s$ .



**Figure 19.116**

For Prob. 19.69.

\* An asterisk indicates a challenging problem.

**Chapter 19, Solution 69.**

We first determine the y parameters for the upper network  $N_a$ .

To get  $y_{11}$  and  $y_{21}$ , consider the circuit in Fig. (a).

$$n = \frac{1}{2}, \quad \mathbf{Z}_R = \frac{1/s}{n^2} = \frac{4}{s}$$

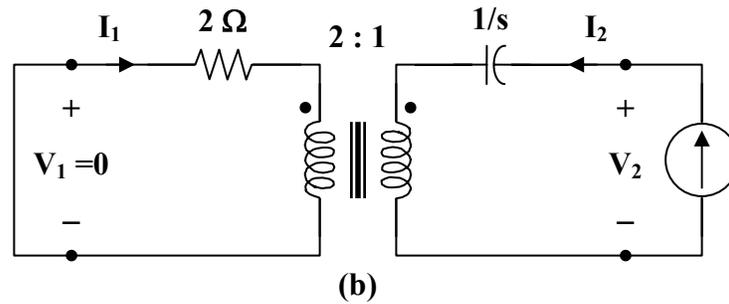
$$\mathbf{V}_1 = (2 + \mathbf{Z}_R) \mathbf{I}_1 = \left(2 + \frac{4}{s}\right) \mathbf{I}_1 = \left(\frac{2s + 4}{s}\right) \mathbf{I}_1$$

$$y_{11} = \frac{\mathbf{I}_1}{\mathbf{V}_1} = \frac{s}{2(s + 2)}$$

$$\mathbf{I}_2 = \frac{-\mathbf{I}_1}{n} = -2\mathbf{I}_1 = \frac{-s\mathbf{V}_1}{s + 2}$$

$$y_{21} = \frac{\mathbf{I}_2}{\mathbf{V}_1} = \frac{-s}{s + 2}$$

To get  $y_{22}$  and  $y_{12}$ , consider the circuit in Fig. (b).



$$Z_R' = (n^2)(2) = \left(\frac{1}{4}\right)(2) = \frac{1}{2}$$

$$V_2 = \left(\frac{1}{s} + Z_R'\right) I_2 = \left(\frac{1}{s} + \frac{1}{2}\right) I_2 = \left(\frac{s+2}{2s}\right) I_2$$

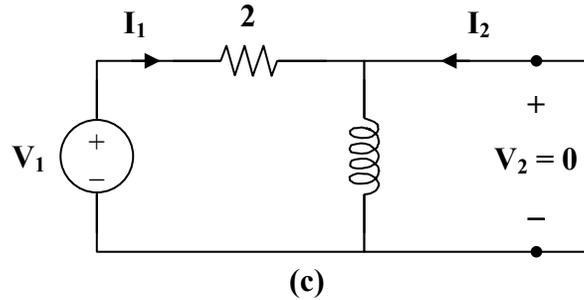
$$y_{22} = \frac{I_2}{V_2} = \frac{2s}{s+2}$$

$$I_1 = -n I_2 = \left(\frac{-1}{2}\right) \left(\frac{2s}{s+2}\right) V_2 = \left(\frac{-s}{s+2}\right) V_2$$

$$y_{12} = \frac{I_1}{V_2} = \frac{-s}{s+2}$$

$$[y_a] = \begin{bmatrix} \frac{s}{2(s+2)} & \frac{-s}{s+2} \\ \frac{-s}{s+2} & \frac{2s}{s+2} \end{bmatrix}$$

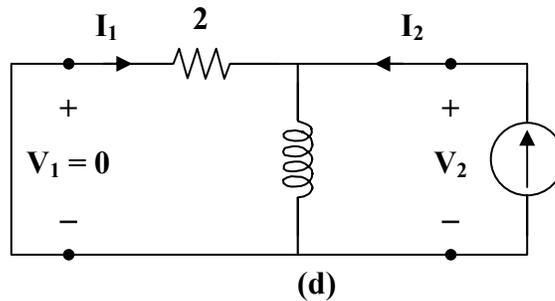
For the lower network  $N_b$ , we obtain  $y_{11}$  and  $y_{21}$  by referring to the network in Fig. (c).



$$V_1 = 2I_1 \longrightarrow y_{11} = \frac{I_1}{V_1} = \frac{1}{2}$$

$$I_2 = -I_1 = \frac{-V_1}{2} \longrightarrow y_{21} = \frac{I_2}{V_1} = \frac{-1}{2}$$

To get  $y_{22}$  and  $y_{12}$ , refer to the circuit in Fig. (d).



$$V_2 = (s \parallel 2)I_2 = \frac{2s}{s+2}I_2 \longrightarrow y_{22} = \frac{I_2}{V_2} = \frac{s+2}{2s}$$

$$I_1 = -I_2 \cdot \frac{-s}{s+2} = \left(\frac{-s}{s+2}\right)\left(\frac{s+2}{2s}\right)V_2 = \frac{-V_2}{2}$$

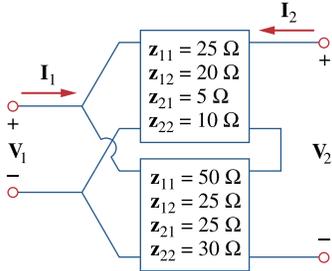
$$y_{12} = \frac{I_1}{V_2} = \frac{-1}{2}$$

$$[y_b] = \begin{bmatrix} 1/2 & -1/2 \\ -1/2 & (s+2)/2s \end{bmatrix}$$

$$[y] = [y_a] + [y_b] = \begin{bmatrix} \frac{s+1}{s+2} & \frac{-(3s+2)}{2(s+2)} \\ \frac{-(3s+2)}{2(s+2)} & \frac{5s^2+4s+4}{2s(s+2)} \end{bmatrix}$$

## Chapter 19, Problem 70.

\* For the parallel-series connection of the two two-ports in Fig. 19.117, find the  $g$  parameters.



**Figure 19.117**

For Prob. 19.70.

\* An asterisk indicates a challenging problem.

## Chapter 19, Solution 70.

We may obtain the  $g$  parameters from the given  $z$  parameters.

$$[\mathbf{z}_a] = \begin{bmatrix} 25 & 20 \\ 5 & 10 \end{bmatrix}, \quad \Delta_{z_a} = 250 - 100 = 150$$

$$[\mathbf{z}_b] = \begin{bmatrix} 50 & 25 \\ 25 & 30 \end{bmatrix}, \quad \Delta_{z_b} = 1500 - 625 = 875$$

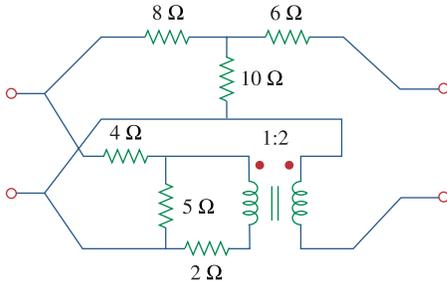
$$[\mathbf{g}] = \begin{bmatrix} \frac{1}{z_{11}} & \frac{-z_{12}}{z_{11}} \\ \frac{z_{21}}{z_{11}} & \frac{\Delta_z}{z_{11}} \end{bmatrix}$$

$$[\mathbf{g}_a] = \begin{bmatrix} 0.04 & -0.8 \\ 0.2 & 6 \end{bmatrix}, \quad [\mathbf{g}_b] = \begin{bmatrix} 0.02 & -0.5 \\ 0.5 & 17.5 \end{bmatrix}$$

$$[\mathbf{g}] = [\mathbf{g}_a] + [\mathbf{g}_b] = \underline{\underline{\begin{bmatrix} 0.06 \text{ S} & -1.3 \\ 0.7 & 23.5 \Omega \end{bmatrix}}}$$

**Chapter 19, Problem 71.**

\* Determine the  $z$  parameters for the network in Fig. 19.118.



**Figure 19.118**

For Prob. 19.71.

\* An asterisk indicates a challenging problem.

**Chapter 19, Solution 71.**

This is a parallel-series connection of two two-ports. We need to add their g parameters together and obtain z parameters from there.

For the transformer,

$$V_1 = \frac{1}{2}V_2, \quad I_1 = -2I_2$$

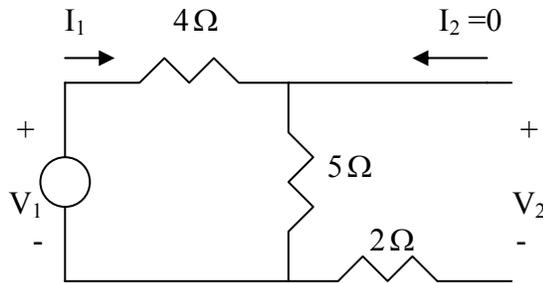
Comparing this with

$$V_1 = AV_2 - BI_2, \quad I_1 = CV_2 - DI_2$$

shows that

$$[T_{b1}] = \begin{bmatrix} 0.5 & 0 \\ 0 & 2 \end{bmatrix}$$

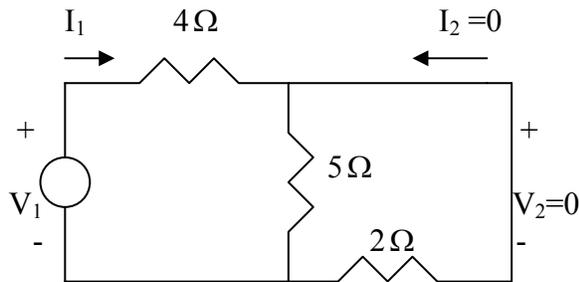
To get A and C for  $T_{b2}$ , consider the circuit below.



$$V_1 = 9I_1, \quad V_2 = 5I_1$$

$$A = \frac{V_1}{V_2} = 9/5 = 1.8, \quad C = \frac{I_1}{V_2} = 1/5 = 0.2$$

We obtain B and D by looking at the circuit below.



$$I_2 = -\frac{5}{7}I_1 \quad \longrightarrow \quad D = -\frac{I_1}{I_2} = 7/5 = 1.4$$

$$V_1 = 4I_1 - 2I_2 = 4\left(-\frac{7}{5}I_2\right) - 2I_2 = -\frac{38}{5}I_2 \quad \longrightarrow \quad B = -\frac{V_1}{I_2} = 7.6$$

$$[T_{b2}] = \begin{bmatrix} 1.8 & 7.6 \\ 0.2 & 1.4 \end{bmatrix}$$

$$[T] = [T_{b1}][T_{b2}] = \begin{bmatrix} 0.9 & 3.8 \\ 0.4 & 2.8 \end{bmatrix}, \quad \Delta_T = 1$$

$$[g_b] = \begin{bmatrix} C/A & -\Delta_T/A \\ 1/A & B/A \end{bmatrix} = \begin{bmatrix} 0.4444 & -1.1111 \\ 1.1111 & 4.2222 \end{bmatrix}$$

From Prob. 19.52,

$$[T_a] = \begin{bmatrix} 1.8 & 18.8 \\ 0.1 & 1.6 \end{bmatrix}$$

$$[g_a] = \begin{bmatrix} C/A & -\Delta_T/A \\ 1/A & B/A \end{bmatrix} = \begin{bmatrix} 0.05555 & -0.5555 \\ 0.5555 & 10.4444 \end{bmatrix}$$

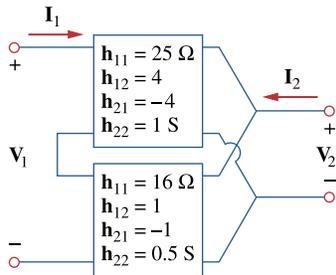
$$[g] = [g_a] + [g_b] = \begin{bmatrix} 0.4999 & -1.6667 \\ 1.6667 & 14.667 \end{bmatrix}$$

Thus,

$$[z] = \begin{bmatrix} 1/g_{11} & -g_{21}/g_{11} \\ g_{21}/g_{11} & \Delta_g/g_{11} \end{bmatrix} = \begin{bmatrix} 2 & -3.334 \\ 3.334 & 20.22 \end{bmatrix} \Omega$$

### Chapter 19, Problem 72.

\* A series-parallel connection of two two-ports is shown in Fig. 19.119. Determine the  $z$  parameter representation of the network.



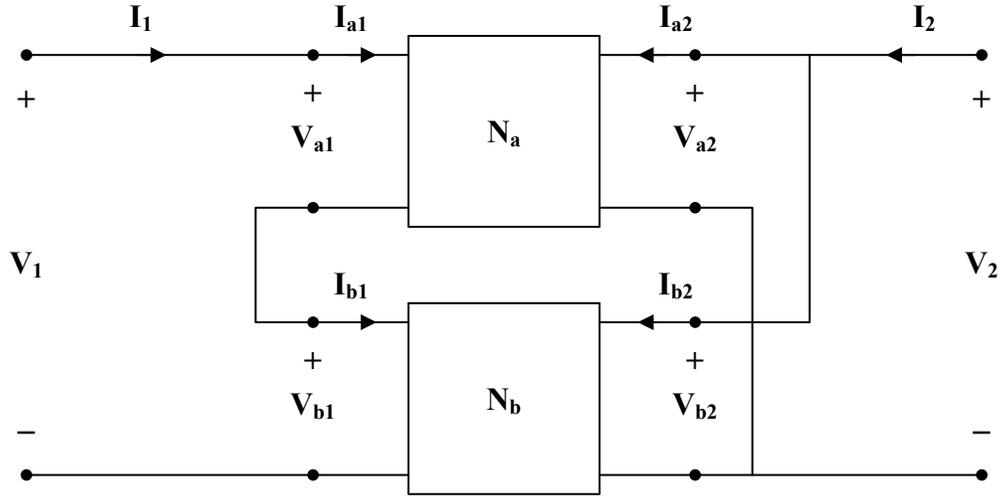
**Figure 19.119**

For Prob. 19.72.

\* An asterisk indicates a challenging problem.

**Chapter 19, Solution 72.**

Consider the network shown below.



$$V_{a1} = 25I_{a1} + 4V_{a2} \quad (1)$$

$$I_{a2} = -4I_{a1} + V_{a2} \quad (2)$$

$$V_{b1} = 16I_{b1} + V_{b2} \quad (3)$$

$$I_{b2} = -I_{b1} + 0.5V_{b2} \quad (4)$$

$$V_1 = V_{a1} + V_{b1}$$

$$V_2 = V_{a2} = V_{b2}$$

$$I_2 = I_{a2} + I_{b2}$$

$$I_1 = I_{a1}$$

Now, rewrite (1) to (4) in terms of  $I_1$  and  $V_2$

$$V_{a1} = 25I_1 + 4V_2 \quad (5)$$

$$I_{a2} = -4I_1 + V_2 \quad (6)$$

$$V_{b1} = 16I_{b1} + V_2 \quad (7)$$

$$I_{b2} = -I_{b1} + 0.5V_2 \quad (8)$$

Adding (5) and (7),

$$V_1 = 25I_1 + 16I_{b1} + 5V_2 \quad (9)$$

Adding (6) and (8),

$$I_2 = -4I_1 - I_{b1} + 1.5V_2 \quad (10)$$

$$I_{b1} = I_{a1} = I_1 \quad (11)$$

Because the two networks  $N_a$  and  $N_b$  are independent,

$$\begin{aligned} \mathbf{I}_2 &= -5\mathbf{I}_1 + 1.5\mathbf{V}_2 \\ \text{or } \mathbf{V}_2 &= 3.333\mathbf{I}_1 + 0.6667\mathbf{I}_2 \end{aligned} \quad (12)$$

Substituting (11) and (12) into (9),

$$\begin{aligned} \mathbf{V}_1 &= 41\mathbf{I}_1 + \frac{25}{1.5}\mathbf{I}_1 + \frac{5}{1.5}\mathbf{I}_2 \\ \mathbf{V}_1 &= 57.67\mathbf{I}_1 + 3.333\mathbf{I}_2 \end{aligned} \quad (13)$$

Comparing (12) and (13) with the following equations

$$\begin{aligned} \mathbf{V}_1 &= \mathbf{z}_{11}\mathbf{I}_1 + \mathbf{z}_{12}\mathbf{I}_2 \\ \mathbf{V}_2 &= \mathbf{z}_{21}\mathbf{I}_1 + \mathbf{z}_{22}\mathbf{I}_2 \end{aligned}$$

indicates that

$$[\mathbf{z}] = \underline{\underline{\begin{bmatrix} 57.67 & 3.333 \\ 3.333 & 0.6667 \end{bmatrix} \Omega}}$$

Alternatively,

$$\begin{aligned} [\mathbf{h}_a] &= \begin{bmatrix} 25 & 4 \\ -4 & 1 \end{bmatrix}, & [\mathbf{h}_b] &= \begin{bmatrix} 16 & 1 \\ -1 & 0.5 \end{bmatrix} \\ [\mathbf{h}] = [\mathbf{h}_a] + [\mathbf{h}_b] &= \begin{bmatrix} 41 & 5 \\ -5 & 1.5 \end{bmatrix} & \Delta_h &= 61.5 + 25 = 86.5 \end{aligned}$$

$$[\mathbf{z}] = \begin{bmatrix} \frac{\Delta_h}{\mathbf{h}_{22}} & \frac{\mathbf{h}_{12}}{\mathbf{h}_{22}} \\ -\frac{\mathbf{h}_{21}}{\mathbf{h}_{22}} & \frac{1}{\mathbf{h}_{22}} \end{bmatrix} = \underline{\underline{\begin{bmatrix} 57.67 & 3.333 \\ 3.333 & 0.6667 \end{bmatrix} \Omega}}$$

as obtained previously.

### Chapter 19, Problem 73.



Three copies of the circuit shown in Fig. 19.70 are connected in cascade. Determine the  $z$  parameters.

### Chapter 19, Solution 73.

From Problem 19.6,

$$[z] = \begin{bmatrix} 25 & 20 \\ 24 & 30 \end{bmatrix}, \quad \Delta Z = 25 \times 30 - 20 \times 24 = 270$$

$$A = \frac{z_{11}}{z_{21}} = \frac{25}{24}, \quad B = \frac{\Delta Z}{z_{21}} = \frac{270}{24}$$

$$C = \frac{1}{z_{21}} = \frac{1}{24}, \quad D = \frac{z_{22}}{z_{21}} = \frac{30}{24}$$

The overall ABCD parameters can be found using MATLAB.

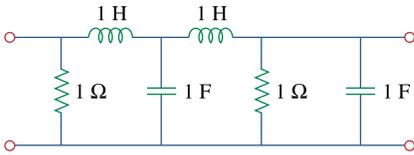
```
>> T=[25/24,270/24;1/24,30/24]
T =
    1.0417    11.2500
    0.0417     1.2500
>> T3=T*T*T
T3 =
    2.6928    49.7070
    0.1841     3.6133
>> Z=[2.693/0.1841,(2.693*3.613-0.1841*49.71)/0.1841;1/0.1841,3.613/0.1841]
Z =
    14.6279     3.1407
     5.4318    19.6252
```

$$Z = \begin{bmatrix} 14.628 & 3.141 \\ 5.432 & 19.625 \end{bmatrix}$$

**Chapter 19, Problem 74.**



\* Determine the **ABCD** parameters of the circuit in Fig. 19.120 as functions of  $s$ . (*Hint:* Partition the circuit into subcircuits and cascade them using the results of Prob. 19.43.)



**Figure 19.120**

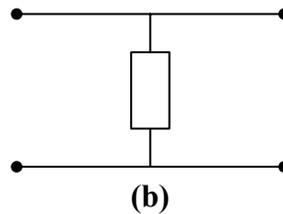
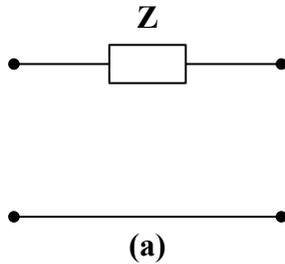
For Prob. 19.74.

\* An asterisk indicates a challenging problem.

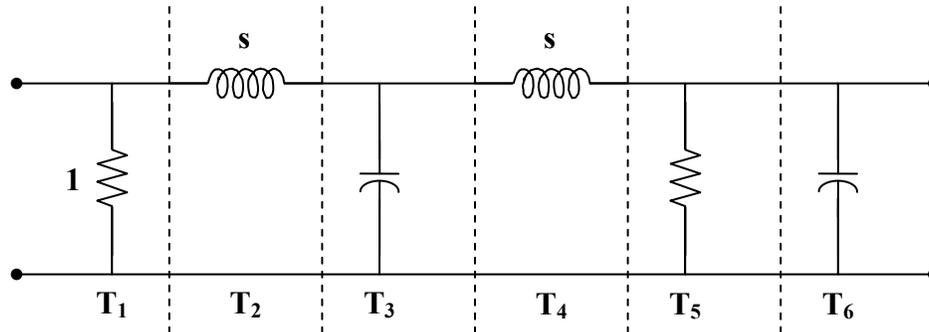
**Chapter 19, Solution 74.**

From Prob. 18.35, the transmission parameters for the circuit in Figs. (a) and (b) are

$$[\mathbf{T}_a] = \begin{bmatrix} 1 & \mathbf{Z} \\ 0 & 1 \end{bmatrix}, \quad [\mathbf{T}_b] = \begin{bmatrix} 1 & 0 \\ 1/\mathbf{Z} & 1 \end{bmatrix}$$



We partition the given circuit into six subcircuits similar to those in Figs. (a) and (b) as shown in Fig. (c) and obtain  $[\mathbf{T}]$  for each.



$$[\mathbf{T}_1] = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}, \quad [\mathbf{T}_2] = \begin{bmatrix} 1 & s \\ 0 & 1 \end{bmatrix}, \quad [\mathbf{T}_3] = \begin{bmatrix} 1 & 0 \\ s & 1 \end{bmatrix}$$

$$[\mathbf{T}_4] = [\mathbf{T}_2], \quad [\mathbf{T}_5] = [\mathbf{T}_1], \quad [\mathbf{T}_6] = [\mathbf{T}_3]$$

$$\begin{aligned} [\mathbf{T}] &= [\mathbf{T}_1][\mathbf{T}_2][\mathbf{T}_3][\mathbf{T}_4][\mathbf{T}_5][\mathbf{T}_6] = [\mathbf{T}_1][\mathbf{T}_2][\mathbf{T}_3][\mathbf{T}_4] \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ s & 1 \end{bmatrix} \\ &= [\mathbf{T}_1][\mathbf{T}_2][\mathbf{T}_3][\mathbf{T}_4] \begin{bmatrix} 1 & 0 \\ s+1 & 1 \end{bmatrix} = [\mathbf{T}_1][\mathbf{T}_2][\mathbf{T}_3] \begin{bmatrix} 1 & s \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ s+1 & 1 \end{bmatrix} \\ &= [\mathbf{T}_1][\mathbf{T}_2] \begin{bmatrix} 1 & 0 \\ s & 1 \end{bmatrix} \begin{bmatrix} s^2 + s + 1 & s \\ s+1 & 1 \end{bmatrix} \\ &= [\mathbf{T}_1] \begin{bmatrix} 1 & s \\ 0 & 1 \end{bmatrix} \begin{bmatrix} s^2 + s + 1 & s \\ s^3 + s^2 + 2s + 1 & s^2 + 1 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} s^4 + s^3 + 3s^2 + 2s + 1 & s^3 + 2s \\ s^3 + s^2 + 2s + 1 & s^2 + 1 \end{bmatrix} \\ [\mathbf{T}] &= \underline{\underline{\begin{bmatrix} s^4 + s^3 + 3s^2 + 2s + 1 & s^3 + 2s \\ s^4 + 2s^3 + 4s^2 + 4s + 2 & s^3 + s^2 + 2s + 1 \end{bmatrix}}} \end{aligned}$$

Note that  $\mathbf{AB} - \mathbf{CD} = 1$  as expected.

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**Chapter 19, Problem 75.**

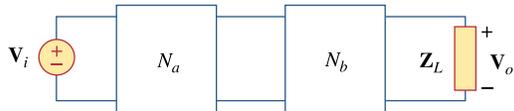
**ML**

\* For the individual two-ports shown in Fig. 19.121 where,

$$[z_a] = \begin{bmatrix} 8 & 6 \\ 4 & 5 \end{bmatrix} \Omega \quad [y_b] = \begin{bmatrix} 8 & -4 \\ 2 & 10 \end{bmatrix} S$$

(a) Determine the y parameters of the overall two-port.

(b) Find the voltage ratio  $V_o/V_i$  when  $Z_L = 2 \Omega$ .



**Figure 19.110**

For Prob. 19.63.

\* An asterisk indicates a challenging problem.

**Chapter 19, Solution 75.**

(a) We convert  $[z_a]$  and  $[z_b]$  to T-parameters. For  $N_a$ ,  $\Delta_z = 40 - 24 = 16$ .

$$[T_a] = \begin{bmatrix} z_{11}/z_{21} & \Delta_z/z_{21} \\ 1/z_{21} & z_{22}/z_{21} \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 0.25 & 1.25 \end{bmatrix}$$

For  $N_b$ ,  $\Delta_y = 80 + 8 = 88$ .

$$[T_b] = \begin{bmatrix} -y_{22}/y_{21} & -1/y_{21} \\ -\Delta_y/y_{21} & -y_{11}/y_{21} \end{bmatrix} = \begin{bmatrix} -5 & -0.5 \\ -44 & -4 \end{bmatrix}$$

$$[T] = [T_a][T_b] = \begin{bmatrix} -186 & -17 \\ -56.25 & -5.125 \end{bmatrix}$$

We convert this to y-parameters.  $\Delta_T = AD - BC = -3$ .

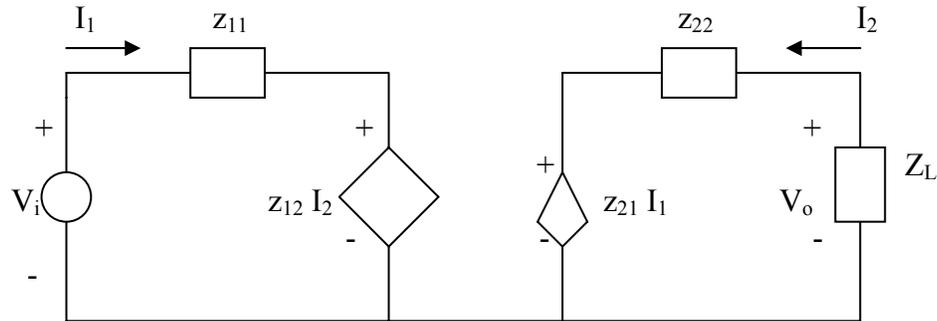
$$[y] = \begin{bmatrix} D/B & -\Delta_T/B \\ -1/B & A/B \end{bmatrix} = \begin{bmatrix} 0.3015 & -0.1765 \\ 0.0588 & 10.94 \end{bmatrix}$$

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(b) The equivalent z-parameters are

$$[z] = \begin{bmatrix} A/C & \Delta_T/C \\ 1/C & D/C \end{bmatrix} = \begin{bmatrix} 3.3067 & 0.0533 \\ -0.0178 & 0.0911 \end{bmatrix}$$

Consider the equivalent circuit below.



$$V_i = z_{11}I_1 + z_{12}I_2 \quad (1)$$

$$V_o = z_{21}I_1 + z_{22}I_2 \quad (2)$$

$$\text{But } V_o = -I_2 Z_L \quad \longrightarrow \quad I_2 = -V_o / Z_L \quad (3)$$

From (2) and (3) ,

$$V_o = z_{21}I_1 - z_{22} \frac{V_o}{Z_L} \quad \longrightarrow \quad I_1 = V_o \left( \frac{1}{z_{21}} + \frac{z_{22}}{Z_L z_{21}} \right) \quad (4)$$

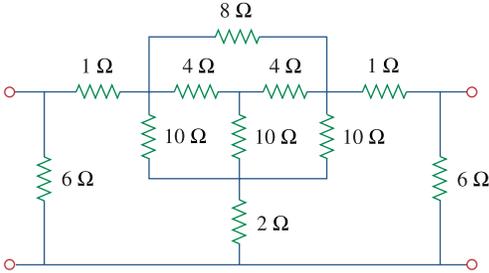
Substituting (3) and (4) into (1) gives

$$\frac{V_i}{V_o} = \left( \frac{z_{11}}{z_{21}} + \frac{z_{11}z_{22}}{z_{21}Z_L} \right) - \frac{z_{12}}{Z_L} = -194.3 \quad \longrightarrow \quad \underline{\underline{\frac{V_o}{V_i} = -0.0051}}$$

**Chapter 19, Problem 76.**



Use *PSpice* to obtain the  $z$  parameters of the network in Fig. 19.122.



**Figure 19.122**

For Prob. 19.76.

**Chapter 19, Solution 76.**

To get  $z_{11}$  and  $z_{21}$ , we open circuit the output port and let  $I_1 = 1\text{A}$  so that

$$z_{11} = \frac{V_1}{I_1} = V_1, \quad z_{21} = \frac{V_2}{I_1} = V_2$$

The schematic is shown below. After it is saved and run, we obtain

$$z_{11} = V_1 = 3.849, \quad z_{21} = V_2 = 1.122$$

Similarly, to get  $z_{22}$  and  $z_{12}$ , we open circuit the input port and let  $I_2 = 1\text{A}$  so that

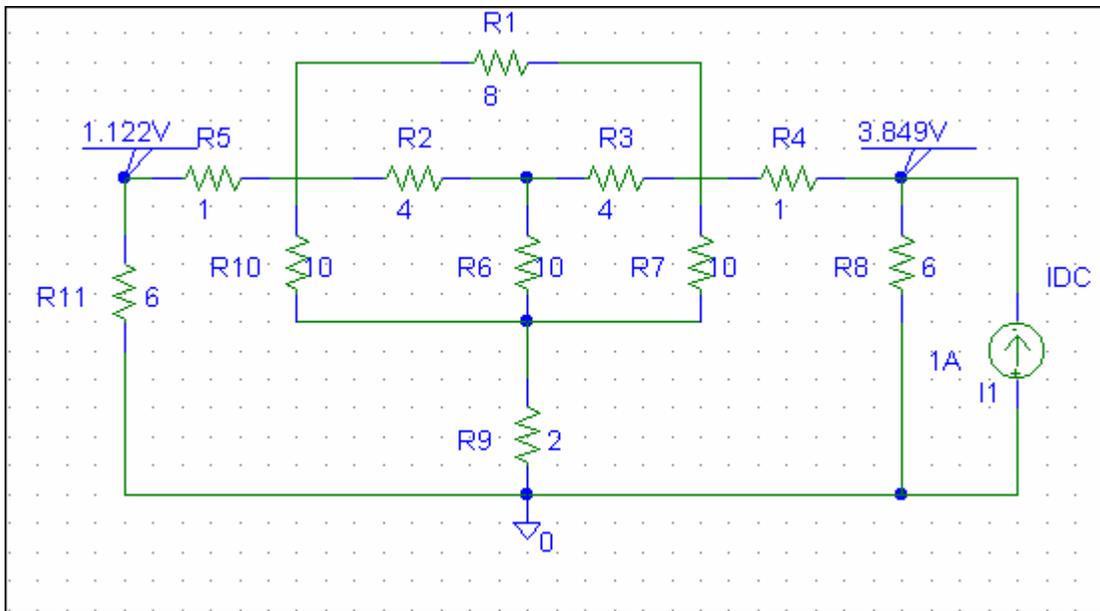
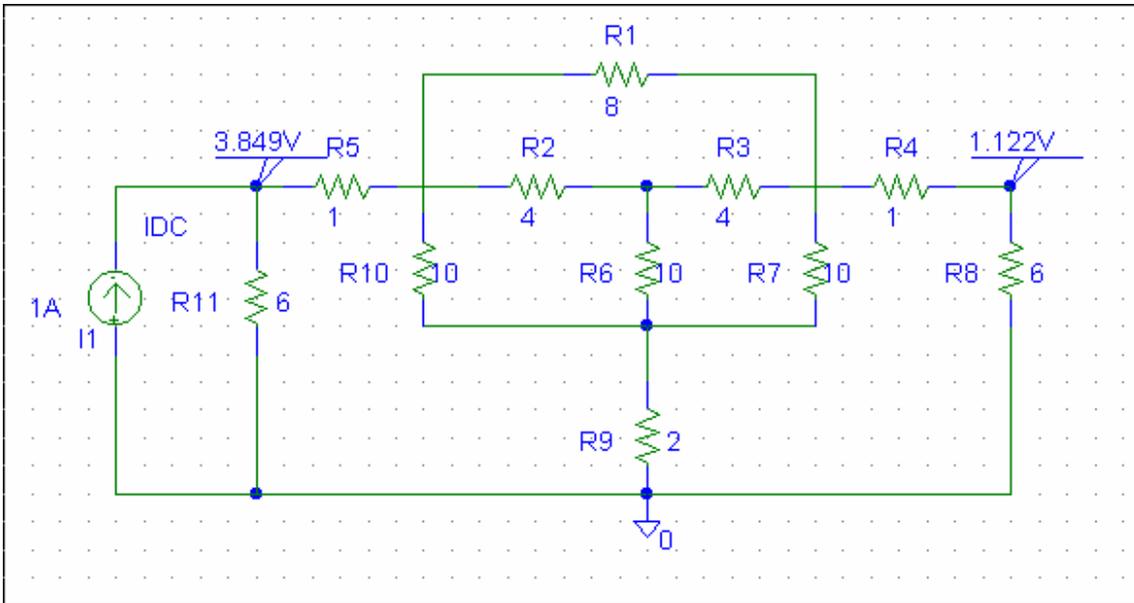
$$z_{12} = \frac{V_1}{I_2} = V_1, \quad z_{22} = \frac{V_2}{I_2} = V_2$$

The schematic is shown below. After it is saved and run, we obtain

$$z_{12} = V_1 = 1.122, \quad z_{22} = V_2 = 3.849$$

Thus,

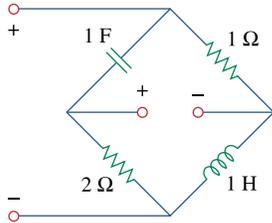
$$[z] = \begin{bmatrix} 3.849 & 1.122 \\ 1.122 & 3.849 \end{bmatrix} \Omega$$



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**Chapter 19, Problem 77.**

Using *PSpice*, find the  $h$  parameters of the network in Fig. 19.123. Take  $\omega = 1$  rad/s



**Figure 19.123**

For Prob. 19.77.

**Chapter 19, Solution 77.**

We follow Example 19.15 except that this is an AC circuit.

(a) We set  $V_2 = 0$  and  $I_1 = 1$  A. The schematic is shown below. In the AC Sweep Box, set Total Pts = 1, Start Freq = 0.1592, and End Freq = 0.1592. After simulation, the output file includes

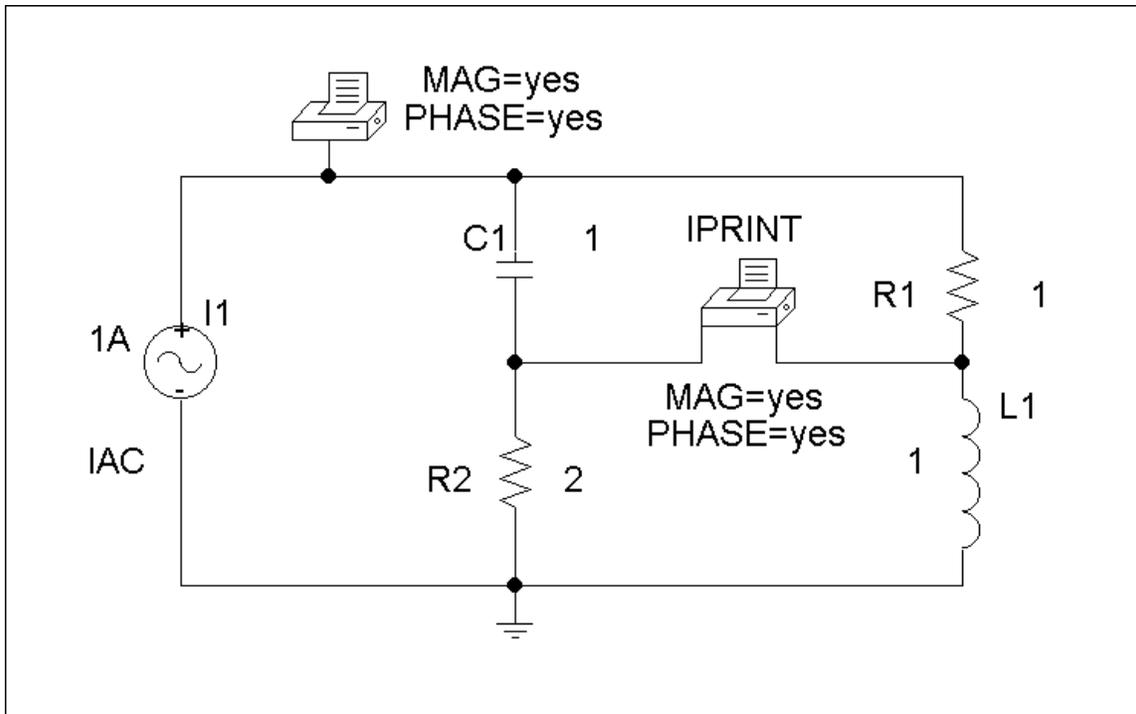
FREQ	IM(V_PRINT2)	IP(V_PRINT2)
1.592 E-01	3.163 E-01	-1.616 E+02

FREQ	VM(\$N_0001)	VP(\$N_0001)
1.592 E-01	9.488 E-01	-1.616 E+02

From this we obtain

$$h_{11} = V_1/1 = 0.9488\angle-161.6^\circ$$

$$h_{21} = I_2/1 = 0.3163\angle-161.6^\circ.$$



(b) In this case, we set  $I_1 = 0$  and  $V_2 = 1V$ . The schematic is shown below. In the AC Sweep box, we set Total Pts = 1, Start Freq = 0.1592, and End Freq = 0.1592. After simulation, we obtain an output file which includes

FREQ	VM(\$N_0001)	VP(\$N_0001)
1.592 E-01	3.163 E-01	1.842 E+01

FREQ	IM(V_PRINT2)	IP(V_PRINT2)
1.592 E-01	9.488 E-01	-1.616 E+02

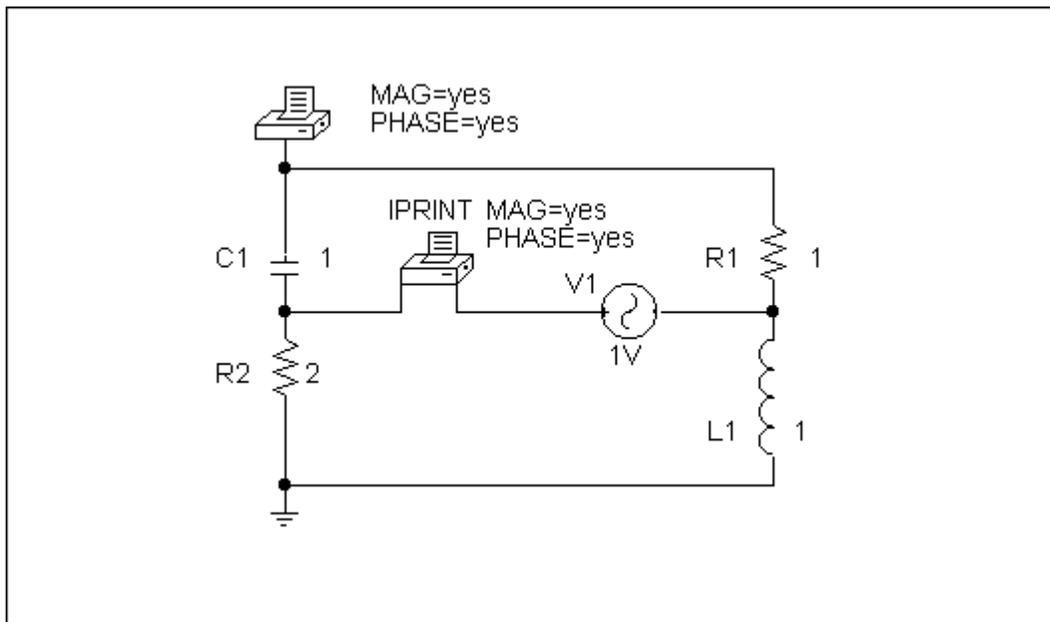
From this,

$$h_{12} = V_1/I_1 = 0.3163 \angle 18.42^\circ$$

$$h_{21} = I_2/I_1 = 0.9488 \angle -161.6^\circ.$$

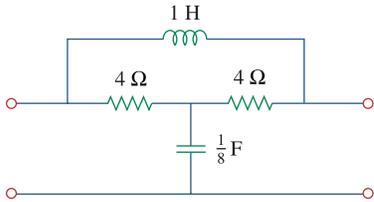
Thus,

$$[h] = \begin{bmatrix} 0.9488 \angle -161.6^\circ & 0.3163 \angle 18.42^\circ \\ 0.3163 \angle -161.6^\circ & 0.9488 \angle -161.6^\circ \end{bmatrix}$$



**Chapter 19, Problem 78.**

Obtain the  $h$  parameters at  $\omega = 4$  rad/s for the circuit in Fig. 19.124 using *PSpice*.



**Figure 19.124**

For Prob. 19.78.

## Chapter 19, Solution 78

For  $h_{11}$  and  $h_{21}$ , short-circuit the output port and let  $I_1 = 1\text{ A}$ .  $f = \omega / 2\pi = 0.6366$ . The schematic is shown below. When it is saved and run, the output file contains the following:

```
FREQ    IM(V_PRINT1)IP(V_PRINT1)
```

```
6.366E-01  1.202E+00  1.463E+02
```

```
FREQ    VM($N_0003) VP($N_0003)
```

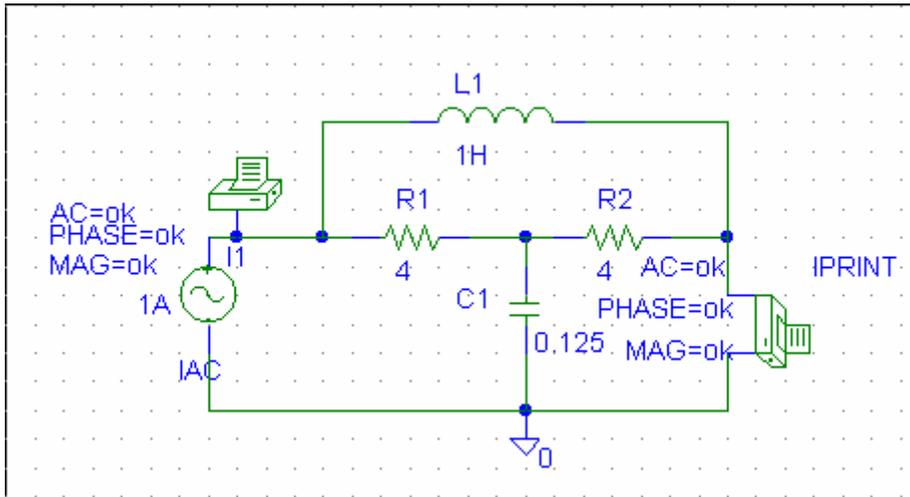
```
6.366E-01  3.771E+00 -1.350E+02
```

From the output file, we obtain

$$I_2 = 1.202 \angle 146.3^\circ, \quad V_1 = 3.771 \angle -135^\circ$$

so that

$$h_{11} = \frac{V_1}{I_1} = 3.771 \angle -135^\circ, \quad h_{21} = \frac{I_2}{I_1} = 1.202 \angle 146.3^\circ$$



For  $h_{12}$  and  $h_{22}$ , open-circuit the input port and let  $V_2 = 1V$ . The schematic is shown below. When it is saved and run, the output file includes:

```
FREQ      VM($N_0003) VP($N_0003)
```

```
6.366E-01  1.202E+00 -3.369E+01
```

```
FREQ      IM(V_PRINT1)IP(V_PRINT1)
```

```
6.366E-01  3.727E-01 -1.534E+02
```

From the output file, we obtain

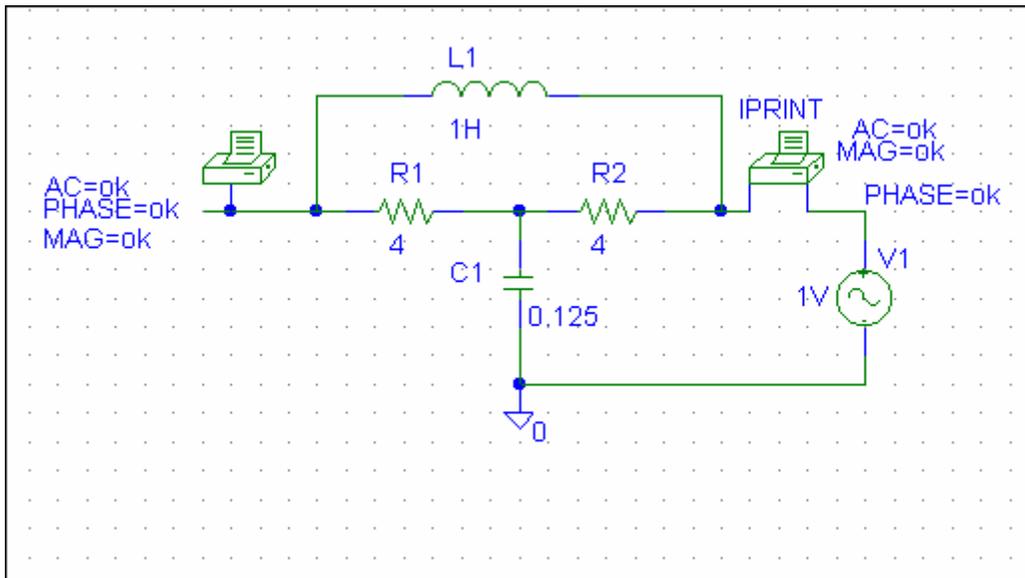
$$I_2 = 0.3727 \angle -153.4^\circ, \quad V_1 = 1.202 \angle -33.69^\circ$$

so that

$$h_{12} = \frac{V_1}{1} = 1.202 \angle -33.69^\circ, \quad h_{22} = \frac{I_2}{1} = 0.3727 \angle -153.4^\circ$$

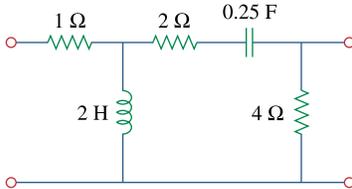
Thus,

$$[h] = \begin{bmatrix} 3.771 \angle -135^\circ & 1.202 \angle -33.69^\circ \\ 1.202 \angle 146.3^\circ & 0.3727 \angle -153.4^\circ \end{bmatrix}$$



**Chapter 19, Problem 79.**

Use *PSpice* to determine the  $z$  parameters of the circuit in Fig. 19.125. Take  $\omega = 2$  rad/s.



**Figure 19.125**

For Prob. 19.79.

## Chapter 19, Solution 79

We follow Example 19.16.

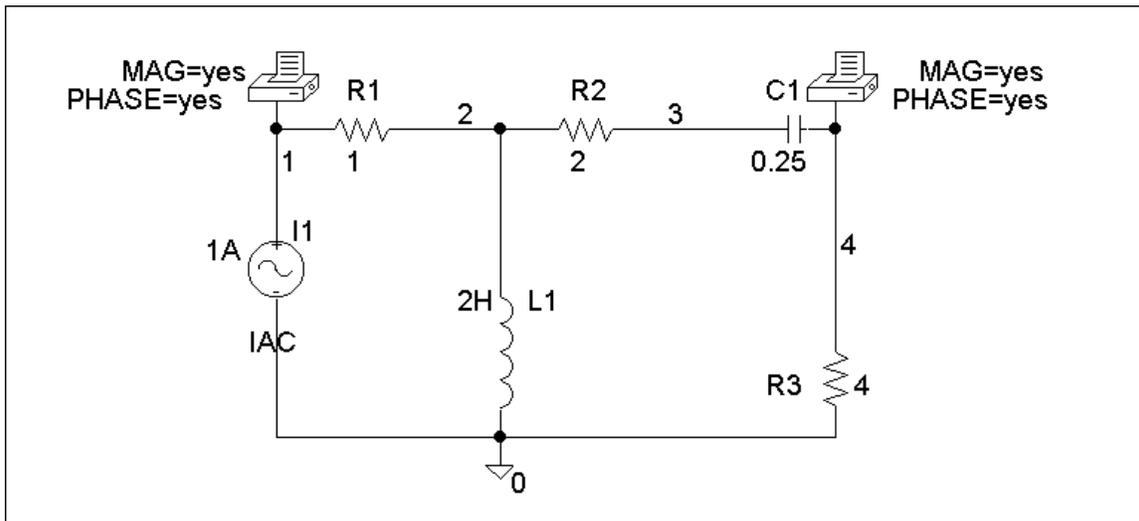
(a) We set  $I_1 = 1$  A and open-circuit the output-port so that  $I_2 = 0$ . The schematic is shown below with two VPRINT1s to measure  $V_1$  and  $V_2$ . In the AC Sweep box, we enter Total Pts = 1, Start Freq = 0.3183, and End Freq = 0.3183. After simulation, the output file includes

FREQ	VM(1)	VP(1)
3.183 E-01	4.669 E+00	-1.367 E+02
FREQ	VM(4)	VP(4)
3.183 E-01	2.530 E+00	-1.084 E+02

From this,

$$z_{11} = V_1/I_1 = 4.669\angle-136.7^\circ/1 = 4.669\angle-136.7^\circ$$

$$z_{21} = V_2/I_1 = 2.53\angle-108.4^\circ/1 = 2.53\angle-108.4^\circ.$$



(b) In this case, we let  $I_2 = 1$  A and open-circuit the input port. The schematic is shown below. In the AC Sweep box, we type Total Pts = 1, Start Freq = 0.3183, and End Freq = 0.3183. After simulation, the output file includes

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FREQ	VM(1)	VP(1)
3.183 E-01	2.530 E+00	-1.084 E+02

FREQ	VM(2)	VP(2)
3.183 E-01	1.789 E+00	-1.534 E+02

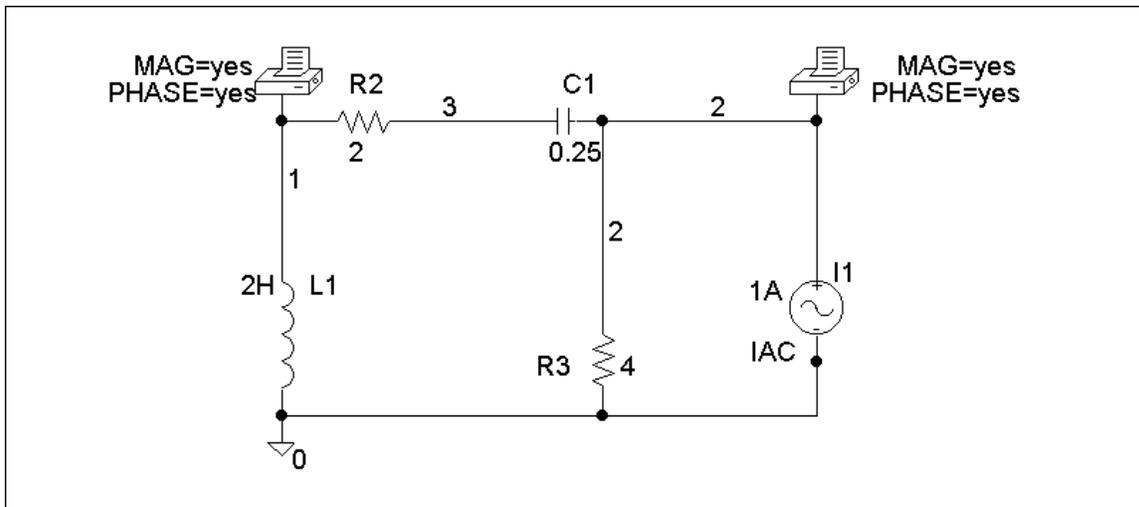
From this,

$$z_{12} = V_1/I_2 = 2.53 \angle -108.4^\circ / 1 = 2.53 \angle -108.4^\circ$$

$$z_{22} = V_2/I_2 = 1.789 \angle -153.4^\circ / 1 = 1.789 \angle -153.4^\circ.$$

Thus,

$$[z] = \begin{bmatrix} 4.669 \angle -136.7^\circ & 2.53 \angle -108.4^\circ \\ 2.53 \angle -108.4^\circ & 1.789 \angle -153.4^\circ \end{bmatrix} \underline{\underline{\Omega}}$$



### Chapter 19, Problem 80.

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Use *PSpice* to find the  $z$  parameters of the circuit in Fig. 19.71.

### Chapter 19, Solution 80

To get  $z_{11}$  and  $z_{21}$ , we open circuit the output port and let  $I_1 = 1\text{A}$  so that

$$z_{11} = \frac{V_1}{I_1} = V_1, \quad z_{21} = \frac{V_2}{I_1} = V_2$$

The schematic is shown below. After it is saved and run, we obtain

$$z_{11} = V_1 = 29.88, \quad z_{21} = V_2 = -70.37$$

Similarly, to get  $z_{22}$  and  $z_{12}$ , we open circuit the input port and let  $I_2 = 1\text{A}$  so that

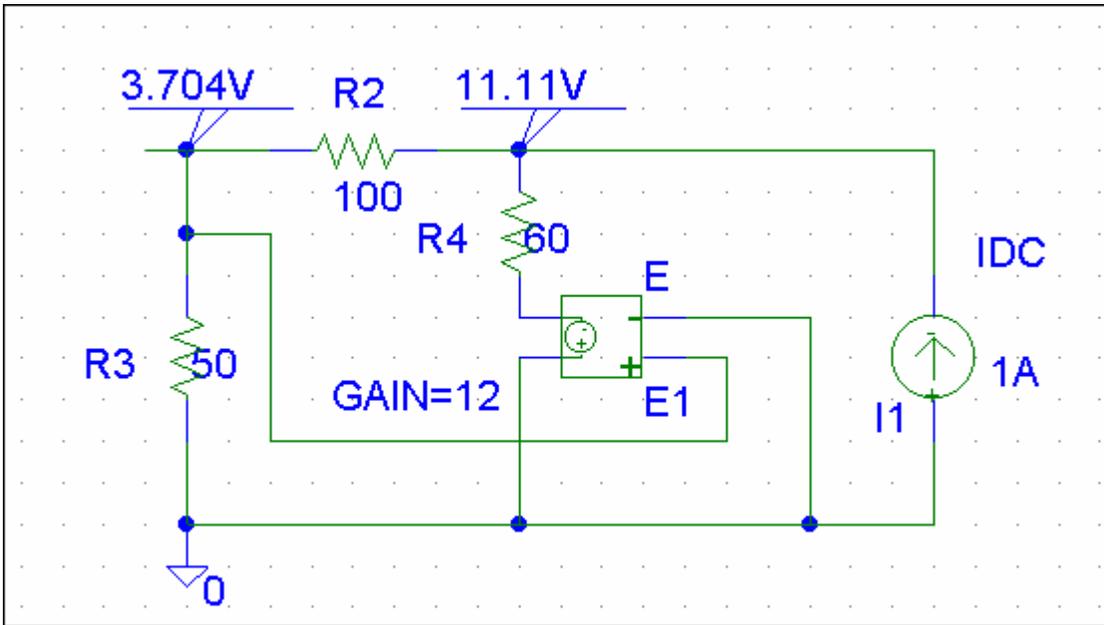
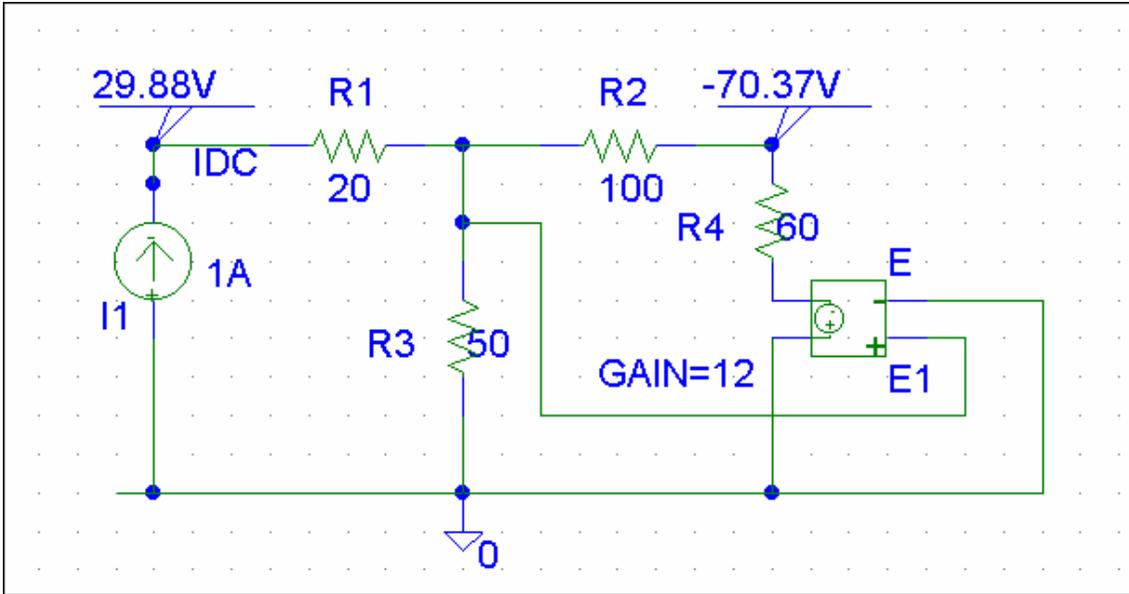
$$z_{12} = \frac{V_1}{I_2} = V_1, \quad z_{22} = \frac{V_2}{I_2} = V_2$$

The schematic is shown below. After it is saved and run, we obtain

$$z_{12} = V_1 = 3.704, \quad z_{22} = V_2 = 11.11$$

Thus,

$$[z] = \begin{bmatrix} 29.88 & 3.704 \\ -70.37 & 11.11 \end{bmatrix} \Omega$$



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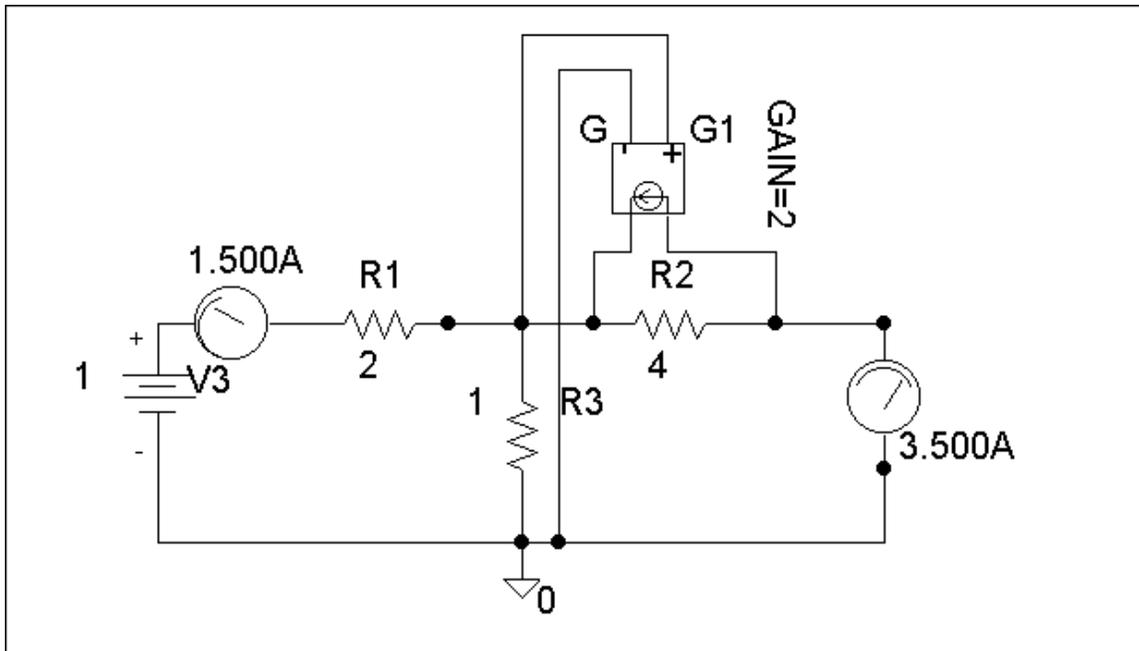
## Chapter 19, Problem 81.

Repeat Prob. 19.26 using *PSpice*.

## Chapter 19, Solution 81

(a) We set  $V_1 = 1$  and short circuit the output port. The schematic is shown below. After simulation we obtain

$$y_{11} = I_1 = 1.5, \quad y_{21} = I_2 = 3.5$$

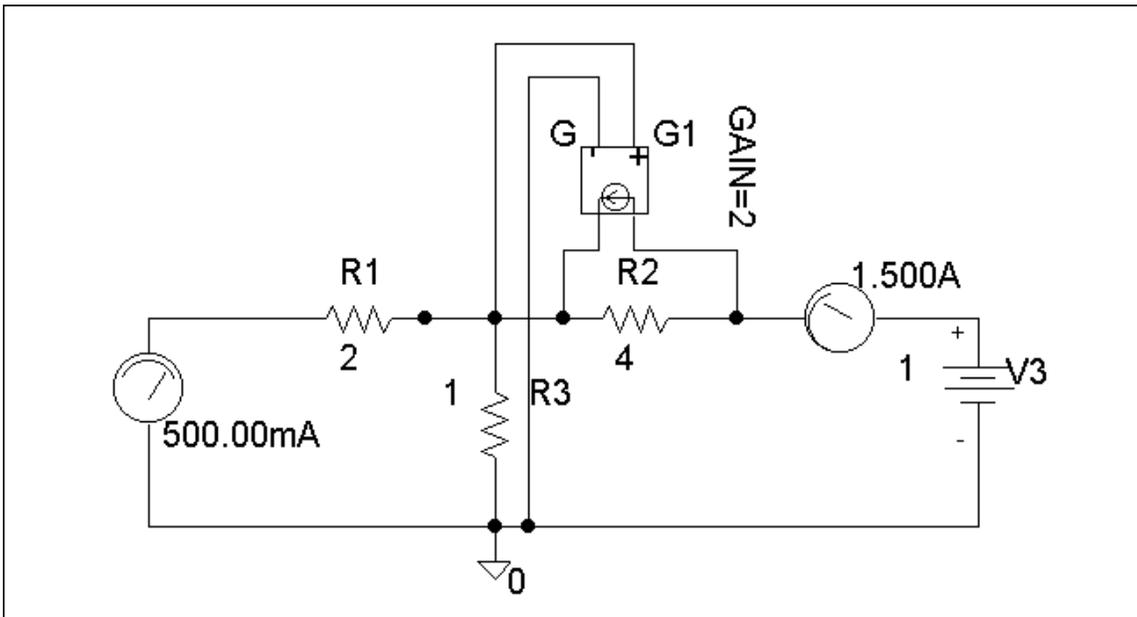


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(b) We set  $V_2 = 1$  and short-circuit the input port. The schematic is shown below. Upon simulating the circuit, we obtain

$$y_{12} = I_1 = -0.5, \quad y_{22} = I_2 = 1.5$$

$$[Y] = \underline{\underline{\begin{bmatrix} 1.5 & -0.5 \\ 3.5 & 1.5 \end{bmatrix} \text{ S}}}$$



## Chapter 19, Problem 82.

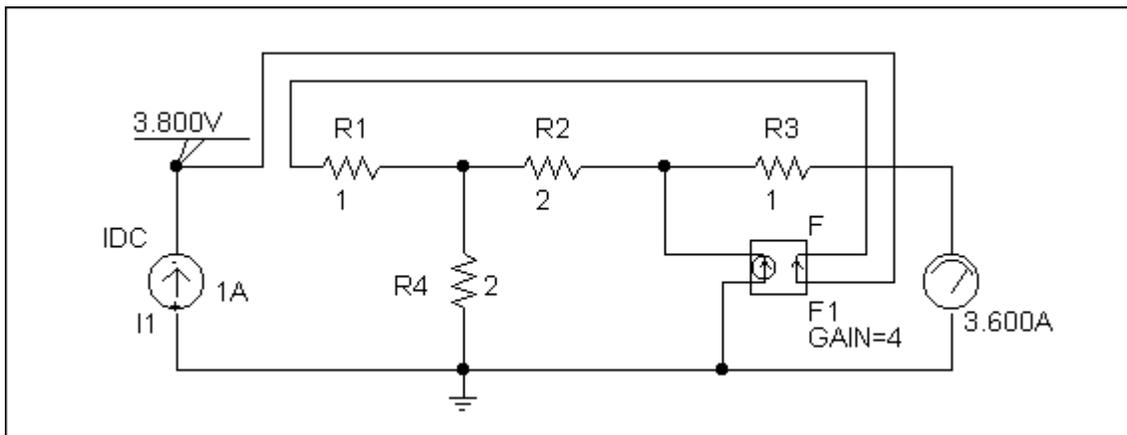
Use *PSpice* to rework Prob. 19.31.

## Chapter 19, Solution 82

We follow Example 19.15.

- (a) Set  $V_2 = 0$  and  $I_1 = 1\text{A}$ . The schematic is shown below. After simulation, we obtain

$$h_{11} = V_1/1 = 3.8, \quad h_{21} = I_2/1 = 3.6$$

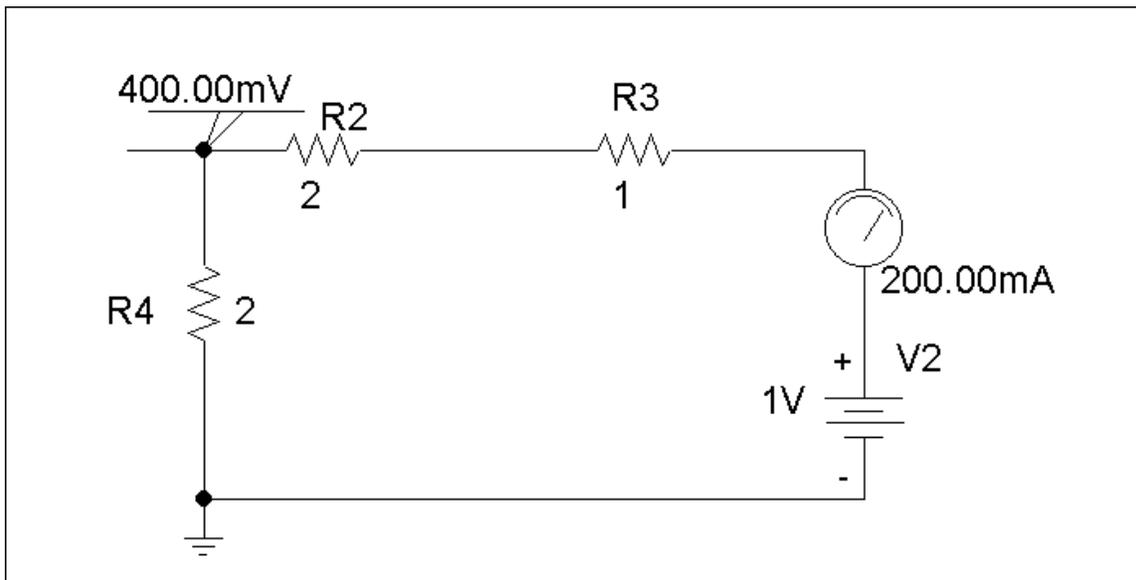


- (b) Set  $V_1 = 1\text{ V}$  and  $I_1 = 0$ . The schematic is shown below. After simulation, we obtain

$$h_{12} = V_1/I_1 = 0.4, \quad h_{22} = I_2/I_1 = 0.25$$

Hence,

$$[h] = \begin{bmatrix} 3.8 & 0.4 \\ 3.6 & 0.25 \end{bmatrix}$$



### Chapter 19, Problem 83.

Rework Prob. 19.47 using *PSpice*.

### Chapter 19, Solution 83

To get A and C, we open-circuit the output and let  $I_1 = 1\text{A}$ . The schematic is shown below. When the circuit is saved and simulated, we obtain  $V_1 = 11$  and  $V_2 = 34$ .

$$A = \frac{V_1}{V_2} = 0.3235, \quad C = \frac{I_1}{V_2} = \frac{1}{34} = 0.02941$$

Similarly, to get B and D, we open-circuit the output and let  $I_1 = 1\text{A}$ . The schematic is shown below. When the circuit is saved and simulated, we obtain  $V_1 = 2.5$  and  $I_2 = -2.125$ .

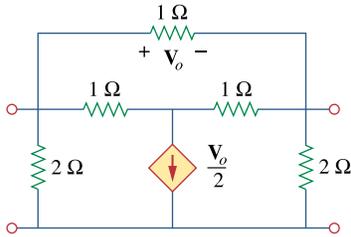
$$B = -\frac{V_1}{I_2} = \frac{2.5}{2.125} = 1.1765, \quad D = -\frac{I_1}{I_2} = \frac{1}{2.125} = 0.4706$$

Thus,

$$[T] = \begin{bmatrix} 0.3235 & 1.1765 \\ 0.02941 & 0.4706 \end{bmatrix}$$

**Chapter 19, Problem 84.**

Using *PSpice*, find the transmission parameters for the network in Fig. 19.126.



**Figure 19.126**

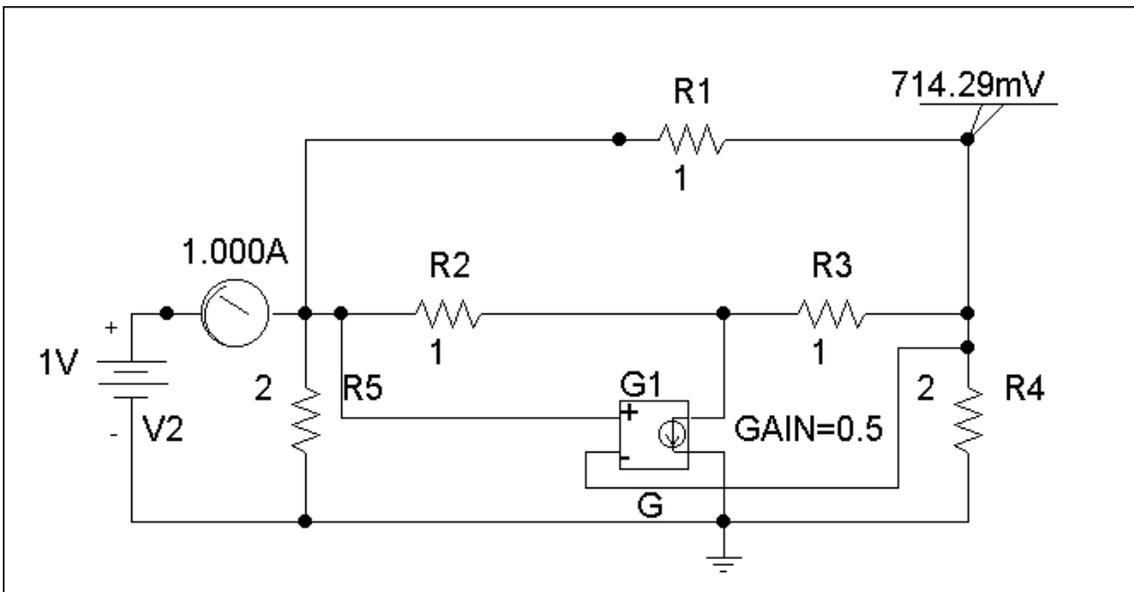
For Prob. 19.84.

**Chapter 19, Solution 84**

(a) Since  $A = \left. \frac{V_1}{V_2} \right|_{I_2=0}$  and  $C = \left. \frac{I_1}{V_2} \right|_{I_2=0}$ , we open-circuit the output port and let  $V_1 = 1$  V. The schematic is as shown below. After simulation, we obtain

$$A = 1/V_2 = 1/0.7143 = 1.4$$

$$C = I_2/V_2 = 1.0/0.7143 = 1.4$$



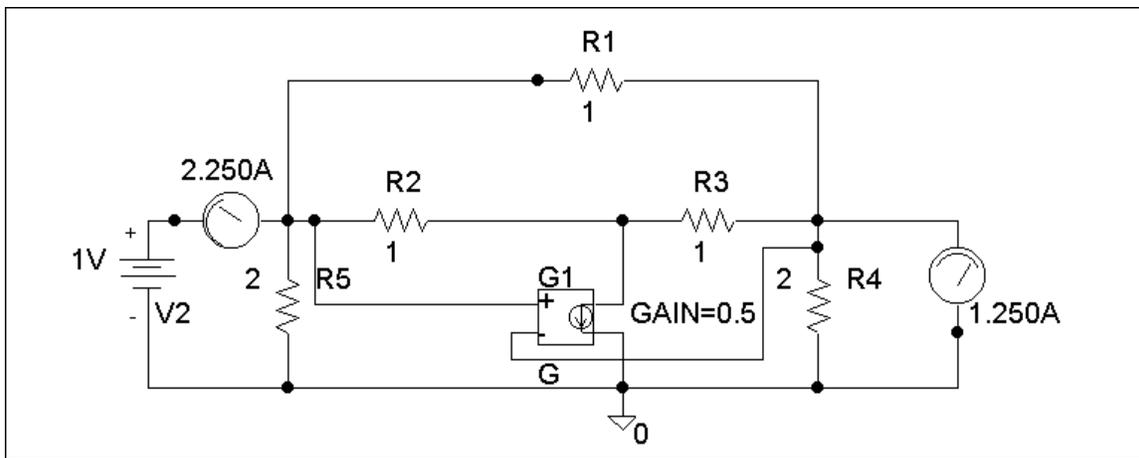
(b) To get B and D, we short-circuit the output port and let  $V_1 = 1$ . The schematic is shown below. After simulating the circuit, we obtain

$$B = -V_1/I_2 = -1/1.25 = -0.8$$

$$D = -I_1/I_2 = -2.25/1.25 = -1.8$$

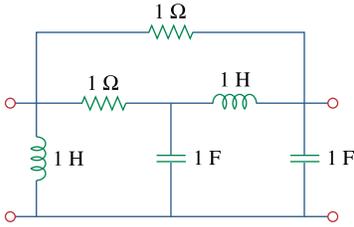
Thus

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \underline{\underline{\begin{bmatrix} 1.4 & -0.8 \\ 1.4 & -1.8 \end{bmatrix}}}$$



**Chapter 19, Problem 85.**

At  $\omega = 1$  rad/s find the transmission parameters of the network in Fig. 19.127 using *PSpice*.



**Figure 19.127**

For Prob. 19.85.

**Chapter 19, Solution 85**

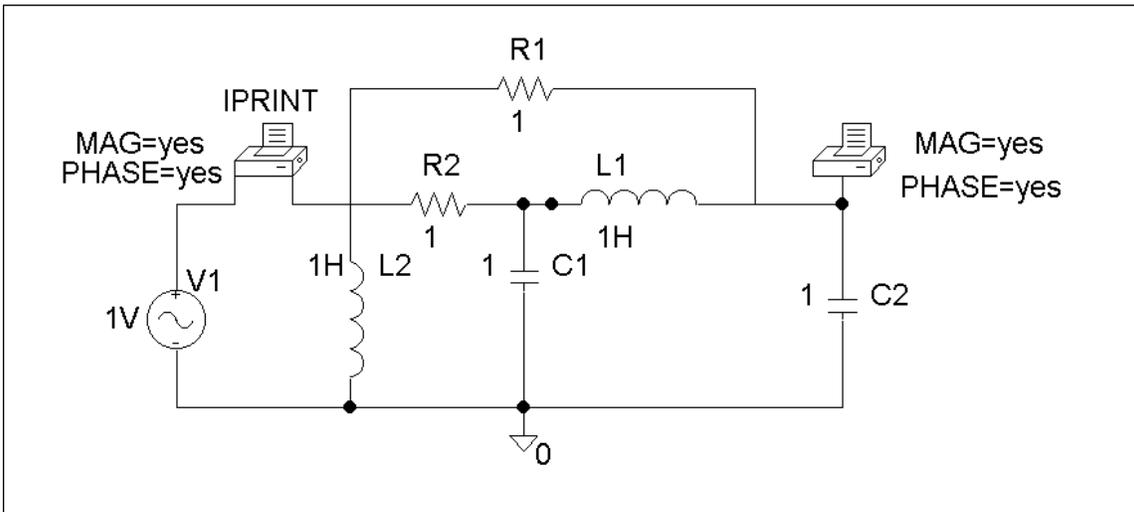
(a) Since  $A = \frac{V_1}{V_2} \Big|_{I_2=0}$  and  $C = \frac{I_1}{V_2} \Big|_{I_2=0}$ , we let  $V_1 = 1$  V and open-circuit the output port. The schematic is shown below. In the AC Sweep box, we set Total Pts = 1, Start Freq = 0.1592, and End Freq = 0.1592. After simulation, we obtain an output file which includes

FREQ	IM(V_PRINT1)	IP(V_PRINT1)
1.592 E-01	6.325 E-01	1.843 E+01
FREQ	VM(\$N_0002)	VP(\$N_0002)
1.592 E-01	6.325 E-01	-7.159 E+01

From this, we obtain

$$A = \frac{1}{V_2} = \frac{1}{0.6325 \angle -71.59^\circ} = 1.581 \angle 71.59^\circ$$

$$C = \frac{I_1}{V_2} = \frac{0.6325 \angle 18.43^\circ}{0.6325 \angle -71.59^\circ} = 1 \angle 90^\circ = j$$



(b) Similarly, since  $B = \left. \frac{V_1}{I_2} \right|_{V_2=0}$  and  $D = -\left. \frac{I_1}{I_2} \right|_{V_2=0}$ , we let  $V_1 = 1$  V and short-circuit the output port. The schematic is shown below. Again, we set Total Pts = 1, Start Freq = 0.1592, and End Freq = 0.1592 in the AC Sweep box. After simulation, we get an output file which includes the following results:

FREQ	IM(V_PRINT1)	IP(V_PRINT1)
1.592 E-01	5.661 E-04	8.997 E+01

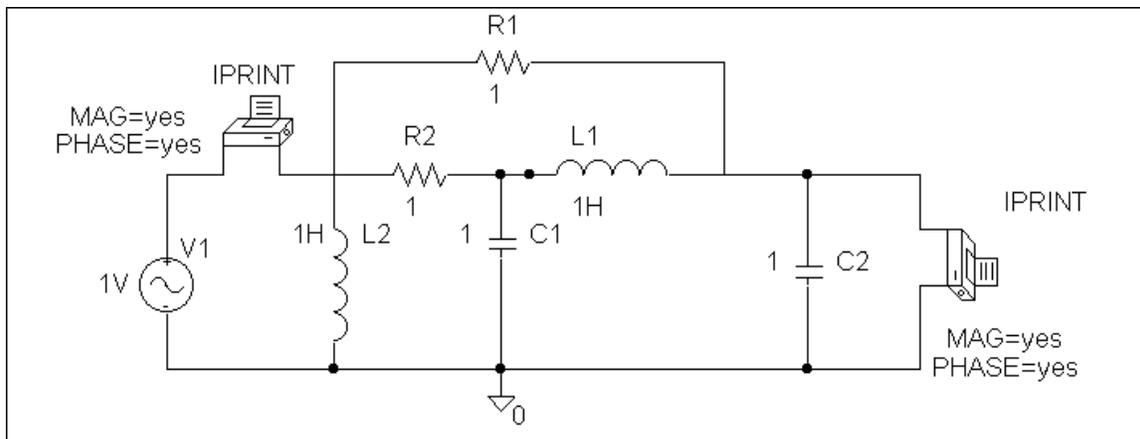
FREQ	IM(V_PRINT3)	IP(V_PRINT3)
1.592 E-01	9.997 E-01	-9.003 E+01

From this,

$$B = -\frac{1}{I_2} = -\frac{1}{0.9997 \angle -90^\circ} = -1 \angle 90^\circ = -j$$

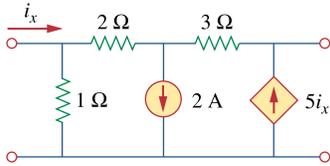
$$D = -\frac{I_1}{I_2} = -\frac{5.661 \times 10^{-4} \angle 89.97^\circ}{0.9997 \angle -90^\circ} = 5.661 \times 10^{-4}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1.581 \angle 71.59^\circ & -j \\ j & 5.661 \times 10^{-4} \end{bmatrix}$$



### Chapter 19, Problem 86.

Obtain the  $g$  parameters for the network in Fig. 19.128 using *PSpice*.



**Figure 19.128**

For Prob. 19.86.

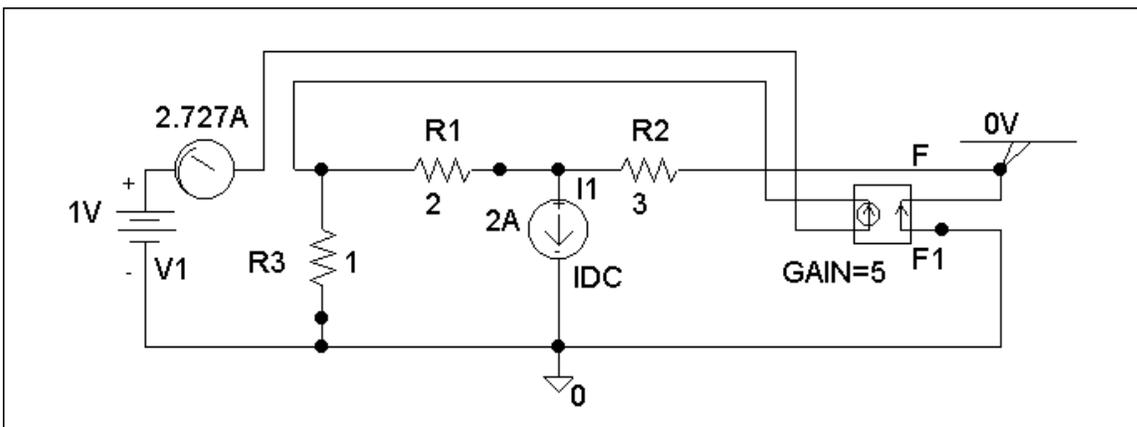
### Chapter 19, Solution 86

(a) By definition,  $g_{11} = \frac{I_1}{V_1} \Big|_{I_2=0}$ ,  $g_{21} = \frac{V_1}{V_2} \Big|_{I_2=0}$ .

We let  $V_1 = 1$  V and open-circuit the output port. The schematic is shown below. After simulation, we obtain

$$g_{11} = I_1 = 2.7$$

$$g_{21} = V_2 = 0.0$$



(b) Similarly,

$$g_{12} = \left. \frac{I_1}{I_2} \right|_{V_1=0}, \quad g_{22} = \left. \frac{V_2}{I_2} \right|_{V_1=0}$$

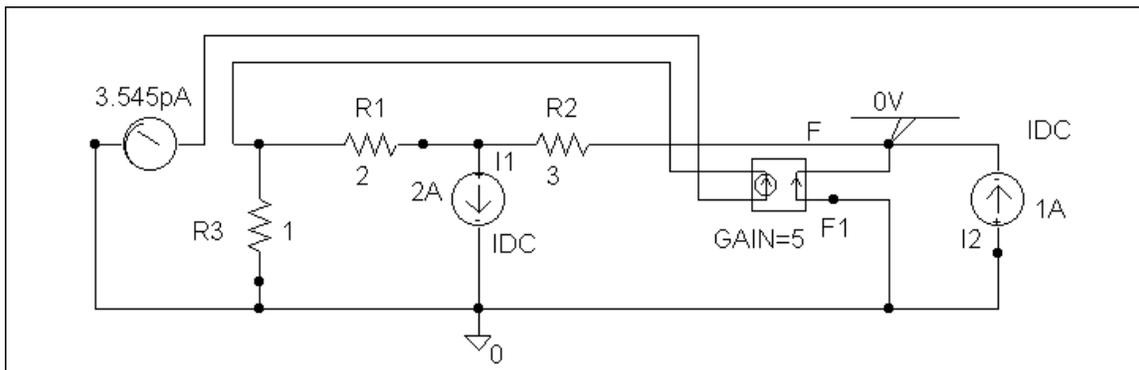
We let  $I_2 = 1$  A and short-circuit the input port. The schematic is shown below. After simulation,

$$g_{12} = I_1 = 0$$

$$g_{22} = V_2 = 0$$

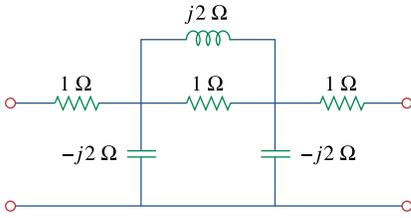
Thus

$$[g] = \begin{bmatrix} 2.727S & 0 \\ 0 & 0 \end{bmatrix}$$



**Chapter 19, Problem 87.**

For the circuit shown in Fig. 19.129, use *PSpice* to obtain the  $t$  parameters. Assume  $\omega = 1$  rad/s.



**Figure 19.129**

For Prob. 19.87.

**Chapter 19, Solution 87**

(a) Since  $a = \left. \frac{V_2}{V_1} \right|_{I_1=0}$  and  $c = \left. \frac{I_2}{V_1} \right|_{I_1=0}$ ,

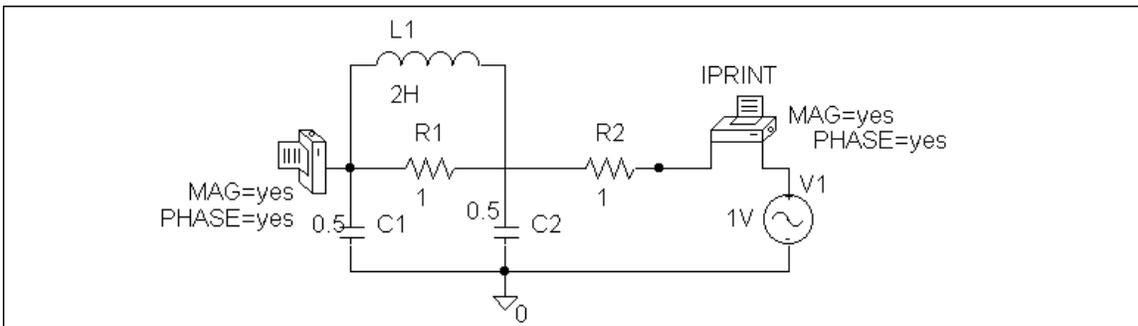
we open-circuit the input port and let  $V_2 = 1$  V. The schematic is shown below. In the AC Sweep box, set Total Pts = 1, Start Freq = 0.1592, and End Freq = 0.1592. After simulation, we obtain an output file which includes

FREQ	IM(V_PRINT2)	IP(V_PRINT2)
1.592 E-01	5.000 E-01	1.800 E+02
FREQ	VM(\$N_0001)	VP(\$N_0001)
1.592 E-01	5.664 E-04	8.997 E+01

From this,

$$a = \frac{1}{5.664 \times 10^{-4} \angle 89.97^\circ} = 1765 \angle -89.97^\circ$$

$$c = \frac{0.5 \angle 180^\circ}{5.664 \times 10^{-4} \angle 89.97^\circ} = -882.28 \angle -89.97^\circ$$



(b) Similarly,

$$b = -\frac{V_2}{I_1} \Big|_{V_1=0} \quad \text{and} \quad d = -\frac{I_2}{I_1} \Big|_{V_1=0}$$

We short-circuit the input port and let  $V_2 = 1 \text{ V}$ . The schematic is shown below. After simulation, we obtain an output file which includes

FREQ	IM(V_PRINT2)	IP(V_PRINT2)
1.592 E-01	5.000 E-01	1.800 E+02
FREQ	IM(V_PRINT3)	IP(V_PRINT3)
1.592 E-01	5.664 E-04	-9.010 E+01

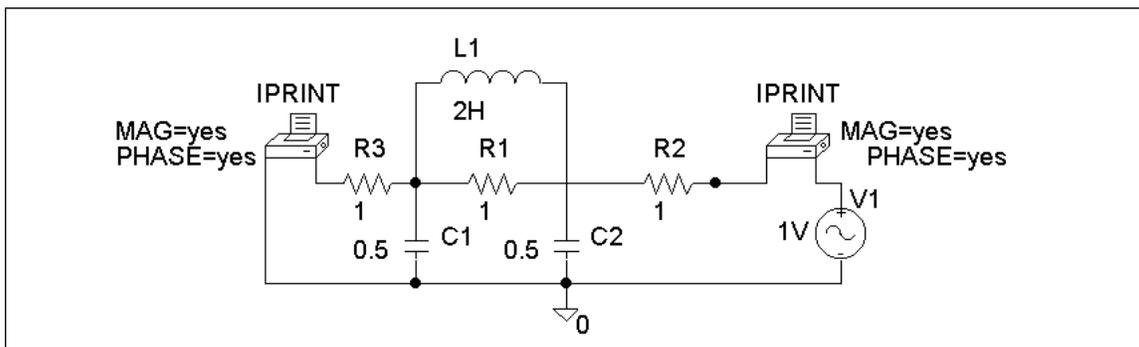
From this, we get

$$b = -\frac{1}{5.664 \times 10^{-4} \angle -90.1^\circ} = -j1765$$

$$d = -\frac{0.5 \angle 180^\circ}{5.664 \times 10^{-4} \angle -90.1^\circ} = j888.28$$

Thus

$$[t] = \begin{bmatrix} -j1765 & -j1765 \\ j888.2 & j888.2 \end{bmatrix}$$

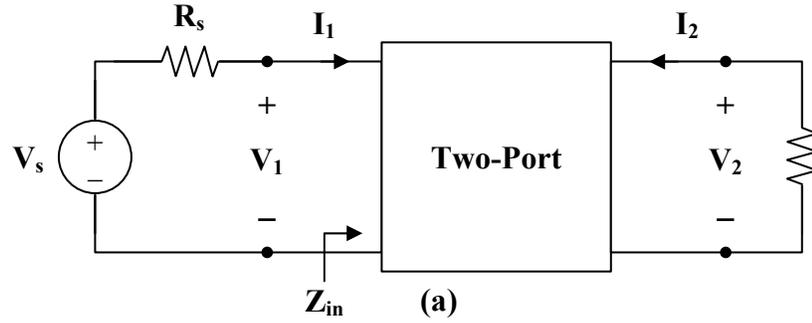


**Chapter 19, Problem 88.**

Using the  $y$  parameters, derive formulas for  $Z_{in}$ ,  $Z_{out}$ ,  $A_i$ , and  $A_v$  for the common-emitter transistor circuit.

**Chapter 19, Solution 88**

To get  $Z_{in}$ , consider the network in Fig. (a).



$$I_1 = y_{11} V_1 + y_{12} V_2 \quad (1)$$

$$I_2 = y_{21} V_1 + y_{22} V_2 \quad (2)$$

But 
$$I_2 = \frac{-V_2}{R_L} = y_{21} V_1 + y_{22} V_2$$

$$V_2 = \frac{-y_{21} V_1}{y_{22} + 1/R_L} \quad (3)$$

Substituting (3) into (1) yields

$$I_1 = y_{11} V_1 + y_{12} \cdot \left( \frac{-y_{21} V_1}{y_{22} + 1/R_L} \right), \quad Y_L = \frac{1}{R_L}$$

$$I_1 = \left( \frac{\Delta_y + y_{11} Y_L}{y_{22} + Y_L} \right) V_1, \quad \Delta_y = y_{11} y_{22} - y_{12} y_{21}$$

or 
$$Z_{in} = \frac{V_1}{I_1} = \frac{y_{22} + Y_L}{\Delta_y + y_{11} Y_L}$$

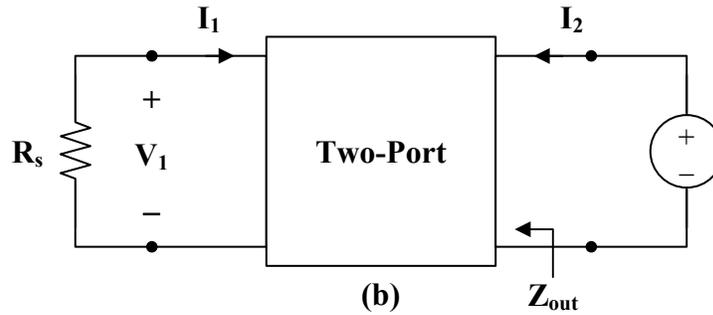
$$\begin{aligned} A_i &= \frac{I_2}{I_1} = \frac{y_{21} V_1 + y_{22} V_2}{I_1} = y_{21} Z_{in} + \left( \frac{y_{22}}{I_1} \right) \left( \frac{-y_{21} V_1}{y_{22} + Y_L} \right) \\ &= y_{21} Z_{in} - \frac{y_{22} y_{21} Z_{in}}{y_{22} + Y_L} = \left( \frac{y_{22} + Y_L}{\Delta_y + y_{11} Y_L} \right) \left( y_{21} - \frac{y_{22} y_{21}}{y_{22} + Y_L} \right) \end{aligned}$$

$$A_i = \frac{y_{21} Y_L}{\Delta_y + y_{11} Y_L}$$

From (3),

$$A_v = \frac{V_2}{V_1} = \frac{-y_{21}}{y_{22} + Y_L}$$

To get  $Z_{out}$ , consider the circuit in Fig. (b).



$$Z_{out} = \frac{V_2}{I_2} = \frac{V_2}{y_{21} V_1 + y_{22} V_2} \quad (4)$$

But  $V_1 = -R_s I_1$

Substituting this into (1) yields

$$I_1 = -y_{11} R_s I_1 + y_{12} V_2$$

$$(1 + y_{11} R_s) I_1 = y_{12} V_2$$

$$I_1 = \frac{y_{12} V_2}{1 + y_{11} R_s} = \frac{-V_1}{R_s}$$

or  $\frac{V_1}{V_2} = \frac{-y_{12} R_s}{1 + y_{11} R_s}$

Substituting this into (4) gives

$$\begin{aligned} Z_{out} &= \frac{1}{y_{22} - \frac{y_{12} y_{21} R_s}{1 + y_{11} R_s}} \\ &= \frac{1 + y_{11} R_s}{y_{22} + y_{11} y_{22} R_s - y_{21} y_{22} R_s} \\ Z_{out} &= \frac{y_{11} + Y_s}{\Delta_y + y_{22} Y_s} \end{aligned}$$

### Chapter 19, Problem 89.

A transistor has the following parameters in a common-emitter circuit:

$$\begin{aligned}h_{ie} &= 2,640 \Omega, & h_{re} &= 2.6 \times 10^{-4} \\h_{fe} &= 72, & h_{oe} &= 16 \mu\text{S}, & R_L &= 100 \text{ k}\Omega\end{aligned}$$

What is the voltage amplification of the transistor? How many decibels gain is this?

### Chapter 19, Solution 89

$$\begin{aligned}A_v &= \frac{-h_{fe} R_L}{h_{ie} + (h_{ie} h_{oe} - h_{re} h_{fe}) R_L} \\A_v &= \frac{-72 \cdot 10^5}{2640 + (2640 \times 16 \times 10^{-6} - 2.6 \times 10^{-4} \times 72) \cdot 10^5} \\A_v &= \frac{-72 \cdot 10^5}{2640 + 1824} = \underline{\underline{-1613}}\end{aligned}$$

$$\text{dc gain} = 20 \log |A_v| = 20 \log(1613) = \underline{\underline{64.15}}$$

### Chapter 19, Problem 90.

**ed**

A transistor with

$$h_{fe} = 120, \quad h_{ie} = 2\text{k}\Omega$$

$$h_{re} = 10^{-4}, \quad h_{oe} = 20\ \mu\text{S}$$

is used for a CE amplifier to provide an input resistance of  $1.5\text{ k}\Omega$ .

- Determine the necessary load resistance  $R_L$ .
- Calculate  $A_v$ ,  $A_i$ , and  $Z_{\text{out}}$  if the amplifier is driven by a 4-mV source having an internal resistance of  $600\ \Omega$ .
- Find the voltage across the load.

**Chapter 19, Solution 90**

$$\begin{aligned}
 \text{(a)} \quad Z_{in} &= h_{ie} - \frac{h_{re} h_{fe} R_L}{1 + h_{oe} R_L} \\
 1500 &= 2000 - \frac{10^{-4} \times 120 R_L}{1 + 20 \times 10^{-6} R_L} \\
 500 &= \frac{12 \times 10^{-3}}{1 + 2 \times 10^{-5} R_L} \\
 500 + 10^{-2} R_L &= 12 \times 10^{-3} R_L \\
 500 \times 10^2 &= 0.2 R_L \\
 R_L &= \underline{\underline{250 \text{ k}\Omega}}
 \end{aligned}$$

$$\begin{aligned}
 \text{(b)} \quad A_v &= \frac{-h_{fe} R_L}{h_{ie} + (h_{ie} h_{oe} - h_{re} h_{fe}) R_L} \\
 A_v &= \frac{-120 \times 250 \times 10^3}{2000 + (2000 \times 20 \times 10^{-6} - 120 \times 10^{-4}) \times 250 \times 10^3} \\
 A_v &= \frac{-30 \times 10^6}{2 \times 10^3 + 7 \times 10^3} = \underline{\underline{-3333}}
 \end{aligned}$$

$$A_i = \frac{h_{fe}}{1 + h_{oe} R_L} = \frac{120}{1 + 20 \times 10^{-6} \times 250 \times 10^3} = \underline{\underline{20}}$$

$$\begin{aligned}
 Z_{out} &= \frac{R_s + h_{ie}}{(R_s + h_{ie}) h_{oe} - h_{re} h_{fe}} = \frac{600 + 2000}{(600 + 2000) \times 20 \times 10^{-6} - 10^{-4} \times 120} \\
 Z_{out} &= \frac{2600}{40} \text{ k}\Omega = \underline{\underline{65 \text{ k}\Omega}}
 \end{aligned}$$

$$\text{(c)} \quad A_v = \frac{V_c}{V_b} = \frac{V_c}{V_s} \longrightarrow V_c = A_v V_s = -3333 \times 4 \times 10^{-3} = \underline{\underline{-13.33 \text{ V}}}$$

### Chapter 19, Problem 91.

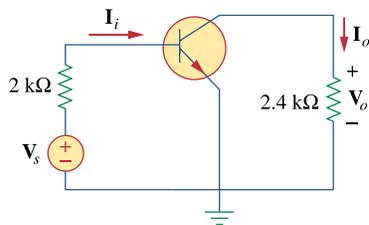
For the transistor network of Fig. 19.130,

$$h_{fe} = 80, \quad h_{ie} = 1.2\text{k}\Omega$$

$$h_{re} = 1.5 \times 10^{-4}, \quad h_{oe} = 20\ \mu\text{S}$$

Determine the following:

- (a) voltage gain  $A_v = V_o/V_s$ ,
- (b) current gain  $A_i = I_o/I_i$ ,
- (c) input impedance  $Z_{in}$ ,
- (d) output impedance  $Z_{out}$ .



**Figure 19.130**

For Prob. 19.91.

**Chapter 19, Solution 91**

$$R_s = 1.2 \text{ k}\Omega, \quad R_L = 4 \text{ k}\Omega$$

$$(a) \quad A_v = \frac{-h_{fe} R_L}{h_{ie} + (h_{ie} h_{oe} - h_{re} h_{fe}) R_L}$$

$$A_v = \frac{-80 \times 4 \times 10^3}{1200 + (1200 \times 20 \times 10^{-6} - 1.5 \times 10^{-4} \times 80) \times 4 \times 10^3}$$

$$A_v = \frac{-32000}{1248} = \underline{\underline{-25.64}} \text{ for the transistor. However, the problem asks for } V_o/V_s.$$

Thus,

$$V_b = V_o/A_{\text{Trans}} = -V_o/25.64$$

$$I_b = V_s/(2000 + 1200) = V_s/3200 \text{ (Note, we used } Z_{in} \text{ from (c) below.)}$$

$$V_b = 1200 I_b = (1200/3200) V_s = 0.375 V_s = -V_o/25.64$$

$$A_v \text{ for the circuit} = V_o/V_s = \underline{\underline{-9.615}}$$

$$(b) \quad A_i = \frac{h_{fe}}{1 + h_{oe} R_L} = \frac{80}{1 + 20 \times 10^{-6} \times 4 \times 10^3} = \underline{\underline{74.07}}$$

$$(c) \quad Z_{in} = h_{ie} - h_{re} A_i$$

$$Z_{in} = 1200 - 1.5 \times 10^{-4} \times 74.074 \cong \underline{\underline{1.2 \text{ k}\Omega}}$$

$$(d) \quad Z_{out} = \frac{R_s + h_{ie}}{(R_s + h_{ie}) h_{oe} - h_{re} h_{fe}}$$

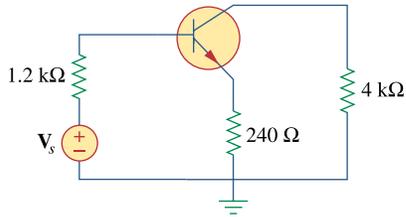
$$Z_{out} = \frac{1200 + 1200}{2400 \times 20 \times 10^{-6} - 1.5 \times 10^{-4} \times 80} = \frac{2400}{0.0468} = \underline{\underline{51.28 \text{ k}\Omega}}$$

**Chapter 19, Problem 92.**

\* Determine  $A_v$ ,  $A_i$ ,  $Z_{in}$ , and  $Z_{out}$  for the amplifier shown in Fig. 19.131. Assume that

$$h_{ie} = 4 \text{ k}\Omega, \quad h_{re} = 10^{-4}$$

$$h_{fe} = 100, \quad h_{oe} = 30 \mu\text{S}$$



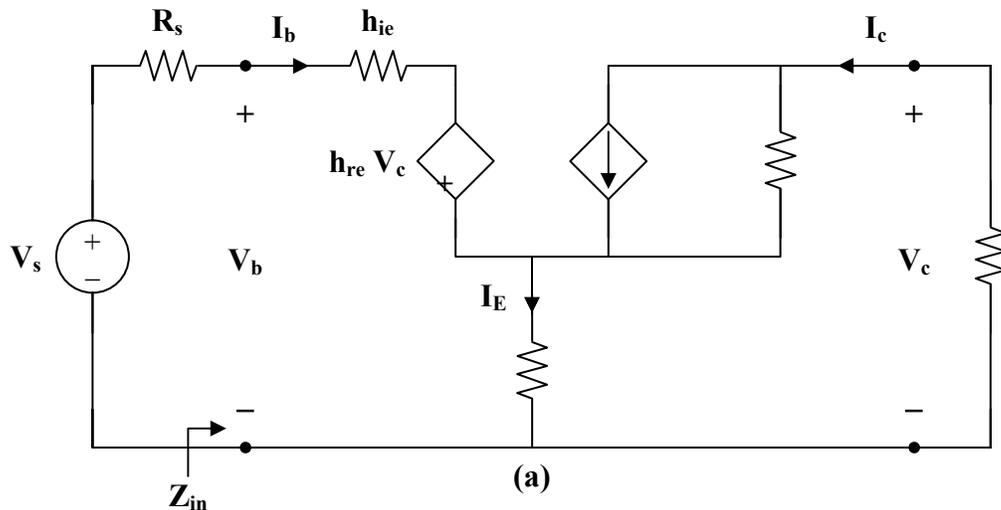
**Figure 19.131**

For Prob. 19.92.

\* An asterisk indicates a challenging problem.

**Chapter 19, Solution 92**

Due to the resistor  $R_E = 240 \Omega$ , we cannot use the formulas in section 18.9.1. We will need to derive our own. Consider the circuit in Fig. (a).



$$\mathbf{I}_E = \mathbf{I}_b + \mathbf{I}_c \quad (1)$$

$$\mathbf{V}_b = h_{ie} \mathbf{I}_b + h_{re} \mathbf{V}_c + (\mathbf{I}_b + \mathbf{I}_c) R_E \quad (2)$$

$$\mathbf{I}_c = h_{fe} \mathbf{I}_b + \frac{\mathbf{V}_c}{R_E + 1/h_{oe}} \quad (3)$$

But  $\mathbf{V}_c = -\mathbf{I}_c R_L \quad (4)$

Substituting (4) into (3),

$$\mathbf{I}_c = h_{fe} \mathbf{I}_b - \frac{R_L}{R_E + 1/h_{oe}} \mathbf{I}_c$$

or  $A_i = \frac{\mathbf{I}_c}{\mathbf{I}_b} = \frac{h_{fe}(1 + R_E h_{oe})}{1 + h_{oe}(R_L + R_E)} \quad (5)$

$$A_i = \frac{100(1 + 240 \times 30 \times 10^{-6})}{1 + 30 \times 10^{-6}(4,000 + 240)}$$

$$A_i = \underline{\underline{79.18}}$$

From (3) and (5),

$$\mathbf{I}_c = \frac{h_{fe}(1 + R_E)h_{oe}}{1 + h_{oe}(R_L + R_E)} \mathbf{I}_b = h_{fe} \mathbf{I}_b + \frac{\mathbf{V}_c}{R_E + 1/h_{oe}} \quad (6)$$

Substituting (4) and (6) into (2),

$$\mathbf{V}_b = (h_{ie} + R_E) \mathbf{I}_b + h_{re} \mathbf{V}_c + \mathbf{I}_c R_E$$

$$\mathbf{V}_b = \frac{\mathbf{V}_c (h_{ie} + R_E)}{\left( R_E + \frac{1}{h_{oe}} \right) \left[ \frac{h_{fe}(1 + R_E h_{oe})}{1 + h_{oe}(R_L + R_E)} - h_{fe} \right]} + h_{re} \mathbf{V}_c - \frac{\mathbf{V}_c}{R_L} R_E$$

$$\frac{1}{A_v} = \frac{\mathbf{V}_b}{\mathbf{V}_c} = \frac{(h_{ie} + R_E)}{\left( R_E + \frac{1}{h_{oe}} \right) \left[ \frac{h_{fe}(1 + R_E h_{oe})}{1 + h_{oe}(R_L + R_E)} - h_{fe} \right]} + h_{re} - \frac{R_E}{R_L} \quad (7)$$

$$\frac{1}{A_v} = \frac{(4000 + 240)}{\left( 240 + \frac{1}{30 \times 10^{-6}} \right) \left[ \frac{100(1 + 240 \times 30 \times 10^{-6})}{1 + 30 \times 10^{-6} \times 4240} - 100 \right]} + 10^{-4} - \frac{240}{4000}$$

$$\frac{1}{A_v} = -6.06 \times 10^{-3} + 10^{-4} - 0.06 = -0.066$$

$$A_v = \underline{\underline{-15.15}}$$

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From (5),

$$I_c = \frac{h_{fe}}{1 + h_{oe} R_L} I_b$$

We substitute this with (4) into (2) to get

$$V_b = (h_{ie} + R_E) I_b + (R_E - h_{re} R_L) I_c$$

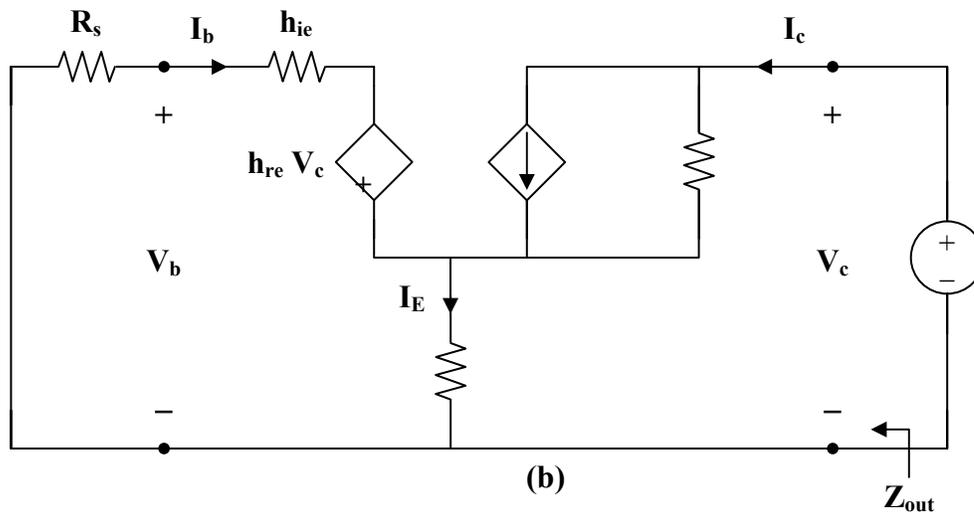
$$V_b = (h_{ie} + R_E) I_b + (R_E - h_{re} R_L) \left( \frac{h_{fe} (1 + R_E h_{oe})}{1 + h_{oe} (R_L + R_E)} I_b \right)$$

$$Z_{in} = \frac{V_b}{I_b} = h_{ie} + R_E + \frac{h_{fe} (R_E - h_{re} R_L) (1 + R_E h_{oe})}{1 + h_{oe} (R_L + R_E)} \quad (8)$$

$$Z_{in} = 4000 + 240 + \frac{(100)(240 \times 10^{-4} \times 4 \times 10^3)(1 + 240 \times 30 \times 10^{-6})}{1 + 30 \times 10^{-6} \times 4240}$$

$$Z_{in} = \mathbf{12.818 \text{ k}\Omega}$$

To obtain  $Z_{out}$ , which is the same as the Thevenin impedance at the output, we introduce a 1-V source as shown in Fig. (b).



From the input loop,

$$I_b (R_s + h_{ie}) + h_{re} V_c + R_E (I_b + I_c) = 0$$

But  $V_c = 1$

So,

$$\mathbf{I}_b (\mathbf{R}_s + h_{ie} + \mathbf{R}_E) + h_{re} + \mathbf{R}_E \mathbf{I}_c = 0 \quad (9)$$

From the output loop,

$$\mathbf{I}_c = \frac{\mathbf{V}_c}{\mathbf{R}_E + \frac{1}{h_{oe}}} + h_{fe} \mathbf{I}_b = \frac{h_{oe}}{\mathbf{R}_E h_{oe} + 1} + h_{fe} \mathbf{I}_b$$

or

$$\mathbf{I}_b = \frac{\mathbf{I}_c}{h_{fe}} - \frac{h_{oe}/h_{fe}}{1 + \mathbf{R}_E h_{oe}} \quad (10)$$

Substituting (10) into (9) gives

$$(\mathbf{R}_s + \mathbf{R}_E + h_{ie}) \left( \frac{\mathbf{I}_c}{h_{fe}} \right) + h_{re} + \mathbf{R}_E \mathbf{I}_c - \frac{(\mathbf{R}_s + \mathbf{R}_E + h_{ie}) \left( \frac{h_{oe}}{h_{fe}} \right)}{1 + \mathbf{R}_E h_{oe}} = 0$$

$$\frac{\mathbf{R}_s + \mathbf{R}_E + h_{ie}}{h_{fe}} \mathbf{I}_c + \mathbf{R}_E \mathbf{I}_c = \frac{\mathbf{R}_s + \mathbf{R}_E + h_{ie}}{1 + \mathbf{R}_E h_{oe}} \left( \frac{h_{oe}}{h_{fe}} \right) - h_{re}$$

$$\mathbf{I}_c = \frac{(h_{oe}/h_{fe}) \left[ \frac{\mathbf{R}_s + \mathbf{R}_E + h_{ie}}{1 + \mathbf{R}_E h_{oe}} \right] - h_{re}}{\mathbf{R}_E + (\mathbf{R}_s + \mathbf{R}_E + h_{ie})/h_{fe}}$$

$$Z_{out} = \frac{1}{\mathbf{I}_c} = \frac{\mathbf{R}_E h_{fe} + \mathbf{R}_s + \mathbf{R}_E + h_{ie}}{\left[ \frac{\mathbf{R}_s + \mathbf{R}_E + h_{ie}}{1 + \mathbf{R}_E h_{oe}} \right] h_{oe} - h_{re} h_{fe}}$$

$$Z_{out} = \frac{240 \times 100 + (1200 + 240 + 4000)}{\left[ \frac{1200 + 240 + 4000}{1 + 240 \times 30 \times 10^{-6}} \right] \times 30 \times 10^{-6} - 10^{-4} \times 100}$$

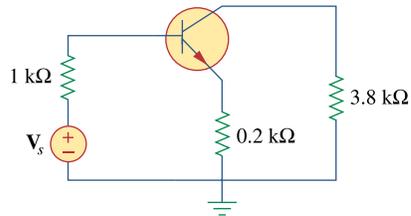
$$Z_{out} = \frac{24000 + 5440}{0.152} = \underline{\underline{193.7 \text{ k}\Omega}}$$

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**\*Chapter 19, Problem 93.**

Calculate  $A_v$ ,  $A_i$ ,  $Z_{in}$ , and  $Z_{out}$ , for the transistor network in Fig. 19.132. Assume that

$$h_{ie} = 2 \text{ k}\Omega, \quad h_{re} = 2.5 \times 10^{-4}$$
$$h_{fe} = 150, \quad h_{oe} = 10 \mu\text{S}$$



**Figure 19.110**

For Prob. 19.63.

\*An asterisk indicates a challenging problem.

## Chapter 19, Solution 93

We apply the same formulas derived in the previous problem.

$$\frac{1}{A_v} = \frac{(h_{ie} + R_E)}{\left(R_E + \frac{1}{h_{oe}}\right) \left[ \frac{h_{fe}(1 + R_E h_{oe})}{1 + h_{oe}(R_L + R_E)} - h_{fe} \right]} + h_{re} - \frac{R_E}{R_L}$$

$$\frac{1}{A_v} = \frac{(2000 + 200)}{(200 + 10^5) \left[ \frac{150(1 + 0.002)}{1 + 0.04} - 150 \right]} + 2.5 \times 10^{-4} - \frac{200}{3800}$$

$$\frac{1}{A_v} = -0.004 + 2.5 \times 10^{-4} - 0.05263 = -0.05638$$

$$A_v = \underline{\underline{-17.74}}$$

$$A_i = \frac{h_{fe}(1 + R_E h_{oe})}{1 + h_{oe}(R_L + R_E)} = \frac{150(1 + 200 \times 10^{-5})}{1 + 10^{-5} \times (200 + 3800)} = \underline{\underline{144.5}}$$

$$Z_{in} = h_{ie} + R_E + \frac{h_{fe}(R_E - h_{re} R_L)(1 + R_E h_{oe})}{1 + h_{oe}(R_L + R_E)}$$

$$Z_{in} = 2000 + 200 + \frac{(150)(200 - 2.5 \times 10^{-4} \times 3.8 \times 10^3)(1.002)}{1.04}$$

$$Z_{in} = 2200 + 28966$$

$$Z_{in} = \underline{\underline{31.17 \text{ k}\Omega}}$$

$$Z_{out} = \frac{R_E h_{fe} + R_s + R_E + h_{ie}}{\left[ \frac{R_s + R_E + h_{ie}}{1 + R_E h_{oe}} \right] h_{oe} - h_{re} h_{fe}}$$

$$Z_{out} = \frac{200 \times 150 + 1000 + 200 + 2000}{\left[ \frac{3200 \times 10^{-5}}{1.002} \right] - 2.5 \times 10^{-4} \times 150} = \frac{33200}{-0.0055}$$

$$Z_{out} = \underline{\underline{-6.148 \text{ M}\Omega}}$$

**Chapter 19, Problem 94.**

**e2d**

A transistor in its common-emitter mode is specified by

$$[\mathbf{h}] = \begin{bmatrix} 200\Omega & 0 \\ 100 & 10^{-6}\text{S} \end{bmatrix}$$

Two such identical transistors are connected in cascade to form a two-stage amplifier used at audio frequencies. If the amplifier is terminated by a 4-k $\Omega$  resistor, calculate the overall  $A_v$  and  $Z_{in}$ .

**Chapter 19, Solution 94**

We first obtain the **ABCD** parameters.

$$\text{Given } [\mathbf{h}] = \begin{bmatrix} 200 & 0 \\ 100 & 10^{-6} \end{bmatrix}, \quad \Delta_h = \mathbf{h}_{11} \mathbf{h}_{22} - \mathbf{h}_{12} \mathbf{h}_{21} = 2 \times 10^{-4}$$

$$[\mathbf{T}] = \begin{bmatrix} \frac{\Delta_h}{\mathbf{h}_{21}} & \frac{-\mathbf{h}_{11}}{\mathbf{h}_{21}} \\ \frac{-\mathbf{h}_{22}}{\mathbf{h}_{21}} & \frac{-1}{\mathbf{h}_{21}} \end{bmatrix} = \begin{bmatrix} -2 \times 10^{-6} & -2 \\ -10^{-8} & -10^{-2} \end{bmatrix}$$

The overall **ABCD** parameters for the amplifier are

$$[\mathbf{T}] = \begin{bmatrix} -2 \times 10^{-6} & -2 \\ -10^{-8} & -10^{-2} \end{bmatrix} \begin{bmatrix} -2 \times 10^{-6} & -2 \\ -10^{-8} & -10^{-2} \end{bmatrix} \cong \begin{bmatrix} 2 \times 10^{-8} & 2 \times 10^{-2} \\ 10^{-10} & 10^{-4} \end{bmatrix}$$

$$\Delta_T = 2 \times 10^{-12} - 2 \times 10^{-12} = 0$$

$$[\mathbf{h}] = \begin{bmatrix} \frac{\mathbf{B}}{\mathbf{D}} & \frac{\Delta_T}{\mathbf{D}} \\ \frac{-1}{\mathbf{D}} & \frac{\mathbf{C}}{\mathbf{D}} \end{bmatrix} = \begin{bmatrix} 200 & 0 \\ -10^4 & 10^{-6} \end{bmatrix}$$

$$\text{Thus, } h_{ie} = 200, \quad h_{re} = 0, \quad h_{fe} = -10^4, \quad h_{oe} = 10^{-6}$$

$$A_v = \frac{(10^4)(4 \times 10^3)}{200 + (2 \times 10^{-4} - 0) \times 4 \times 10^3} = \underline{\underline{2 \times 10^5}}$$

$$Z_{in} = h_{ie} - \frac{h_{re} h_{fe} R_L}{1 + h_{oe} R_L} = 200 - 0 = \underline{\underline{200 \Omega}}$$

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### Chapter 19, Problem 95.

Realize an  $LC$  ladder network such that

$$y_{22} = \frac{s^3 + 5s}{s^4 + 10s^2 + 8}$$

### Chapter 19, Solution 95

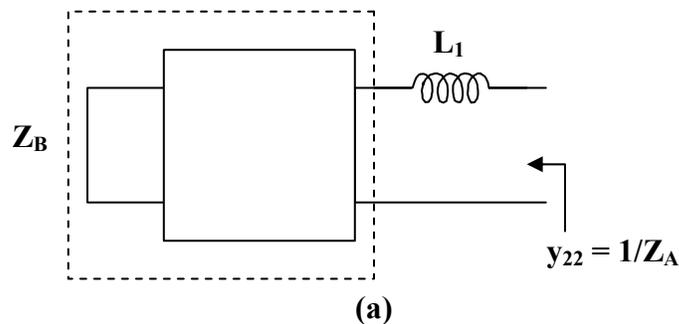
$$\text{Let } \mathbf{Z}_A = \frac{1}{\mathbf{y}_{22}} = \frac{s^4 + 10s^2 + 8}{s^3 + 5s}$$

Using long division,

$$\mathbf{Z}_A = s + \frac{5s^2 + 8}{s^3 + 5s} = s\mathbf{L}_1 + \mathbf{Z}_B$$

i.e.  $\mathbf{L}_1 = 1 \text{ H}$  and  $\mathbf{Z}_B = \frac{5s^2 + 8}{s^3 + 5s}$

as shown in Fig (a).

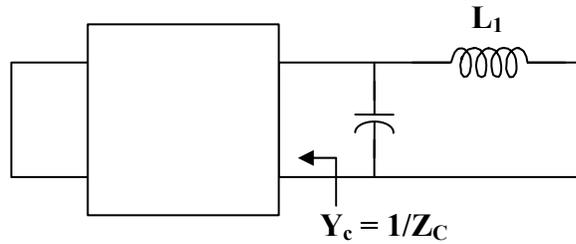


$$\mathbf{Y}_B = \frac{1}{\mathbf{Z}_B} = \frac{s^3 + 5s}{5s^2 + 8}$$

Using long division,

$$Y_B = 0.2s + \frac{3.4s}{5s^2 + 8} = sC_2 + Y_C$$

where  $C_2 = 0.2 \text{ F}$  and  $Y_C = \frac{3.4s}{5s^2 + 8}$   
as shown in Fig. (b).

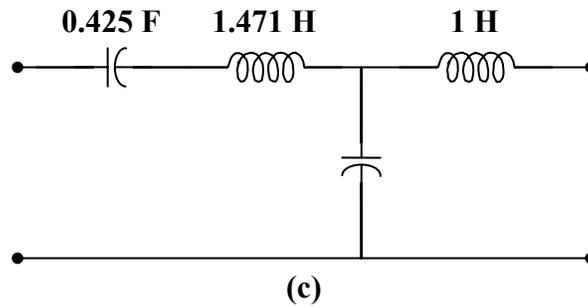


$$Z_C = \frac{1}{Y_C} = \frac{5s^2 + 8}{3.4s} = \frac{5s}{3.4} + \frac{8}{3.4s} = sL_3 + \frac{1}{sC_4}$$

i.e. an inductor in series with a capacitor

$$L_3 = \frac{5}{3.4} = 1.471 \text{ H} \quad \text{and} \quad C_4 = \frac{3.4}{8} = 0.425 \text{ F}$$

Thus, **the LC network is shown in Fig. (c).**



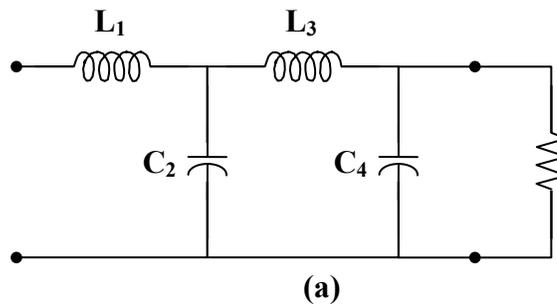
**Chapter 19, Problem 96.**

Design an *LC* ladder network to realize a lowpass filter with transfer function

$$H(s) = \frac{1}{s^4 + 2.613s^2 + 3.414s^2 + 2.613s + 1}$$

**Chapter 19, Solution 96**

This is a fourth order network which can be realized with the network shown in Fig. (a).



$$\Delta(s) = (s^4 + 3.414s^2 + 1) + (2.613s^3 + 2.613s)$$

$$H(s) = \frac{1}{\frac{2.613s^3 + 2.613s}{s^4 + 3.414s^2 + 1} + 1}$$

which indicates that

$$y_{21} = \frac{-1}{2.613s^3 + 2.613s}$$

$$y_{22} = \frac{s^4 + 3.414s + 1}{2.613s^3 + 2.613s}$$

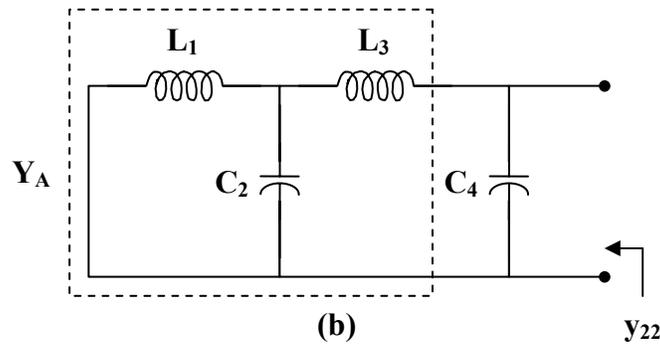
We seek to realize  $y_{22}$ .

By long division,

$$y_{22} = 0.383s + \frac{2.414s^2 + 1}{2.613s^3 + 2.613s} = sC_4 + Y_A$$

i.e.  $C_4 = 0.383 \text{ F}$  and  $Y_A = \frac{2.414s^2 + 1}{2.613s^3 + 2.613s}$

as shown in Fig. (b).



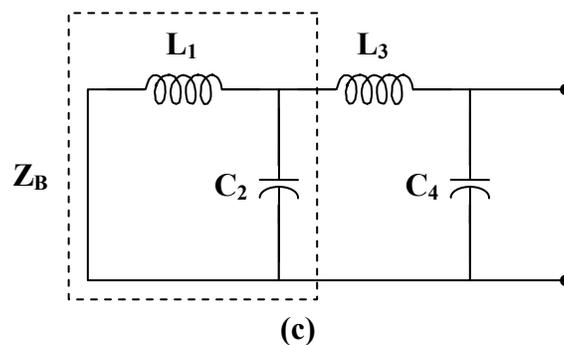
$$Z_A = \frac{1}{Y_A} = \frac{2.613s^3 + 2.613s}{2.414s^2 + 1}$$

By long division,

$$Z_A = 1.082s + \frac{1.531s}{2.414s^2 + 1} = sL_3 + Z_B$$

i.e.  $L_3 = 1.082 \text{ H}$  and  $Z_B = \frac{1.531s}{2.414s^2 + 1}$

as shown in Fig.(c).

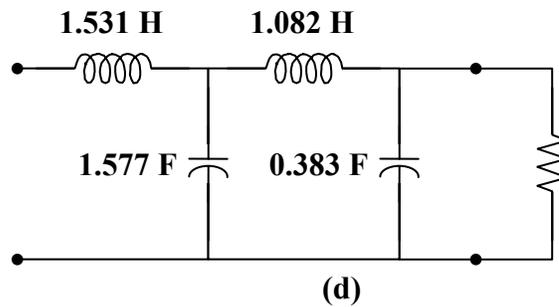


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$$Y_B = \frac{1}{Z_B} = 1.577s + \frac{1}{1.531s} = sC_2 + \frac{1}{sL_1}$$

i.e.  $C_2 = 1.577 \text{ F}$  and  $L_1 = 1.531 \text{ H}$

Thus, **the network is shown in Fig. (d).**

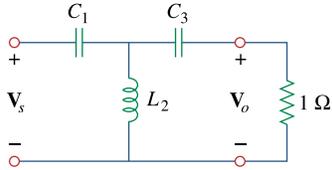


**Chapter 19, Problem 97.**

Synthesize the transfer function

$$H(s) = \frac{V_o}{V_s} = \frac{s^3}{s^3 + 6s + 12s + 24}$$

using the *LC* ladder network in Fig. 19.133.



**Figure 19.133**

For Prob. 19.97.

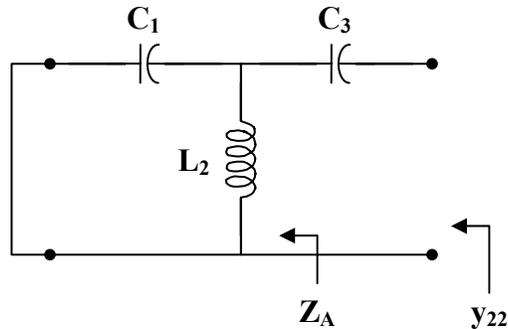
**Chapter 19, Solution 97**

$$H(s) = \frac{s^3}{(s^3 + 12s) + (6s^2 + 24)} = \frac{\frac{s^3}{s^3 + 12s}}{1 + \frac{6s^2 + 24}{s^3 + 12s}}$$

Hence,

$$y_{22} = \frac{6s^2 + 24}{s^3 + 12s} = \frac{1}{sC_3} + Z_A \quad (1)$$

where  $Z_A$  is shown in the figure below.



We now obtain  $C_3$  and  $Z_A$  using partial fraction expansion.

$$\text{Let } \frac{6s^2 + 24}{s(s^2 + 12)} = \frac{A}{s} + \frac{Bs + C}{s^2 + 12}$$
$$6s^2 + 24 = A(s^2 + 12) + Bs^2 + Cs$$

Equating coefficients :

$$s^0: \quad 24 = 12A \quad \longrightarrow \quad A = 2$$

$$s^1: \quad 0 = C$$

$$s^2: \quad 6 = A + B \quad \longrightarrow \quad B = 4$$

Thus,

$$\frac{6s^2 + 24}{s(s^2 + 12)} = \frac{2}{s} + \frac{4s}{s^2 + 12} \quad (2)$$

Comparing (1) and (2),

$$C_3 = \frac{1}{A} = \frac{1}{2} F$$

$$\frac{1}{Z_A} = \frac{s^2 + 12}{4s} = \frac{1}{4}s + \frac{3}{s} \quad (3)$$

$$\text{But } \frac{1}{Z_A} = sC_1 + \frac{1}{sL_2} \quad (4)$$

Comparing (3) and (4),

$$C_1 = \frac{1}{4} F \quad \text{and} \quad L_2 = \frac{1}{3} H$$

Therefore,

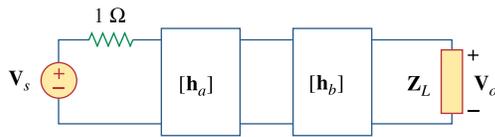
$$C_1 = \underline{\underline{0.25 F}}, \quad L_2 = \underline{\underline{0.3333 H}}, \quad C_3 = \underline{\underline{0.5 F}}$$

### Chapter 19, Problem 98.

A two-stage amplifier in Fig. 19.134 contains two identical stages with

$$[\mathbf{h}] = \begin{bmatrix} 2 \text{ k}\Omega & 0.004 \\ 200 & 500 \mu\text{S} \end{bmatrix}$$

If  $Z_L = 20 \text{ k}\Omega$ , find the required value of  $V_s$  to produce  $V_o = 16 \text{ V}$ .



**Figure 19.134**

For Prob. 19.98.

### Chapter 19, Solution 98

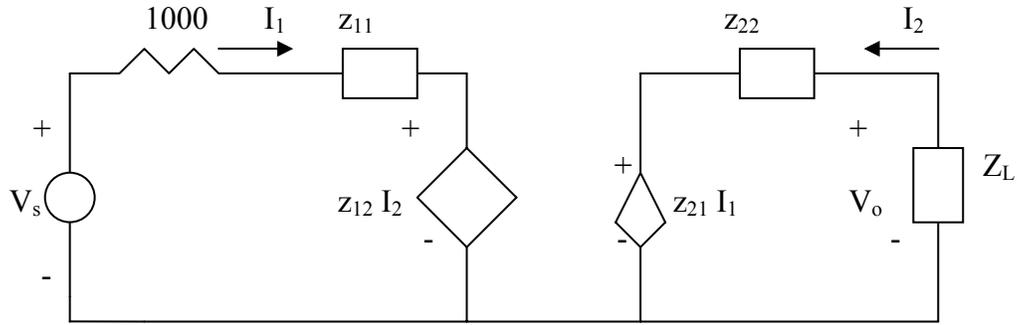
$$\Delta_h = 1 - 0.8 = 0.2$$

$$[T_a] = [T_b] = \begin{bmatrix} -\Delta_h/h_{21} & -h_{11}/h_{21} \\ -h_{22}/h_{21} & -1/h_{21} \end{bmatrix} = \begin{bmatrix} -0.001 & -10 \\ -2.5 \times 10^{-6} & -0.005 \end{bmatrix}$$

$$[T] = [T_a][T_b] = \begin{bmatrix} 2.6 \times 10^{-5} & 0.06 \\ 1.5 \times 10^{-8} & 5 \times 10^{-5} \end{bmatrix}$$

We now convert this to z-parameters

$$[z] = \begin{bmatrix} A/C & \Delta_T/C \\ 1/C & D/C \end{bmatrix} = \begin{bmatrix} 1.733 \times 10^3 & 0.0267 \\ 6.667 \times 10^7 & 3.33 \times 10^3 \end{bmatrix}$$



$$V_s = (1000 + z_{11})I_1 + z_{12}I_2 \quad (1)$$

$$V_o = z_{22}I_2 + z_{21}I_1 \quad (2)$$

$$\text{But } V_o = -I_2 Z_L \quad \longrightarrow \quad I_2 = -V_o / Z_L \quad (3)$$

Substituting (3) into (2) gives

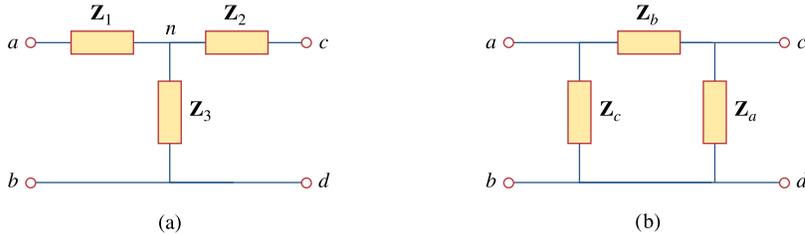
$$I_1 = V_o \left( \frac{1}{z_{21}} + \frac{z_{22}}{z_{21}Z_L} \right) \quad (4)$$

We substitute (3) and (4) into (1)

$$\begin{aligned} V_s &= (1000 + z_{11}) \left( \frac{1}{z_{21}} + \frac{z_{22}}{z_{21}Z_L} \right) V_o - \frac{z_{12}}{Z_L} V_o \\ &= 7.653 \times 10^{-4} - 2.136 \times 10^{-5} = \underline{744 \mu\text{V}} \end{aligned}$$

### Chapter 19, Problem 99.

Assume that the two circuits in Fig. 19.135 are equivalent. The parameters of the two circuits must be equal. Using this factor and the  $z$  parameters, derive Eqs. (9.67) and (9.68).



**Figure 19.135**  
For Prob. 19.99.

### Chapter 19, Solution 99

$$\begin{aligned} Z_{ab} &= Z_1 + Z_3 = Z_c \parallel (Z_b + Z_a) \\ Z_1 + Z_3 &= \frac{Z_c(Z_a + Z_b)}{Z_a + Z_b + Z_c} \end{aligned} \quad (1)$$

$$\begin{aligned} Z_{cd} &= Z_2 + Z_3 = Z_a \parallel (Z_b + Z_c) \\ Z_2 + Z_3 &= \frac{Z_a(Z_b + Z_c)}{Z_a + Z_b + Z_c} \end{aligned} \quad (2)$$

$$\begin{aligned} Z_{ac} &= Z_1 + Z_2 = Z_b \parallel (Z_a + Z_c) \\ Z_1 + Z_2 &= \frac{Z_b(Z_a + Z_c)}{Z_a + Z_b + Z_c} \end{aligned} \quad (3)$$

Subtracting (2) from (1),

$$Z_1 - Z_2 = \frac{Z_b(Z_c - Z_a)}{Z_a + Z_b + Z_c} \quad (4)$$

Adding (3) and (4),

$$Z_1 = \frac{Z_b Z_c}{Z_a + Z_b + Z_c} \quad (5)$$

Subtracting (5) from (3),

$$\mathbf{Z}_2 = \frac{\mathbf{Z}_a \mathbf{Z}_b}{\mathbf{Z}_a + \mathbf{Z}_b + \mathbf{Z}_c} \quad (6)$$

Subtracting (5) from (1),

$$\mathbf{Z}_3 = \frac{\mathbf{Z}_c \mathbf{Z}_a}{\mathbf{Z}_a + \mathbf{Z}_b + \mathbf{Z}_c} \quad (7)$$

Using (5) to (7)

$$\begin{aligned} \mathbf{Z}_1 \mathbf{Z}_2 + \mathbf{Z}_2 \mathbf{Z}_3 + \mathbf{Z}_3 \mathbf{Z}_1 &= \frac{\mathbf{Z}_a \mathbf{Z}_b \mathbf{Z}_c (\mathbf{Z}_a + \mathbf{Z}_b + \mathbf{Z}_c)}{(\mathbf{Z}_a + \mathbf{Z}_b + \mathbf{Z}_c)^2} \\ \mathbf{Z}_1 \mathbf{Z}_2 + \mathbf{Z}_2 \mathbf{Z}_3 + \mathbf{Z}_3 \mathbf{Z}_1 &= \frac{\mathbf{Z}_a \mathbf{Z}_b \mathbf{Z}_c}{\mathbf{Z}_a + \mathbf{Z}_b + \mathbf{Z}_c} \end{aligned} \quad (8)$$

Dividing (8) by each of (5), (6), and (7),

$$\begin{aligned} \mathbf{Z}_a &= \frac{\mathbf{Z}_1 \mathbf{Z}_2 + \mathbf{Z}_2 \mathbf{Z}_3 + \mathbf{Z}_3 \mathbf{Z}_1}{\mathbf{Z}_1} \\ \mathbf{Z}_b &= \frac{\mathbf{Z}_1 \mathbf{Z}_2 + \mathbf{Z}_2 \mathbf{Z}_3 + \mathbf{Z}_3 \mathbf{Z}_1}{\mathbf{Z}_3} \\ \mathbf{Z}_c &= \frac{\mathbf{Z}_1 \mathbf{Z}_2 + \mathbf{Z}_2 \mathbf{Z}_3 + \mathbf{Z}_3 \mathbf{Z}_1}{\mathbf{Z}_2} \end{aligned}$$

as required. Note that the formulas above are not exactly the same as those in Chapter 9 because the locations of  $\mathbf{Z}_b$  and  $\mathbf{Z}_c$  are interchanged in Fig. 18.122.