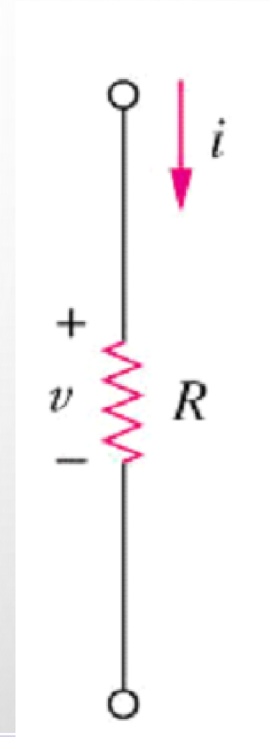
EE 220

CHAPTER 2 – RESISTIVE CIRCUITS

OVERVIEW

- Ohm's law
- Kirchhoff's laws
- Equivalent circuits
- Wye-Delta transformations
- Wheatstone bridge
- Linear vs. nonlinear i-v relationships
- Light emitting diodes (LEDs)
- Superconductivity

OHM'S LAW



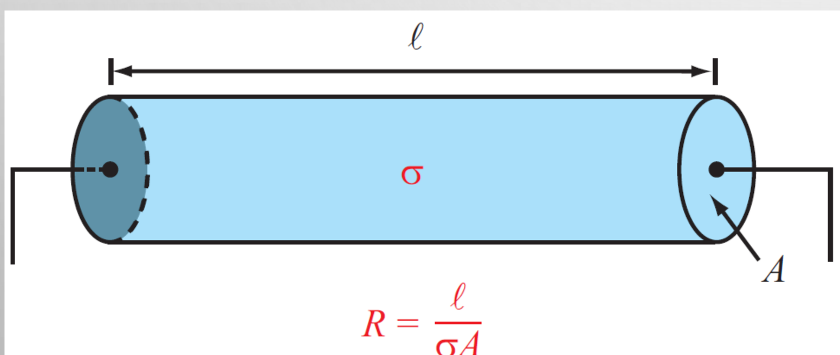
Voltage across resistor is proportional to current

$$v = iR$$

Power dissipated (or absorbed) by a resistor

$$p = vi = i^2 R = \frac{v^2}{R}$$

Resistance: ability to resist flow of electric current



$$R = \rho \frac{\ell}{A} \qquad (\Omega),$$

ρ = resistivity (Ω.m)
ℓ = length (m)
A = cross section area (m^2)

Table 2-1: Conductivity and resistivity of some common materials at 20 °C.

Material	Conductivity σ (S/m)	Resistivity ρ (Ω-m)
Conductors		
Silver	6.17×10^{7}	1.62×10^{-8}
Copper	5.81×10^{7}	1.72×10^{-8}
Gold	4.10×10^{7}	2.44×10^{-8}
Aluminum	3.82×10^{7}	2.62×10^{-8}
Iron	1.03×10^{7}	9.71×10^{-8}
Mercury (liquid)	1.04×10^{6}	9.58×10^{-7}
Semiconductors		
Carbon (graphite)	7.14×10^{4}	1.40×10^{-5}
Pure germanium	2.13	0.47
Pure silicon	4.35×10^{-4}	2.30×10^{3}
Insulators		
Paper	$\sim 10^{-10}$	$\sim 10^{10}$
Glass	$\sim 10^{-12}$	$\sim 10^{12}$
Teflon	$\sim 3.3 \times 10^{-13}$	$\sim 3 \times 10^{12}$
Porcelain	$\sim 10^{-14}$	$\sim 10^{14}$
Mica	$\sim 10^{-15}$	$\sim 10^{15}$
Polystyrene	$\sim 10^{-16}$	$\sim 10^{16}$
Fused quartz	$\sim 10^{-17}$	$\sim 10^{17}$
Common materials		
Distilled water	5.5×10^{-6}	1.8×10^{5}
Drinking water	$\sim 5 \times 10^{-3}$	~ 200
Sea water	4.8	0.2
Graphite	1.4×10^{-5}	71.4×10^{3}
Rubber	1×10^{-13}	1×10^{13}

Conductance: Ability to allow the flow of electric current

$$G = \sigma \frac{A}{l} = \frac{1}{R} \quad (S)$$

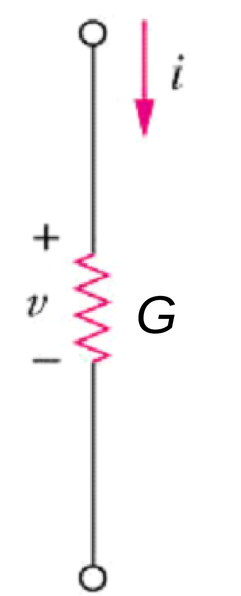
$\sigma = 1/\rho$ = conductivity (S/m)

ℓ = length (m)

A = cross section area (m²)

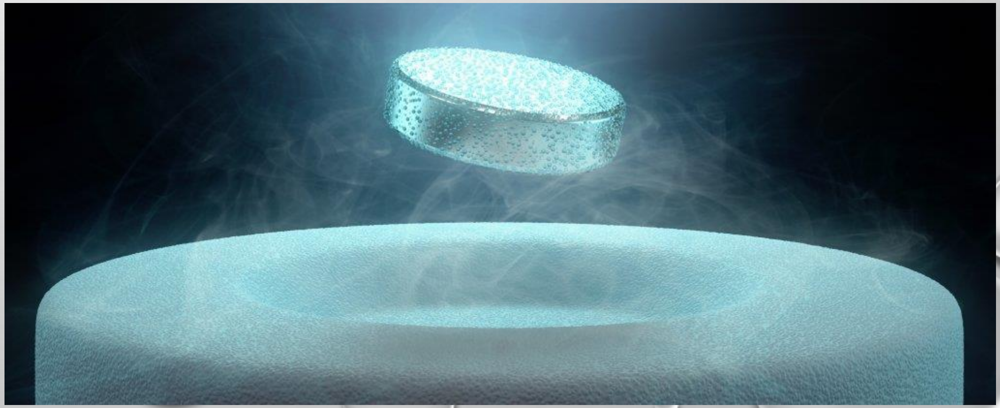
$$i = Gv$$

$$p = vi = Gv^2 = \frac{i^2}{G}$$



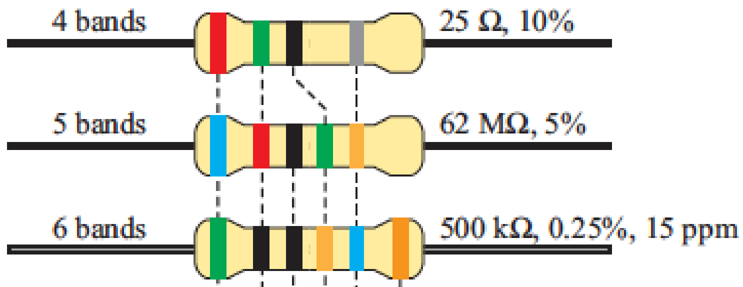
SUPERCONDUCTIVITY

- In 1911, Dutch physicist H.K. Onnes discovered that the resistance of a solid piece of mercury at 4.2 K (0 ◦K = −273 ◦C) was zero! This phenomenon is referred to as **superconductivity**.
- Superconductors have unique properties. The current in a superconductor can persist with no external voltage applied. Some applications include ultra high speed trains, and the medical field (such as MRI).
- Can room-temperature superconductors exist? The race is on! Recently, scientists in Germany have hit a new superconductivity milestone - achieving a resistance-free electrical current at the highest temperature yet - just 250 ◦K , or -23 ◦C (*)



(*) Sciencealert.com (May 2019)

RESISTOR CODES



Color	b_1	b_2	b_3	b_4	b_5	b_6
Silver				0.01	10%	
Gold				0.1	5%	
Black	0	0	0	1		
Brown	1	1	1	10	1%	100 ppm
Red	2	2	2	100	2%	50 ppm
Orange	3	3	3	1K		15 ppm
Yellow	4	4	4	10K		25 ppm
Green	5	5	5	100K	0.5%	
Blue	6	6	6	1M	0.25%	10 ppm
Purple	7	7	7	10M	0.1%	5 ppm
Gray	8	8	8			
White	9	9	9			
	1st digit	2nd digit	3rd digit	Multiplier $\times 10^{b_4}$ # of zeros	Tolerance	Temperature coefficient ppm/°C

4-, 5-, and 6-band color code system

The power rating of a color coded resistors usually ranges between 1/8 W and 1 W. Resistors with power ratings of more than 1 W are usually referred to as power Resistors.

RESISTOR CODES

(a) 4-Band color code: $\boldsymbol{b_1}$ $\boldsymbol{b_2}$ $\boldsymbol{b_4}$ $\boldsymbol{b_5}$

Note that a wider spacing exists between b_3 and b_5 than between the earlier bands. The resistor value is given by

$$R = (b_1 b_2) \times 10^{b_4} \pm b_5,$$

with the values of b_1, b_2, b_4, and b_5 specified by the color code shown in Fig. 2-2. For example,

 $= 25 \times 10^0 \pm 10\% = 25 \pm 10\% \ \Omega.$

(b) 5-Band color code: $\boldsymbol{b_1}$ $\boldsymbol{b_2}$ $\boldsymbol{b_3}$ $\boldsymbol{b_4}$ $\boldsymbol{b_5}$

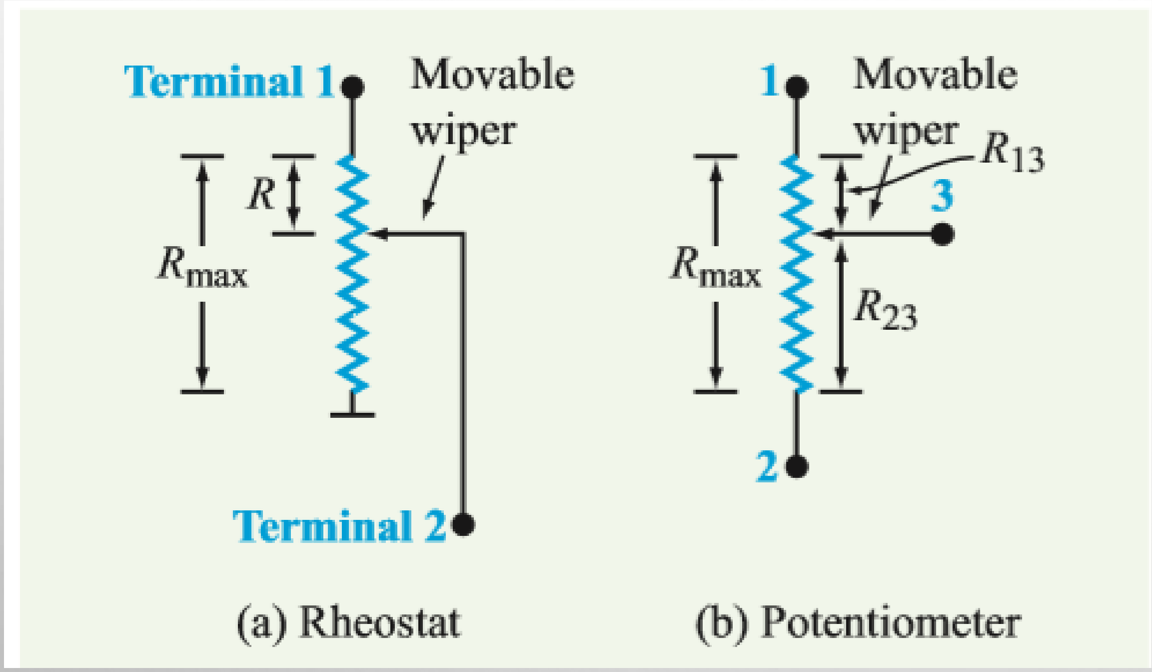
In this case

$$R = (b_1 b_2 b_3) \times 10^{b_4} \pm b_5.$$

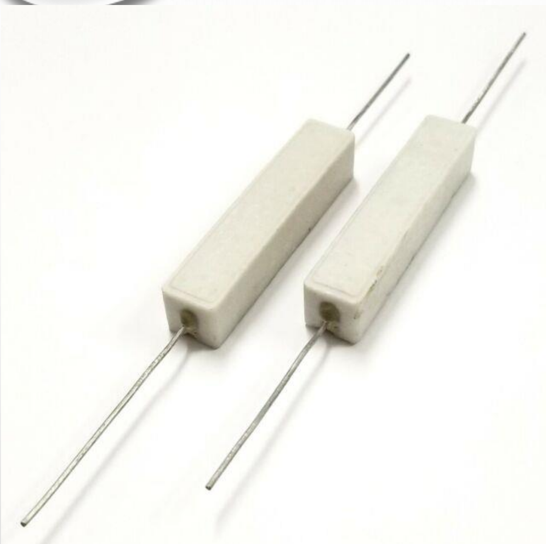
(c) 6-Band color code: $\boldsymbol{b_1}$ $\boldsymbol{b_2}$ $\boldsymbol{b_3}$ $\boldsymbol{b_4}$ $\boldsymbol{b_5}$ $\boldsymbol{b_6}$

VARIABLE RESISTORS

- **Rheostat:** 2-terminal variable resistor
- **Potentiometer:** 3-terminal variable resistor.



POWER RESISTORS



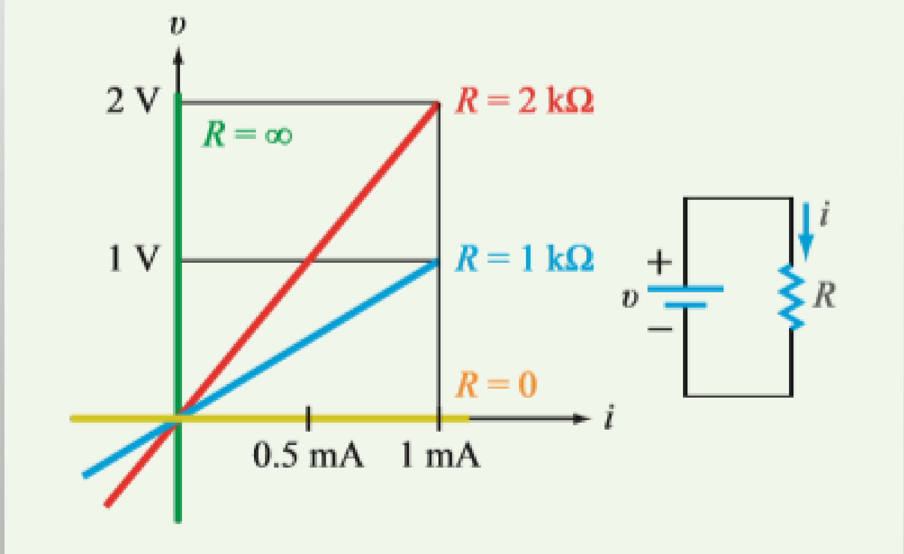
4 Ohm 120W

10 Ohm 10 Watt

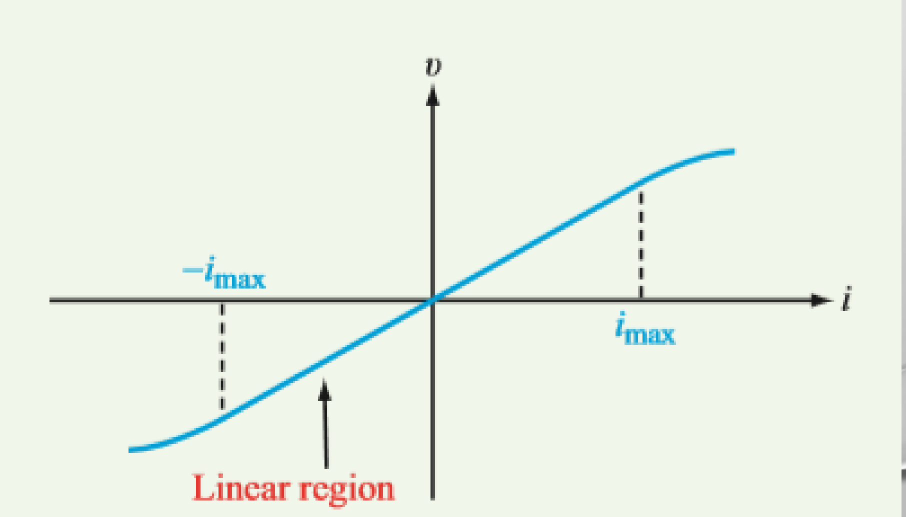
190 Ohm 75 Watt

LINEAR RESISTORS

- An ideal linear resistor is one whose resistance is constant and independent of the magnitude of the current flowing through it, in which case its i–v response is a straight line.
- Unless noted otherwise, the common use of the term resistor in circuit analysis and design usually refers to the linear resistor exclusively.



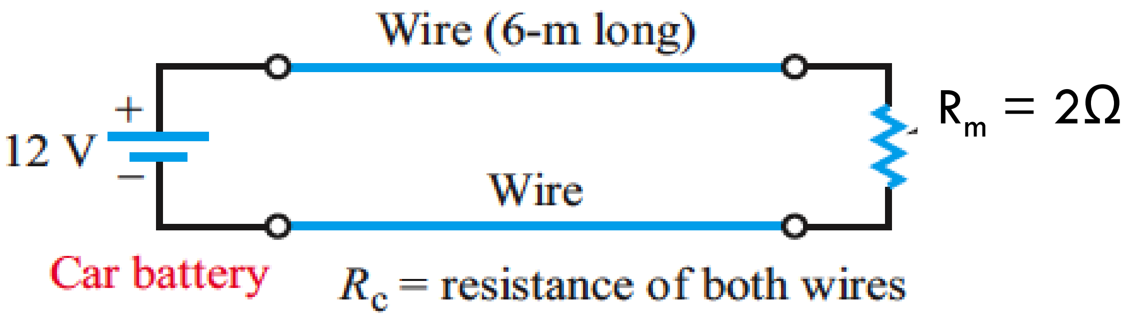
(a) Ideal resistor



(b) Real resistor

EXAMPLE

What fraction of power supplied by the battery is supplied to the 2 Ω resistor? Assume copper wire with a radius of 1.3 mm.



Solution:

$$R_c = \rho \frac{\ell}{A}$$

$$= 1.72 \times 10^{-8} \times \frac{12}{\pi (1.3 \times 10^{-3})^2}$$

$$= 0.04\ \Omega.$$

$$R = R_c + R_m$$

$$= 0.04 + 2$$

$$= 2.04\ \Omega.$$

$$I = \frac{V}{R}$$

$$= \frac{12}{2.04}$$

$$= 5.88\ \text{A}$$

$$P = IV$$

$$= 5.88 \times 12$$

$$= 70.56\ \text{W}$$

$$P_m = I^2 R_m$$

$$= (5.88)^2 \times 2$$

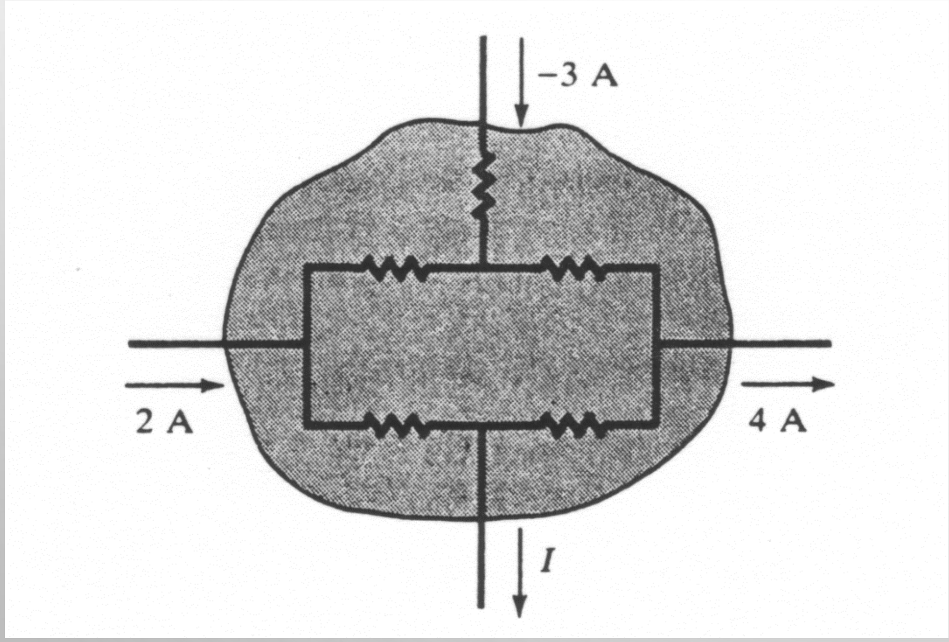
$$= 69.15\ \text{W},$$

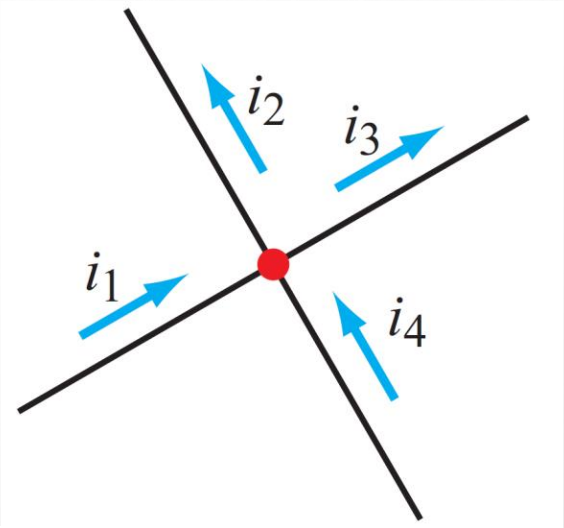
$$= 98\%\ \text{of}\ P$$

KIRCHHOFF'S CURRENT LAW (KCL)

Algebraic sum of currents entering a node is zero.

Also holds for closed boundary.





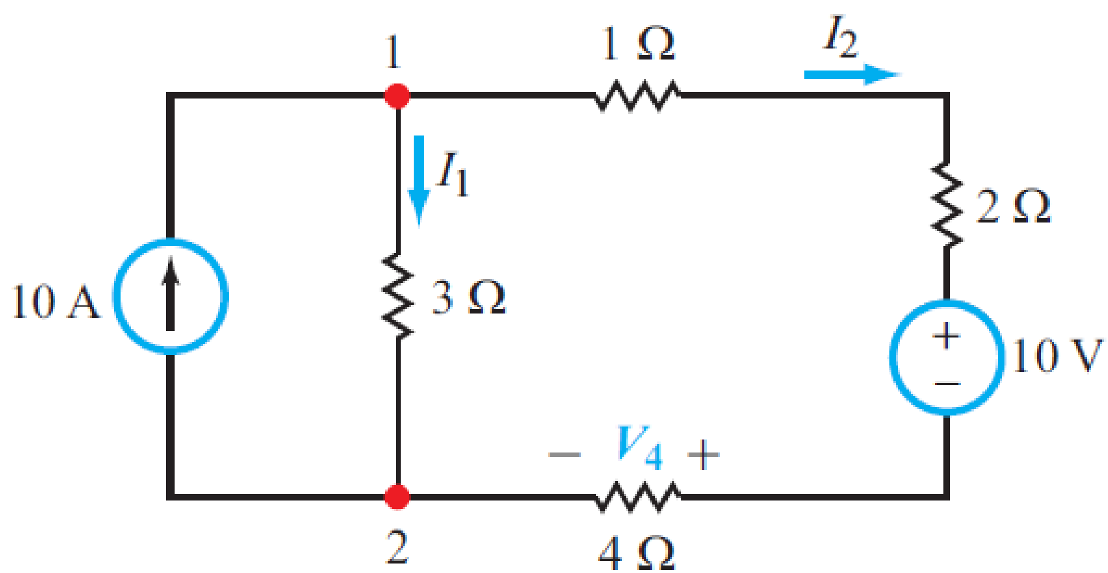
$$i_1 - i_2 - i_3 + i_4 = 0$$

$$i_1 + i_4 = i_2 + i_3$$

$$\sum_{n=1}^{N} i_n = 0 \qquad \text{(KCL)},$$

EXAMPLE

If V_4, the voltage across the 4–Ω resistor in Fig. 2-12, is 8 V, determine I_1 and I_2.



Solution: Applying Ohm's law,

$$I_2 = \frac{V_4}{4} = \frac{8}{4} = 2\text{ A}.$$

At node 1: $10 - I_1 - I_2 = 0.$

Hence,

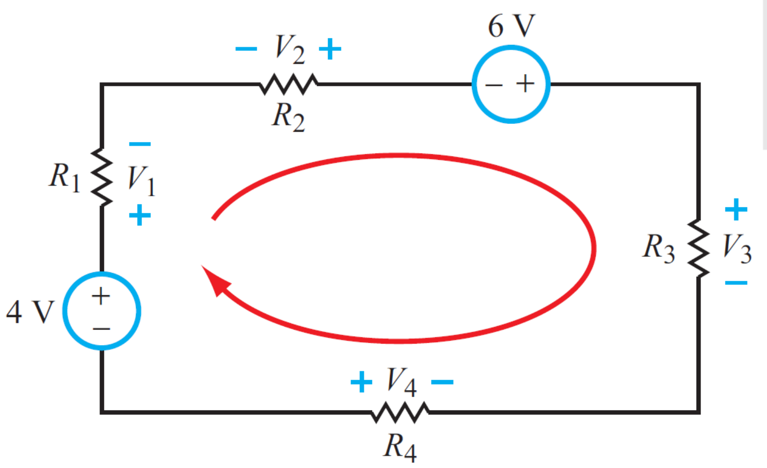
$$I_1 = 10 - I_2 = 10 - 2 = 8\text{ A}.$$

KIRCHHOFF'S VOLTAGE LAW (KVL)

- **The algebraic sum of voltages around a closed path is zero.**

Sum of voltage drops = sum of voltage rises

$$\sum_{n=1}^{N} v_n = 0 \qquad \text{(KVL)},$$



Sign Convention

- Add up the voltages in a systematic clockwise movement around the loop.
- Assign a positive sign to the voltage across an element if the (+) side of that voltage is encountered first, and assign a negative sign if the (−) side is encountered first.

$$-4 + V_1 - V_2 - 6 + V_3 - V_4 = 0$$

KCL/KVL RULES

Table 2-4: **Equally valid, multiple statements of Kirchhoff's Current Law (KCL) and Kirchhoff's Voltage Law (KVL).**

KCL

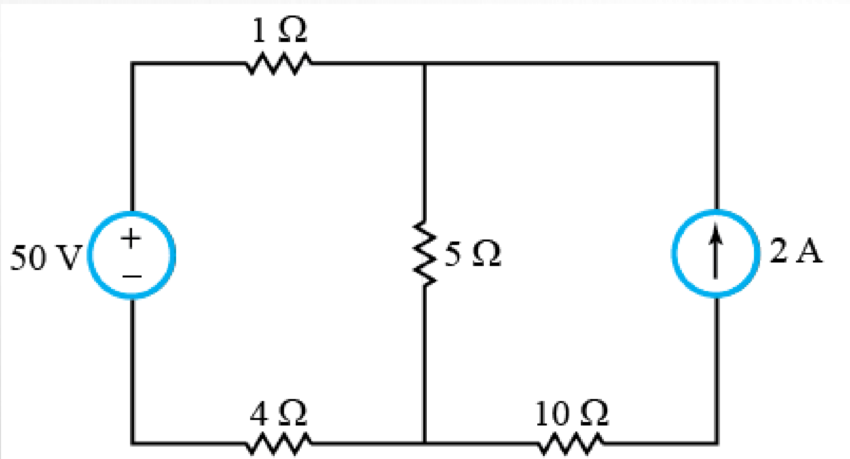
- Sum of all currents entering a node = 0
  [i = "+" if entering; i = "−" if leaving]
- Sum of all currents leaving a node = 0
  [i = "+" if leaving; i = "−" if entering]
- Total of currents entering = Total of currents leaving

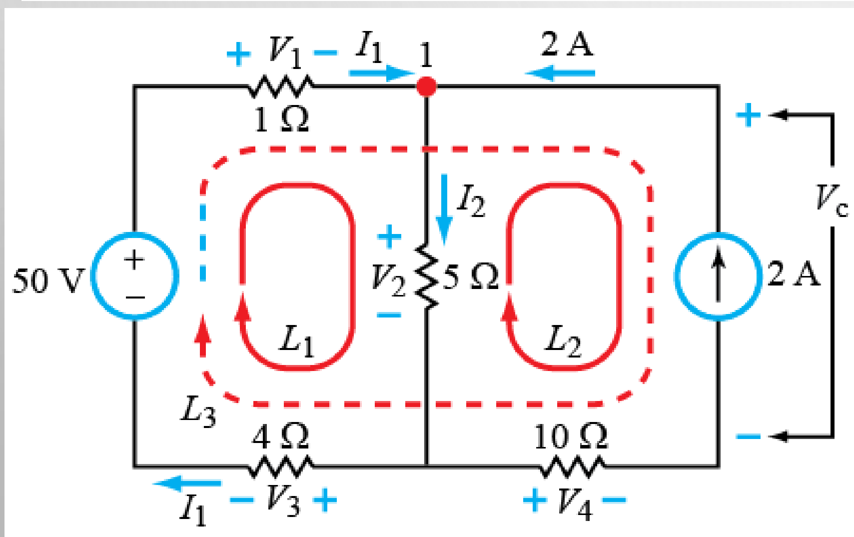
KVL

- Sum of voltages around closed loop = 0
  [υ = "+" if + side encountered first in clockwise direction]
- Total voltage rise = Total voltage drop

EXAMPLE : KCL/KVL

- Determine the currents flowing through the 1 Ohm and 5 Ohm resistors, and the voltage across the 2 A current source.





Solution:

$$-50 + I_1 + 5I_2 + 4I_1 = 0,$$ Loop 1

$$-5I_2 + V_c - (10 \times 2) = 0.$$ Loop 2

Next, we apply KCL at node 1 which gives

$$I_1 - I_2 + 2 = 0.$$

Three equations with three unknowns:

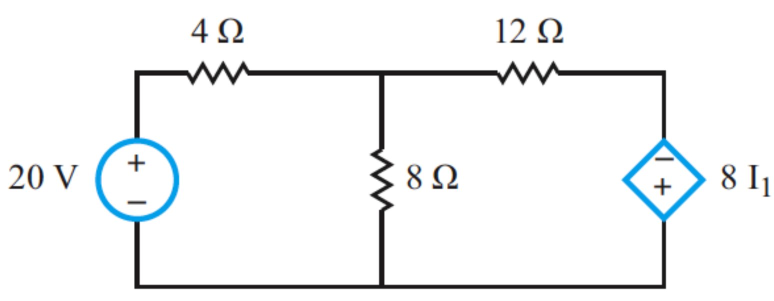
$$I_1 = 4 \text{ A},$$

$$I_2 = 6 \text{ A},$$

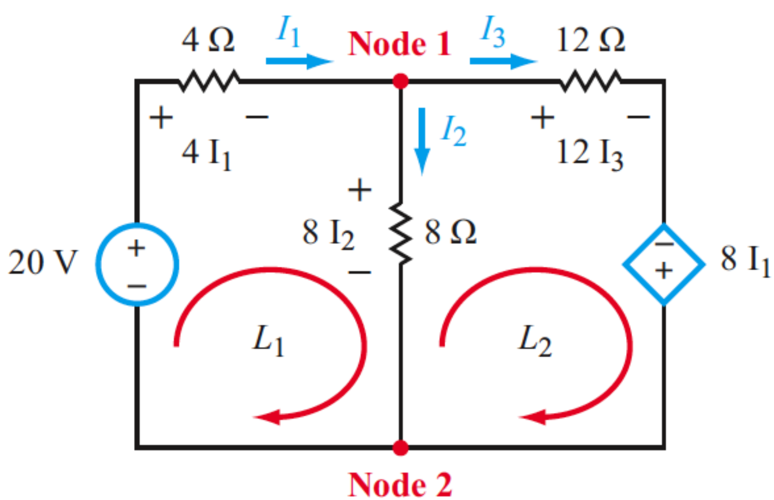
$$V_c = 50 \text{ V}.$$

EXAMPLE

Apply KCL and KVL to determine the amount of power consumed by the 12 Ω resistor.



(a) Given circuit



(b) After assigning currents at node 1

$$-20 + 4I_1 + 8I_2 = 0, \qquad \textbf{(KVL for Loop 1)}$$

$$-8I_2 + 12I_3 - 8I_1 = 0. \qquad \textbf{(KVL for Loop 2)}$$

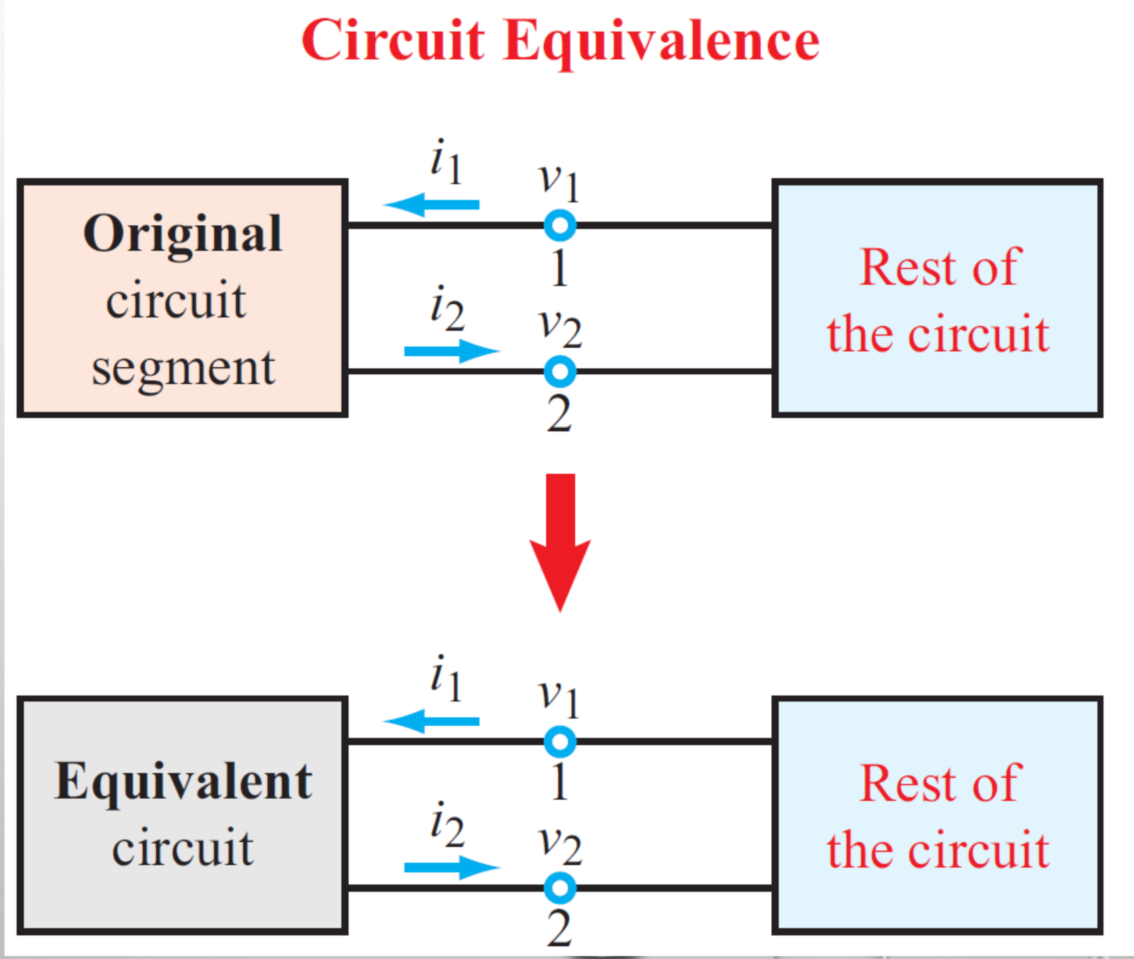
$$I_1 = I_2 + I_3.$$

$$I_1 = \frac{25}{7}\text{ A}, \qquad I_2 = \frac{5}{7}\text{ A}, \qquad I_3 = \frac{20}{7}\text{ A}.$$

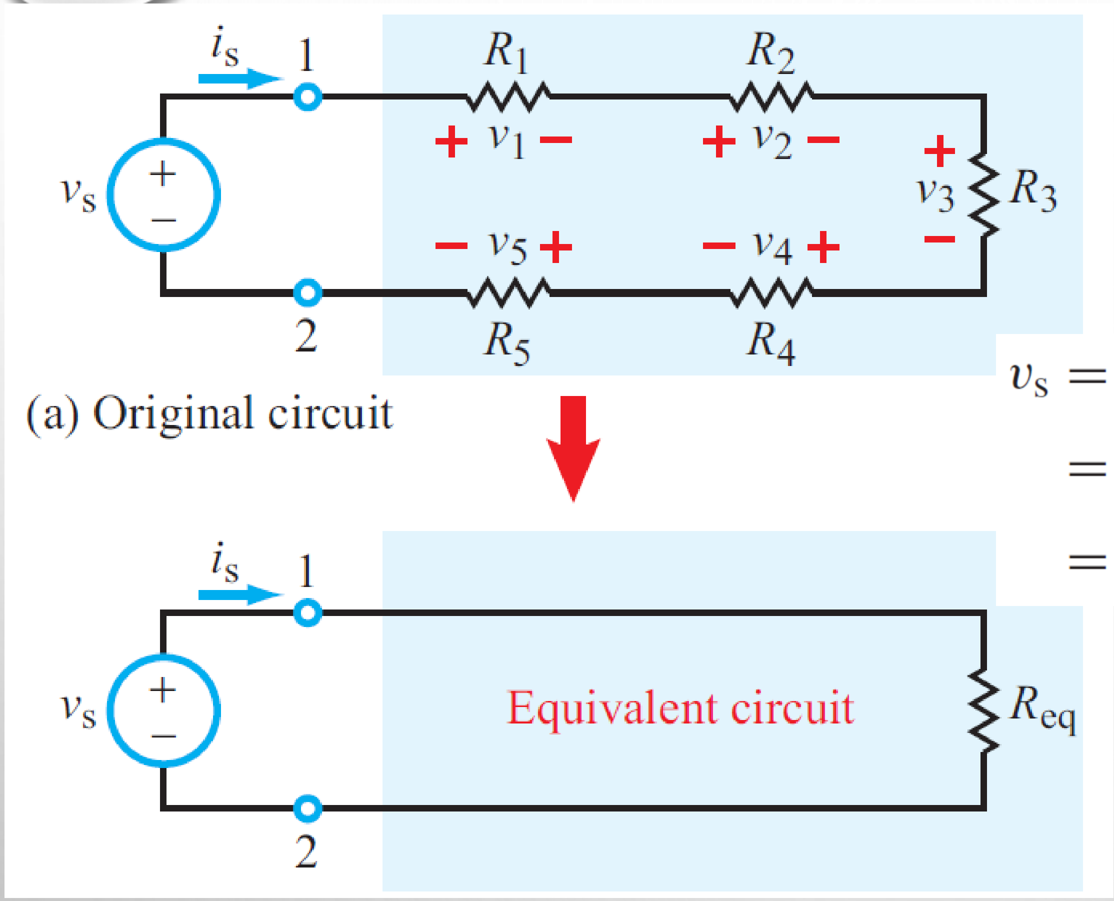
$$P = I_3^2 R = \left(\frac{20}{7}\right)^2 \times 12 = 97.96\text{ W}.$$

EQUIVALENT CIRCUITS

- If the current and voltage characteristics at nodes are identical, the circuits are considered "equivalent".
- Identifying equivalent circuits simplifies analysis.



RESISTORS IN SERIES



(a) Original circuit

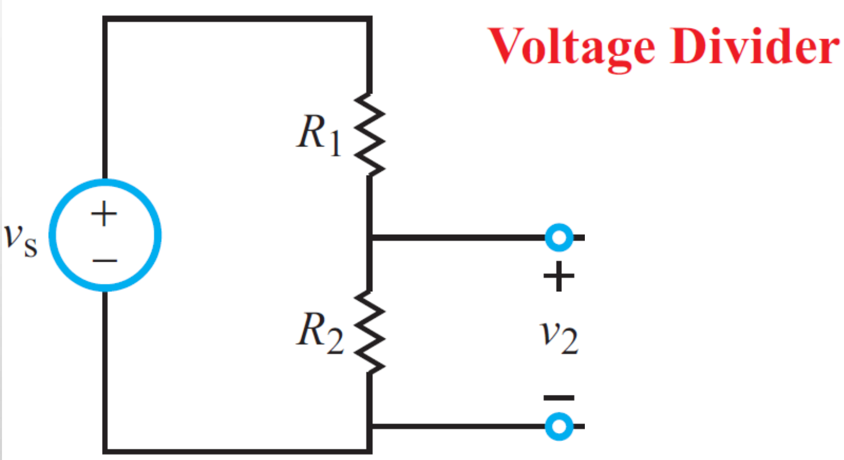
$$v_s = R_1 i_s + R_2 i_s + R_3 i_s + R_4 i_s + R_5 i_s$$
$$= (R_1 + R_2 + R_3 + R_4 + R_5) i_s$$
$$= R_{eq} i_s,$$

Equivalent resistance of N series resistors:

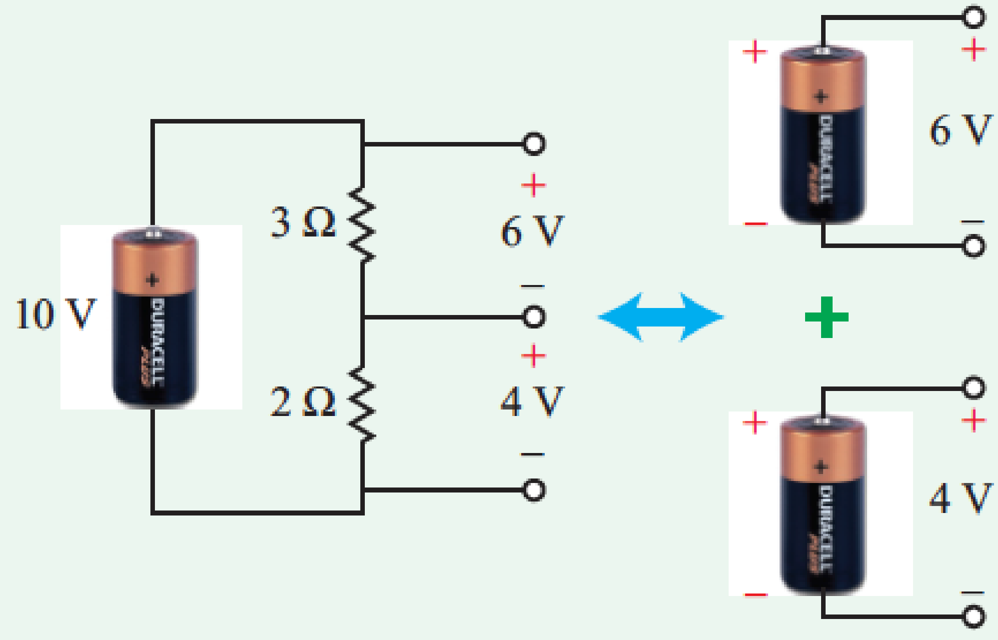
$$R_{eq} = R_1 + R_2 + \cdots + R_N = \sum_{n=1}^{N} R_n$$

VOLTAGE DIVIDER

Voltage divider: voltage is divided over series resistors.



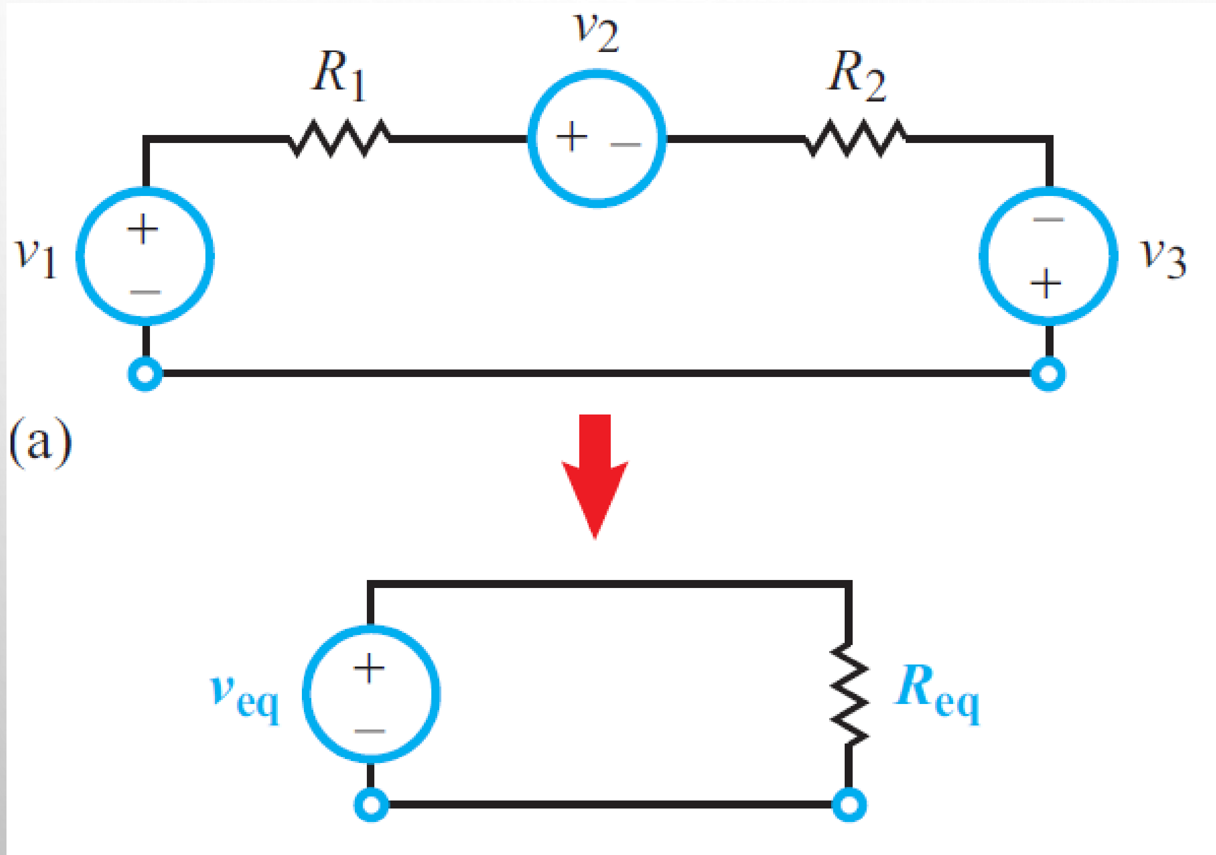
$$v_2 = \left(\frac{R_2}{R_1 + R_2}\right) v_s$$



(b) Voltage divider is equivalent to subdividing a battery into two separate batteries

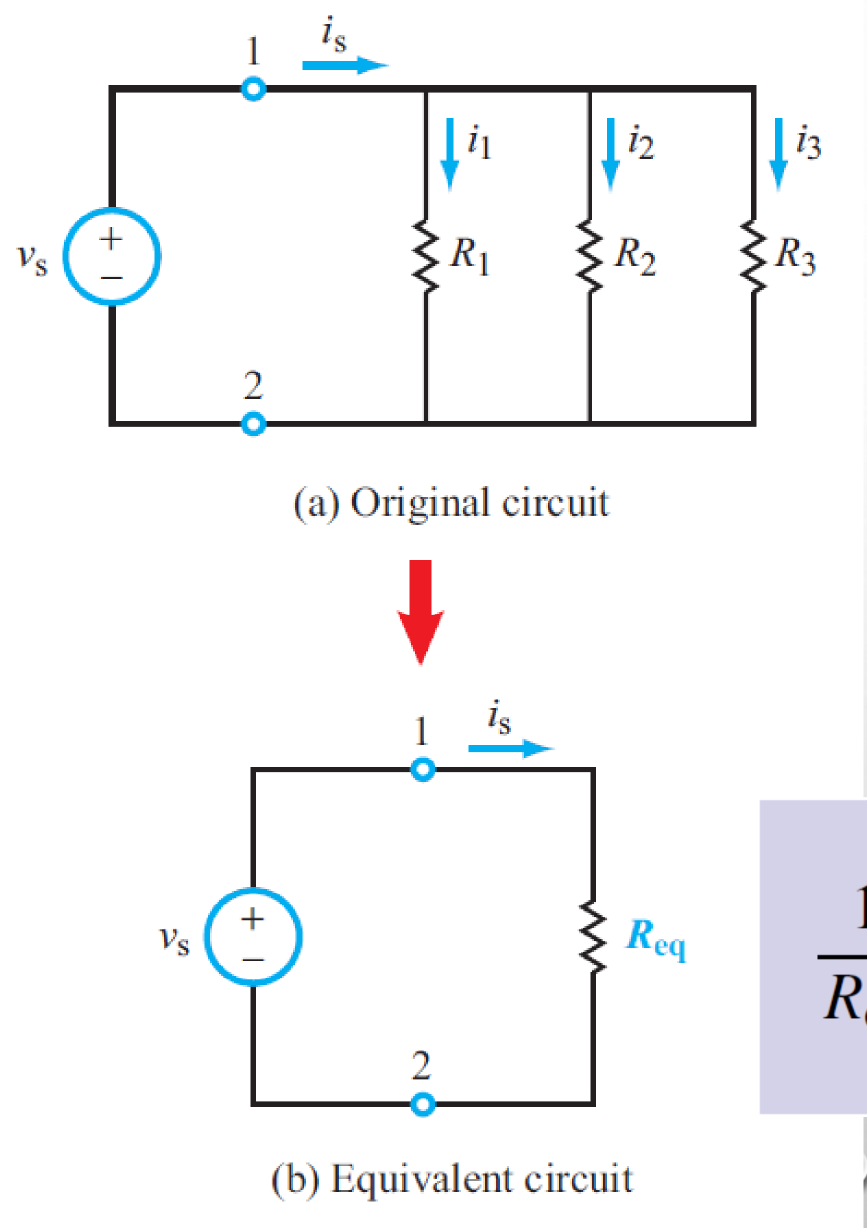
ADDING SOURCES IN SERIES

Combining voltage sources



(b) $v_{eq} = v_1 - v_2 + v_3$ $R_{eq} = R_1 + R_2$

RESISTORS IN PARALLEL



$$i_s = \frac{v_s}{R_1} + \frac{v_s}{R_2} + \frac{v_s}{R_3}$$

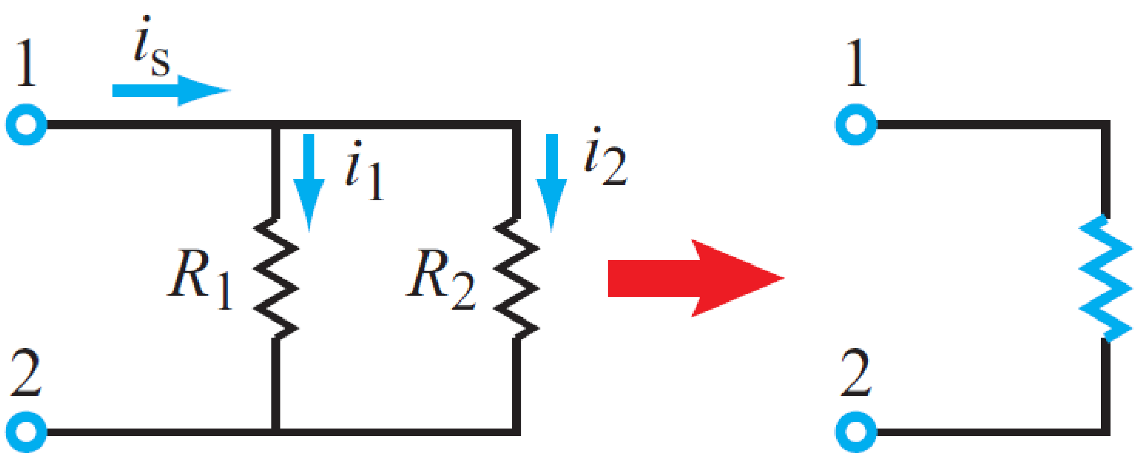
$$= \frac{v_s}{R_{eq}}.$$

$$R_{eq} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right)^{-1}$$

$$\frac{1}{R_{eq}} = \sum_{i=1}^{N} \frac{1}{R_i} \qquad \text{(resistors in parallel).}$$

CURRENT DIVIDER

Current Division



$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

$$i_1 = \left(\frac{R_2}{R_1 + R_2}\right) i_s \qquad i_2 = \left(\frac{R_1}{R_1 + R_2}\right) i_s$$

EXAMPLE

Use the equivalent-resistance approach to determine V_2, I_1, I_2, and I_3 in the circuit of Fig. 2-28(a).

$$I_1 = \frac{24}{10+2} = 2 \text{ A}$$

and

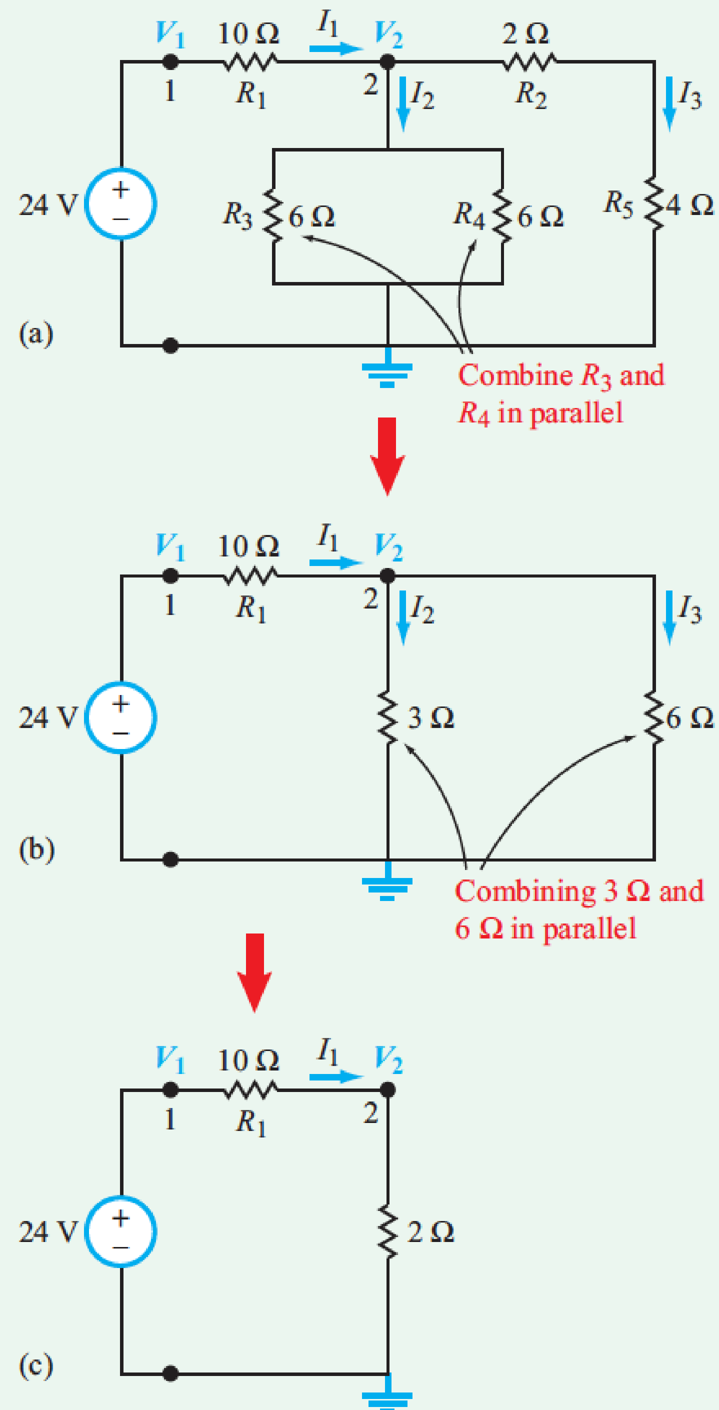
$$V_2 = 2I_1 = 2 \times 2 = 4 \text{ V}.$$

Returning to Fig. 2-28(b), we apply Ohm's law to find I_2 and I_3.

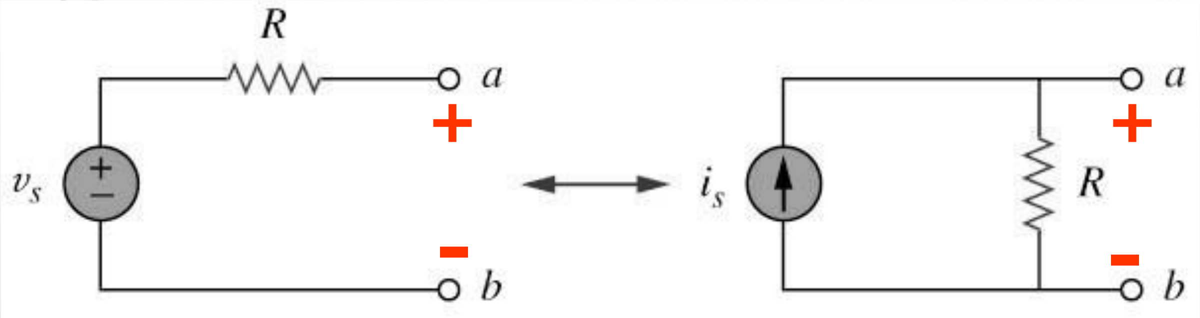
$$I_2 = \frac{V_2}{3} = \frac{4}{3} = 1.33 \text{ A},$$

and

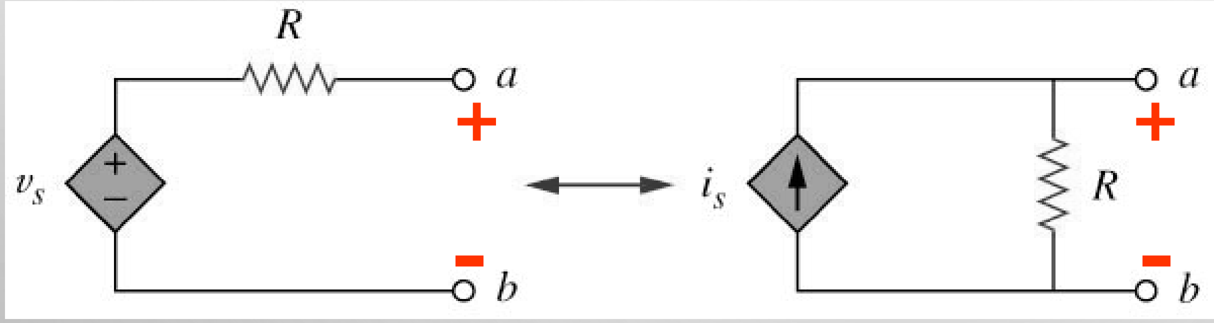
$$I_3 = \frac{V_2}{6} = \frac{4}{6} = 0.67 \text{ A}.$$



SOURCE TRANSFORMATION



(a) Independent source transform



(b) Dependent source transform

- The arrow of the current source is directed toward the positive terminal of the voltage source.
- The source transformation is not possible when R = 0 for voltage source and R = ∞ for current source.

SOURCE TRANSFORMATION

$$-v_s + iR_1 + v_{12} = 0$$

$$i = \frac{v_s}{R_1} - \frac{v_{12}}{R_1}.$$

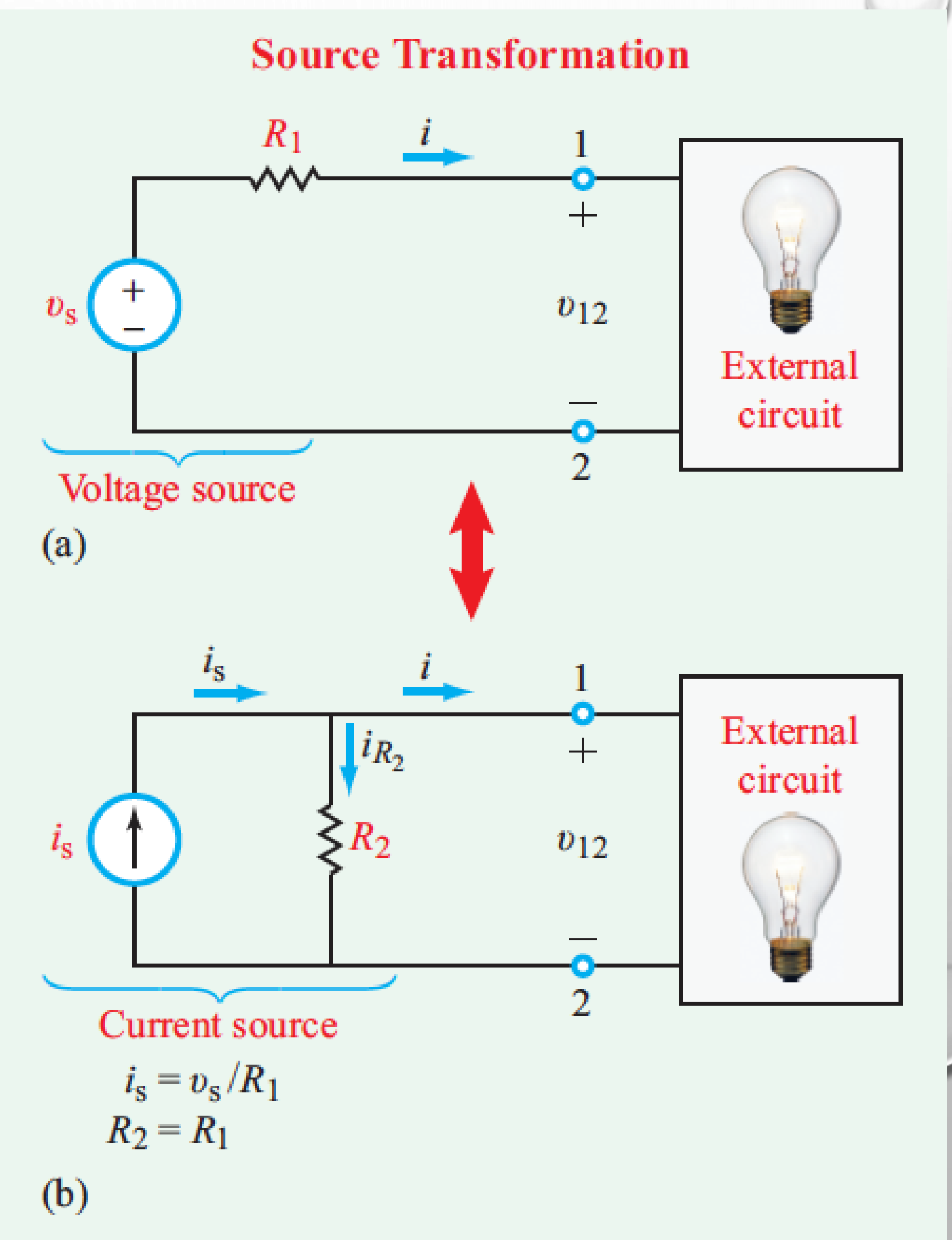
$$i = i_s - i_{R_2}$$

$$= i_s - \frac{v_{12}}{R_2}$$

For the two circuits to be equivalent :

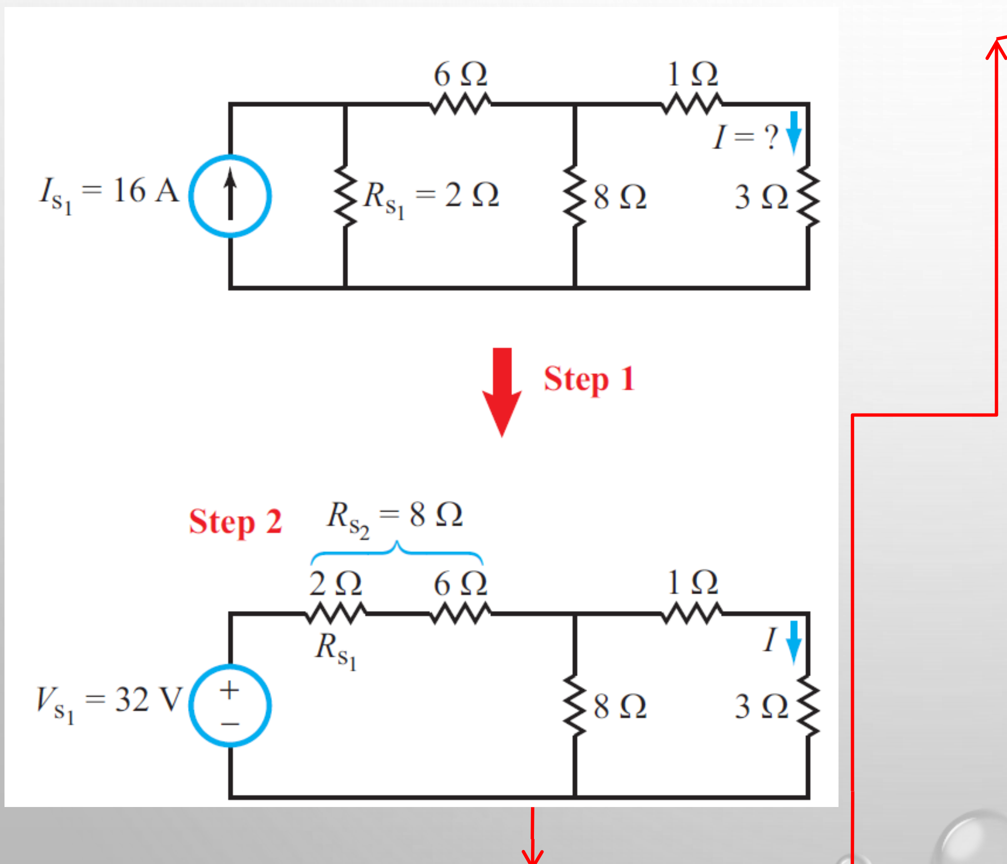
$$R_1 = R_2$$

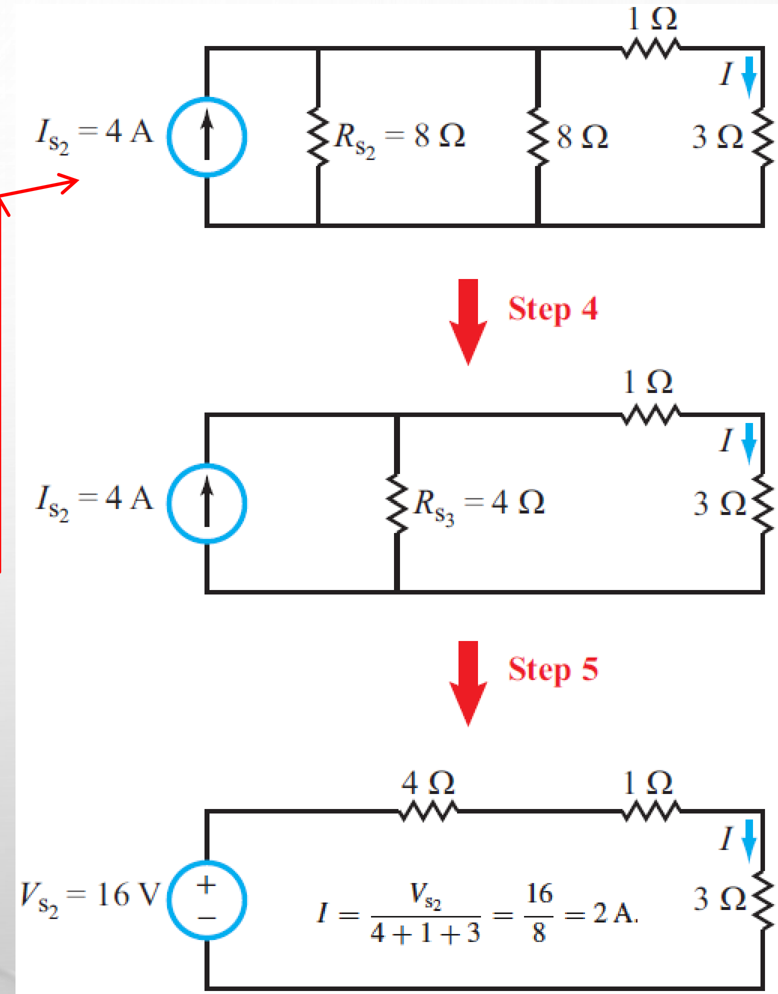
$$i_s = \frac{v_s}{R_1}.$$



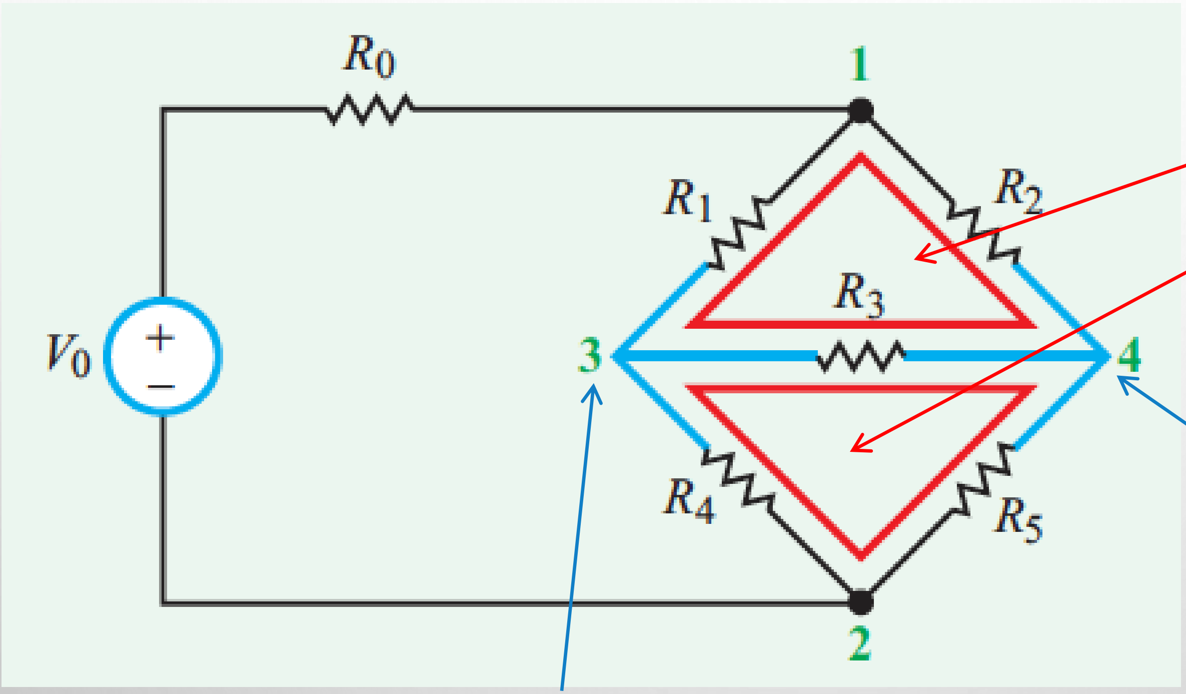
EXAMPLE

Use source transformation to determine
The current *I*.





Y-Δ TRANSFORMATION



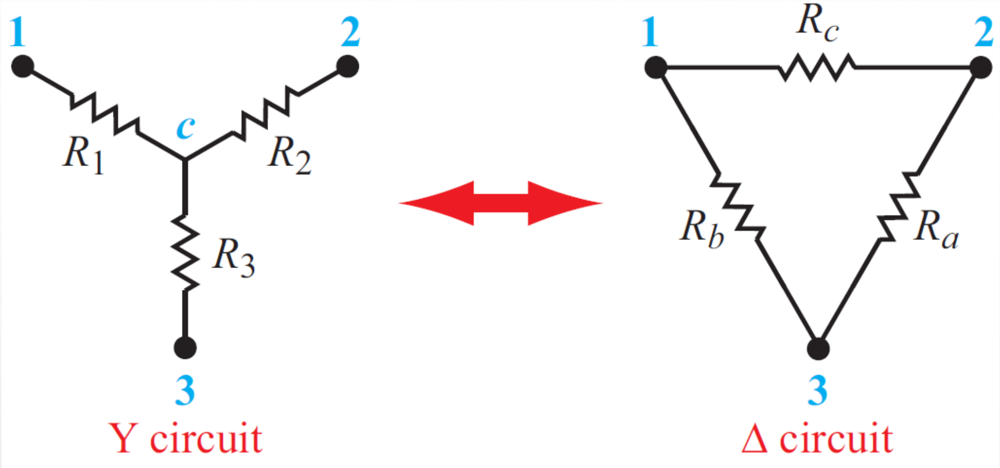
R1,R2,R3=Δ

R3,R4,R5=Δ

R2,R3,R5=Y

R1,R3,R4=Y

Y-Δ TRANSFORMATION



Δ →Y Transformation

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$$

$$R_2 = \frac{R_a R_c}{R_a + R_b + R_c}$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

Y→ Δ Transformation

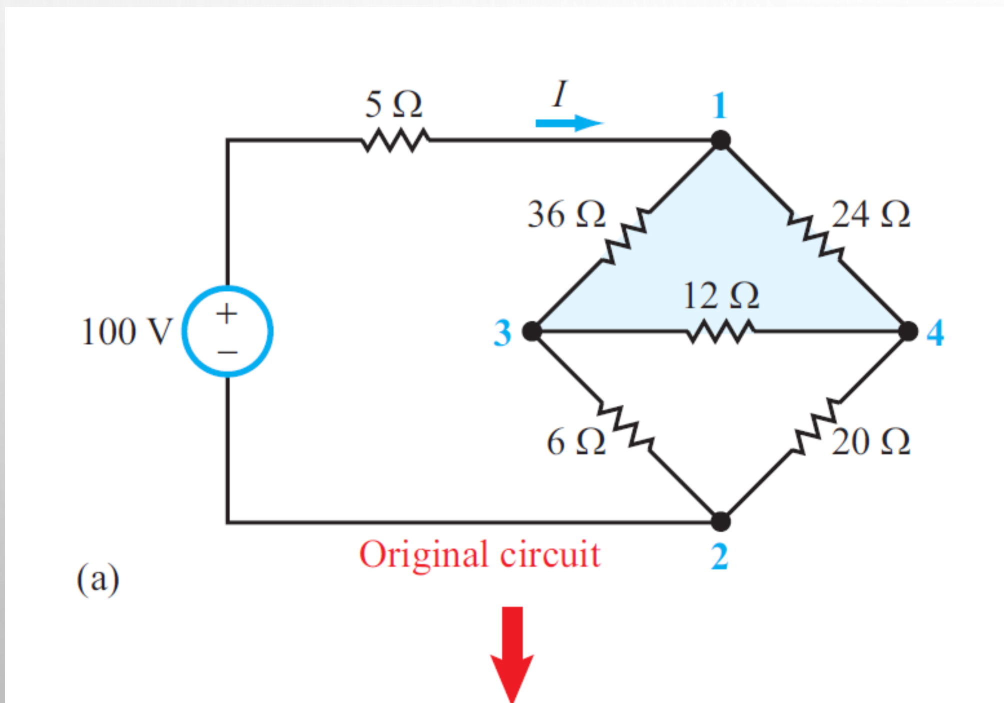
$$R_a = \frac{R_1 R_2 + R_2 R_3 + R_1 R_3}{R_1}$$

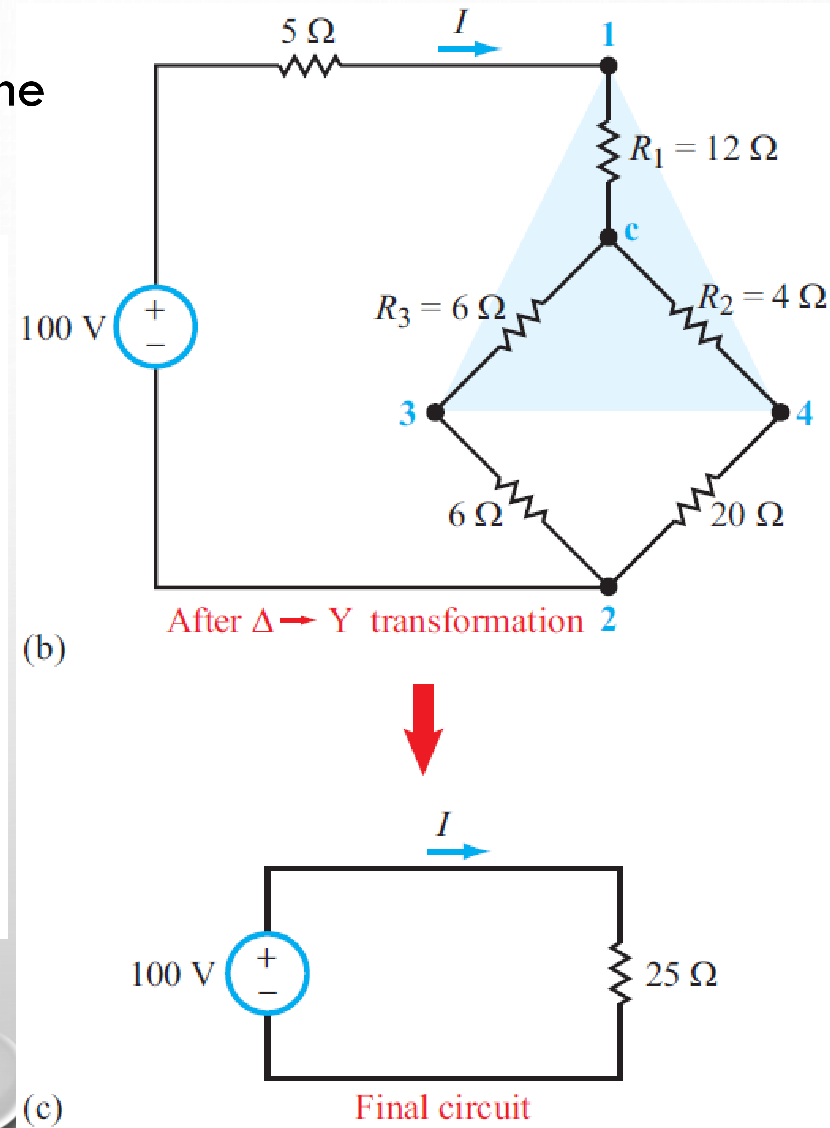
$$R_b = \frac{R_1 R_2 + R_2 R_3 + R_1 R_3}{R_2}$$

$$R_c = \frac{R_1 R_2 + R_2 R_3 + R_1 R_3}{R_3}$$

EXAMPLE

Use Delta-Wye transformation to determine The current *I*.





Mathematical and Physical Models

Linear resistor

$$R = \rho\ell/A$$
$$p = i^2 R$$

Kirchhoff current law (KCL)

$$\sum_{n=1}^{N} i_n = 0$$

i_n = current entering node n

Kirchhoff voltage law (KVL)

$$\sum_{n=1}^{N} v_n = 0$$

v_n = voltage across branch n

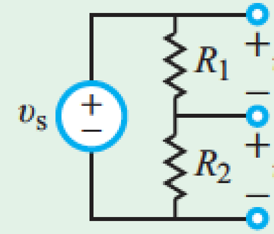
Resistor combinations

In series $$R_{eq} = \sum_{i=1}^{N} R_i$$

In parallel $$\frac{1}{R_{eq}} = \sum_{i=1}^{N} \frac{1}{R_i}$$

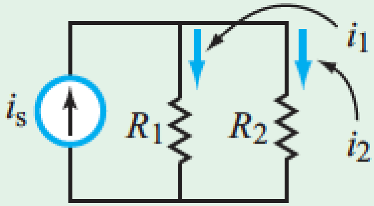
or $$G_{eq} = \sum_{i=1}^{N} G_i$$

Voltage division



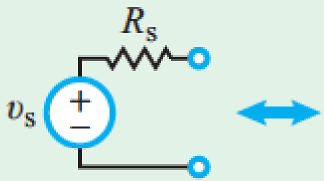
$$v_1 = \left(\frac{R_1}{R_1 + R_2}\right) v_s$$
$$v_2 = \left(\frac{R_2}{R_1 + R_2}\right) v_s$$

Current division

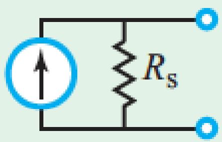


$$i_1 = \left(\frac{R_2}{R_1 + R_2}\right) i_s = \frac{G_1}{G_{eq}} i_s$$
$$i_2 = \left(\frac{R_1}{R_1 + R_2}\right) i_s = \frac{G_2}{G_{eq}} i_s$$

Source transformation

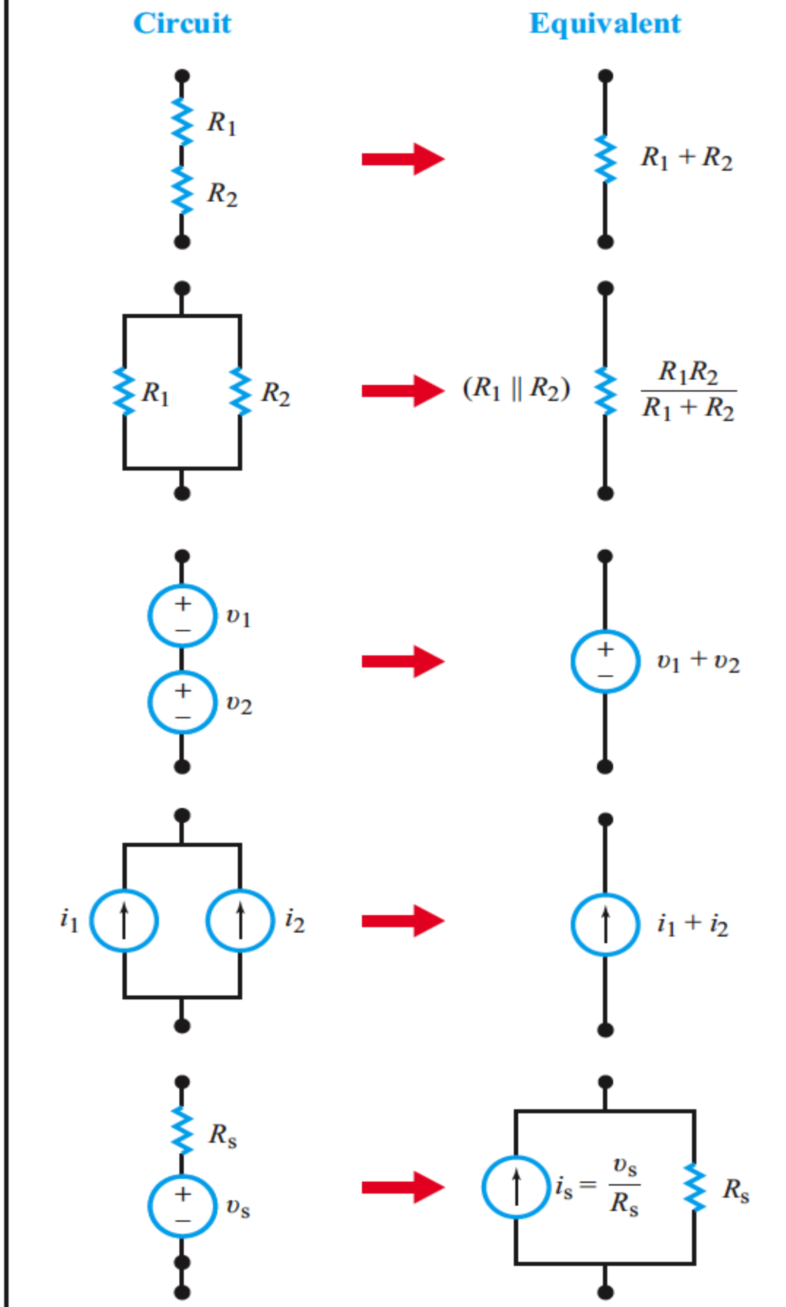


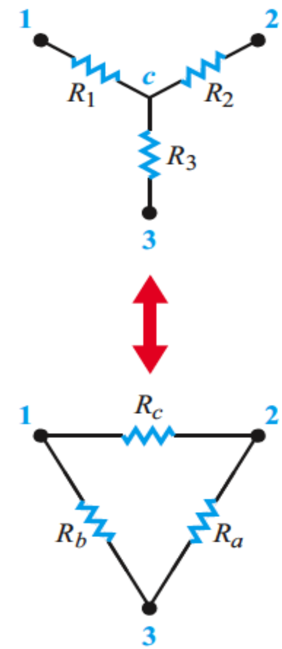
$$i_s = \frac{v_s}{R_s}$$



Y–Δ transformation Table 2-5

Table 2-5: Equivalent circuits.





$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$$

$$R_2 = \frac{R_a R_c}{R_a + R_b + R_c}$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

$$R_a = \frac{R_1 R_2 + R_2 R_3 + R_1 R_3}{R_1}$$

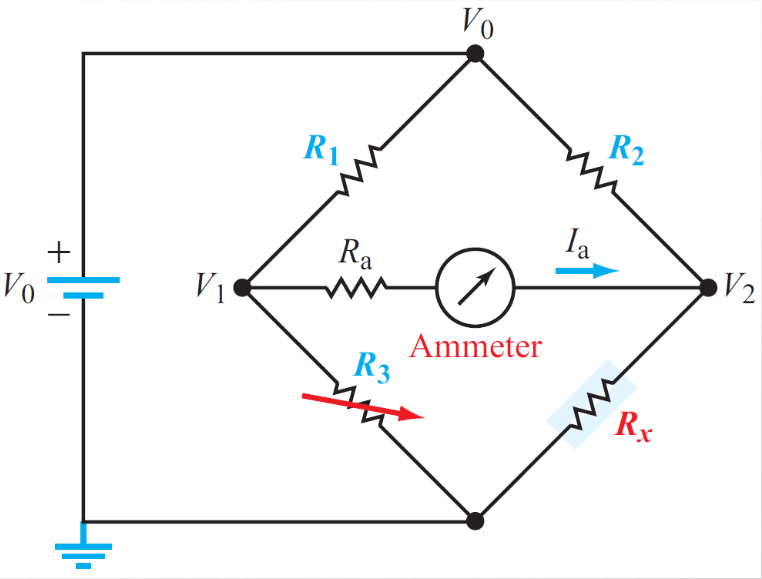
$$R_b = \frac{R_1 R_2 + R_2 R_3 + R_1 R_3}{R_2}$$

$$R_c = \frac{R_1 R_2 + R_2 R_3 + R_1 R_3}{R_3}$$

- For $R_a = R_b = R_c$ ➔ $R_1 = R_2 = R_3 = R_a/3$
- For $R_1 = R_2 = R_3$ ➔ $R_a = R_b = R_c = 3R_1$

WHEATSTONE-BRIDGE

- The device was initially developed as an accurate Ohm-meter for measuring resistance. Today, the circuit is integral to numerous sensing devices, including strain gauges, force and torque sensors.
  - R_1 and R_2 are fixed with known values
  - R_3 is variable with known value
  - R_x is unknown.
- Determining R_x is achieved by adjusting R_3 so as to make $I_a = 0$ (i.e., balanced condition).



$$I_a = 0 \longrightarrow V_1 = V_2$$

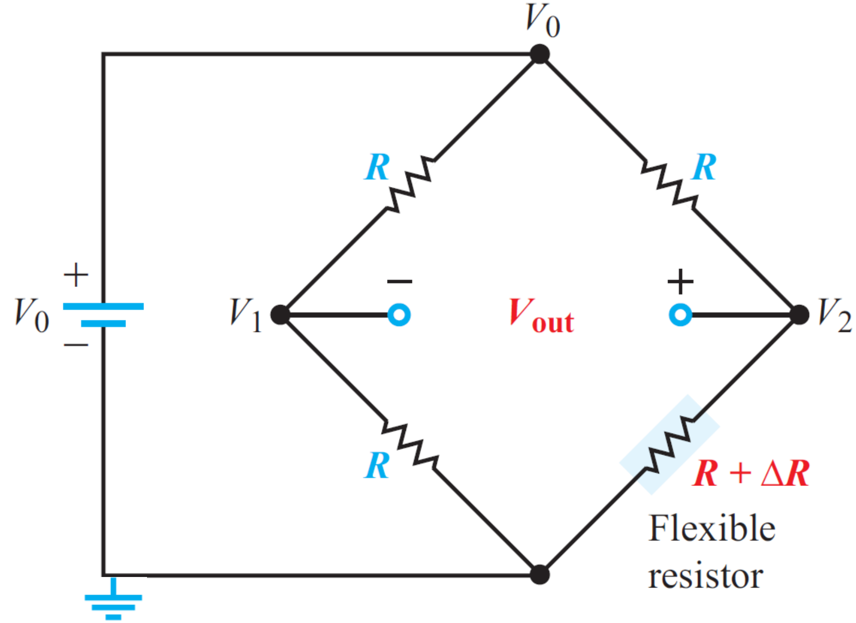
- Circuit equations in balanced condition:

$$\frac{R_3 V_0}{R_1 + R_3} = \frac{R_x V_0}{R_2 + R_x} \qquad \frac{R_1 V_0}{R_1 + R_3} = \frac{R_2 V_0}{R_2 + R_x}. \longrightarrow R_x = \left(\frac{R_2}{R_1}\right) R_3$$

EXAMPLE OF WHEATSTONE SENSOR

(Measuring small deviations from a reference condition)

Determine V_{out} in the circuit below in terms of R and ΔR.



Solution: Voltage division gives

$$V_1 = \frac{V_0 R}{R + R} = \frac{V_0}{2}$$

and

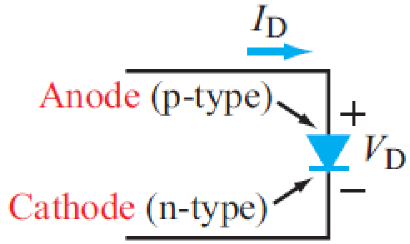
$$V_2 = \frac{V_0(R + \Delta R)}{R + (R + \Delta R)} = \frac{V_0(R + \Delta R)}{2R + \Delta R}.$$

$$\begin{aligned} V_{out} &= V_2 - V_1 \\ &= \frac{V_0(R + \Delta R)}{2R + \Delta R} - \frac{V_0}{2} \\ &= \frac{2V_0(R + \Delta R) - V_0(2R + \Delta R)}{2(2R + \Delta R)} \\ &= \frac{V_0\, \Delta R}{4R + 2\,\Delta R} = \frac{V_0\, \Delta R}{4R(1 + \Delta R/2R)}. \end{aligned}$$

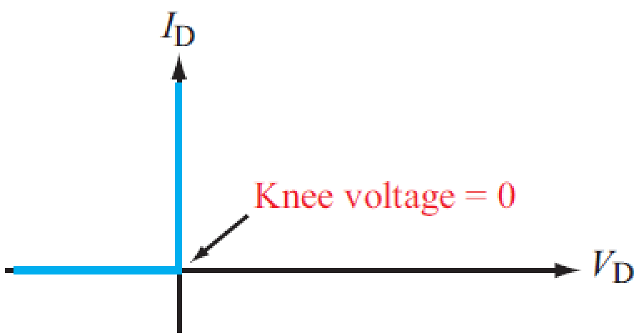
Since $\Delta R/R \ll 1$, ignoring the second term in the denominator would incur negligible error. Such an approximation leads to

$$V_{out} \simeq \frac{V_0}{4}\left(\frac{\Delta R}{R}\right), \qquad (2.64)$$

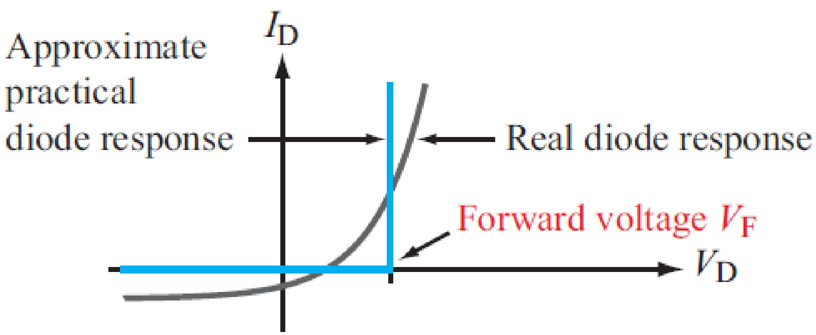
THE DIODE: SOLID-STATE NONLINEAR ELEMENT



(a) Diode symbol



(b) *i*–*v* of an ideal diode



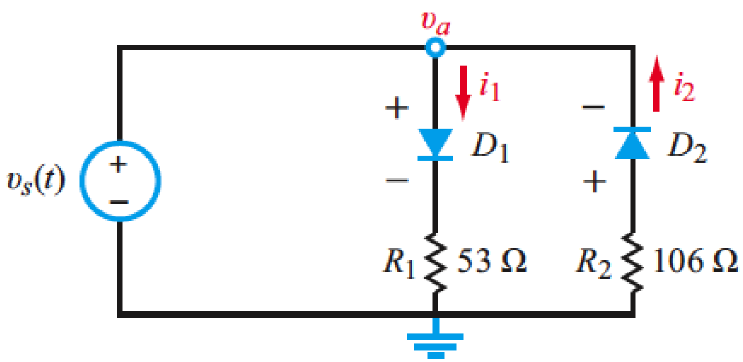
(c) *i*–*v* of a real diode

- The main use of the diode is as a one-way valve for current. The current can only flow from the anode to the cathode.
- An ideal diode looks like a short circuit for positive values of V_D (forward bias), and like an open circuit for negative values of V_D (reverse bias).
- The voltage level at which the diode switches from reverse bias to forward bias is called the forward-bias voltage.
  - For an ideal diode, V_F =0 V
  - For a practical diode, $V_F \approx 0.7$ V

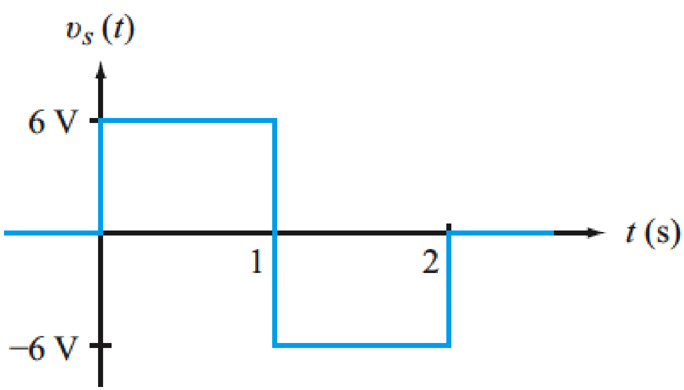
EXAMPLE: CIRCUIT WITH DIODES

The circuit in Fig. 2-36 contains two diodes, both with $V_F = 0.7$ V.

The waveform of the voltage source consists of a single cycle of a square wave. Generate plots for $i_1(t)$ and $i_2(t)$.



(a) Diode circuit



(b) Source voltage waveform

Solution:

$$i_1(t) = \frac{v_a - 0.7}{R_1}$$

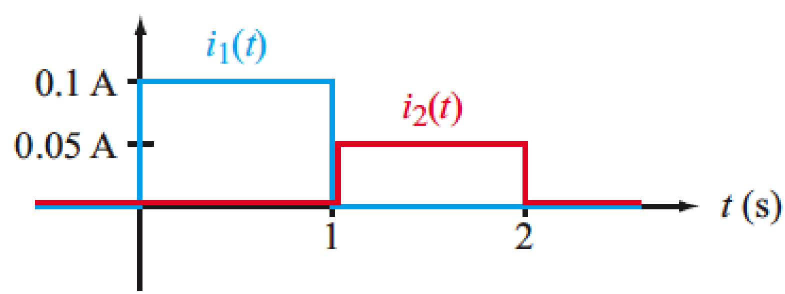
$$= \frac{6 - 0.7}{53} = 0.1 \text{ A} \qquad \text{for } 0 \leq t \leq 1 \text{ s.}$$

$$i_2(t) = 0 \qquad \text{for } 0 \leq t \leq 1 \text{ s.}$$

$$i_1(t) = 0 \qquad \text{for } 1 \leq t \leq 2 \text{ s,}$$

$$i_2(t) = \frac{6 - 0.7}{R_2}$$

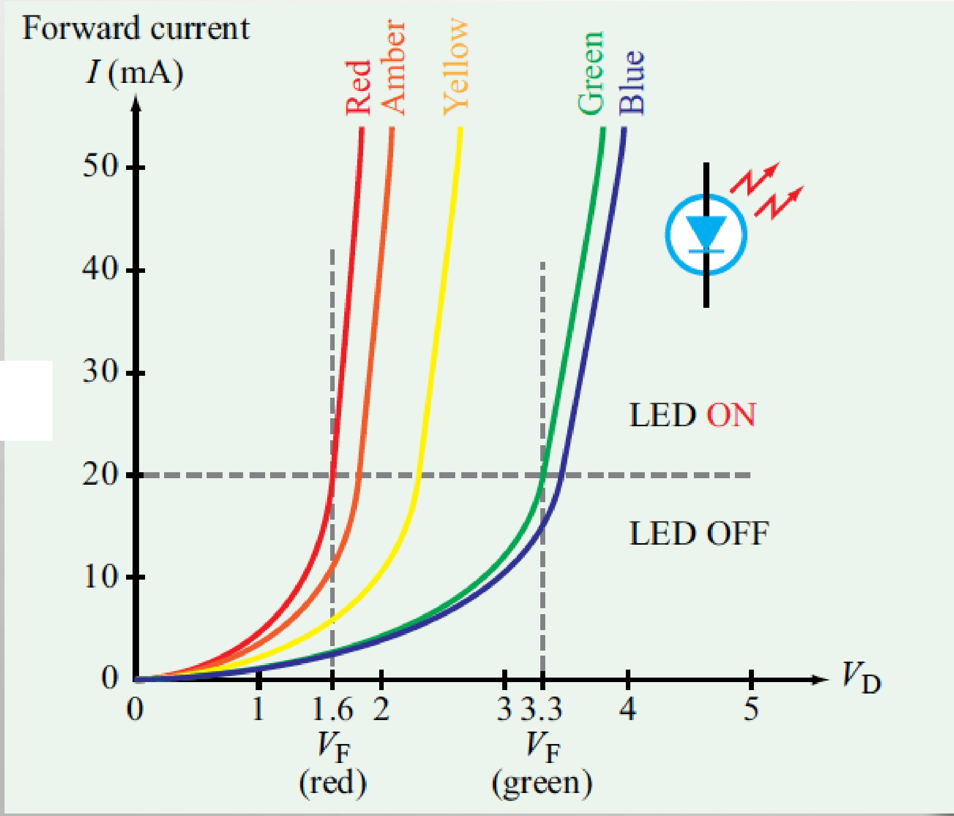
$$= \frac{6 - 0.7}{106} = 0.05\text{A} \qquad \text{for } 1 \leq t \leq 2 \text{ s.}$$

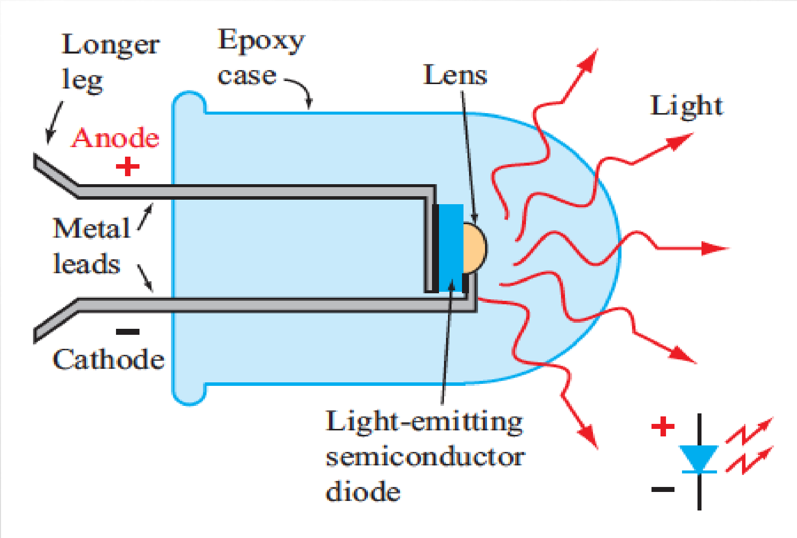


(c) Current waveforms

LIGHT EMITTING DIODE (LED)

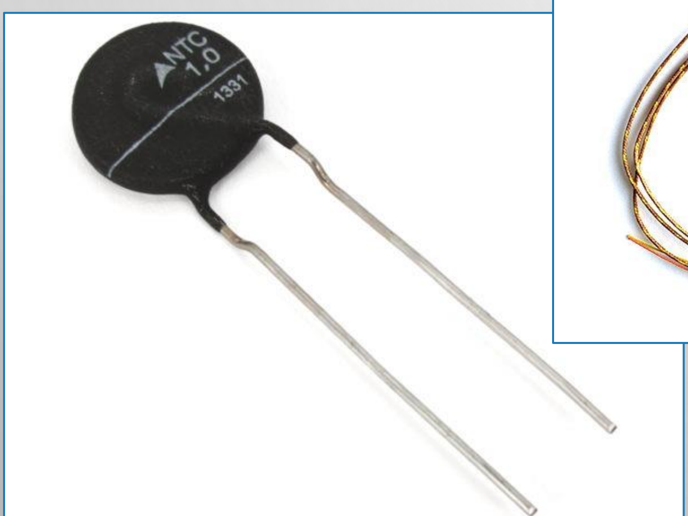
- A light emitting diode is a special kind of diode in that it emits light if the current exceeds a certain threshold that corresponds to a voltage threshold (V_F).
- The figure below displays plots of current versus V_D for five LEDs of different colors. The color of light emitted by an LED depends on the semiconductor compounds from which it is constructed.

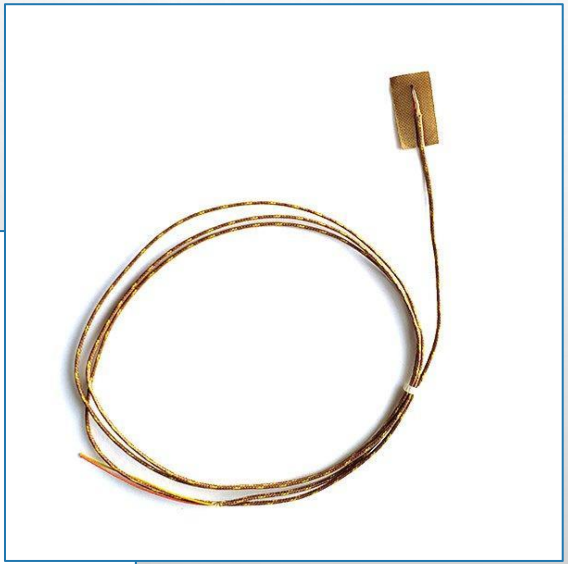




THE THERMISTOR

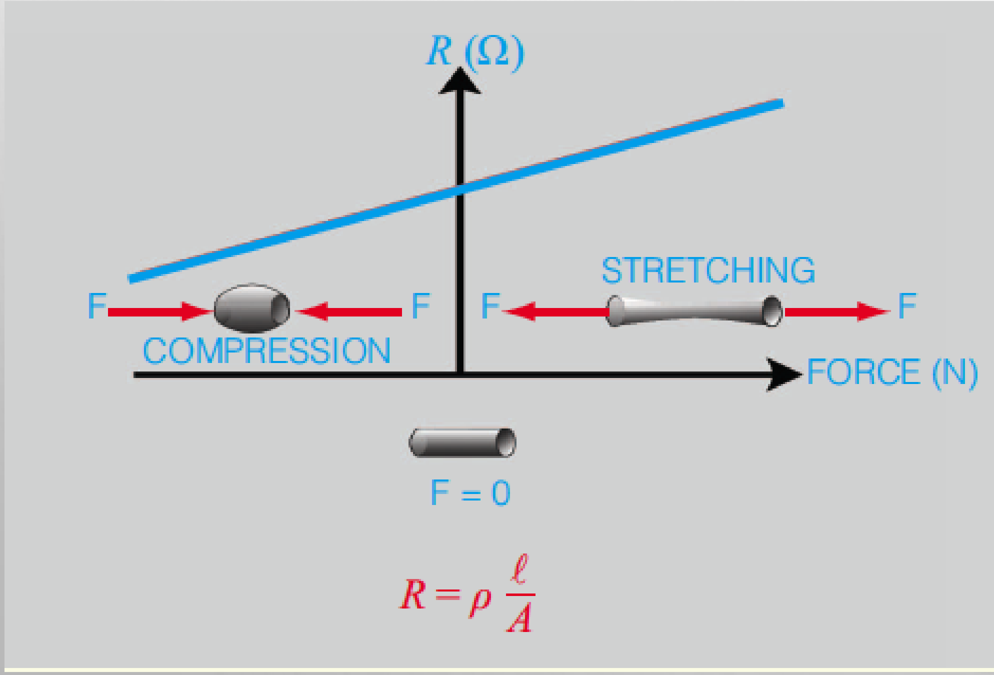
- For some metal oxides, the resistivity ρ exhibits a strong sensitivity to temperature.
- A resistor manufactured of such materials is called a **thermistor (or thermocouple)**, and it is used for temperature measurement, temperature compensation, and related applications.

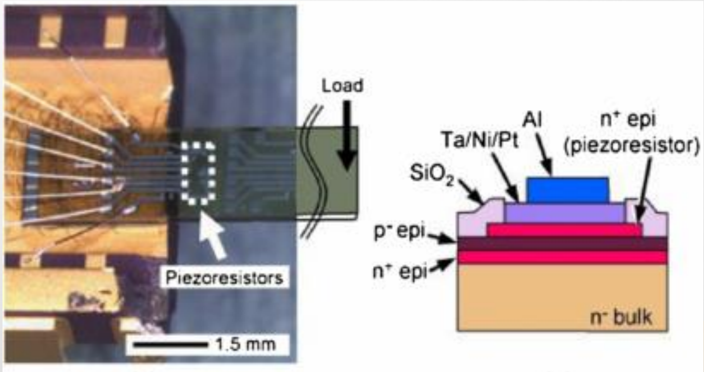





THE PIEZORESISTOR

- In 1856, Lord Kelvin discovered that applying a mechanical load on a bar of metal changed its resistance.
- The phenomenon is referred to as the **piezoresistive effect**. Today, its is used in many practical devices to convert a mechanical signal into an electrical one. Such sensors are called strain gauges.
- In its simplest form, a resistance change ΔR occurs when a mechanical pressure is applied along the axis of the resistor.





HOMEWORK ASSIGNMENT

SOLVE THE FOLLOWING PROBLEMS:

1, 3, 5, 7, 11, 13, 15, 19, 23, 25, 29, 33, 37, 41, 43, 47, 49, 51, 53.