EE 220

CHAPTER 3 – ANALYSIS TECHNIQUES

OVERVIEW

- Nodal analysis
- Mesh analysis
- Linearity
- Superposition
- Thevenin/Norton Equivalent Circuits
- Maximum Power Transfer
- Bipolar Junction Transistor (BJT)

NODAL ANALYSIS

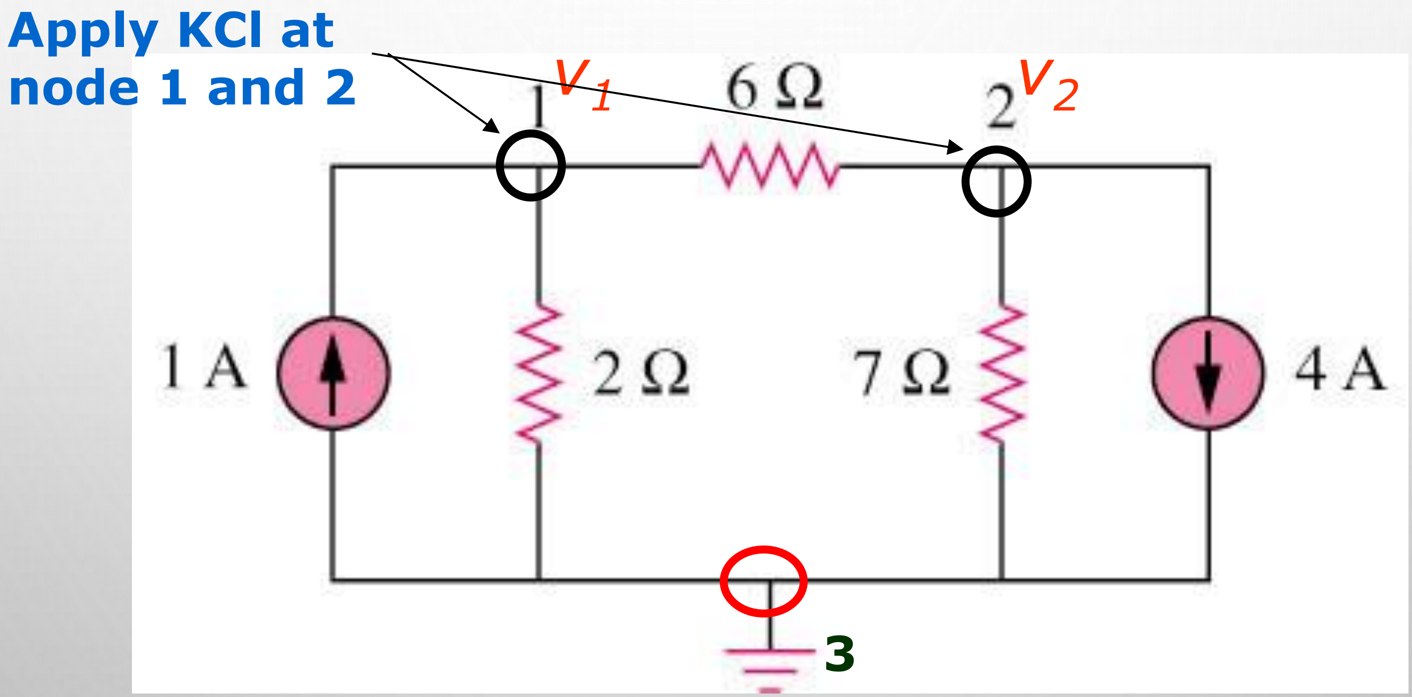
Nodal Analysis provides a general procedure for analyzing circuits using node voltages as the circuit variables.

Steps to determine the node voltages:

1. Select a node as the reference node.
2. Assign voltages $V_1, V_2, \ldots, V_{n-1}$ to the remaining n-1 nodes. These voltages are referenced with respect to the reference node.
3. Apply KCL to each of the n-1 non-reference nodes. Use Ohm's law to express the branch currents in terms of node voltages.
4. Solve the resulting simultaneous equations to obtain the unknown node voltages.

NODAL ANALYSIS

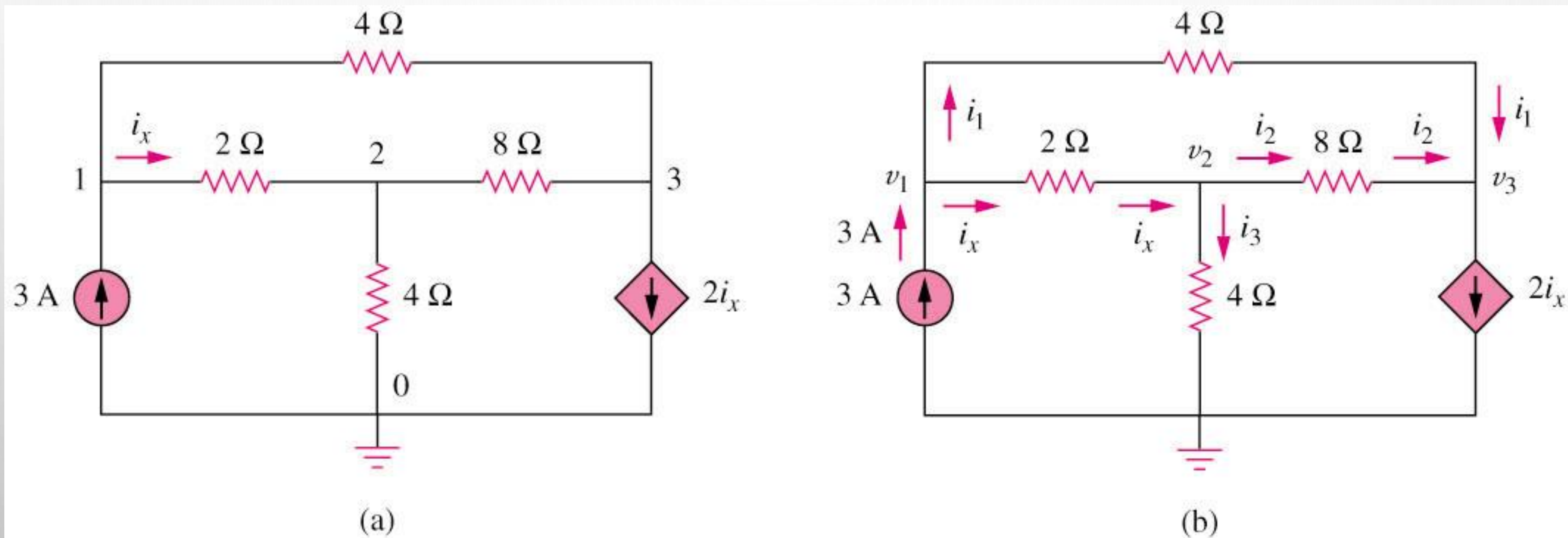
Example 1 – circuit with independent current source only



Answer: V_1 = -2V, V_2 = -14V

NODAL ANALYSIS

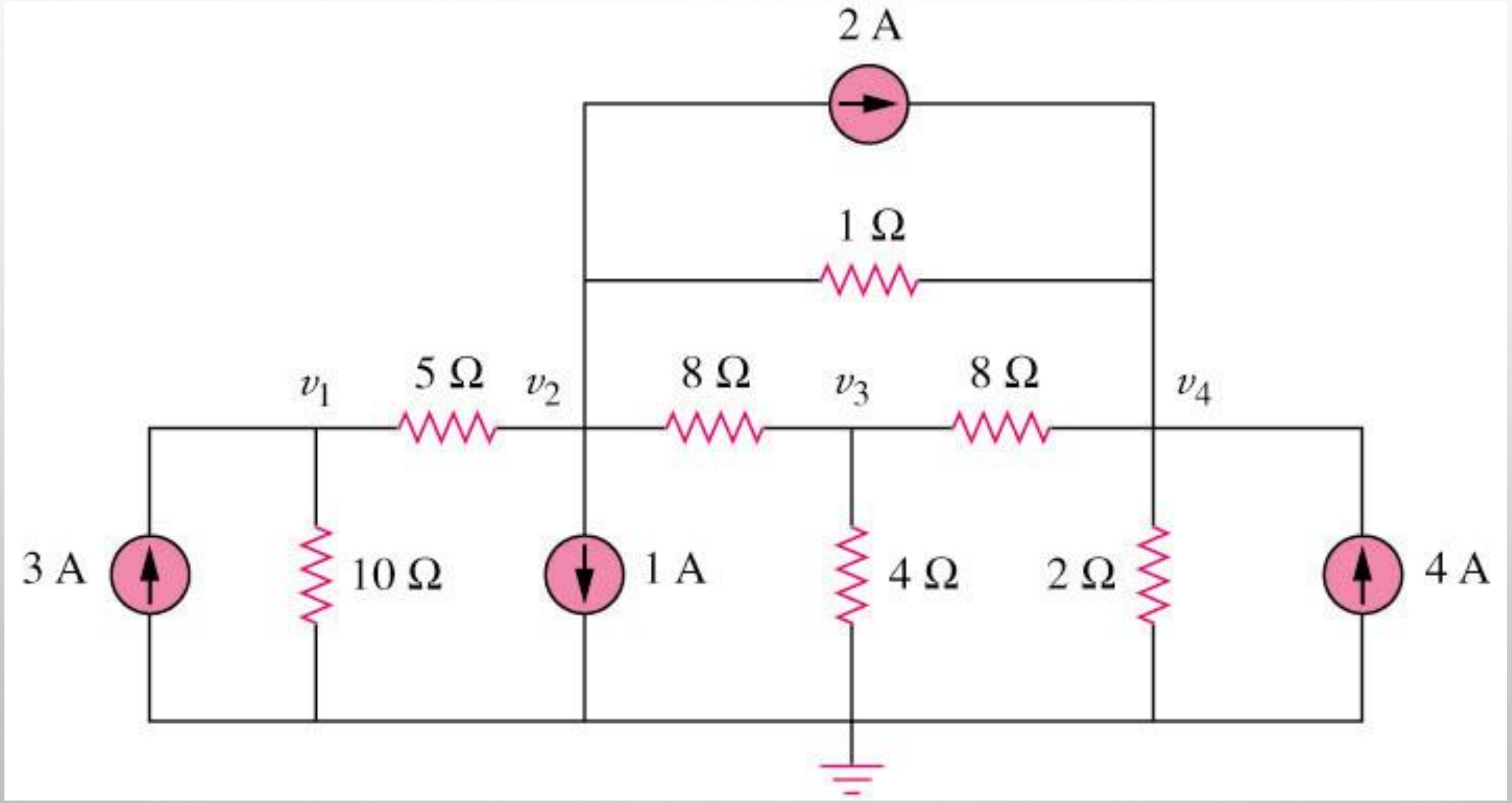
Example 2 – Circuit with dependent current sources



Answer: V_1= 4.8V, V_2 = 2.4V, V_3 = -2.4V

NODAL ANALYSIS

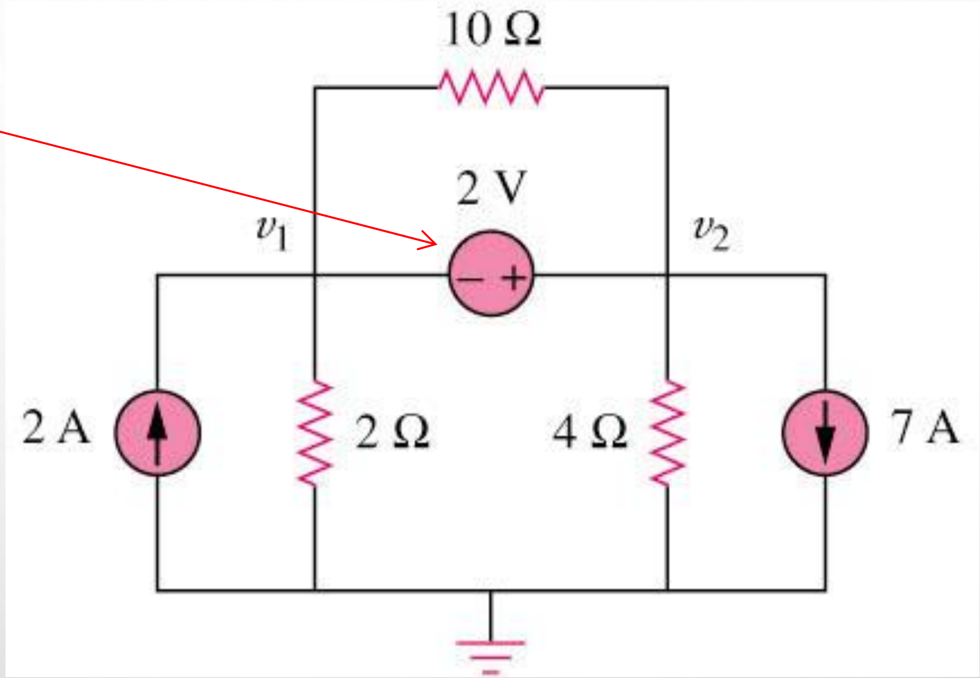
Example 3 – Write the nodal voltage equations for the circuit



NODAL ANALYSIS

Example 4 –circuit with independent voltage source

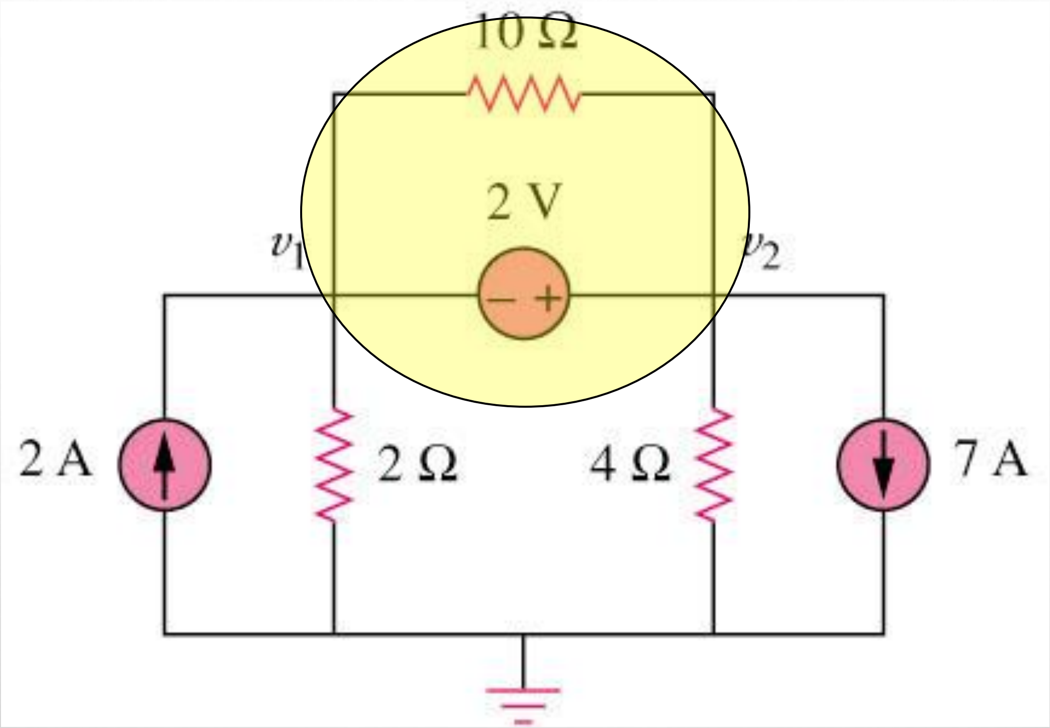
How to handle the 2V voltage source?



- A super-node is formed by enclosing a (dependent or independent) voltage source connected between two non-reference nodes and any elements connected in parallel with it.
- Note: We analyze a circuit with super-nodes using the same three steps mentioned above except that the super-nodes are treated differently.

NODAL ANALYSIS

Example 4 –circuit with independent voltage source

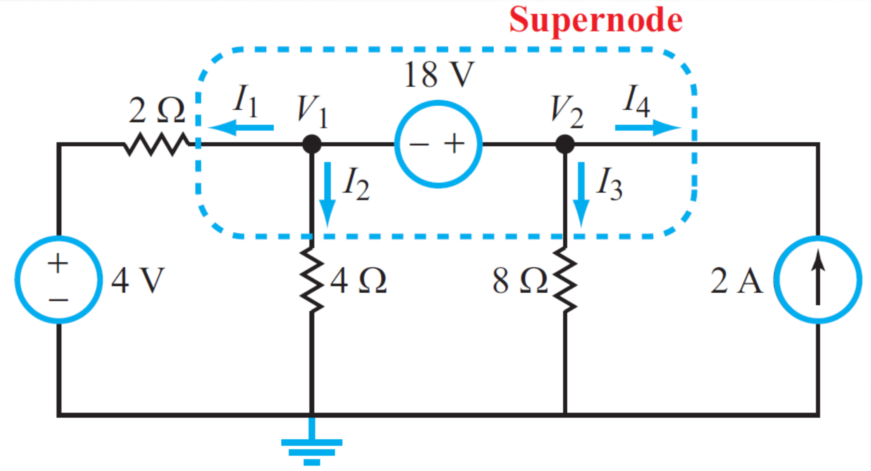


Super-node equation: 2= $V_1/2 + V_2/4 + 7$

Relation between node voltages: $V_2 - V_1 = 2$

NODAL ANALYSIS

<u>Example 5</u> –circuit with independent voltage source



Solution:

$$\frac{V_1 - 4}{2} + \frac{V_1}{4} + \frac{V_2}{8} - 2 = 0,$$

which may be simplified to

$$6V_1 + V_2 = 32.$$

Additionally, the supernode KVL equation is

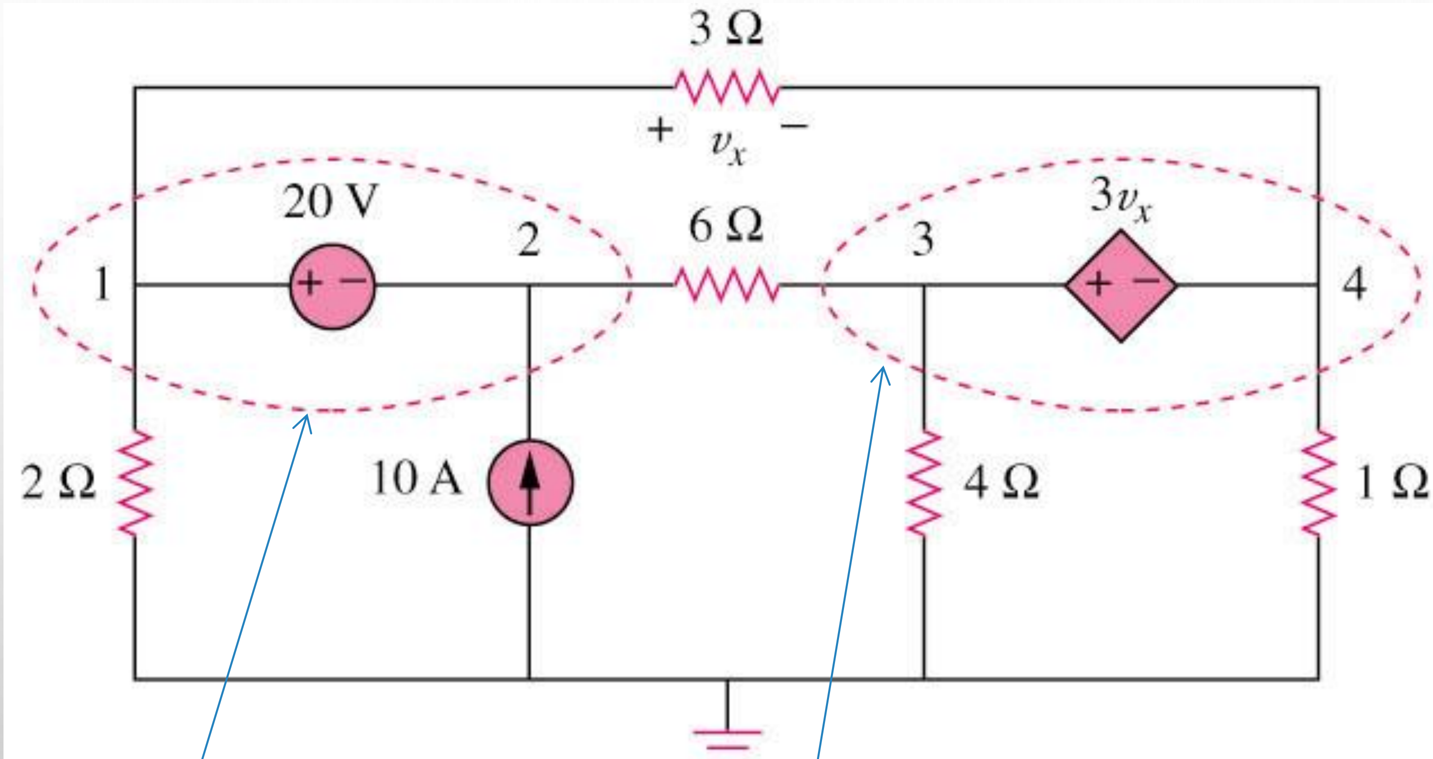
$$V_2 - V_1 = 18.$$

Simultaneous solution of the two equations yields

$$V_1 = 2 \text{ V}, \qquad V_2 = 20 \text{ V}.$$

NODAL ANALYSIS

Example 6 – circuit with current and voltage sources



$V_1/2 + (V_2 - V_3)/6 + (V_1 - V_4)/3 = 10$

$V_1 - V_2 = 20$

$V_2 - V_3)/6 + (V_1 - V_4)/3 = V_3/4 + V_4/1$

$V_3 - V_4 = 3\{(V_1 - V_4)/3\}$

MESH ANALYSIS

- Mesh analysis provides another general procedure for analyzing circuits using mesh currents as the circuit variables.

- Nodal analysis applies KCL to find unknown voltages in a given circuit, while mesh analysis applies KVL to find unknown currents.

- Recall that a mesh is a loop which does not contain any other loops within it.

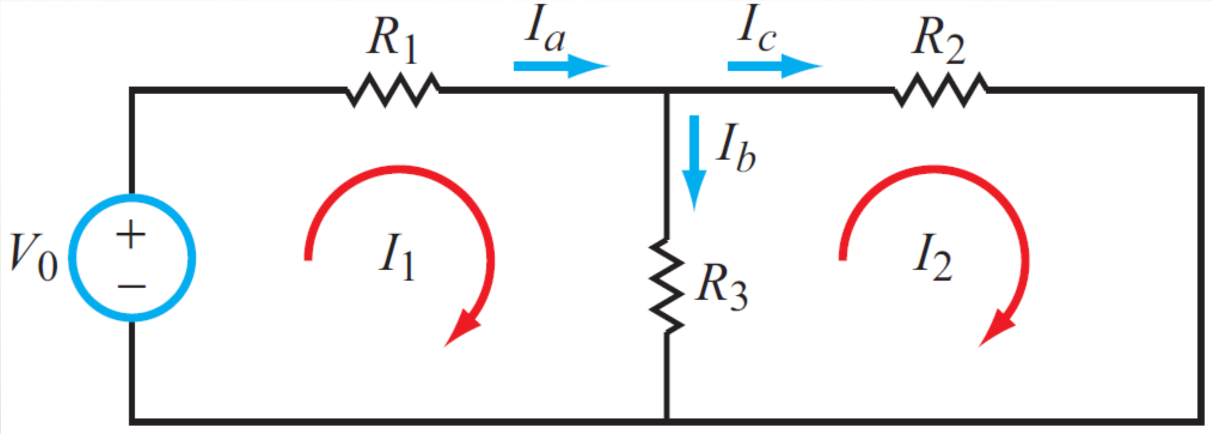
MESH ANALYSIS

Steps to determine the mesh currents:

1. Assign mesh currents i_1, i_2, …, in to the n meshes.
2. Apply KCL to each of the n meshes. Use Ohm's law to express the voltages in terms of the mesh currents.
3. Solve the resulting n simultaneous equations to get the mesh currents.

MESH ANALYSIS

<u>Example 1</u> – Circuit with independent voltage source



$$-V_0 + I_1 R_1 + (I_1 - I_2) R_3 = 0 \qquad \text{(mesh 1)}$$

$$(I_2 - I_1) R_3 + I_2 R_2 = 0 \qquad \text{(mesh 2)}$$

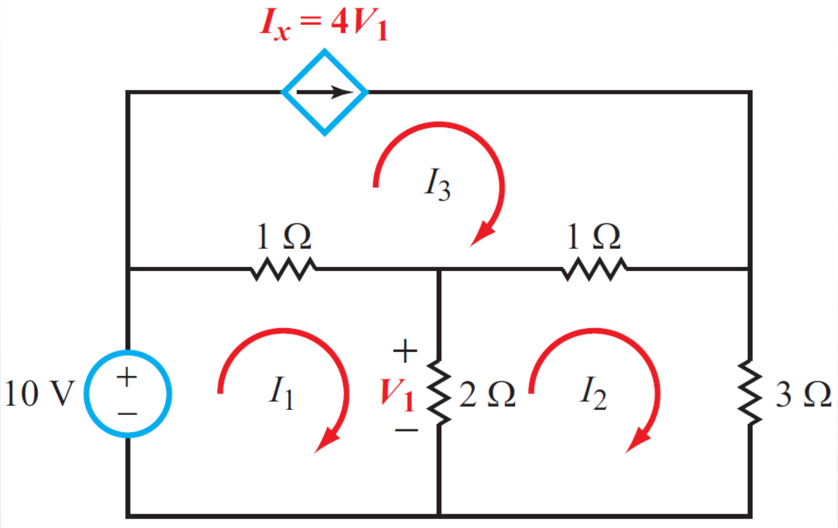
Two equations in 2 unknowns:
Solve using Cramer's rule, matrix
inversion, or MATLAB

$$(R_1 + R_3) I_1 - I_2 R_3 = V_0 \qquad \text{(mesh 1)}$$

$$-R_3 I_1 + (R_2 + R_3) I_2 = 0 \qquad \text{(mesh 2)}$$

MESH ANALYSIS

<u>Example 2</u> – circuit with dependent current source



$$(1+2)I_1 - 2I_2 - I_3 = 10,$$ Mesh 1

$$-2I_1 + (2+1+3)I_2 - I_3 = 0.$$ Mesh 2

$$I_3 = I_x = 4V_1.$$ Mesh 3

But $V_1 = 2(I_1 - I_2).$

Hence

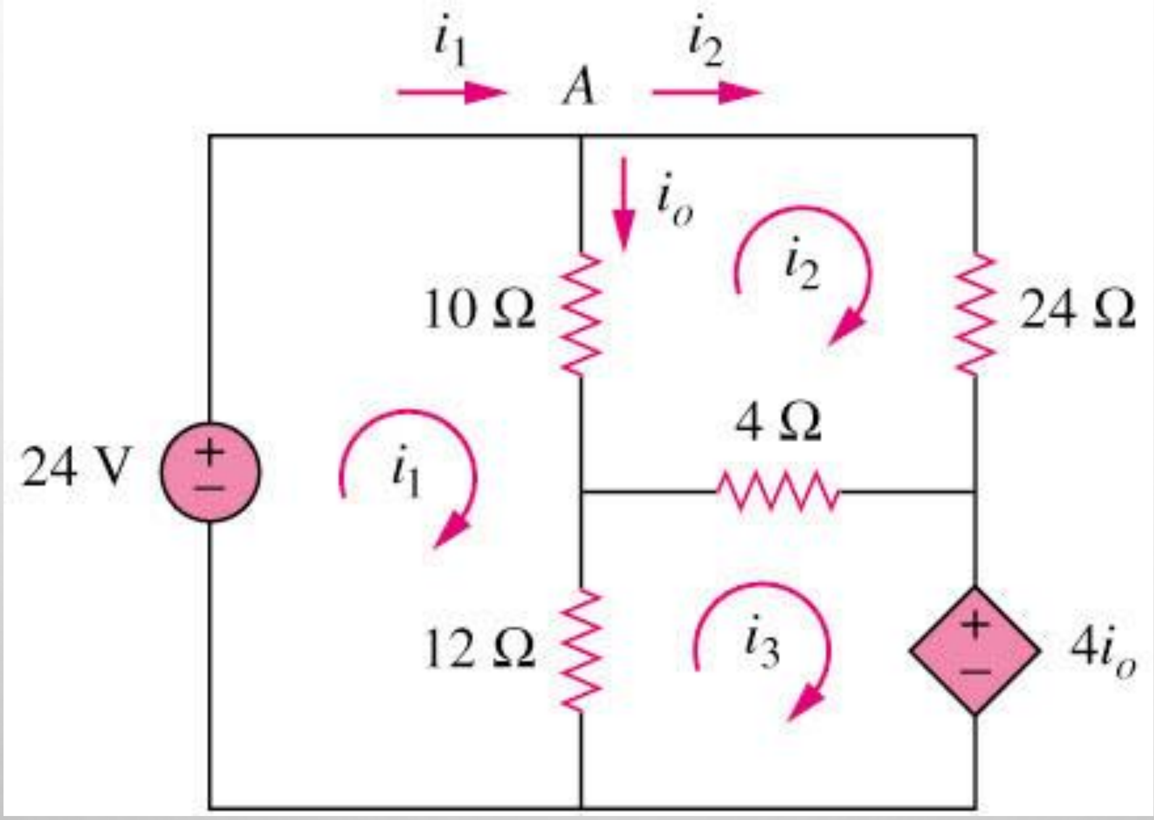
$$-5I_1 + 6I_2 = 10,$$
$$-10I_1 + 14I_2 = 0.$$

$$I_1 = -14 \text{ A}, \qquad I_2 = -10 \text{ A}.$$

$$\begin{aligned} I_x &= 8(I_1 - I_2) \\ &= 8(-14+10) \\ &= -32 \text{ A}. \end{aligned}$$

MESH ANALYSIS

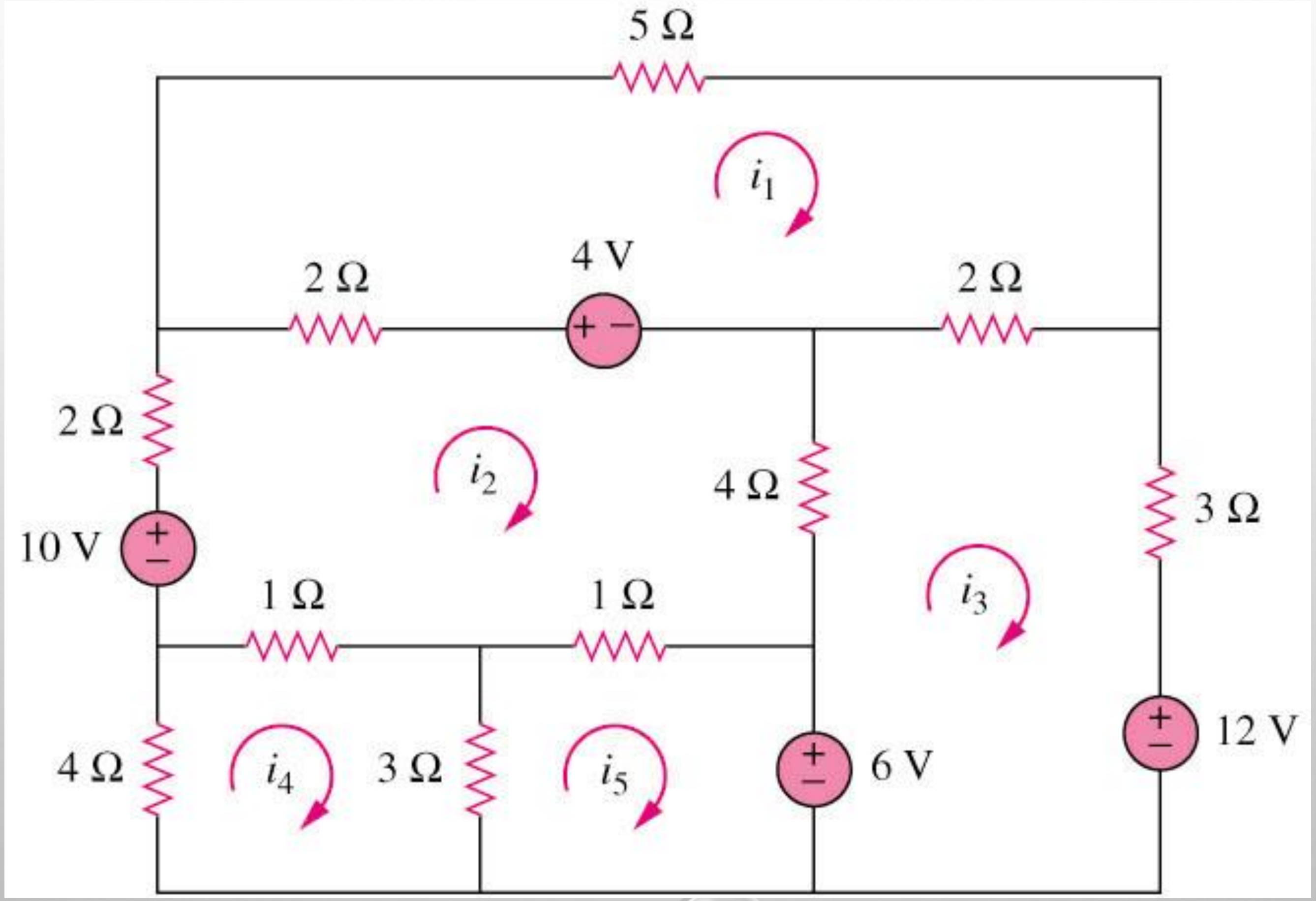
<u>Example 3</u> – circuit with dependent voltage source



Answer: i_o = 1.5A

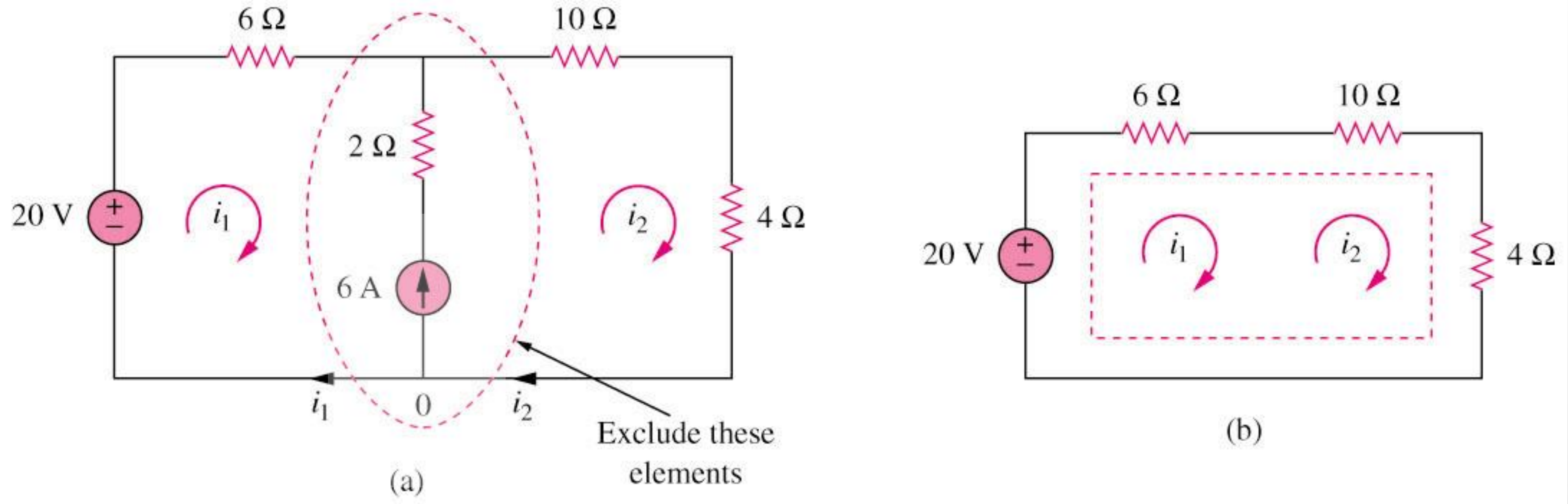
MESH ANALYSIS

<u>Example 4</u>: Write the mesh-current equations for the circuit



MESH ANALYSIS

Example 5: Circuit with current source



A **super-mesh** results when two meshes have a (dependent or independent) current source in common as shown in (a). We create a super-mesh by excluding the current source and any elements connected in series with it as shown in (b).

$$-20 + 6i_1 + (10+4)i_2 = 0$$

$$i_2 - i_1 = 6$$

MESH ANALYSIS

Properties of a super-mesh:

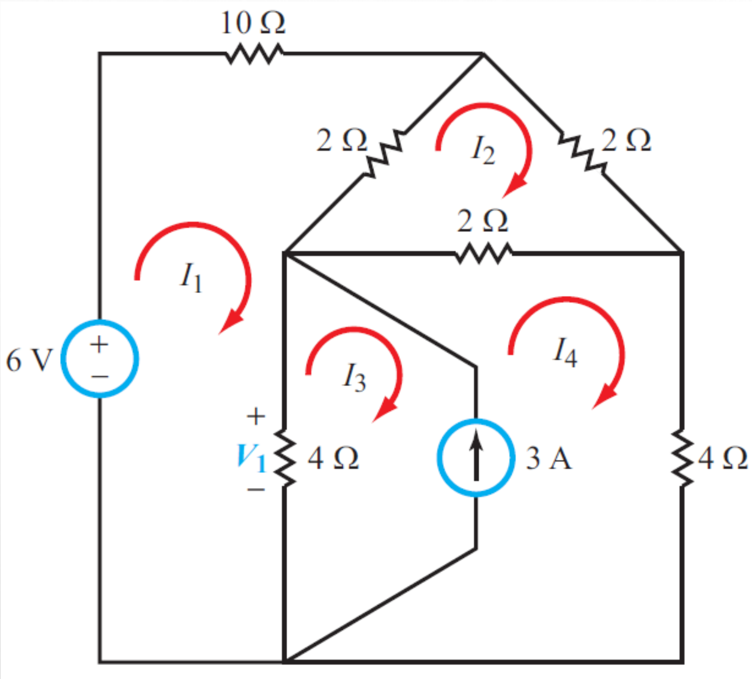
1. The current source inside the super-mesh is not completely ignored; it provides the constraint equation necessary to solve for the mesh currents.

2. A super-mesh has no current of its own.

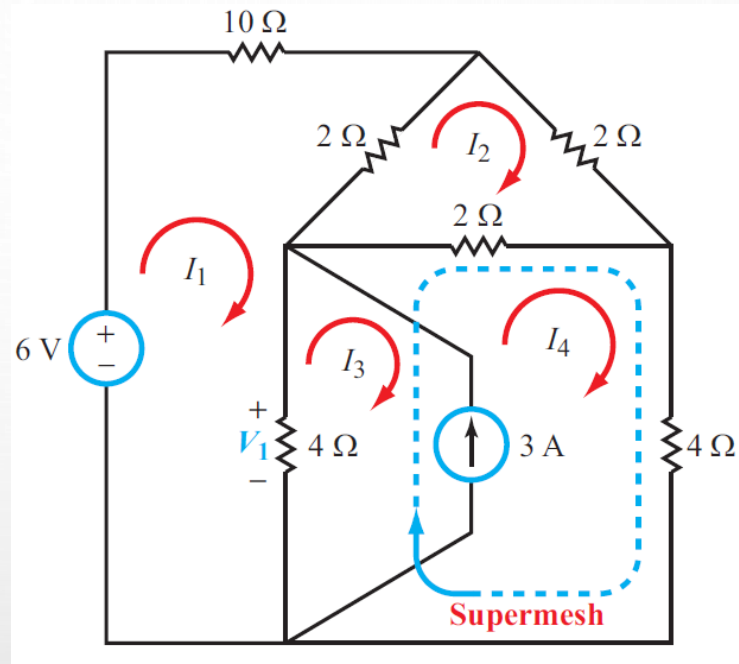
3. A super-mesh requires the application of both KVL and KCL.

MESH ANALYSIS

Example 6: super-mesh



(a) Original circuit



(b) Meshes 3 and 4 constitute a supermesh

$$(10+2+4)I_1 - 2I_2 - 4I_3 = 6,$$ Mesh 1

$$-2I_1 + (2+2+2)I_2 - 2I_4 = 0,$$ Mesh 2

$$-4I_1 - 2I_2 + 4I_3 + (2+4)I_4 = 0.$$ Super-mesh 3/4

$$I_4 - I_3 = 3.$$ Super-mesh Auxiliary Equation

Solution gives:

NODAL VERSUS MESH ANALYSIS

To select the method that results in the smaller number of equations. For example:

1. Choose nodal analysis for circuit with fewer nodes than meshes.
   *Choose mesh analysis for circuit with fewer meshes than nodes.
   *Networks that contain many series connected elements, voltage sources, or supermeshes are more suitable for mesh analysis.
   *Networks with parallel-connected elements, current sources, or supernodes are more suitable for nodal analysis.
2. If node voltages are required, it may be expedient to apply nodal analysis. If branch or mesh currents are required, it may be better to use mesh analysis.

LINEARITY

- A circuit is linear if output is proportional to input
- A function $f(x)$ is linear if $f(ax) = af(x)$
- All circuit elements will be assumed to be linear or can be modeled by linear equivalent circuits
  - resistors $v = iR$
  - linearly dependent sources
  - capacitors
  - inductors
- We will examine theorems and principles that apply to linear circuits to simplify analysis

SUPERPOSITION

If a circuit contains more than one independent source, the voltage (or current) response of any element in the circuit is equal to the algebraic sum of the individual responses associated with the individual independent sources, as if each had been acting alone.

Solution Procedure: Source Superposition

Step 1: Set all independent sources equal to zero (by replacing voltage sources with short circuits and current sources with open circuits), except for source 1.

Step 2: Apply node-voltage, mesh-current, or any other convenient analysis technique to solve for the response υ_1 due to source 1.

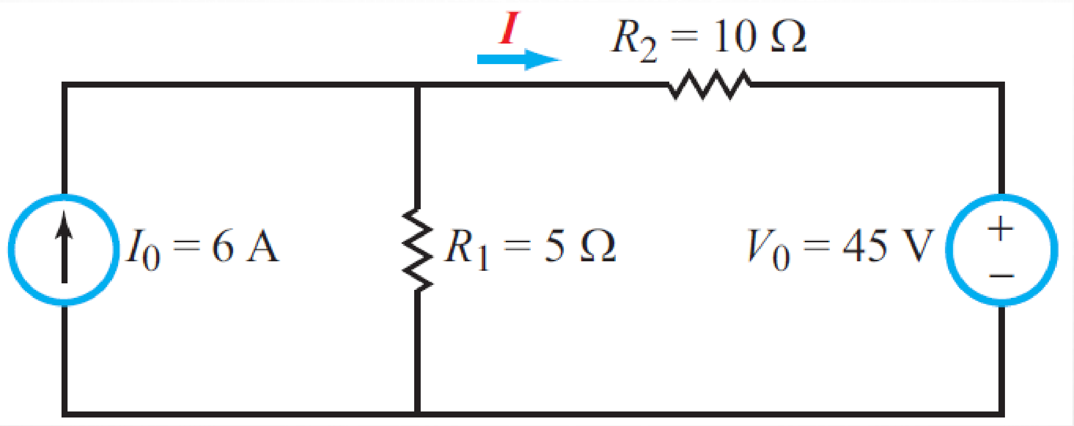
Step 3: Repeat the process for sources 2 through n, calculating in each case the response due to that one source acting alone.

Step 4: Use Eq. (3.30) to determine the total response υ.

Alternatively, the procedure can be used to find currents i_1 to i_n and then to add them up algebraically to find the total current i using Eq. (3.31).

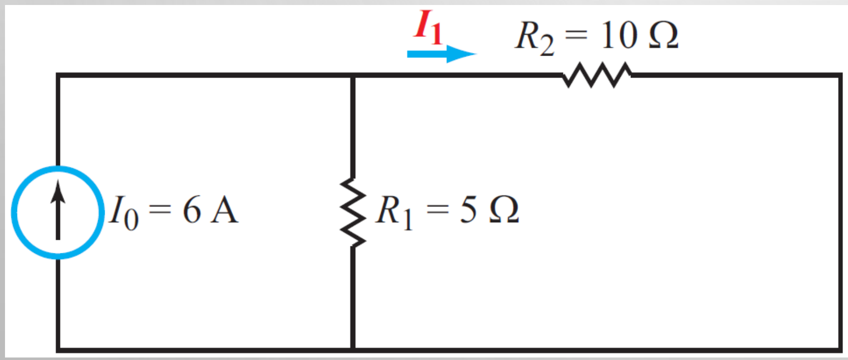
SUPERPOSITION

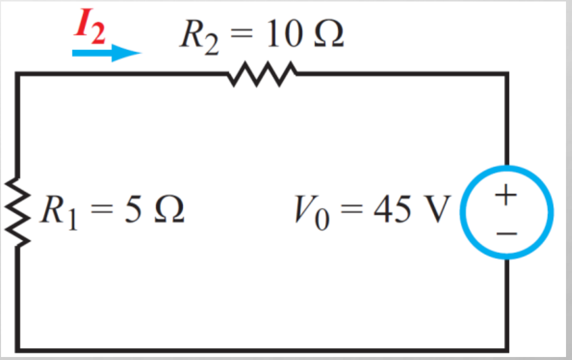
Example 8: find I using superposition



Contribution from I_0 alone

Contribution from V_0 alone





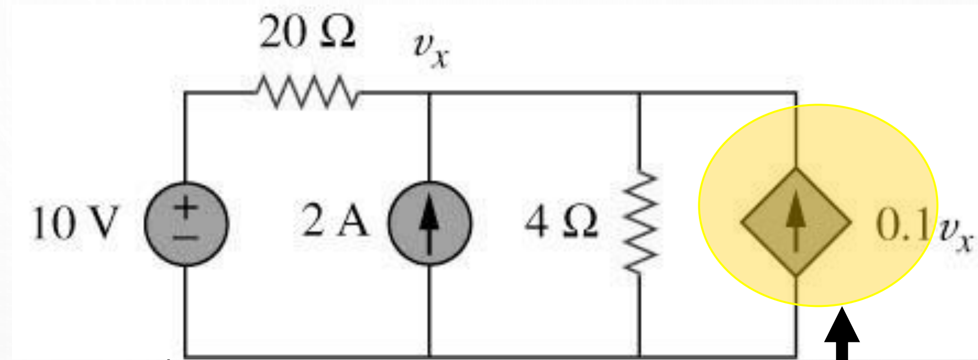
$I_1 = 2$ A

$I_2 = -3$ A

$$\boxed{I = I_1 + I_2 = 2 - 3 = -1 \text{ A}}$$

SUPERPOSITION

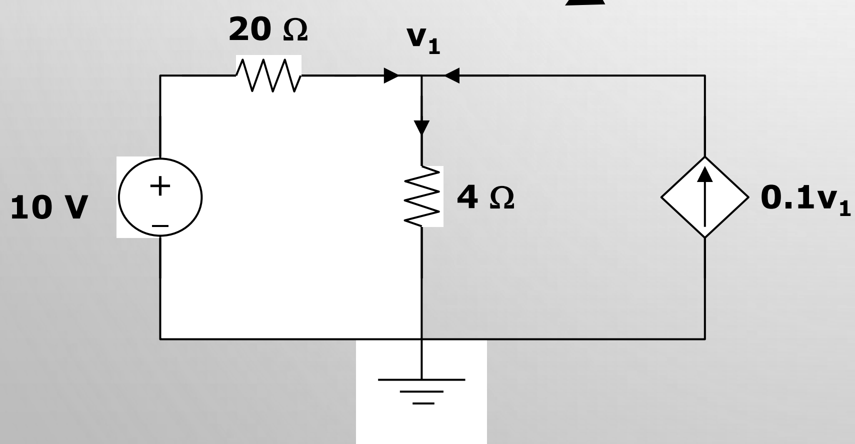
<u>Example 9:</u>Use superposition to find v_x.



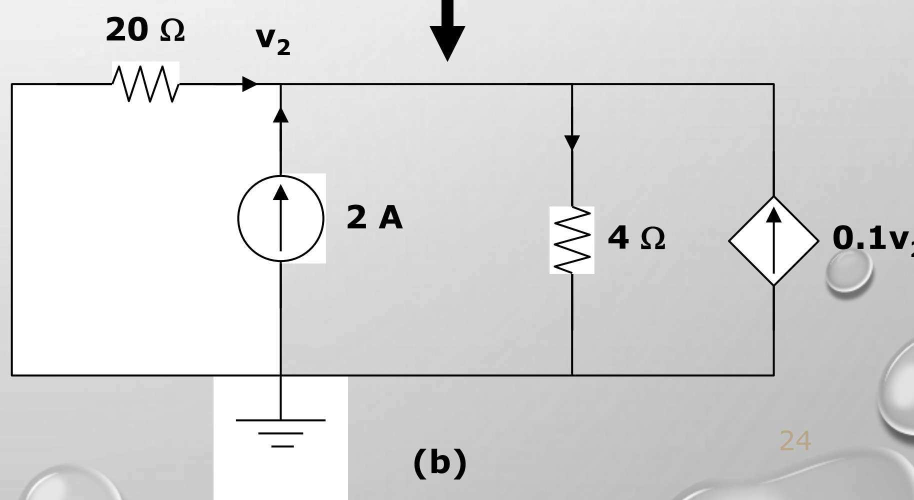
2A is discarded by open-circuit

10V is discarded by open-circuit

Dependent source keep unchanged



(a)

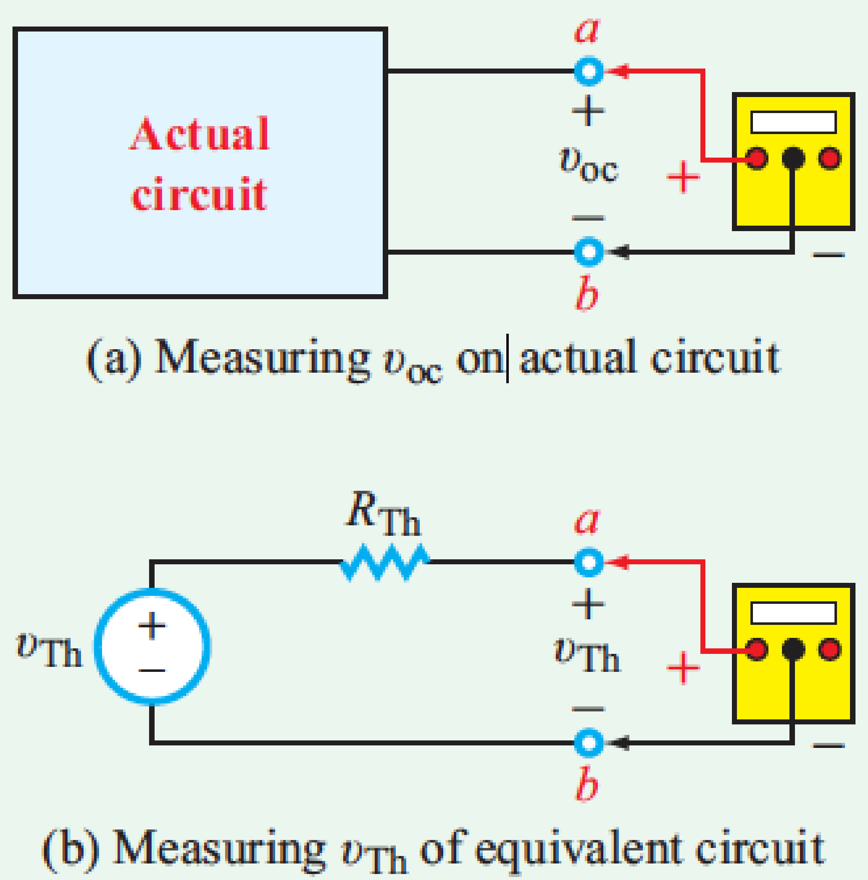


(b)

Answer: v_x = 12.5 V

THÉVENIN'S THEOREM

A linear two-terminal circuit can be replaced by an equivalent circuit that is composed of a voltage source and a series resistor



Voltage across output with no load (open circuit)

$$v_{Th} = v_{oc}$$

Resistance at terminals with all independent circuit sources set to zero

$$R_{Th} = R_{in}$$

NORTON'S THEOREM

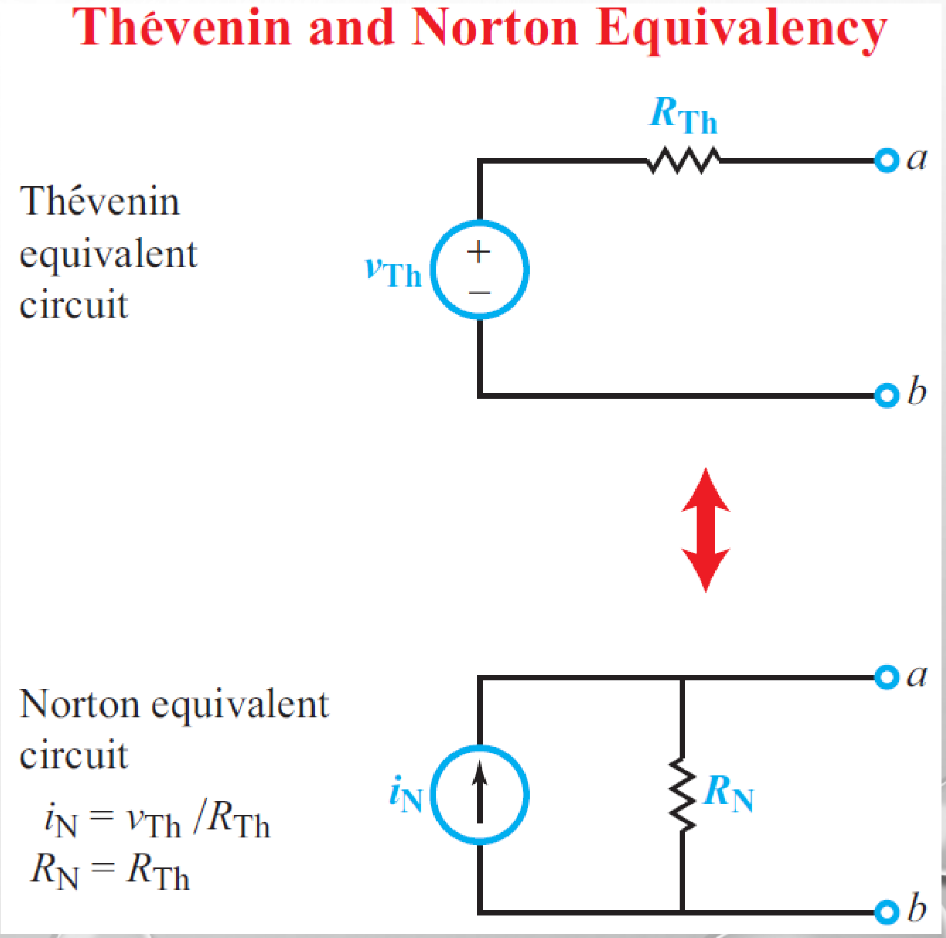
A linear two-terminal circuit can be replaced by an equivalent circuit composed of a current source and parallel resistor

Current through output with short circuit

$$i_N = \frac{v_{Th}}{R_{Th}}$$

Resistance at terminals with all circuit set to zero sources

$$R_N = R_{Th}.$$



HOW DO WE FIND THÉVENIN/NORTON EQUIVALENT CIRCUITS ?

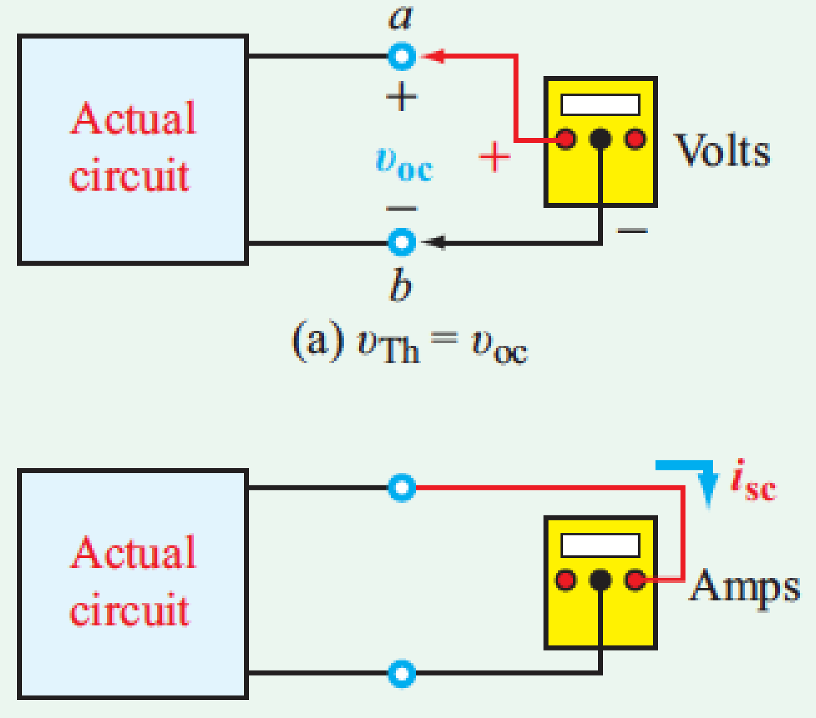
Method 1: open circuit/short circuit

1. Analyze circuit to find v_{oc}
2. Analyze circuit to find i_{sc}

$$v_{Th} = v_{OC}$$

$$R_{Th} = \frac{v_{Th}}{i_{sc}}$$

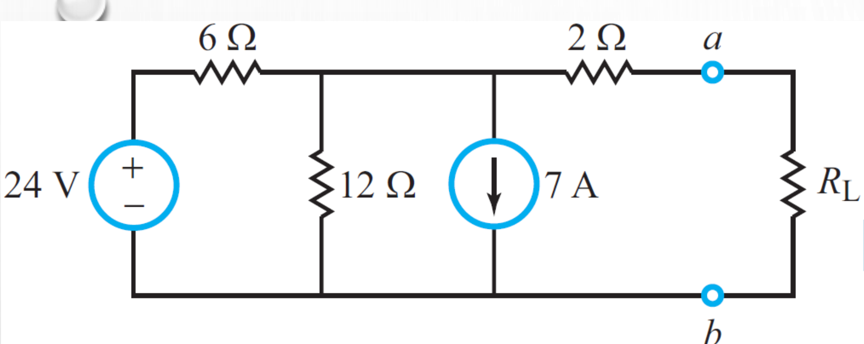
Note: This method is applicable to any circuit, whether or not it contains dependent sources.



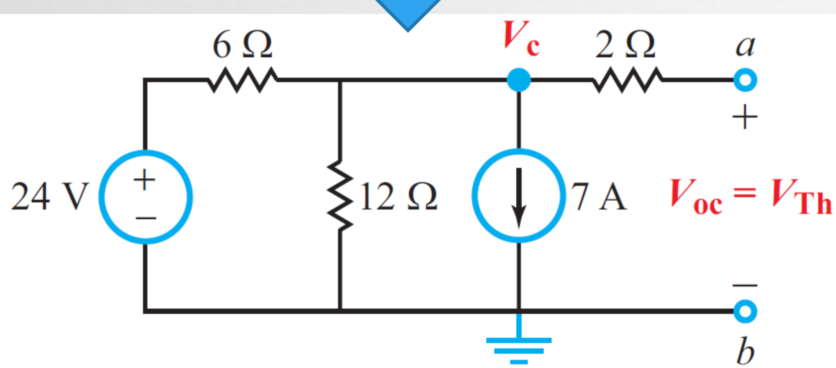
(a) $v_{Th} = v_{oc}$

(b) $R_{Th} = v_{oc} / i_{sc}$

EXAMPLE 10. THÉVENIN EQUIVALENT



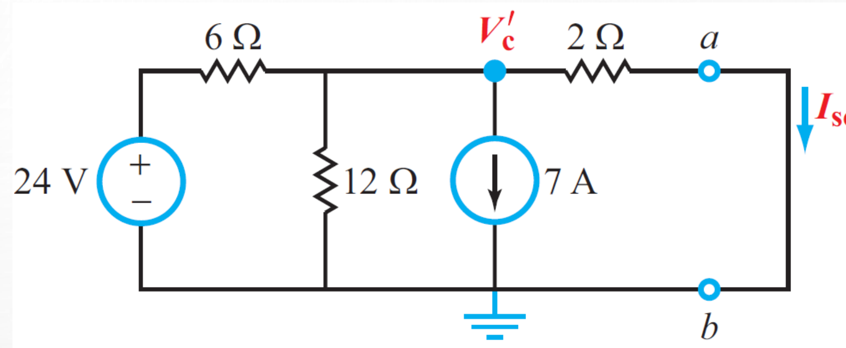
(a) Original circuit



(b) Replacing R_L with open circuit

$$\frac{V_c - 24}{6} + \frac{V_c}{12} + 7 = 0$$

$$V_{Th} = -12 \text{ V}$$

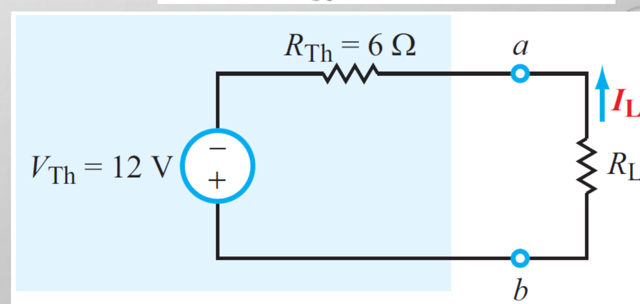


(c) Replacing R_L with short circuit

$$\frac{V'_c - 24}{6} + \frac{V'_c}{12} + 7 + \frac{V'_c}{2} = 0 \rightarrow V'_c = -4 \text{ V},$$

$$I_{sc} = \frac{V'_c}{2} = -\frac{4}{2} = -2 \text{ A}$$

$$R_{Th} = \frac{V_{Th}}{I_{sc}} = \frac{-12}{-2} = 6 \ \Omega$$



(d) Thévenin equivalent circuit

HOW DO WE FIND THÉVENIN/NORTON EQUIVALENT CIRCUITS?

Method 2: equivalent resistance

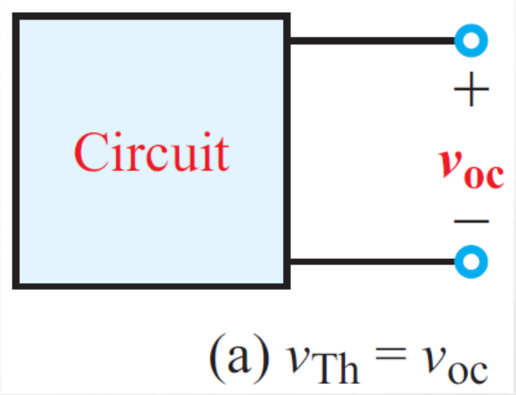
1. Analyze circuit to find either

v_{oc} or i_{sc}

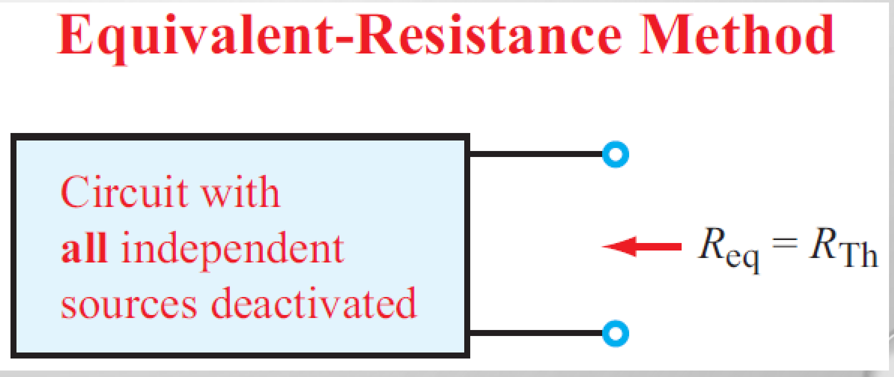
2. Deactivate all independent sources by replacing voltage sources with short circuits and current sources with open circuits.

3. Simplify circuit to find equivalent resistance.

Note: This method does not apply to circuits that contain dependent sources.



(a) $v_{Th} = v_{oc}$

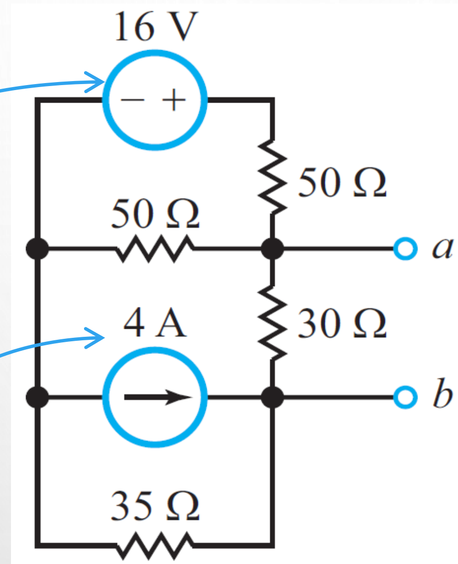


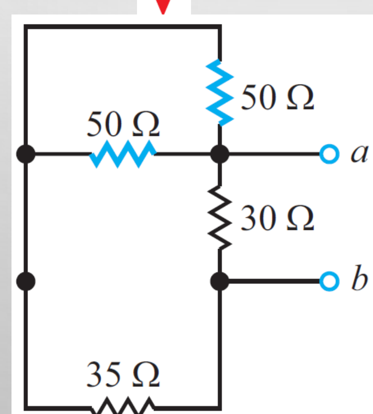
EXAMPLE 11: FINDING R_{th}

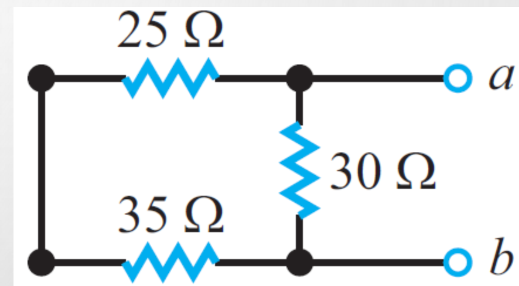
(Circuit with no dependent sources)

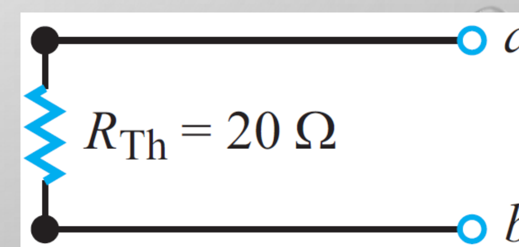
Replace with SC

Replace with OC



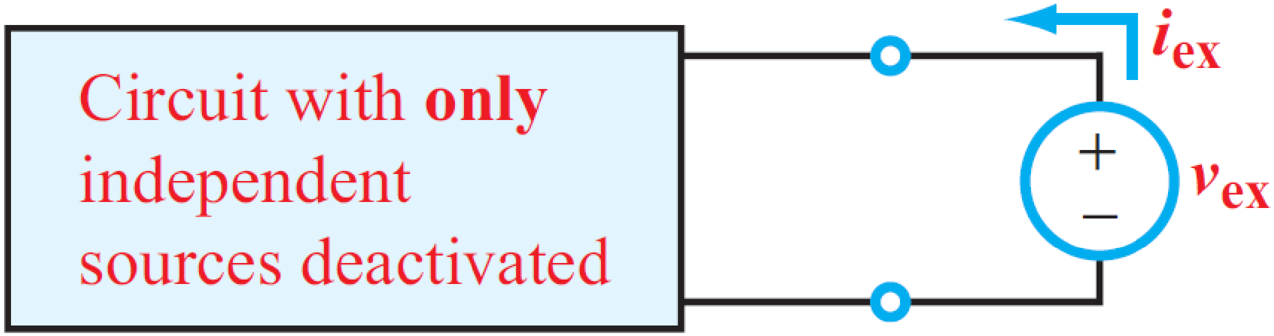






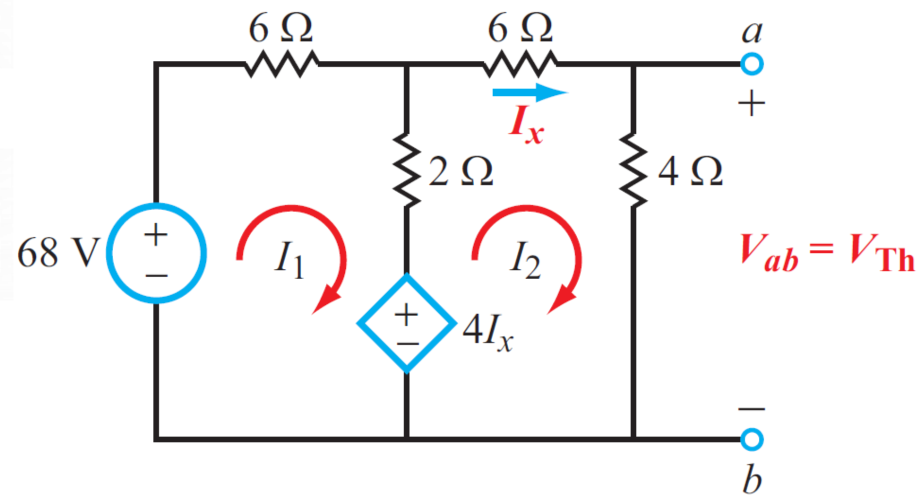
HOW DO WE FIND THÉVENIN/NORTON EQUIVALENT CIRCUITS?

Method 3: External Source Method



If a circuit contains both dependent and independent sources, R_{Th} can be determined by (a) deactivating independent sources (only), (b) adding an external source v_{ex}, and then (c) solving the circuit to determine i_{ex}. The solution is $R_{Th} = v_{ex}/i_{ex}$.

EXAMPLE 12: FINDING V_{th}



(a) Solving for V_{Th}

Solution:

Mesh analysis results in

$$-68 + 6I_1 + 2(I_1 - I_2) + 4I_x = 0$$

and

$$-4I_x + 2(I_2 - I_1) + 6I_2 + 4I_2 = 0.$$

Recognizing that $I_x = I_2$, solution of these two simultaneous equations leads to

$$I_1 = 8\text{ A},$$

and

$$I_2 = 2\text{ A}.$$

The Thévenin voltage is V_{ab}. Hence,

$$\begin{aligned} V_{Th} &= V_{ab} \\ &= 4I_2 \\ &= 8\text{ V}. \end{aligned}$$

EXAMPLE 12: FINDING R_{th}

Solution: Using the external source method,

$$6I_1' + 2(I_1' - I_2') + 4I_x = 0,$$
$$-4I_x + 2(I_2' - I_1') + 6I_2' + 4(I_2' - I_3') = 0,$$
$$4(I_3' - I_2') + V_{\text{ex}} = 0.$$

After replacing I_x with I_2' and solving the three simultaneous equations, we obtain

$$I_1' = \frac{1}{18} V_{\text{ex}},$$

$$I_2' = -\frac{2}{9} V_{\text{ex}},$$

and

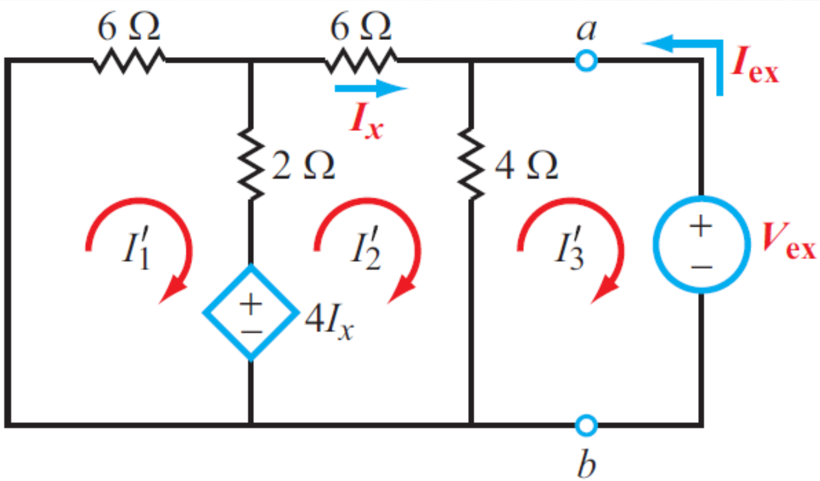
$$I_3' = -\frac{17}{36} V_{\text{ex}}.$$

For the equivalent circuit shown in Fig. 3-23(c),

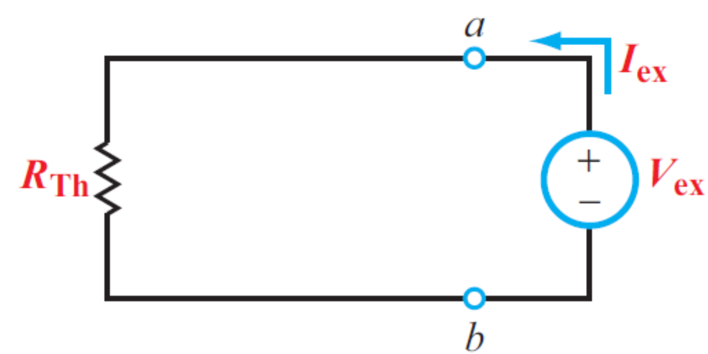
$$R_{\text{Th}} = \frac{V_{\text{ex}}}{I_{\text{ex}}}.$$

In terms of our solution, $I_{\text{ex}} = -I_3'$. Hence,

$$R_{\text{Th}} = -\frac{V_{\text{ex}}}{I_3'} = \frac{36}{17}\ \Omega.$$



(b) Solving for I_{ex}

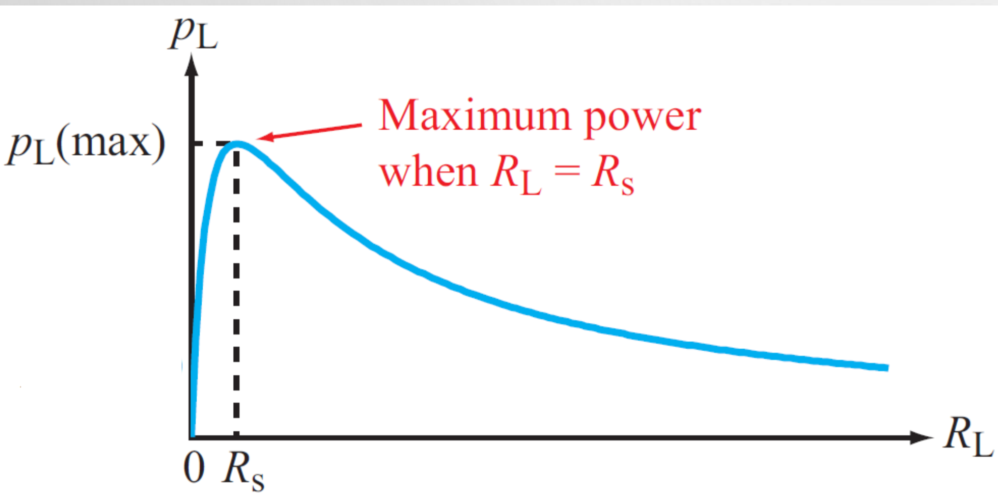


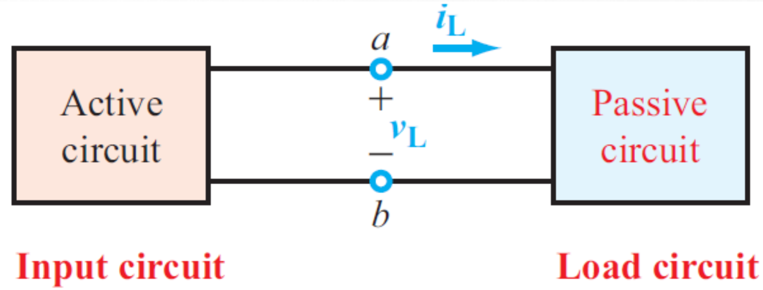
(c) Equivalent circuit for calculating R_{Th}

MAXIMUM POWER TRANSFER

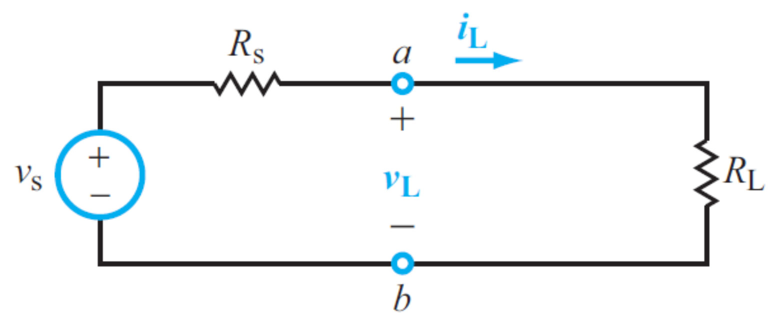
In many situations, we want to maximize power transfer to the load

$$p_L = i_L v_L = \frac{v_s^2 R_L}{(R_s + R_L)^2}.$$





(a) Source and load circuits

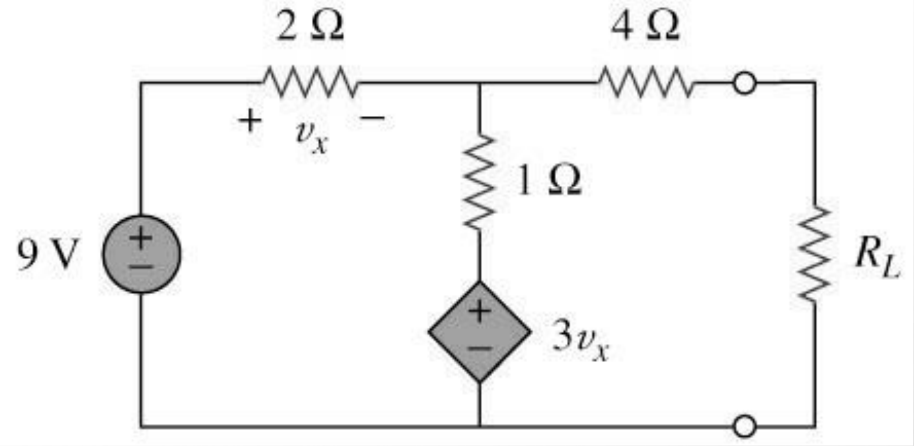


(b) Replacing source and load circuits with their Thévenin equivalents

$$p_L(\max) = \frac{v_s^2 R_L}{(R_L + R_L)^2} = \frac{v_s^2}{4R_L},$$

MAXIMUM POWER TRANSFER

<u>Example 13</u> Determine the value of R_L that will draw the maximum power from the rest of the circuit shown below. Calculate the maximum power.



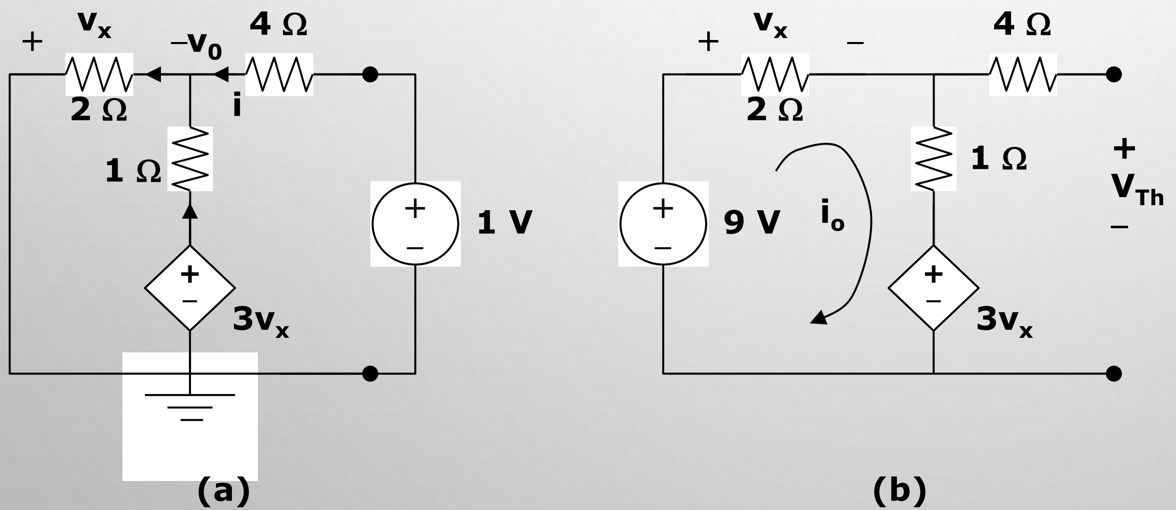


Fig. a

=> To determine R_{TH}

Fig. b

=> To determine V_{TH}

Answer: $R_L = 4.22\Omega$, $P_m = 2.901\text{W}$

EXAMPLE 14: MAXIMUM POWER TRANSFER

In the bridge circuit shown in Fig. 3-27(a), choose R_L so that the power delivered to it is a maximum. How much power will that be?

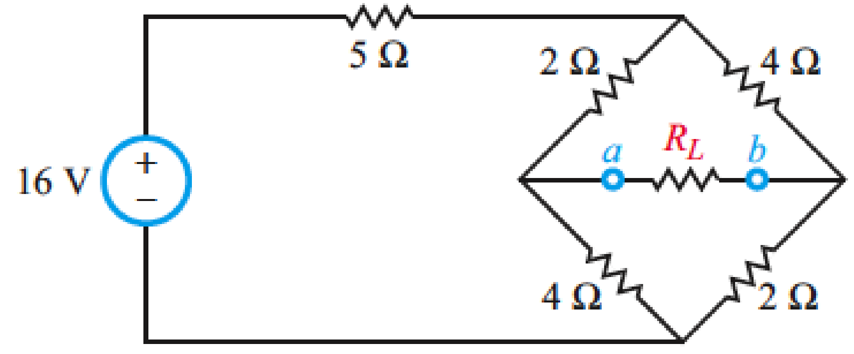
$$V_1 = 6\text{ V}.$$

$$V_a = \left(\frac{4}{2+4}\right) V_1 = 4\text{ V},$$

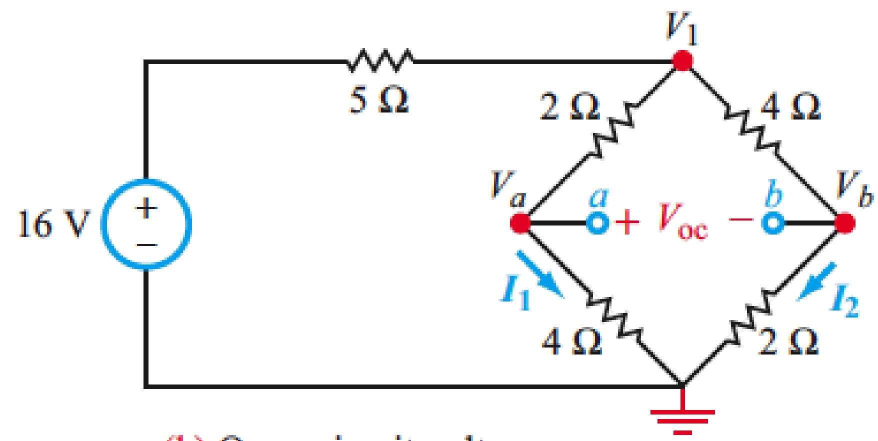
$$V_b = \left(\frac{2}{2+4}\right) V_1 = 2\text{ V}.$$

Hence,

$$V_{\text{Th}} = V_{\text{oc}} = V_a - V_b = 4 - 2 = 2\text{ V}.$$



(a) Original circuit



(b) Open-circuit voltage

EXAMPLE 14: MAXIMUM POWER TRANSFER

Short-Circuit Current: In the circuit configuration shown in Fig. 3-27(c), terminals (a, b) are connected by a short circuit. Application of the mesh-analysis by-inspection method (Section 3-3.2) leads to the matrix equation

$$\begin{bmatrix} 11 & -2 & -4 \\ -2 & 6 & 0 \\ -4 & 0 & 6 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 16 \\ 0 \\ 0 \end{bmatrix}.$$

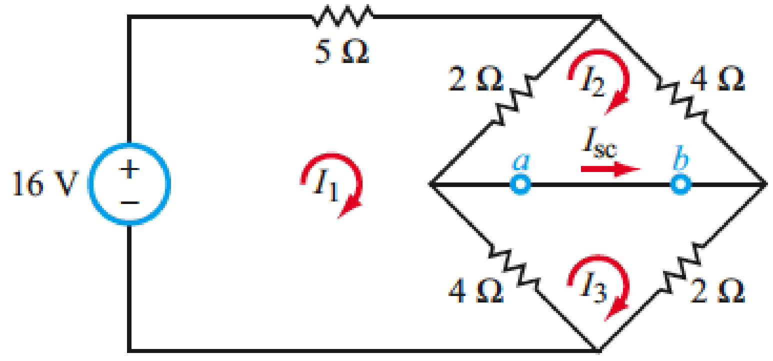
$$I_1 = \frac{96}{46} \text{ A}, \quad I_2 = \frac{32}{46} \text{ A}, \quad I_3 = \frac{64}{46} \text{ A}.$$

$$I_{sc} = I_3 - I_2 = \frac{64}{46} - \frac{32}{46} = \frac{32}{46} = 0.7,$$

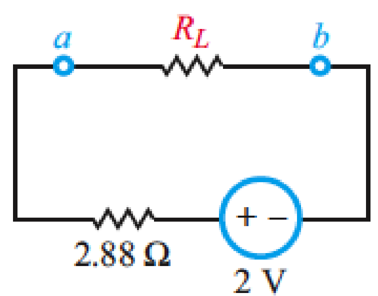
and

$$R_{Th} = \frac{V_{oc}}{I_{sc}} = \frac{2}{0.7} = 2.88\ \Omega.$$

$$p_{max} = \frac{v_s^2}{4R_L} = \frac{(2)^2}{4 \times 2.88} = 0.35 \text{ W}.$$

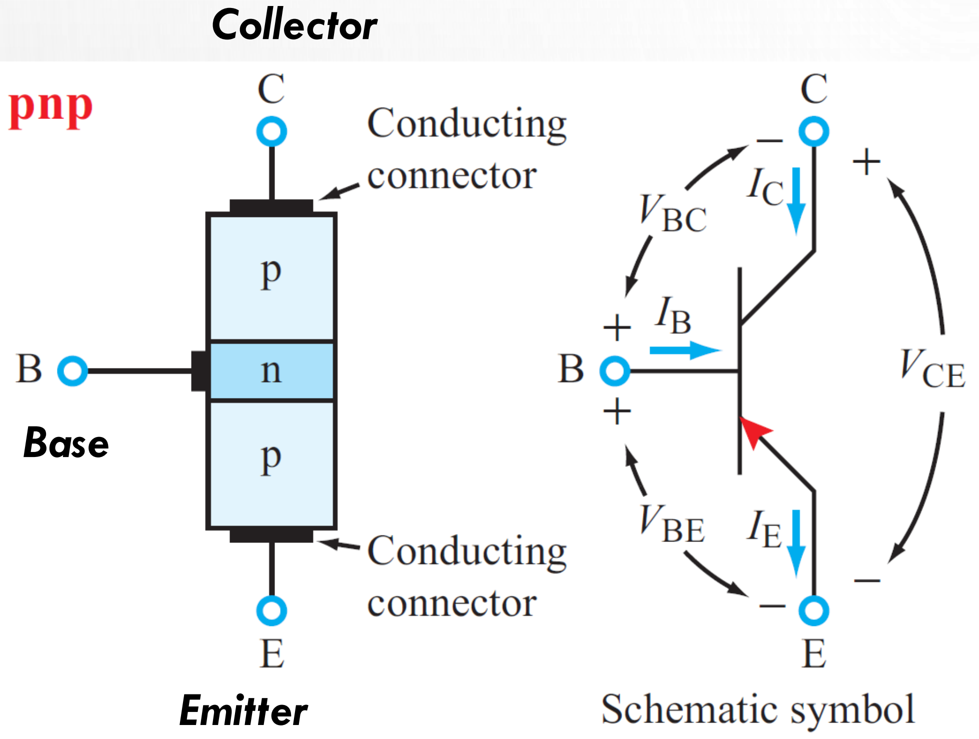


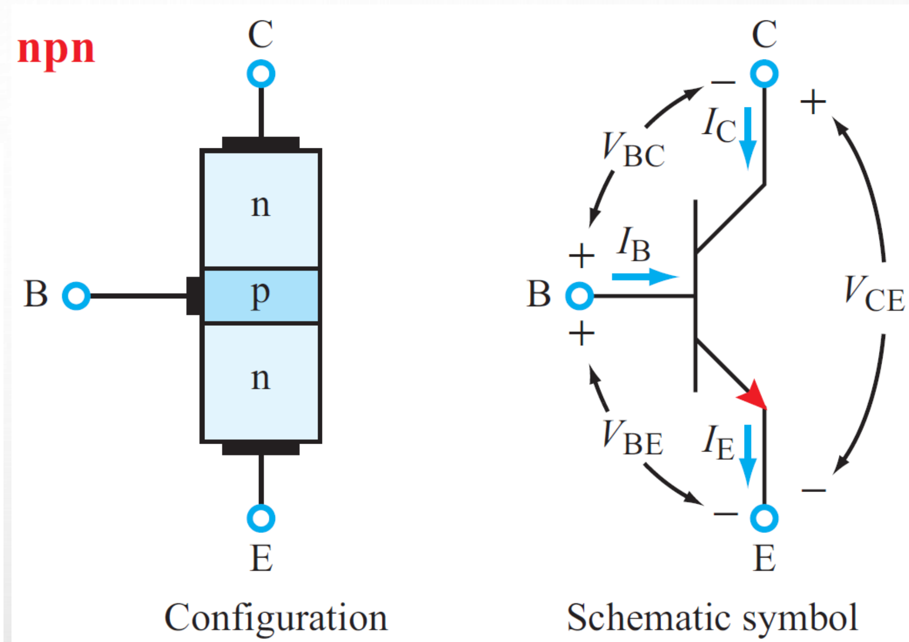
(c) Short-circuit current



(d) Thévenin equivalent circuit

BJT: 3 TERMINAL DEVICE

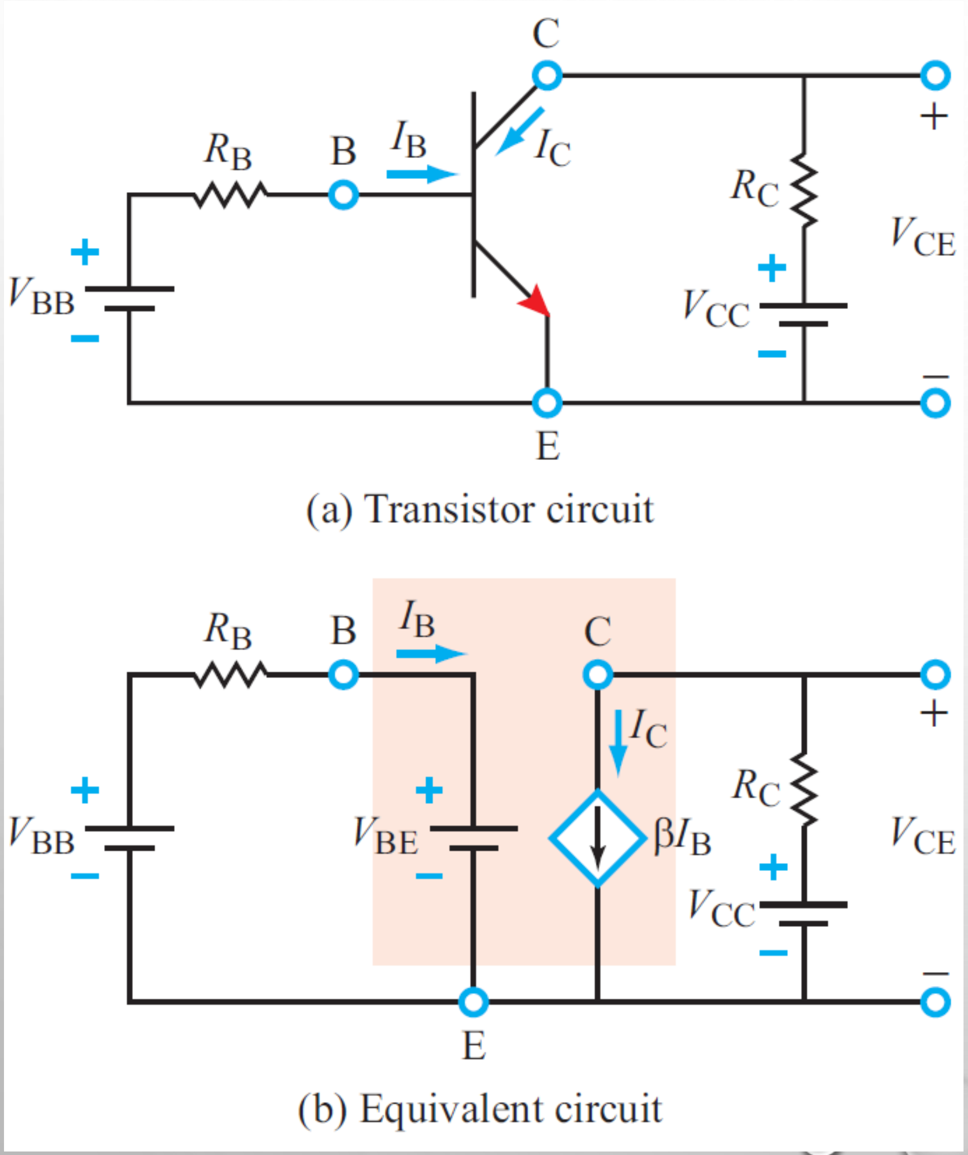




$$I_E = I_B + I_C.$$

The lead containing the arrow identifies the emitter terminal and whether the transistor is a pnp or npn. The arrow always points towards an n-type material.

BJT EQUIVALENT CIRCUIT



Looks like a current amplifier with gain β

SUMMARY

Node-voltage method

$\sum$ of all current leaving a node $= 0$
[current entering a node is $(-)$]

Mesh-current method

$\sum$ of all voltages around a loop $= 0$
[passive sign convention applied to mesh currents in clockwise direction]

Thévenin equivalent circuit	$v_{\text{Th}} = v_{\text{oc}}$ $R_{\text{Th}} = v_{\text{oc}}/i_{\text{sc}}$
Norton equivalent circuit	$i_{\text{N}} = i_{\text{sc}}$ $R_{\text{N}} = R_{\text{Th}}$
Maximum power transfer	$R_{\text{L}} = R_{\text{s}}$ $P_{\text{L}}(\text{max}) = \dfrac{v_{\text{s}}^2}{4R_{\text{L}}}$

HOMEWORK ASSIGNMENT – CHAP. 3

Solve problems 5,10, 15,….,85.