Magnetic Circuits

EE 340

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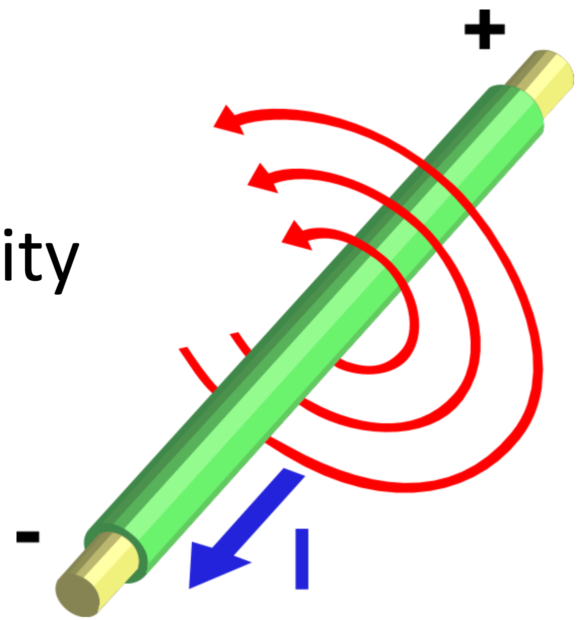
Ampere's Law

- Ampère's law (discovered by André-Marie Ampère in 1826) relates the integrated magnetic field around a closed loop to the electric current passing through the loop.

$$\oint H.dl = I$$

where H is the magnetic field intensity (measured in At/m)



- At a distance *r* from the wire,

$$\oint H.dl = H.(2\pi r) = I$$

Magnetic Flux Density

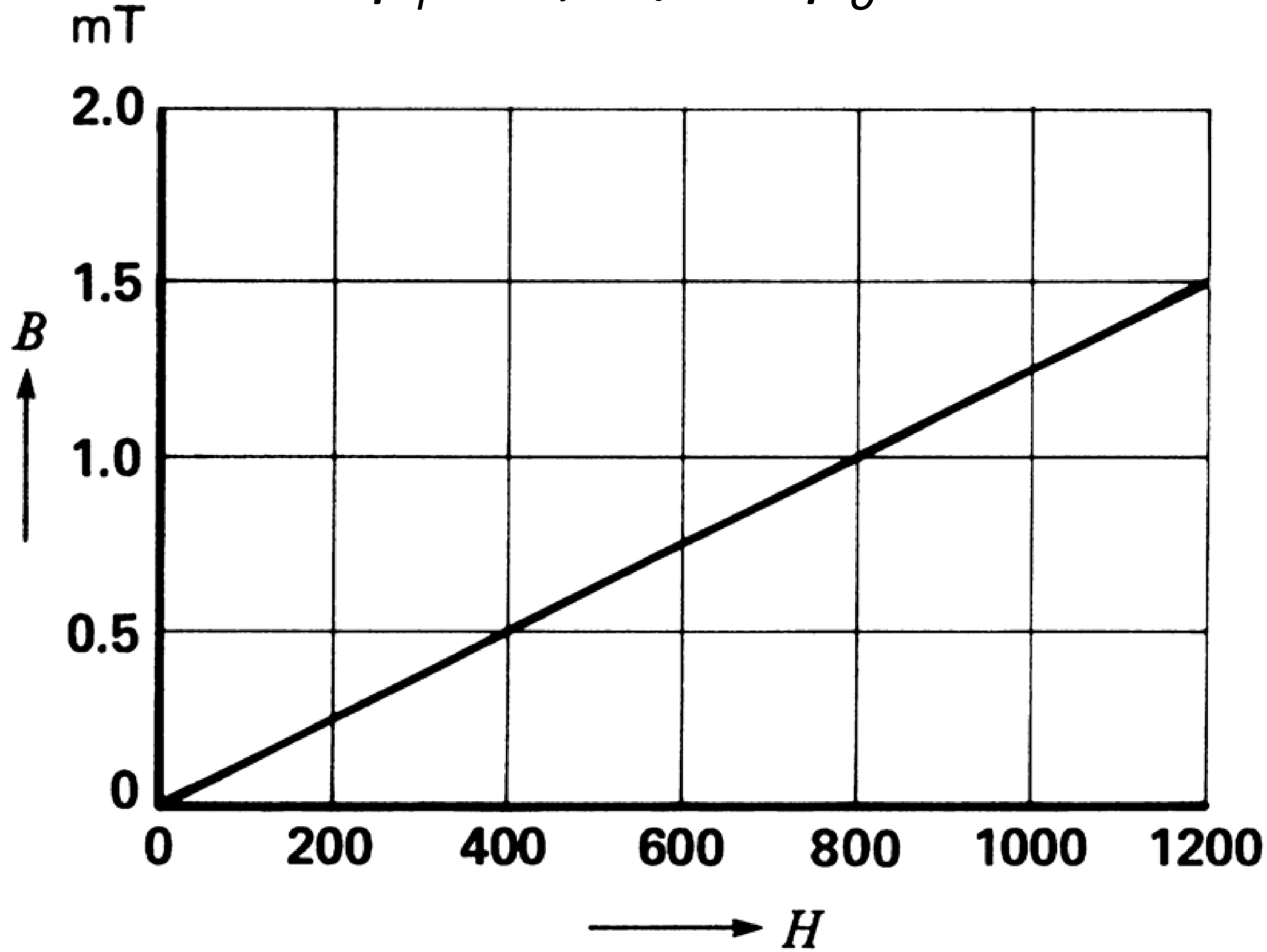
- Relation between magnetic field intensity H and magnetic field density B (measured in Tesla):

$$B = \mu H = (\mu_r \mu_0) H$$

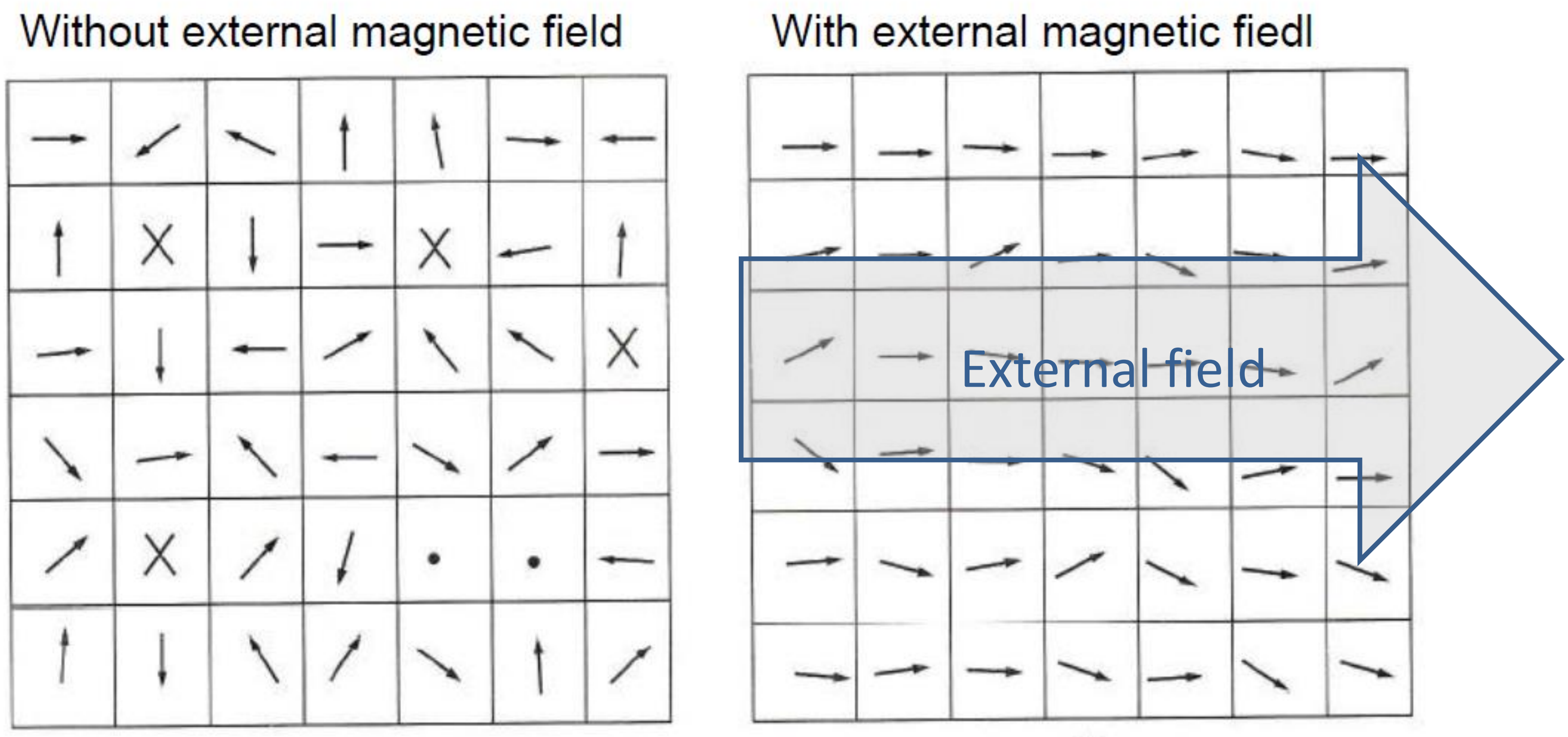
where is μ_r is the relative permeability of the medium (unit-less), is μ_o is the permeability of free space ($4\pi \times 10^{-7}$ H/m).

B-H Curve in air and non-ferromagnetic material

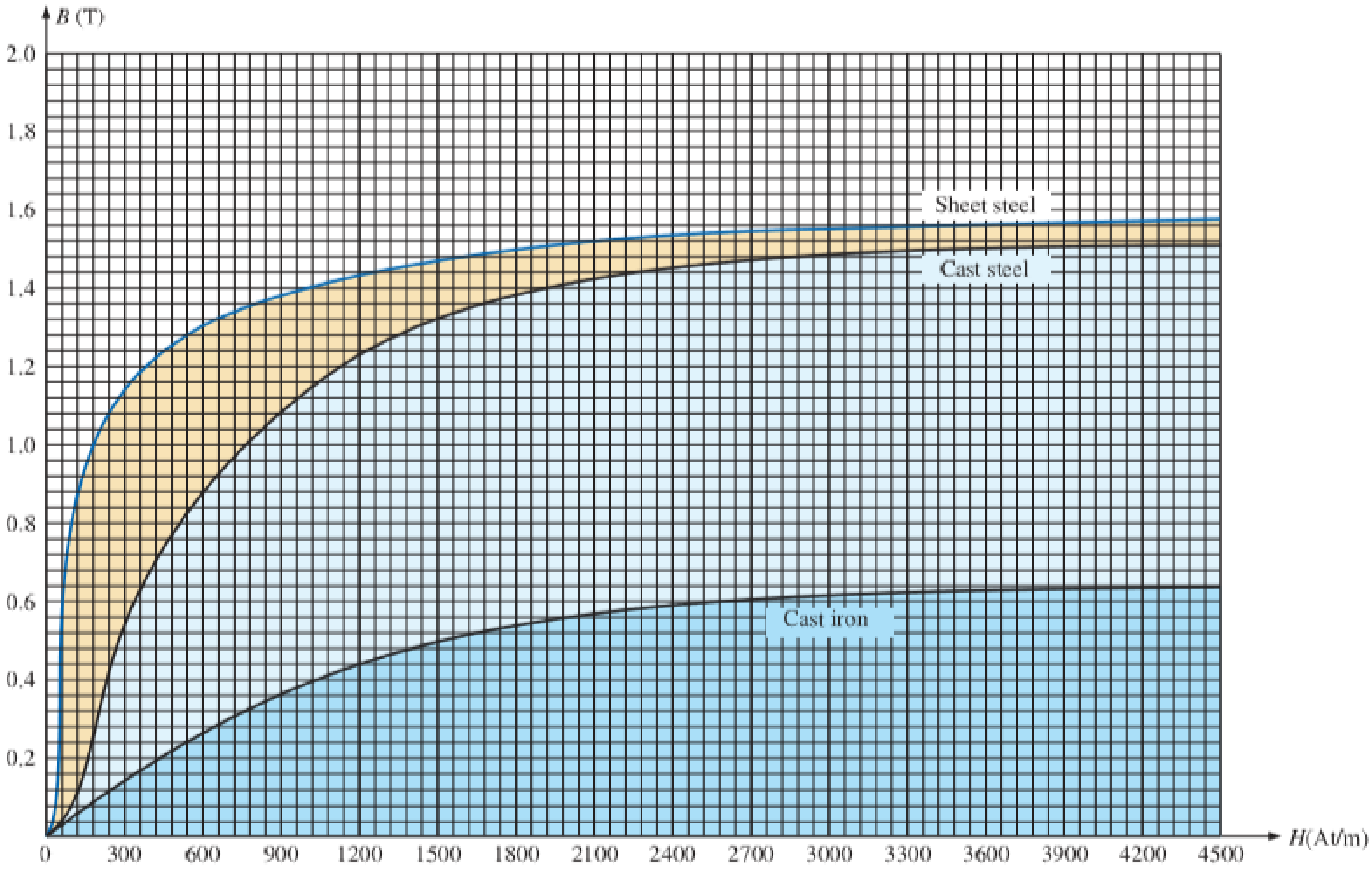
$\mu_r = 1$, $B/H = \mu_o = 4\pi x 10^{-7}$



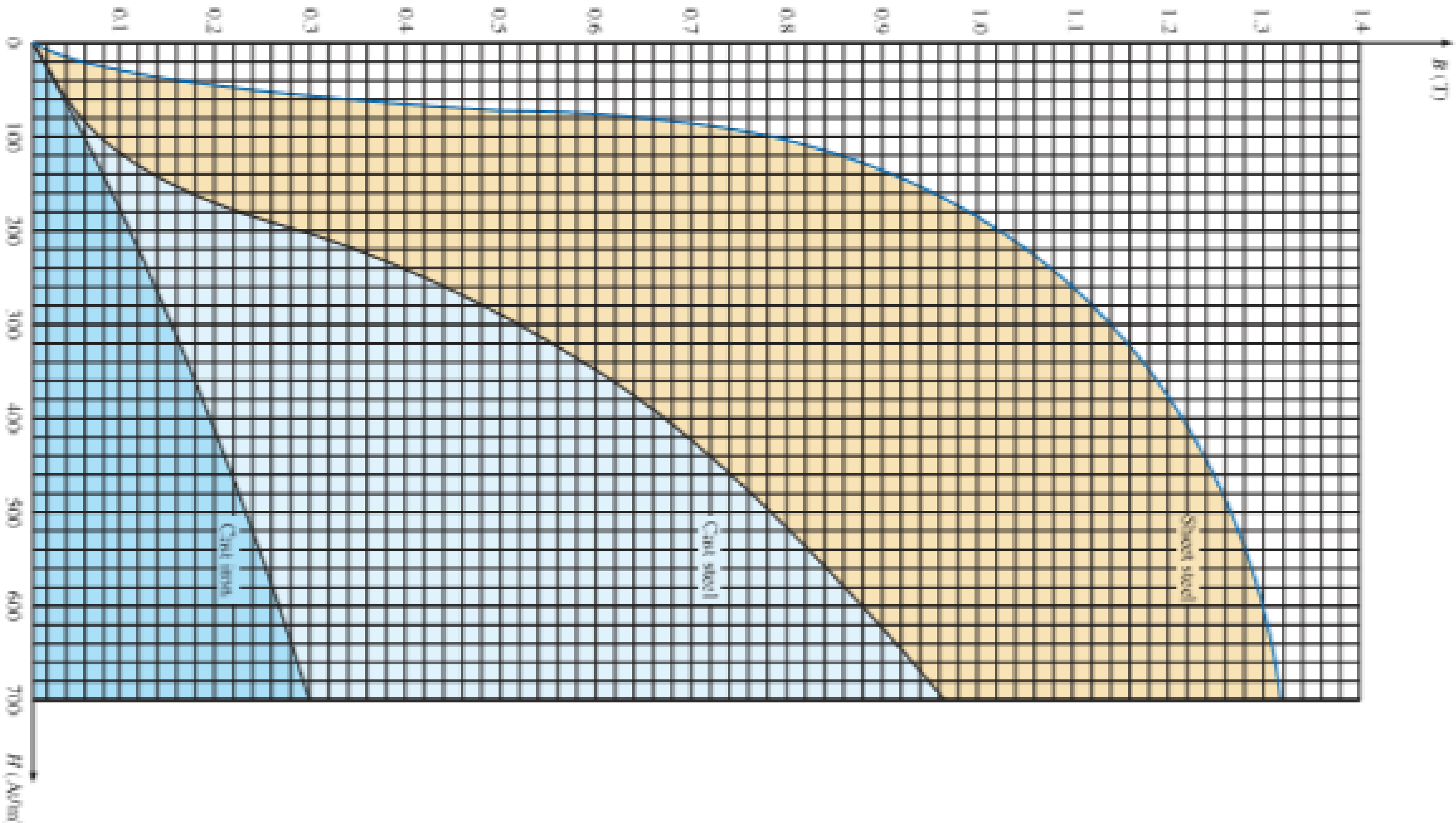
Orientation of magnetic domains without and with the presence of an external magnetic field



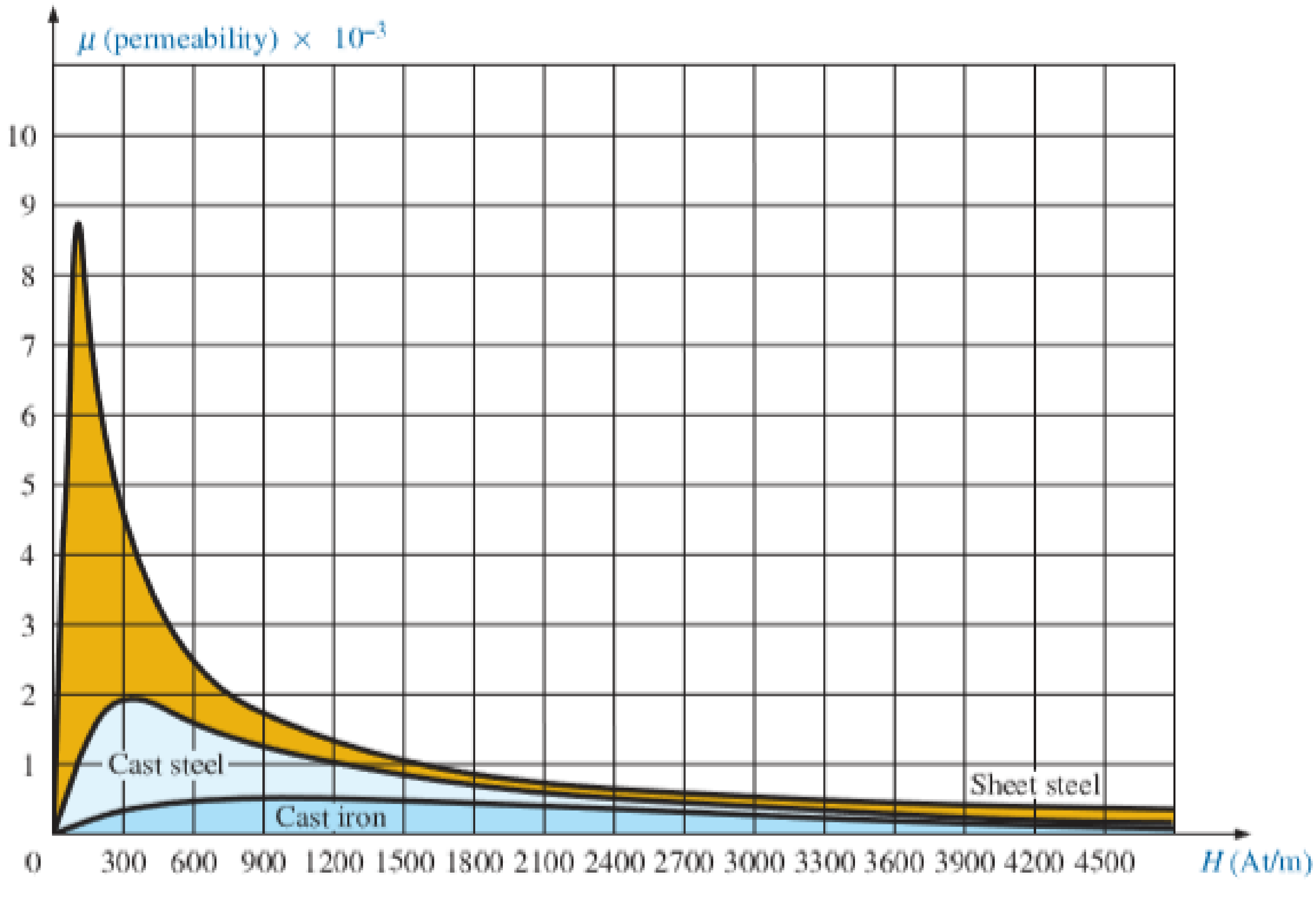
B-H Curve of 3 Ferromagnetic Materials



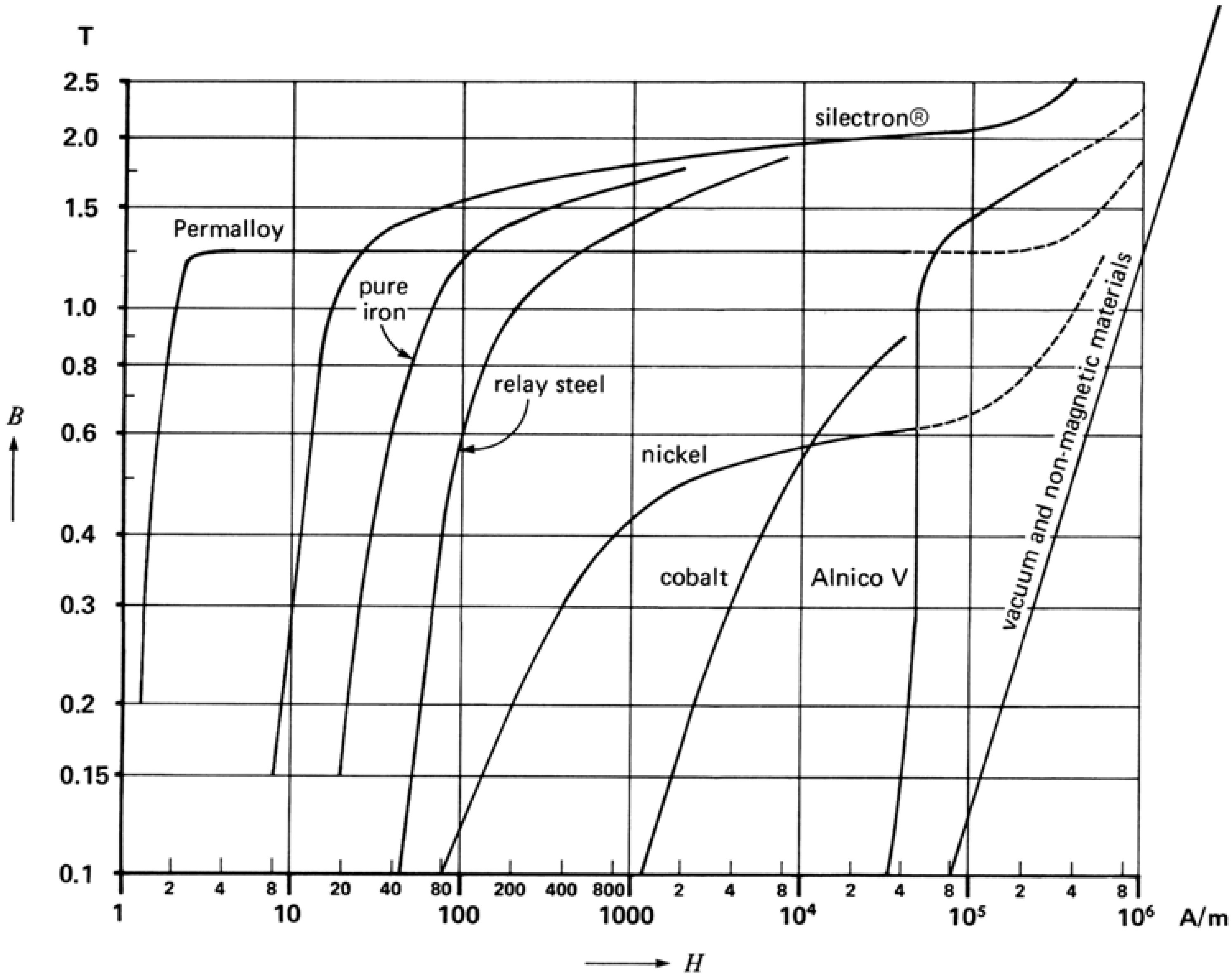
B-H Curve of 3 Ferromagnetic Materials



Variation of μ with Flux Intensity H



Saturation curves of other magnetic materials



Magnetic Flux

- Magnetic flux is the total flux within a given area. It is obtained by integrating the flux density over this area:

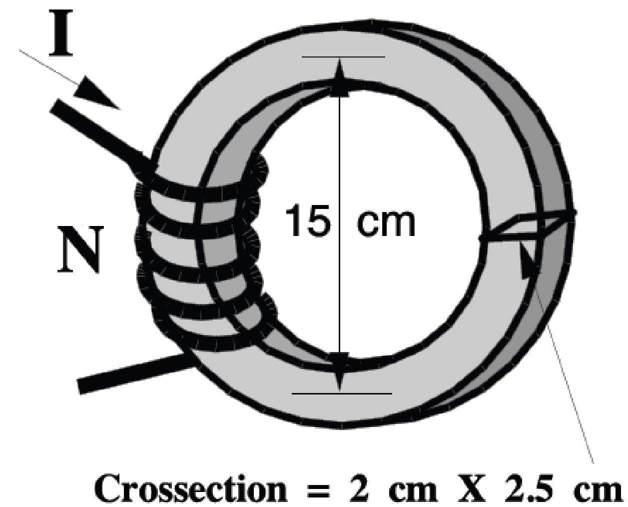
$$\phi = \int BdA$$

- If the flux density is constant throughout the area, then,

$$\phi = BA$$

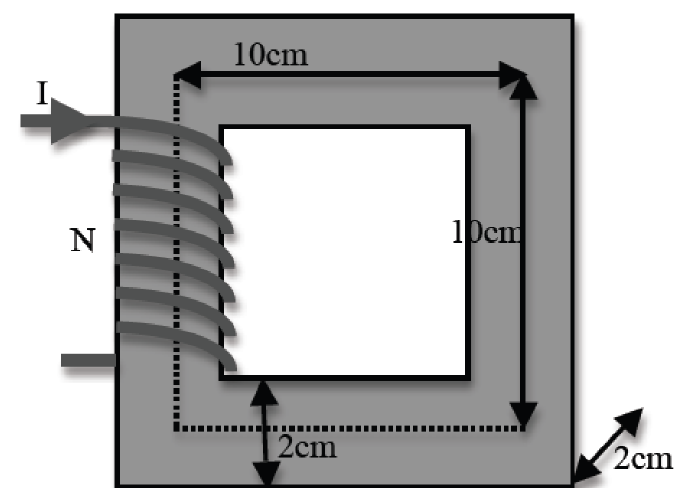
Ampere's Law applied to a magnetic circuit (solid core)

$$\oint H.dl = Hl = \frac{B}{\mu} l = NI$$



- Magnetic flux (Wb):

$$\phi = \int BdA = BA$$



- Hence, $$NI = \phi(\frac{l}{\mu A})$$

$$= \phi \Re$$

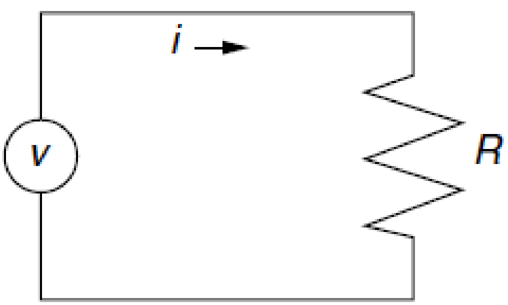
Analogy between electric and magnetic circuits

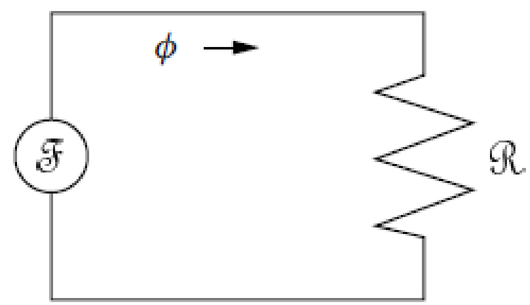
Electrical	Magnetic	Magnetic Units
Voltage v	Magnetomotive force $\mathfrak{F} = Ni$	Amp-turns
Current i	Magnetic flux ϕ	Webers Wb
Resistance R	Reluctance $\mathfrak{R}$	Amp-turns/Wb
Conductivity $1/\rho$	Permeability μ	Wb/A-t-m
Current density J	Magnetic flux density B	Wb/m^2 = teslas T
Electric field E	Magnetic field intensity H	Amp-turn/m

Electrical

Magnetic

EQUIVALENT CIRCUITS



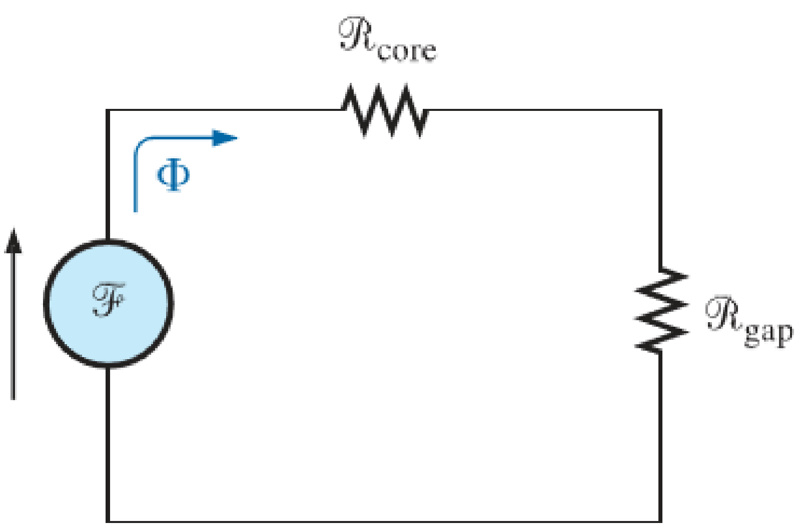


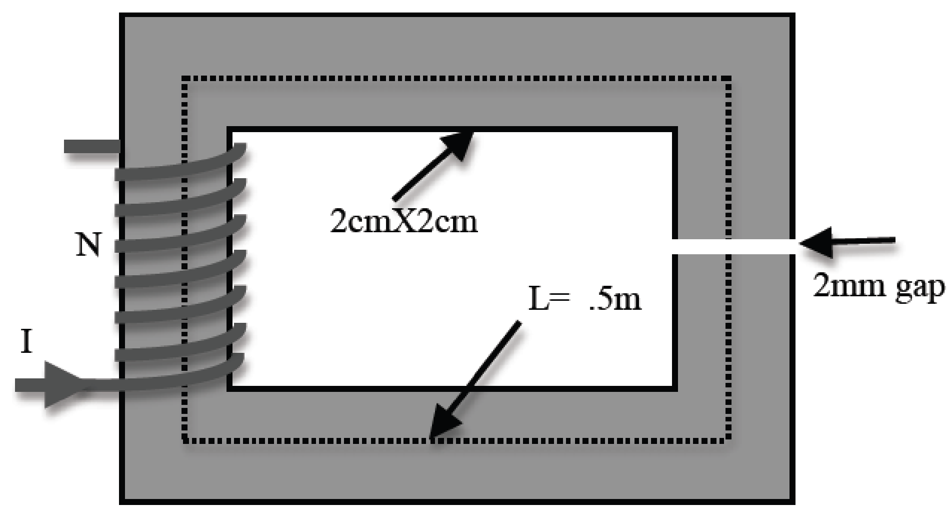
Ampere's Law applied to a magnetic circuit (core with air gap)

$$\oint H.dl = H_c l_c + H_a l_a = \frac{B}{\mu_r \mu_o} l_c + \frac{B}{\mu_o} l_a = NI$$

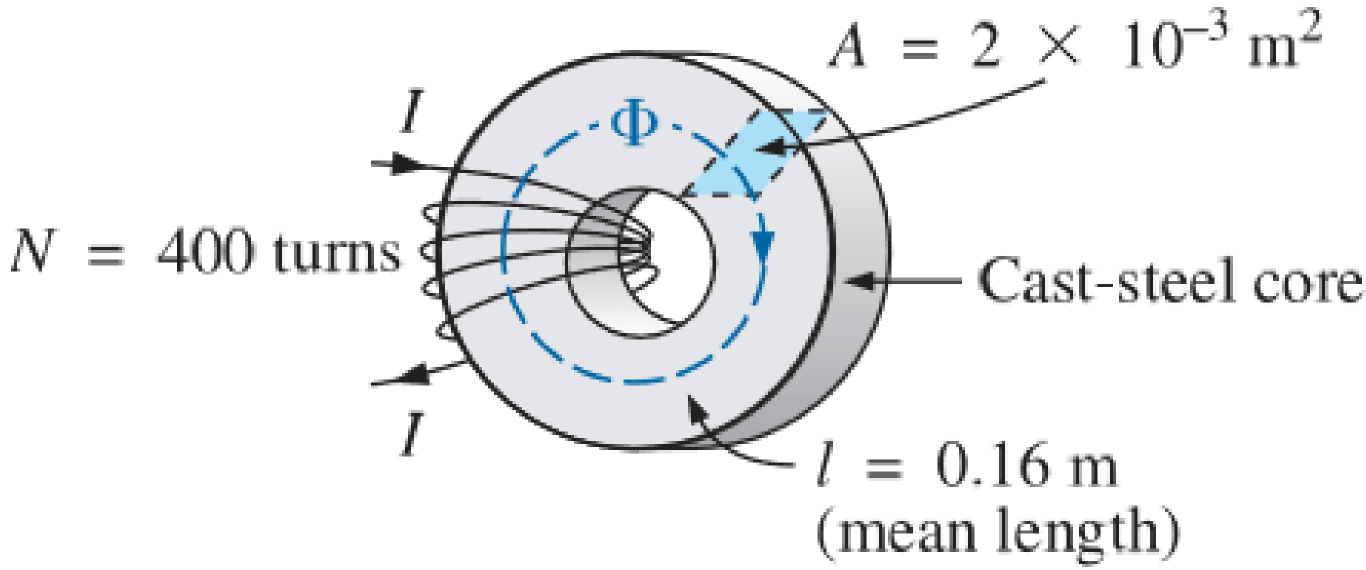
$$NI = \phi \Re$$

where $\Re = \left(\dfrac{l_c}{\mu_r \mu_o A} + \dfrac{l_a}{\mu_o A} \right)$





Exercise 1



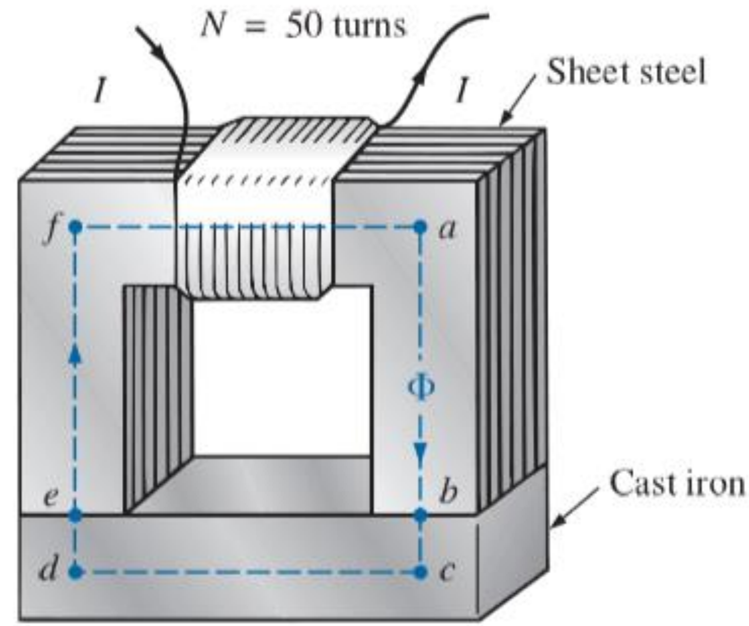
1. Find the value of I that will develop a magnetic flux of 0.4 mWb.
2. Determine μ_r of the material under the above conditions.

Answer:

1. B =0.2 T, H = 170 At/m, I = 68 mA
2. $\mu = 1.176 \times 10^{-3}$, $\mu_r = 935.8$

Exercise 2

The electromagnet to the right has picked up a piece of cast iron (bottom section). Calculate the current required to establish the indicated flux in the core.



$l_{ab} = l_{cd} = l_{ef} = l_{fa} = 4$ in.

$l_{bc} = l_{de} = 0.5$ in.

Area (throughout) $= 1$ in.^{2}

$\Phi = 3.5 \times 10^{-4}$ Wb

Answer:

(convert lengths to m and area to m^2)

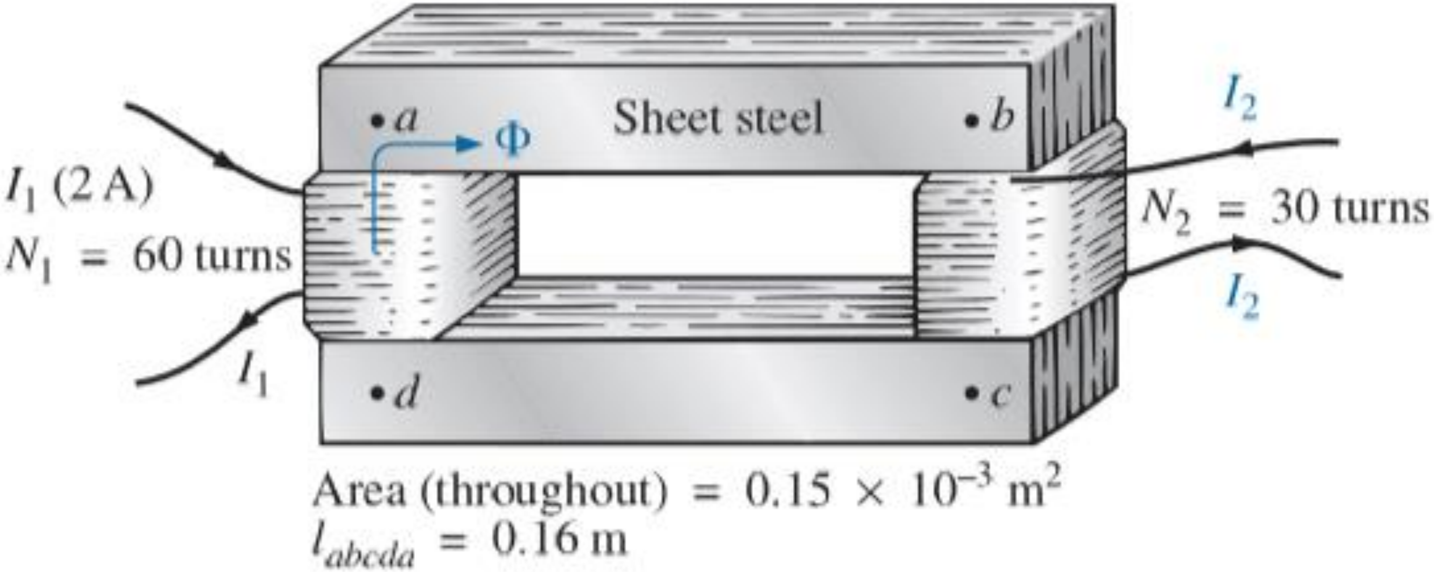
B = 0.542 T

H(steel) = 70 At/m

H(cast iron) = 1600 At/m

I = 4.49 A

Exercise 3



Determine the current I_2 of the resultant clockwise flux is 15 µWb. Assume both current flow in a counterclockwise direction.

Answer:

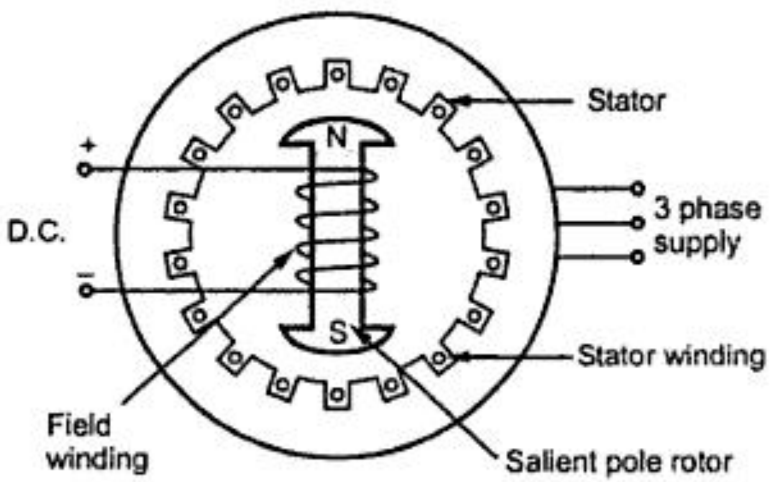
B = 0.1 T

H(steel) = 20 At/m

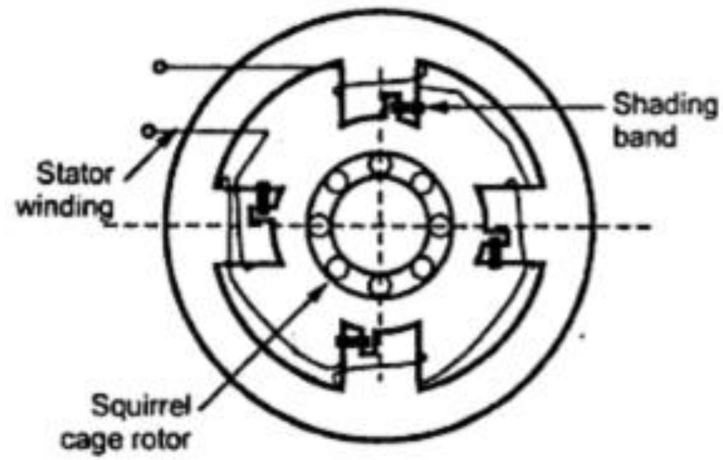
I = 3.89 A

Complex Magnetic Cores

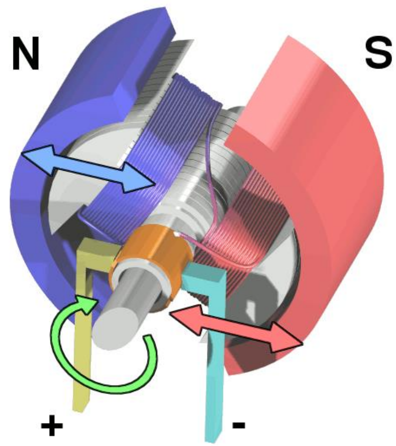
Synchronous Machine



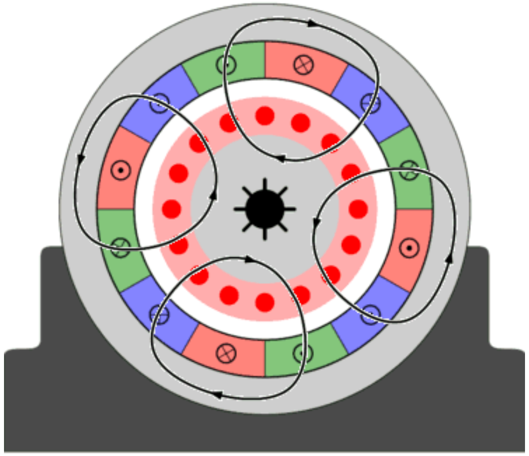
Shaded Pole Induction Motor



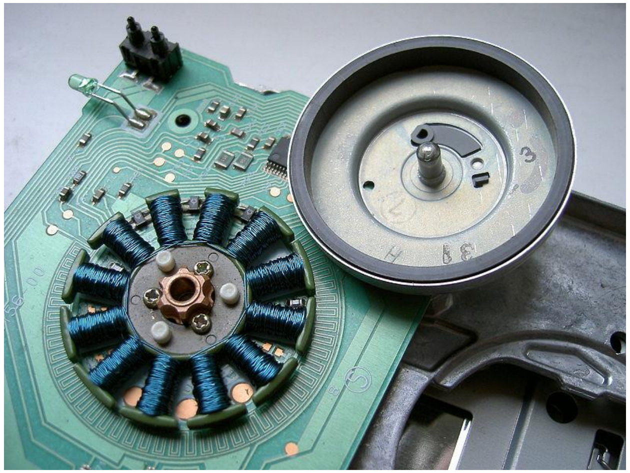
Simple Brushed DC Motor



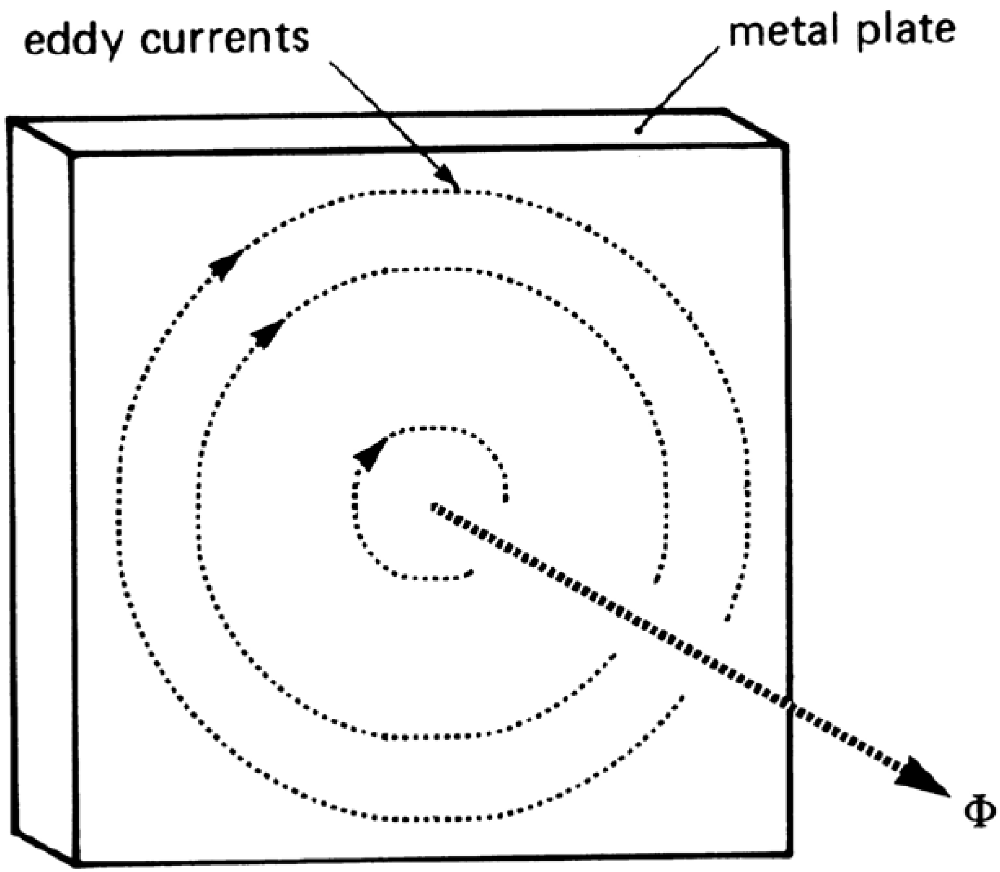
AC Induction Motor



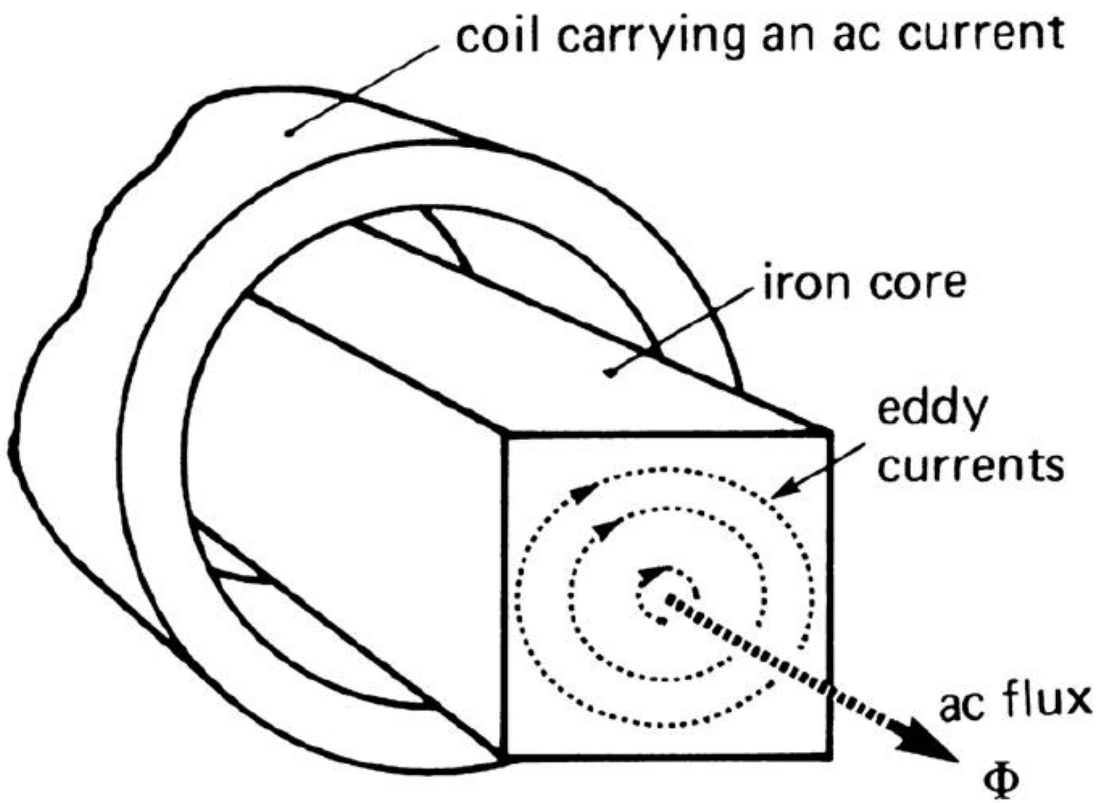
Brushless DC Motor



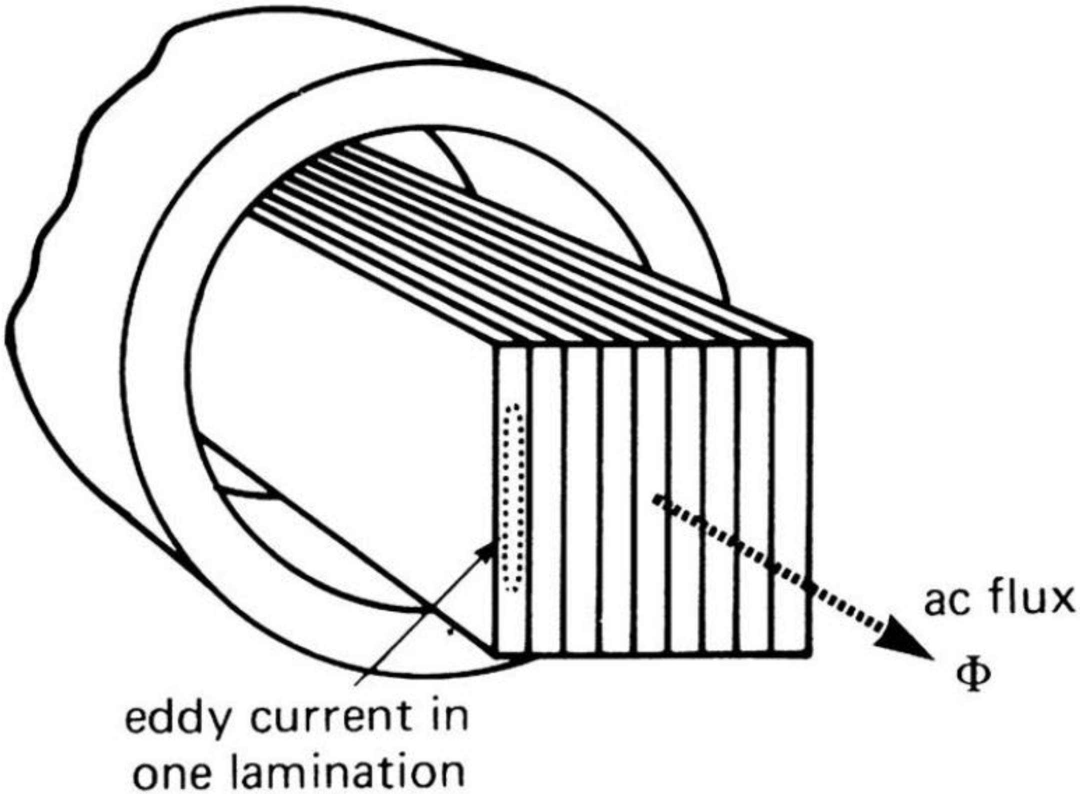
Eddy currents are induced in a solid metal plate under the presence of a varying magnetic field

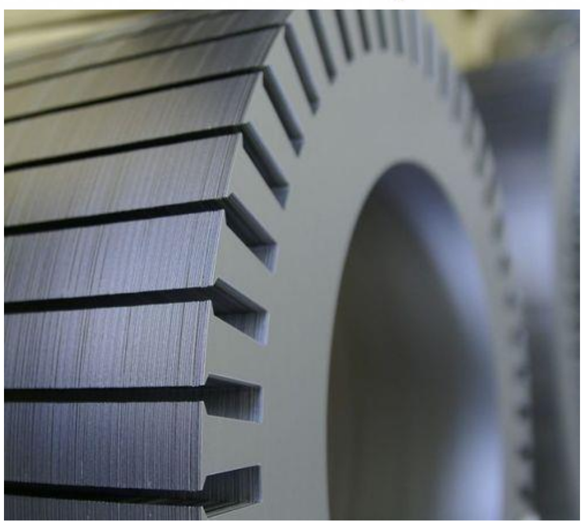


Solid iron core carrying an AC flux (significant eddy current flow)

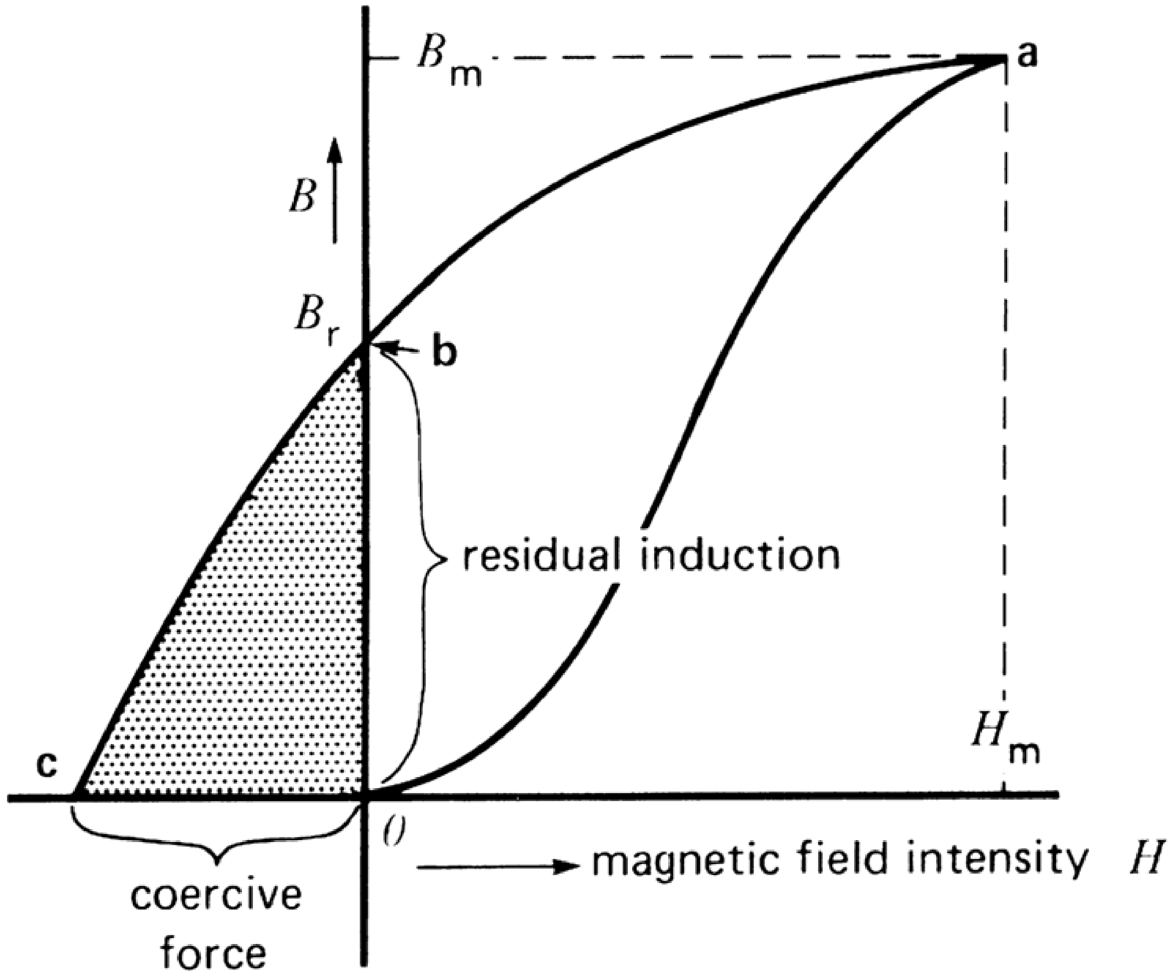


Core built up of insulated laminations minimizes eddy currents (and eddy current losses)

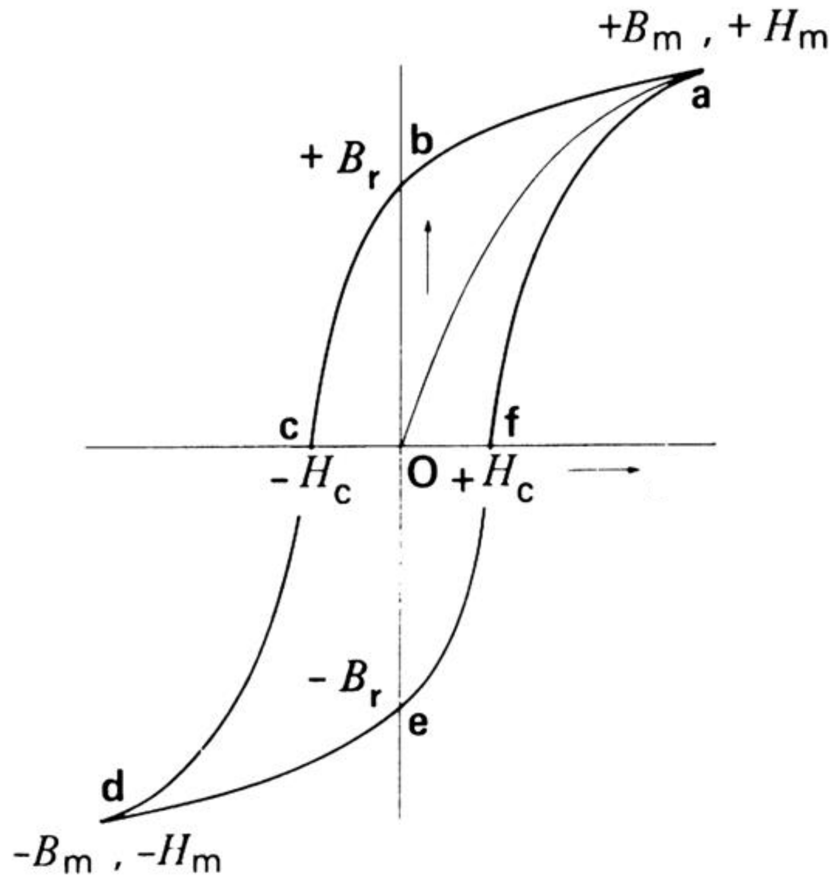




Residual induction and Coercive Force



Hysteresis Loop (AC Current)



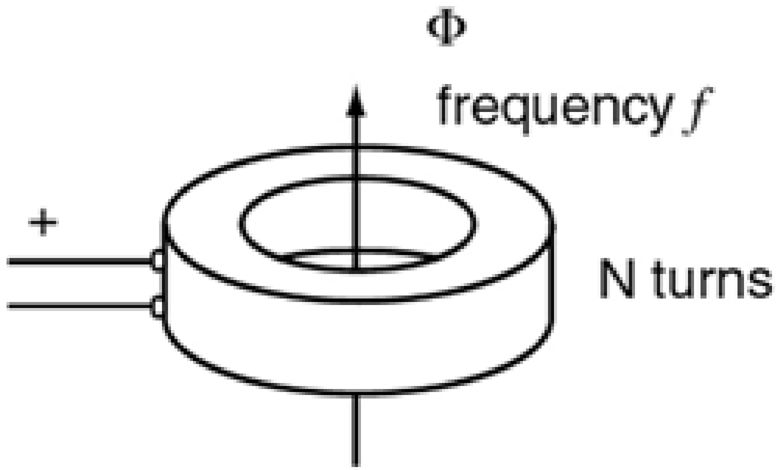
Faraday’s Law

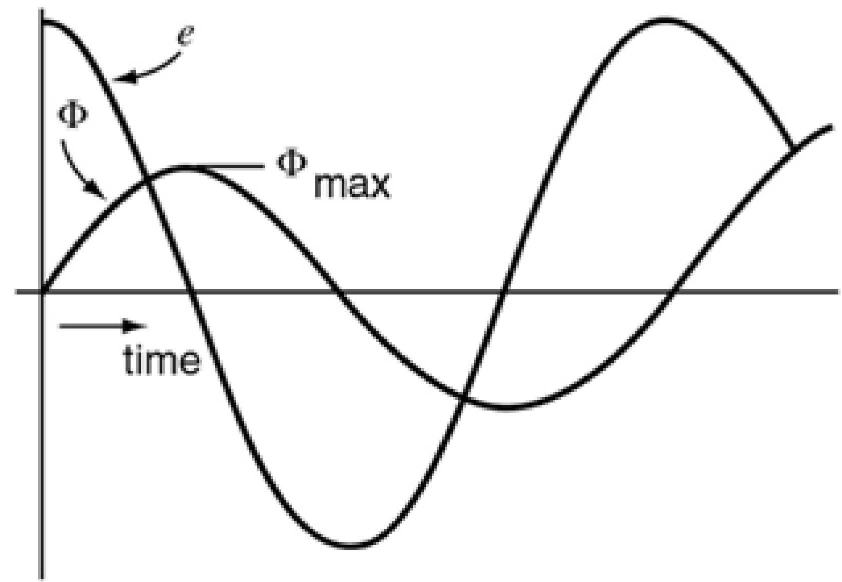
- Faraday's law of induction is a basic law of electromagnetism relating to the operating principles of transformers, inductors, electrical motors and generators. The law states that:

“The induced electromotive force (EMF) in any closed circuit is proportional to the rate of change of the magnetic flux through that circuit”

$$e = -N\frac{d\phi}{dt}$$

Voltage induced in a coil when it encircles a variable flux in the form of a sinusoid

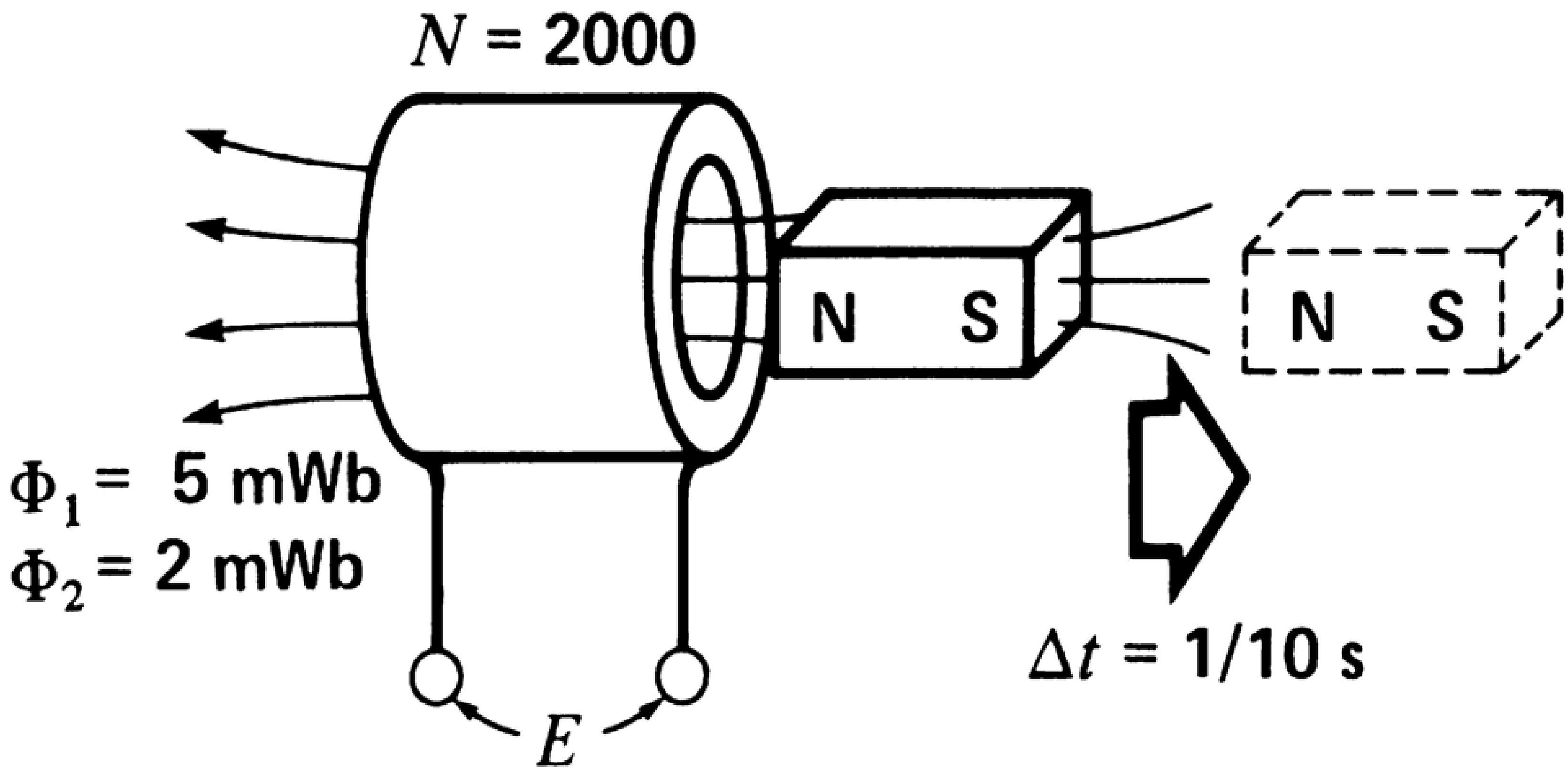




$$\phi(t) = \Phi \sin(2\pi f t)$$

$$e(t) = -N d\phi(t) / dt = -[N\Phi 2\pi f]\cos(2\pi f t)$$

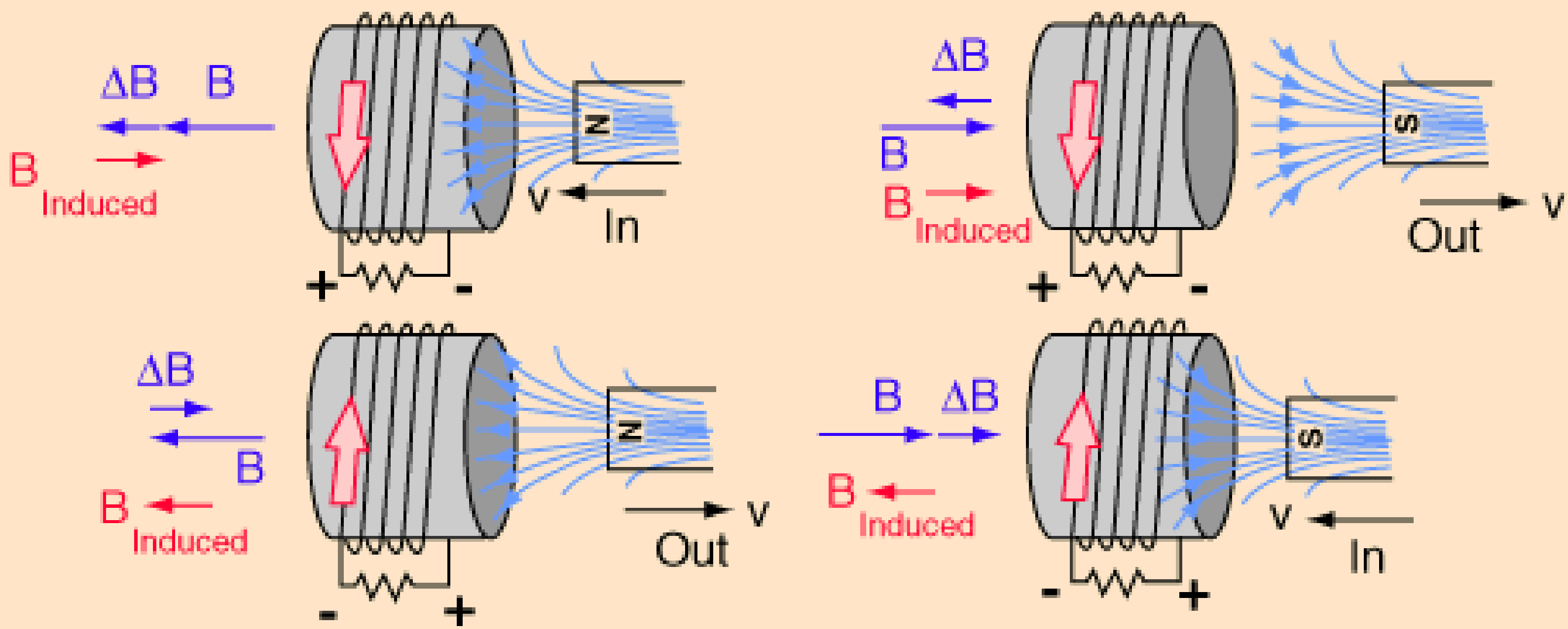
Example: voltage induced in a coil by a moving magnet



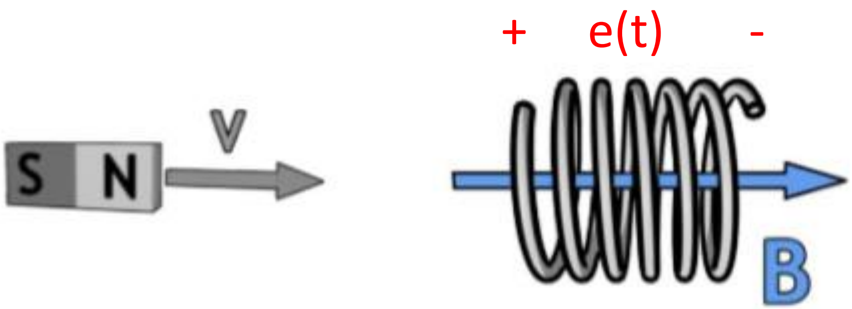
$E = -N\Delta\varphi/\Delta t = 2000(-3/0.1) = 60{,}000$ mV or 60 V

Polarity of induced voltage: Lenz's Law

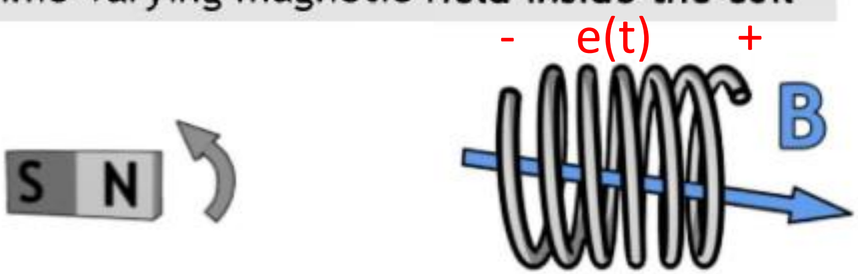
When an emf is generated by a change in magnetic flux according to Faraday's Law, the polarity of the induced emf is such that it produces a current whose magnetic field opposes the change which produces it. The induced magnetic field inside any loop of wire always acts to keep the magnetic flux in the loop constant. In the examples below, if the B field is increasing, the induced field acts in opposition to it. If it is decreasing, the induced field acts in the direction of the applied field to try to keep it constant.



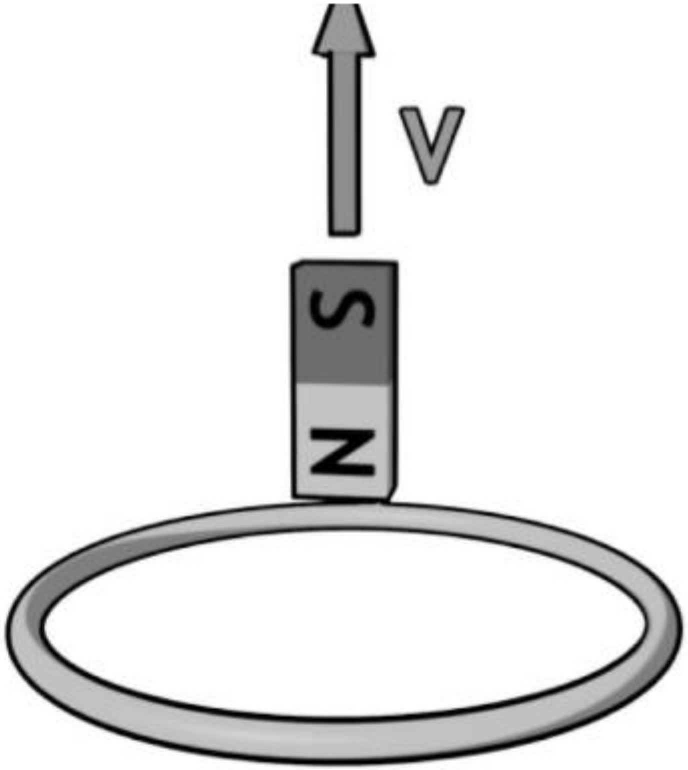
Voltage Polarity and Direction of Induced Current



Moving a magnet towards a coil produces a time-varying magnetic field inside the coil



Rotating a bar of magnet (or the coil) produces a time-varying magnetic field inside the coil



Will the current run CLOCKWISE or ANTICLOCKWISE ?

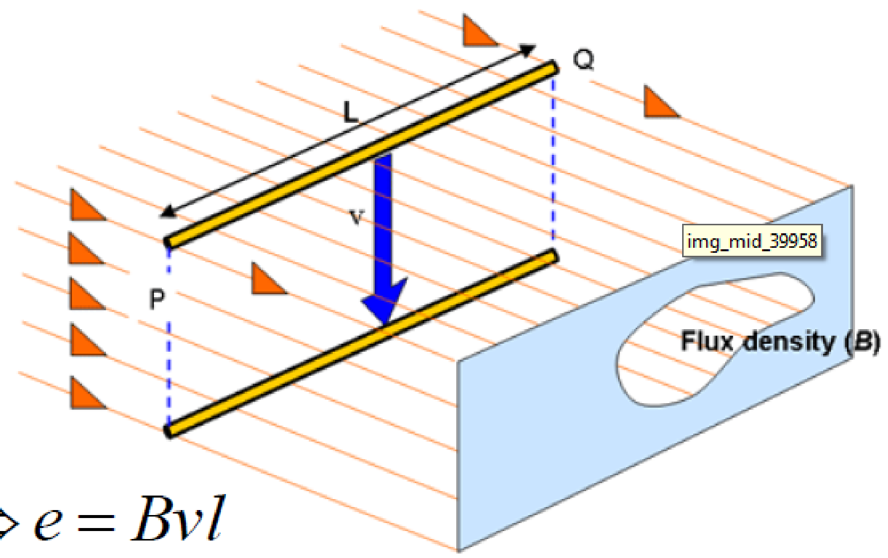
Answer: Clockwise

Induced voltage in a conductor moving in a magnetic field

- The voltage induced in a conductor of length *l* that is moving in a magnetic field with flux density *B*, at a speed *v* is given by

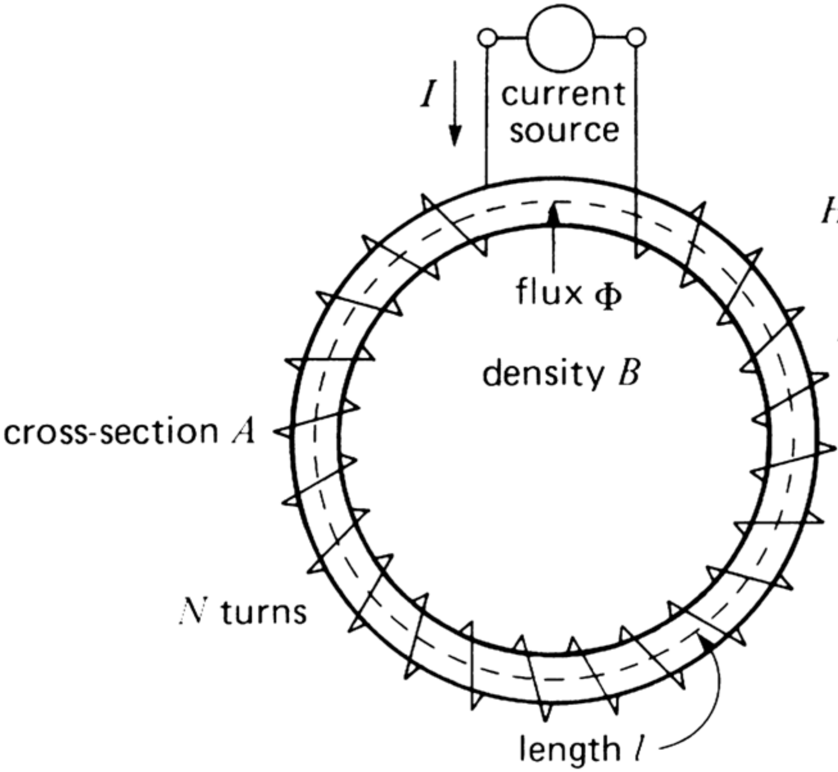
$$e = (vB\sin\theta)l\cos\phi$$

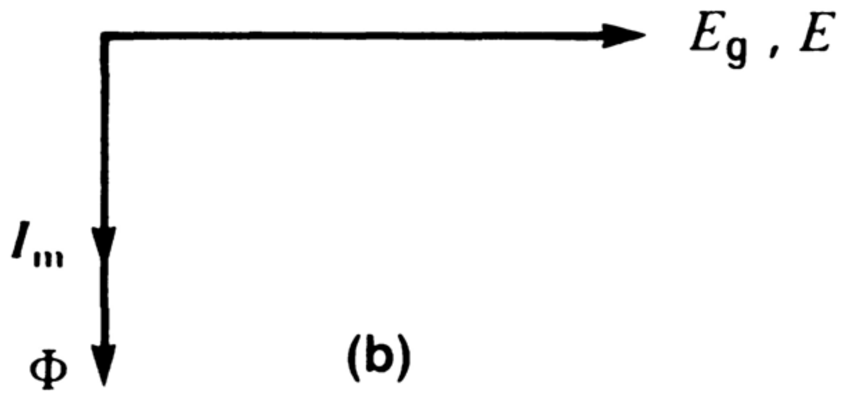
where θ is the angle between vxB and the velocity vector, and ϕ is the angle between vxB and the wire. The polarity of the induced voltage is determined by Lenz's Law.



$$\theta = 90\deg. \quad and \quad \phi = 0\deg \Rightarrow e = Bvl$$

Inductance of a coil and energy storage





$$e = L\frac{di}{dt} = N\frac{d\phi}{dt} = N\frac{d(Ni\mu A/l)}{dt} = (N^2\mu A/l)\frac{di}{dt} \longrightarrow L = \frac{N^2\mu A}{l}$$

Energy stored in an inductor

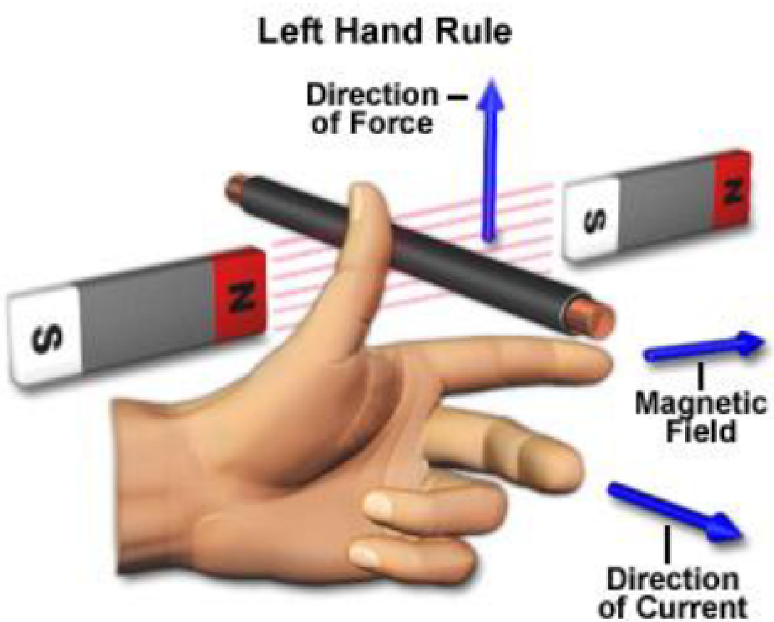
$$W = \int i.e.dt = \int i.L\frac{di}{dt}.dt = \frac{1}{2}Li^2, \quad W = \int eidt = \int i.N\frac{d\phi}{dt}.dt = \int \phi\frac{l}{\mu A}d\phi = \frac{l}{2\mu A}\phi^2$$

Induced force on a current-carrying conductor

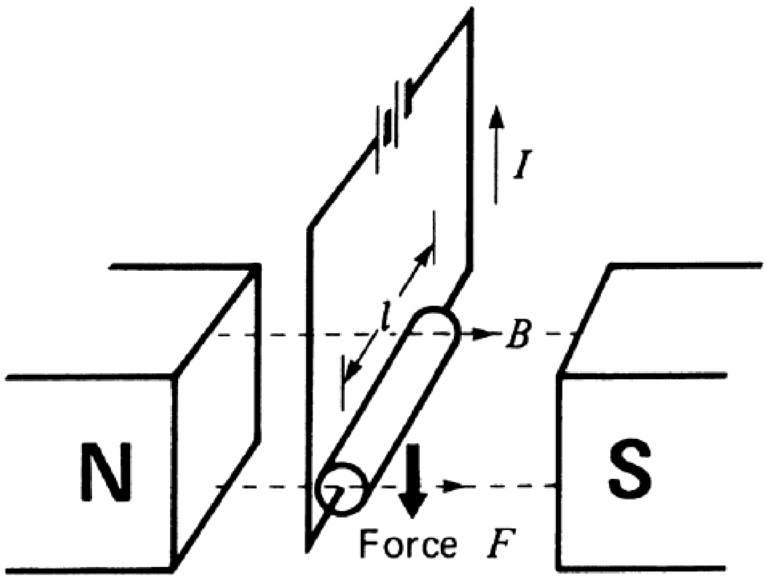
- The force on a wire of length *l* and carrying a current *i* under the presence of a magnetic flux B is given by

$$F = Bil \sin\theta$$

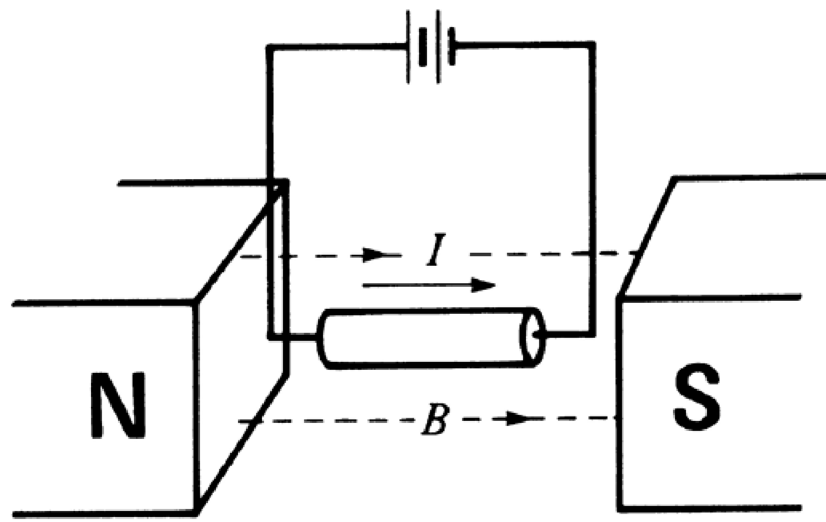
where θ is the angle between the wire and flux density vector. The direction of the force is determined by the right hand rule



Induced force on a current-carrying conductor

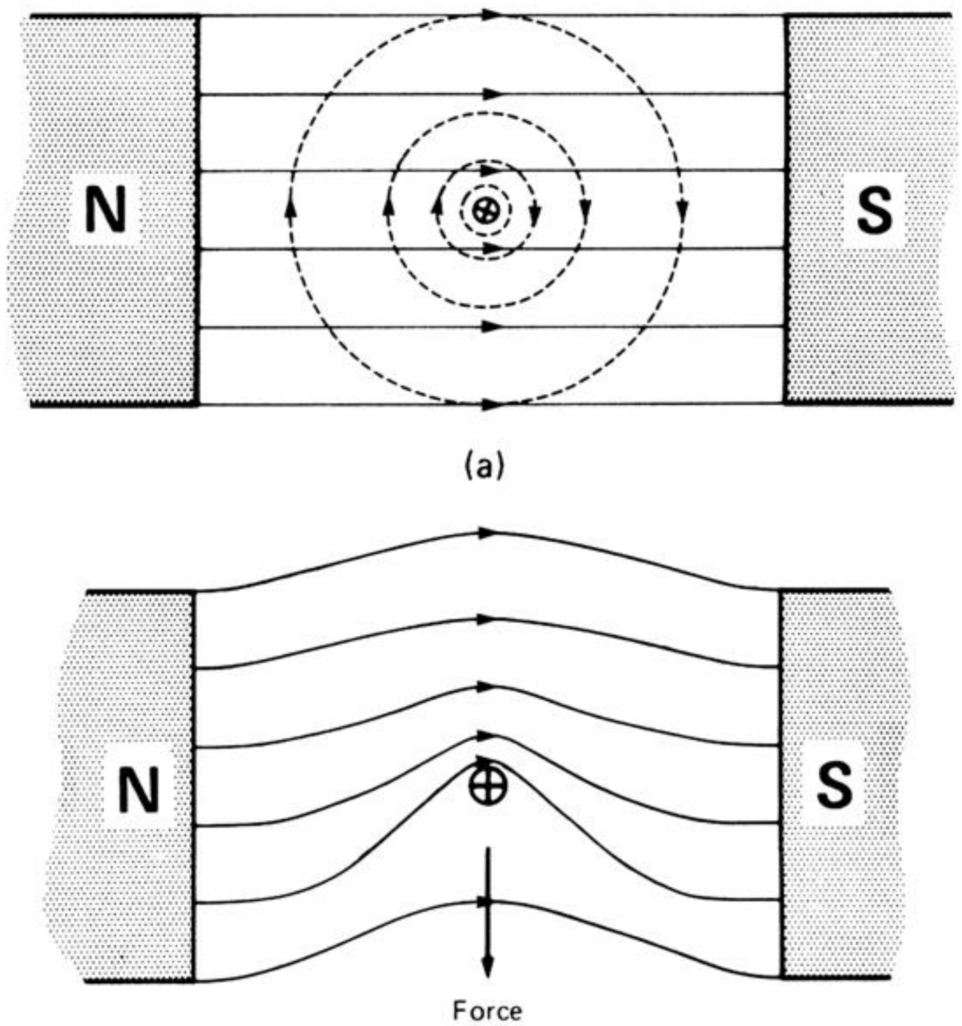


$$\theta = 90\,\text{deg.} \Rightarrow F = BIl$$



$$\theta = 0\,\text{deg.} \Rightarrow F = 0$$

Induced Force on a Current Carrying Conductor

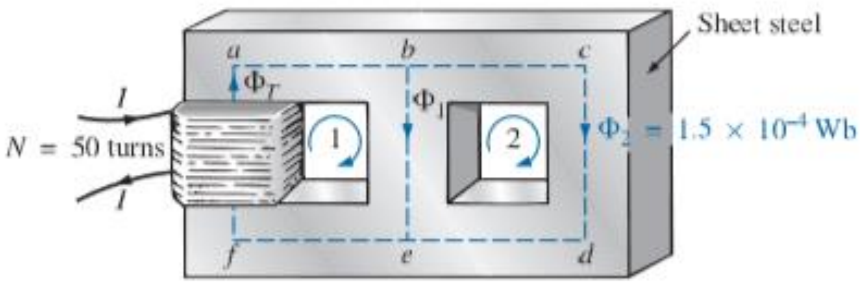


Problems (Chap. 1)

- 5, 6, 8, 10, 12, 14

Practice Problems

Determine the current I required to establish a flux of 1.5 $\times 10^{-4}$ Wb in the section of the core indicated



$l_{bcde} = l_{efab} = 0.2$ m
$l_{be} = 0.05$ m
Cross-sectional area = 6×10^{-4} m^2 throughout

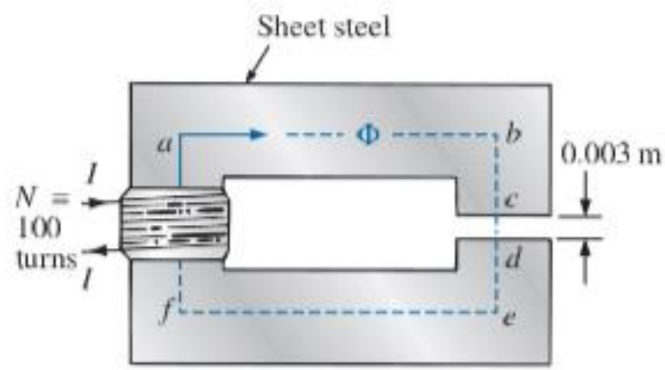
Problem 1

Sol: I = 1.76 A

Problem 2

a. Find the current I required to establish a flux Φ = 2.4×10^{-4} Wb in the magnetic circuit

b. Compare the mmf drop across the air gap to that across the rest of the magnetic circuit. Discuss your results using the value of μ for each material.



Area (throughout) = 2×10^{-4} m^2
$l_{ab} = l_{ef} = 0.05$ m
$l_{af} = l_{be} = 0.02$ m
$l_{bc} = l_{de}$

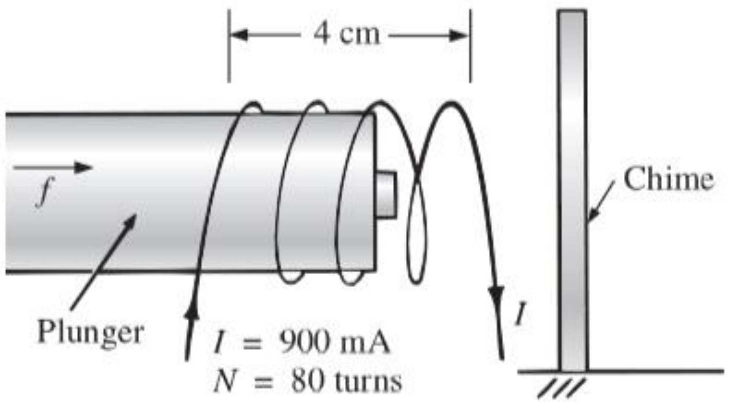
Sol: I = 29.16 A
Hc.lc = 52 At/m
Hg.lg = 2,864 At/m
μ/μ_o = 2,500

Problem 3

The force carried by the plunger of the door chime is determined by

$$f = \frac{1}{2} NI \frac{d\phi}{dx} \qquad \text{(newtons)}$$

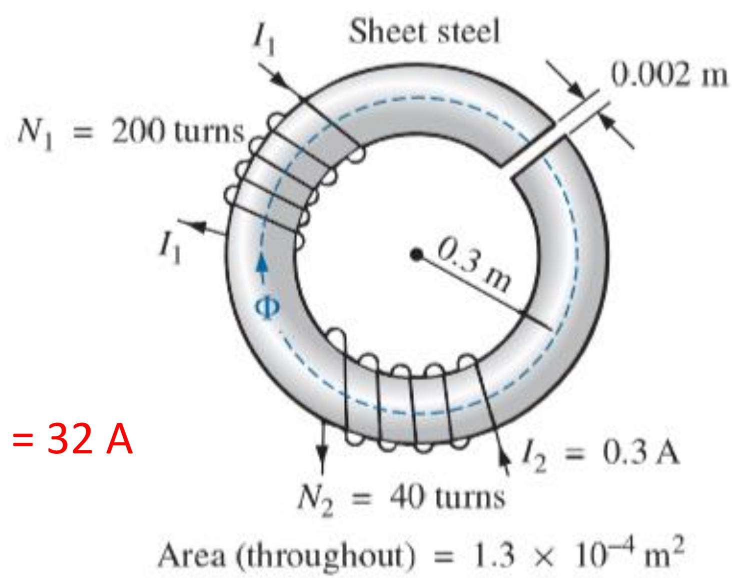
where $d\phi/dx$ is the rate of change of flux linking the coil as the core is drawn into the coil. The greatest rate of change of flux occurs when the core is 1/4 to 3/4 the way through. In this region, if Φ changes from 0.5×10^{-4} Wb to 8×10^{-4} Wb, what is the force carried by the plunger?



Sol: f = 1.35 N

Problem 4

Determine the current I_1 required to establish a flux of $\Phi = 2 \times 10^{-4}$ Wb in the magnetic circuit



Sol: I = 32 A

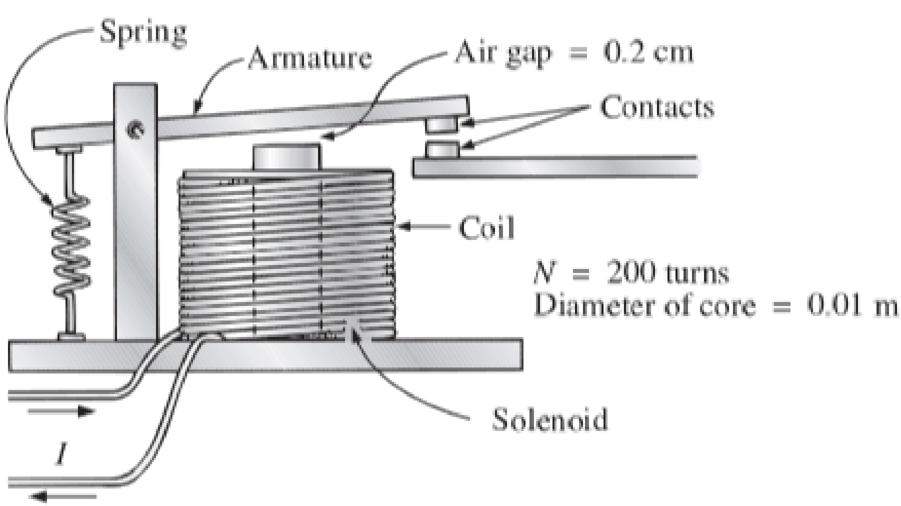
Problem 5

a. A flux of 0.2×10^{-4} Wb will establish sufficient attractive force for the armature of the relay to close the contacts. Determine the required current to establish this flux level if we assume that the total mmf drop is across the air gap.

b. The force exerted on the armature is determined by the equation

$$F(\text{newtons}) = \frac{1}{2} \cdot \frac{B_g^2 A}{\mu_o}$$

where B_g is the flux density within the air gap and A is the common area of the air gap. Find the force in newtons exerted when the flux Φ specified in part (a) is established.



Sol: I = 2.02 A
f = 2 N