Review of 3-Phase AC Circuits

EE 340

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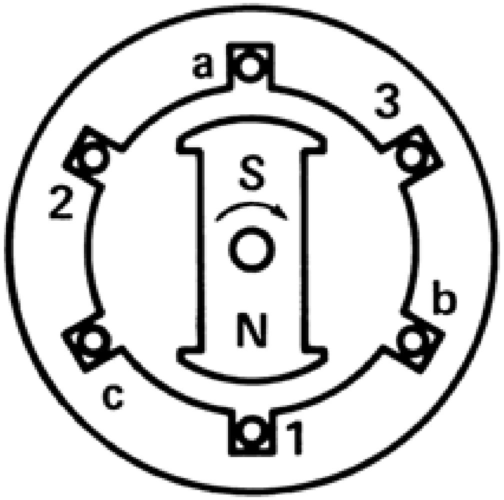
Advantages of 3-Phase Systems

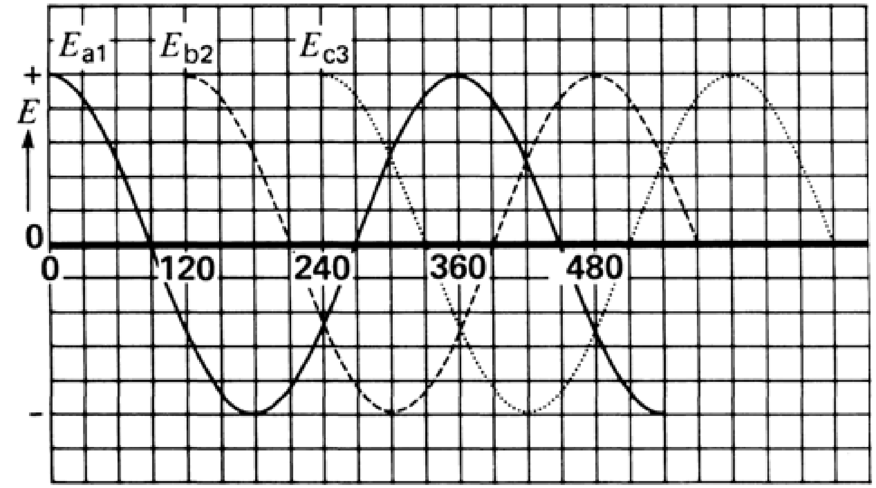
- Can transmit more power for same amount of wire (twice as much as single phase)
- Torque produced by 3ϕ machines is constant
- Three phase machines use less material for same power rating
- Three phase machines start more easily than single phase machines

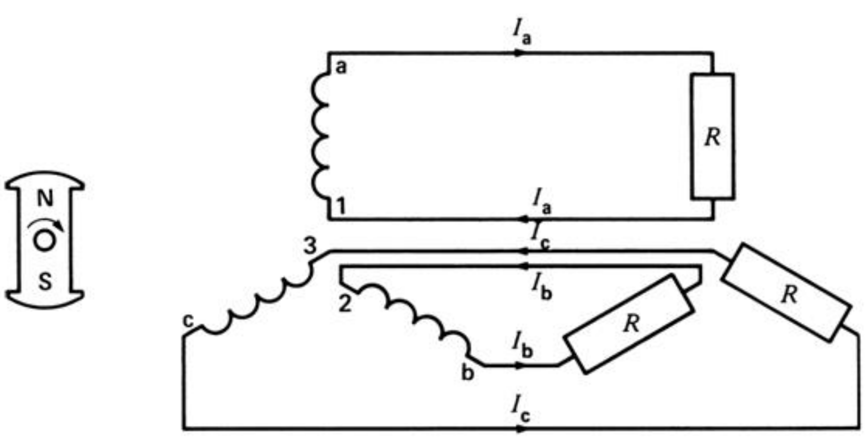
Balanced 3-Phase Systems

- A balanced 3 phase (ϕ) system has
  - three voltage sources with equal magnitude, but with an angle shift of 120°
  - equal loads on each phase
  - equal impedance on the lines connecting the generators to the loads
- Bulk power systems are almost exclusively 3ϕ
- Single phase is used primarily only in low voltage, low power settings, such as residential and some commercial

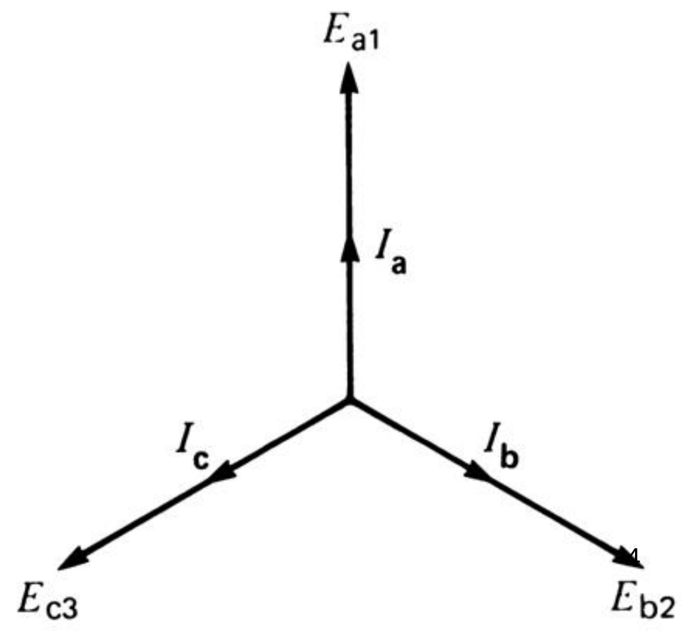
3-Phase Voltage Source



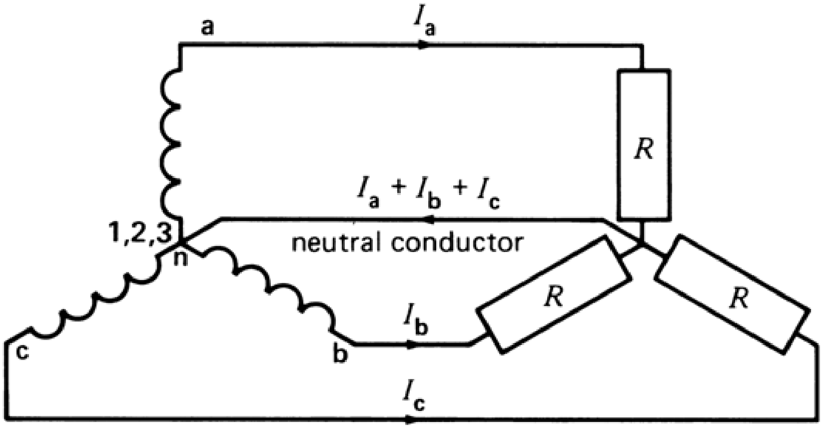




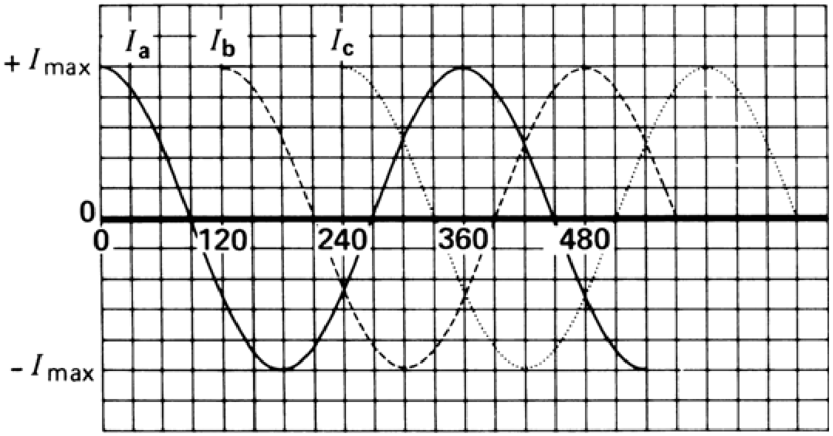
(a)



Neutral Wire Sharing

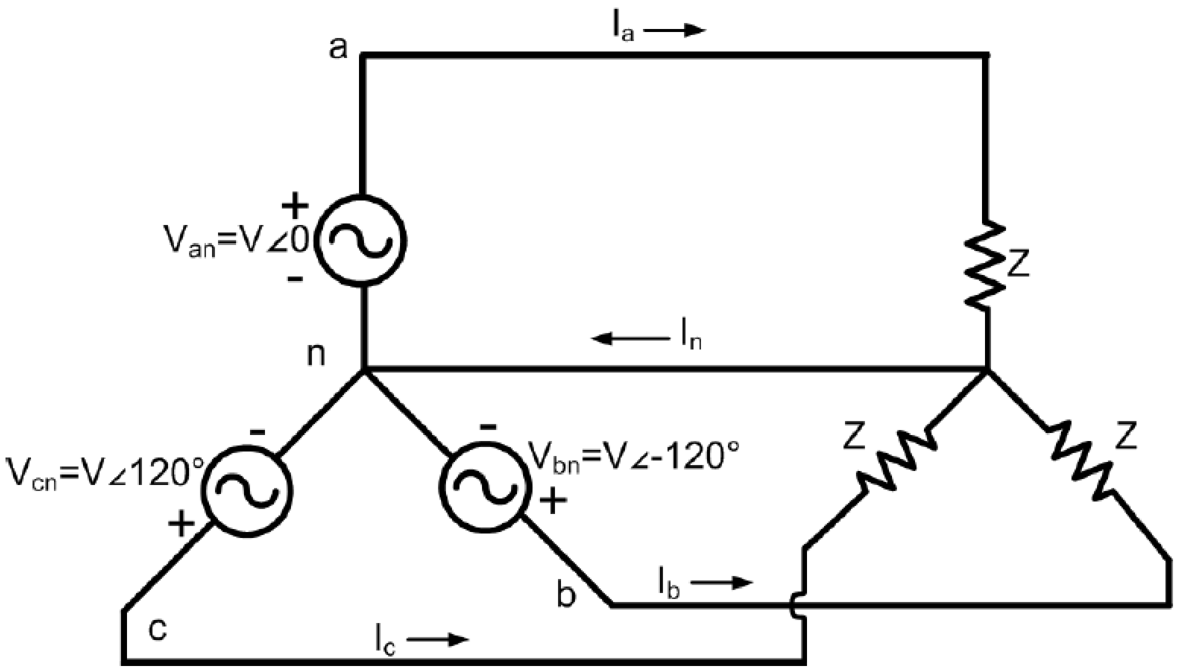


(a)



(b)

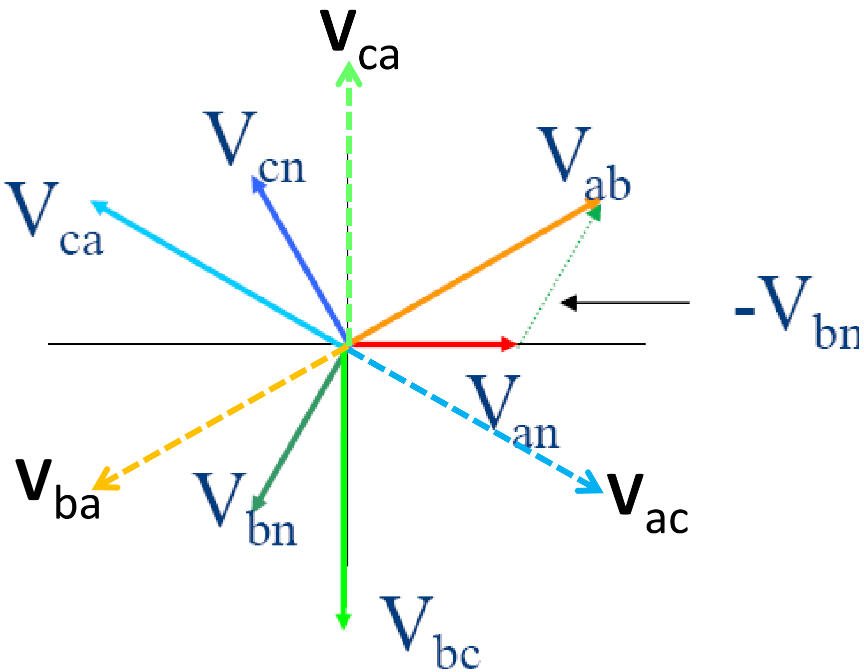
Neutral Current in Balanced Circuit = 0



$$I_n = I_a + I_b + I_c$$

$$I_n = \frac{V}{Z}(1\angle 0° + 1\angle -120° + 1\angle 120°) = 0$$

Phase Voltages and Line Voltages



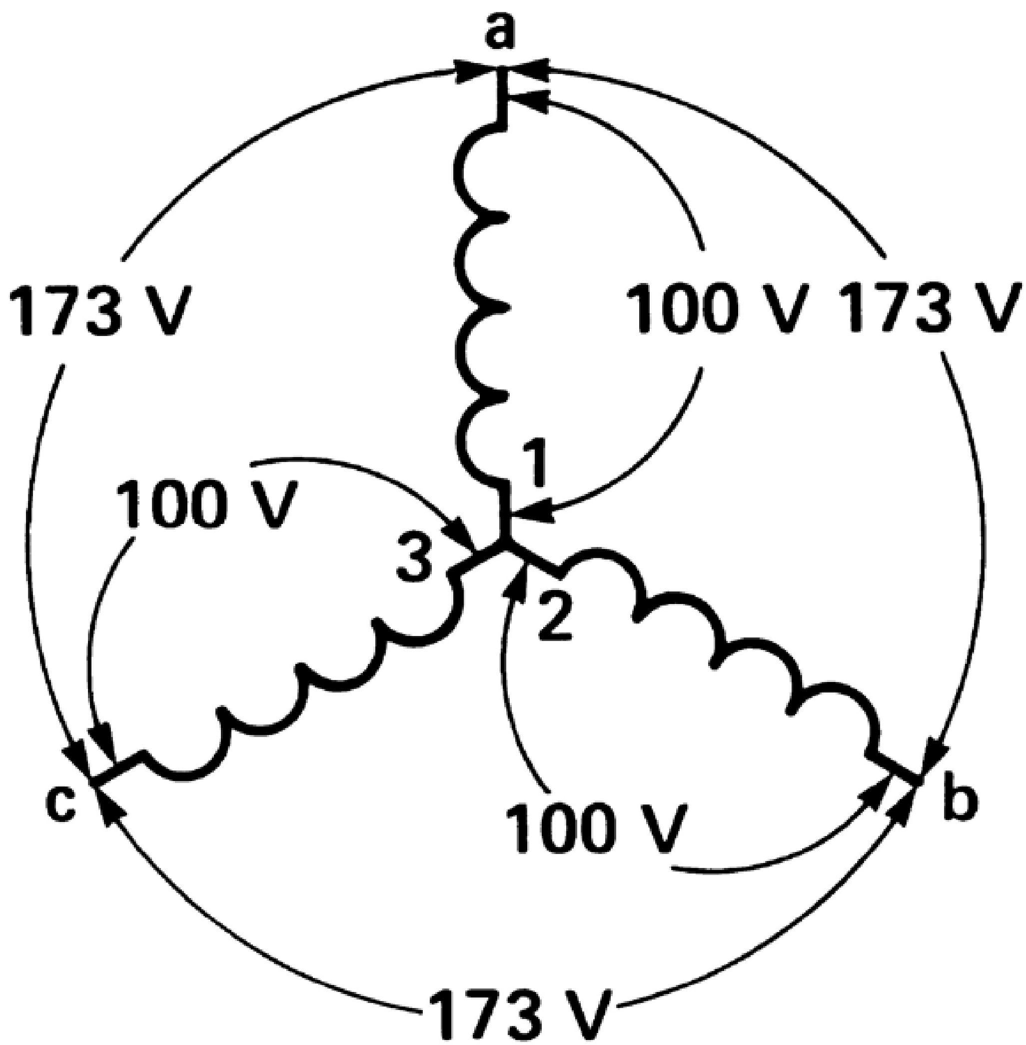
$$V_{an} = |V|\angle\alpha^\circ$$
$$V_{bn} = |V|\angle\alpha^\circ - 120^\circ$$
$$V_{cn} = |V|\angle\alpha^\circ + 120^\circ$$

($\alpha = 0$ in this case)

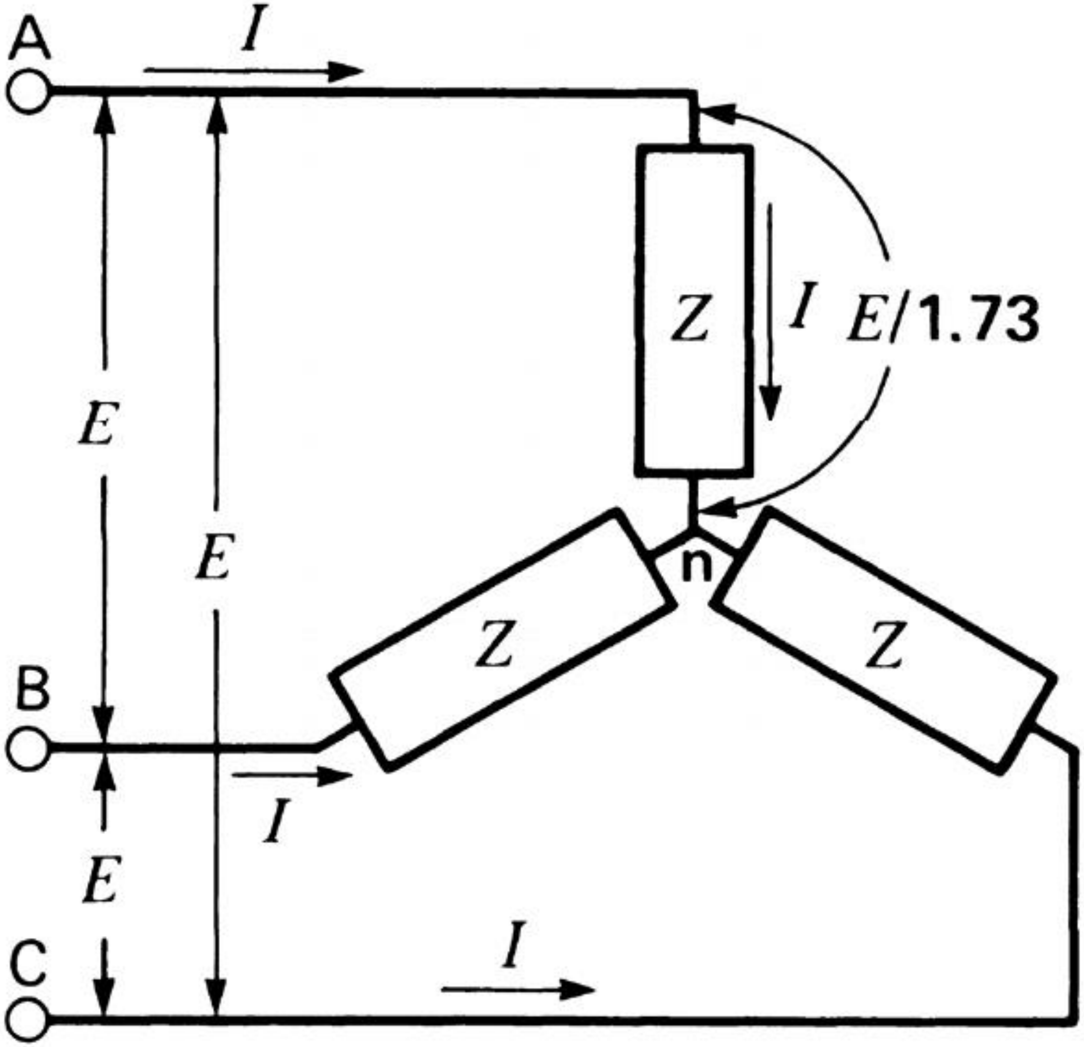
$$V_{ab} = V_{an} - V_{bn} = |V|(1\angle\alpha - 1\angle\alpha + 120^\circ)$$
$$= \sqrt{3}\,|V|\angle\alpha + 30^\circ$$
$$V_{bc} = \sqrt{3}\,|V|\angle\alpha - 90^\circ$$
$$V_{ca} = \sqrt{3}\,|V|\angle\alpha + 150^\circ$$

Line to line voltages are also balanced

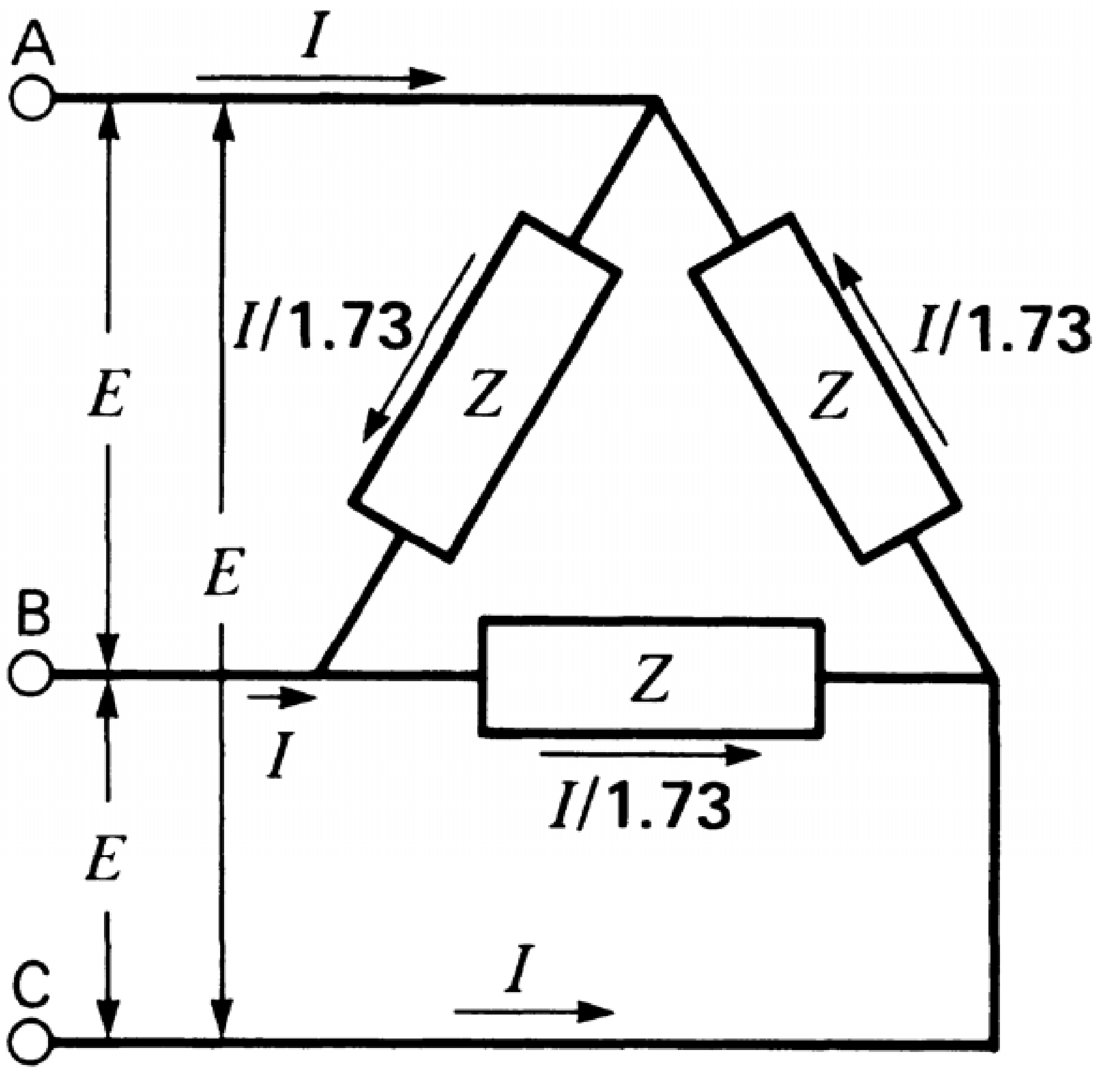
Example of Phase and Line Voltages



Y-Connected Load



Δ-Connected Load



Power in Balanced 3-Phase Circuits

- The real power, reactive power, apparent power, complex power and power factor are the same in each phase.

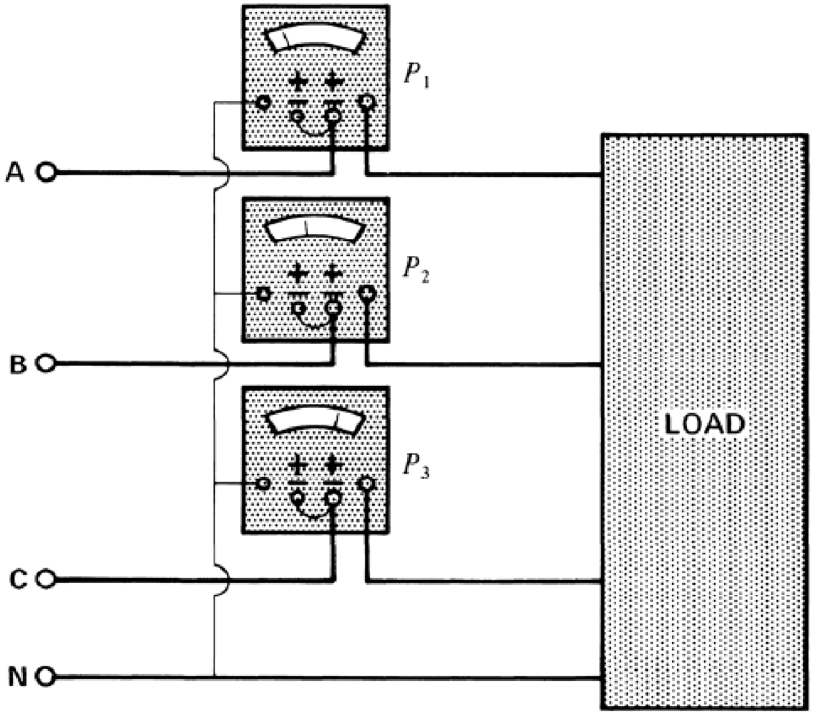
$$\boxed{P = 3V_p I\cos(\theta) = \sqrt{3}V_L I\cos(\theta)}$$

$$\boxed{Q = 3V_p I\sin(\theta) = \sqrt{3}V_L I\sin(\theta)}$$

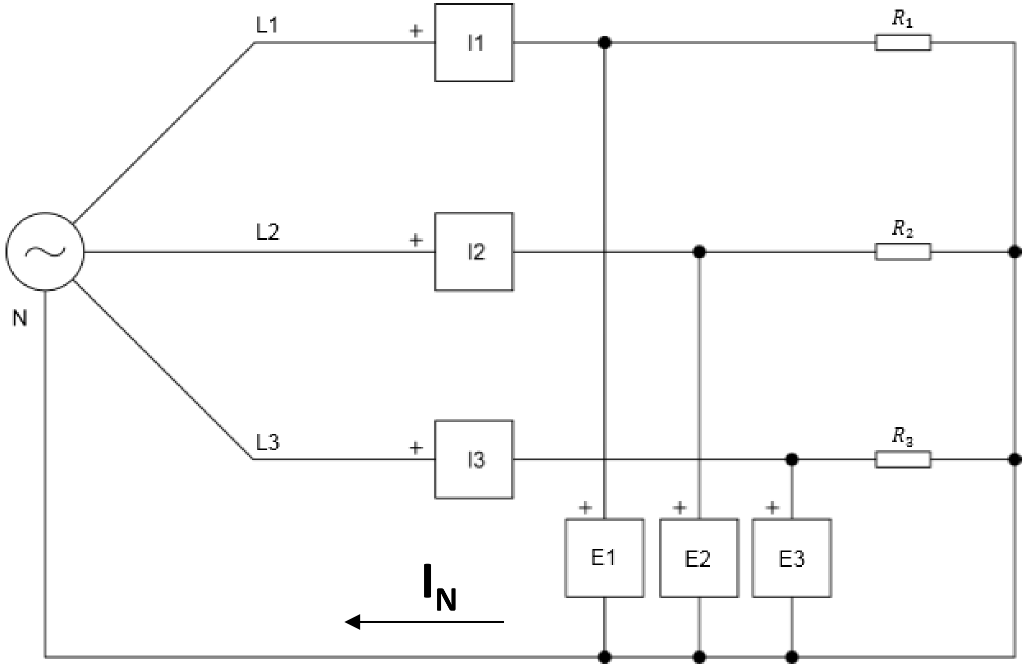
$$\boxed{S = 3V_p I = \sqrt{3}V_L I}$$

Power measurement in 3-phase 4-wire circuit

- If the load is balanced, then $P_1 = P_2 = P_3$. Hence, $P_T = 3P_1$
- If the load is unbalanced, $P_1 \neq P_2 \neq P_3$, Hence, $P_T = P_1 + P_2 + P_3$



Example: 1



The phase voltage of the positive-sequence balanced source is 120 V. Let R1 = 100 Ω, R2 = 200 Ω, R3 = 300 Ω.

Compute the real power supplied by each phase, the neutral current, and the ***current imbalance*** which is defined as the maximum percentage deviation of any pair of phase currents relative to the average of the three phases.

Answer:

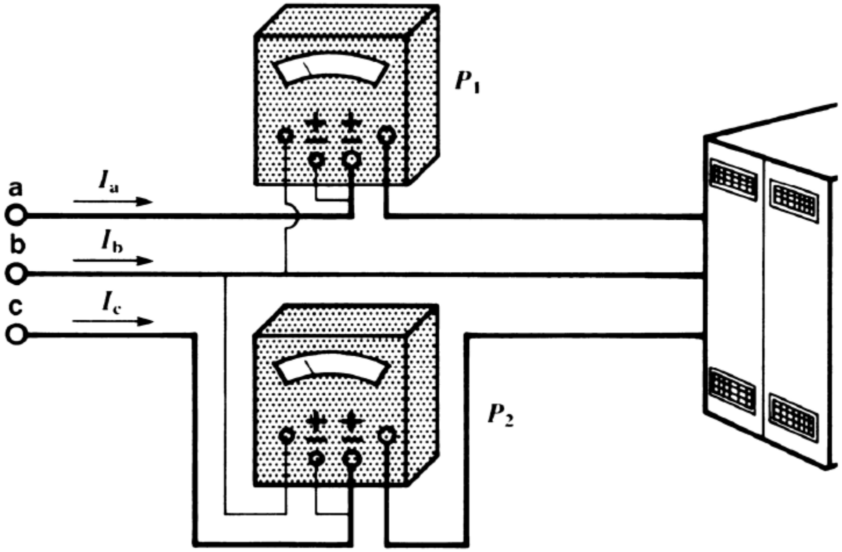
I1 = 1.2 A, I2 = 0.6 A, I3 = 0.4 A,

P1 = 144 W, P2 = 72 W, P3 = 48 W,

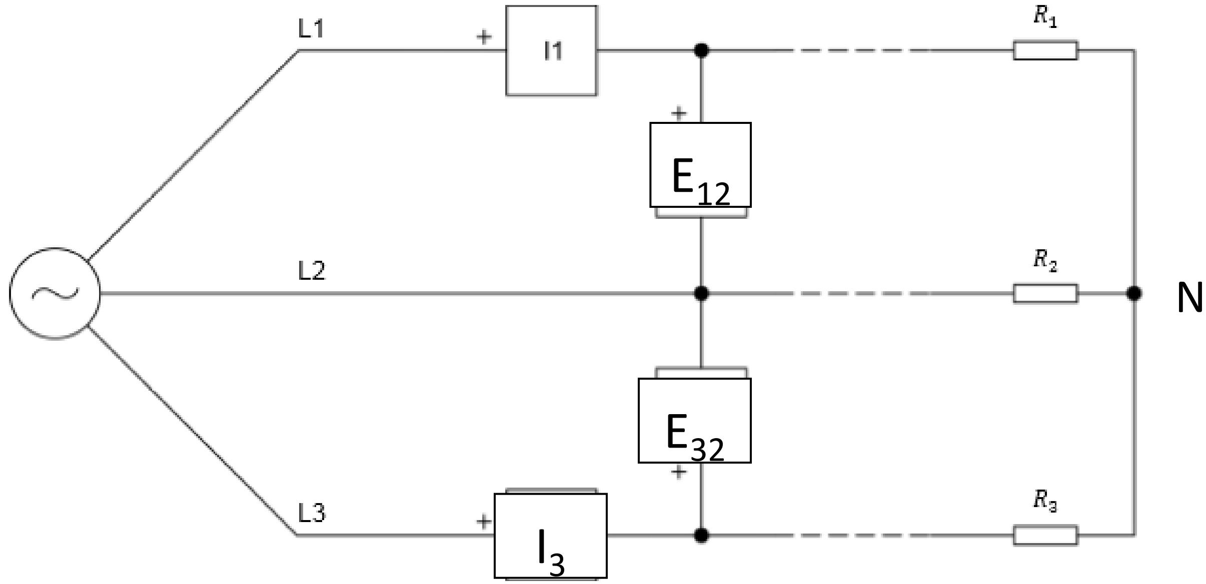
I_N = 0.72 A, I_{im}= 63.7 %

Power measurement in 3-phase 3-wire circuit: the 2-wattmeter method

- $P_T = P_1 + P_2$
- The load can be Y-connected, Δ-connected, balanced, or unbalanced.
- Any one of the 3 phases can be used as a reference.
- If the load is balanced, then $P_1 = P_2 = P_T/2$,



Example 2



The phase voltage of the positive-sequence balanced source is 120 V. Let R1 = 100 Ω, R2 = 200 Ω, R3 = 300 Ω.

Compute the real power supplied by each of the two wattmeters, the power supplied by each phase, and the ***current imbalance.***

Answer:

$E_1 = 120\angle 0^o$ V, $E_2 = 120\angle -120^o$ V, $E_3 = 120\angle +120^o$ V, E12 = $208\angle 30^o$ V, E32 = $208\angle 90^o$ V, $E_N = 39.33\angle -13.9^o$ V

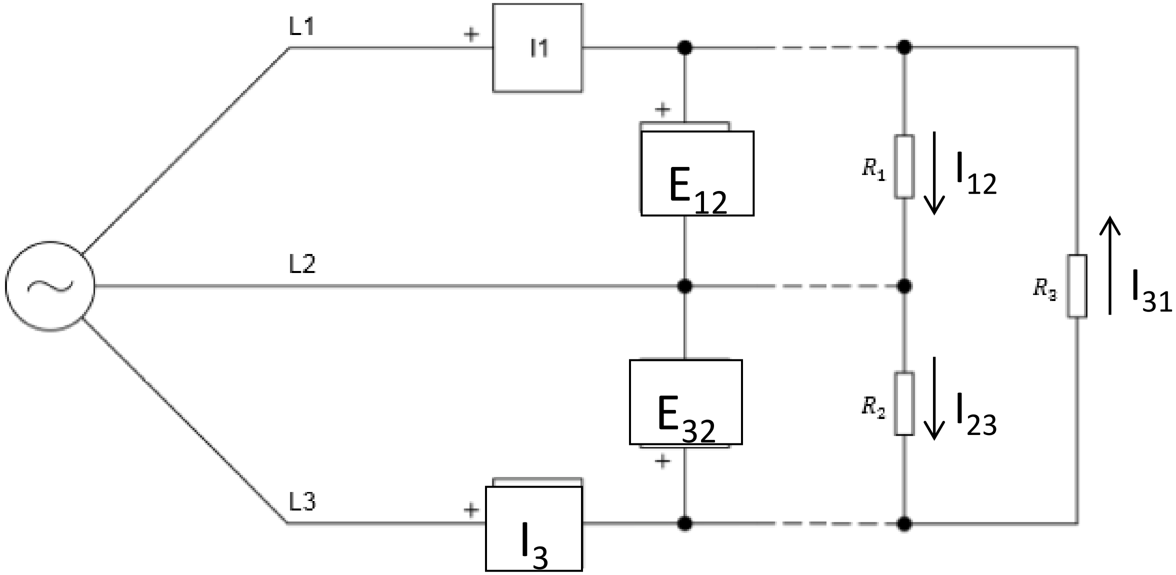
I1 = $0.823\angle 6.6^o$ A, I2 = $0.68\angle -135^o$ A, I3 = $0.50\angle 130.9^o$ A,

P12 = 157 W, P32 = 79 W, P_{tot} = 236 W.

P1 = 98 W, P2 = 79 W, P3 = 59 W, P_{tot} = 236 W

I_{im} = 25%

Example 3



The phase voltage of the positive-sequence balanced source is 120 V. Let R1 = 100 Ω, R2 = 200 Ω, R3 = 300 Ω.

Compute the real power measured by the 2 wattmeter method.

Answer:

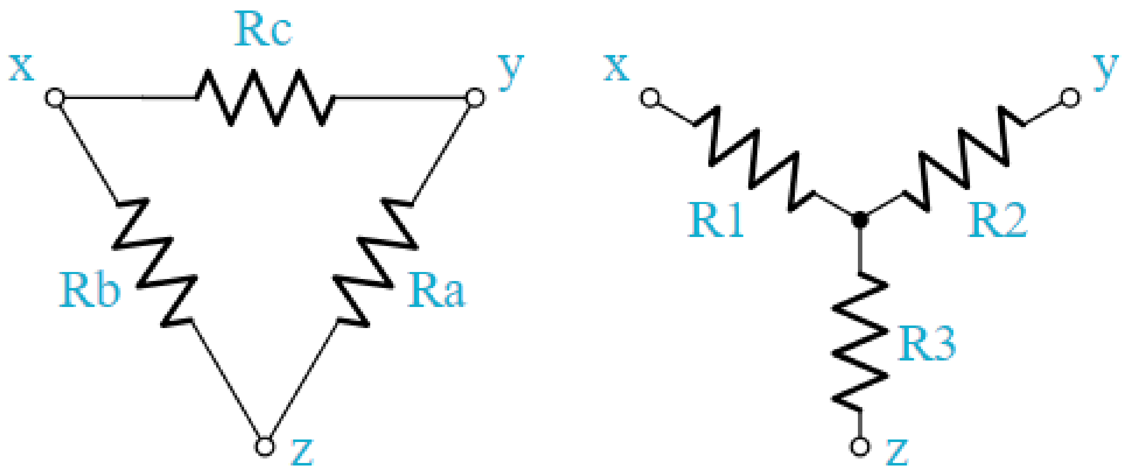
I12 = 2.08 ∟30° A, I23 = 1.04 ∟-90° A, I31 = 0.69 ∟150° A,

I1 = 2.5 ∟16.2° A, I3 = 1.51 ∟113.3° A,

P12 = 505 W, P32 = 288 W, P_{tot} = 793 W,

Check: P_{tot} = 208^2/100 + 208^2/200+ 208^2/300 = 793 W,

Delta-Wye Transformation



$\Delta \rightarrow Y$ transformation

$$R1 = \frac{Rb\,Rc}{Ra + Rb + Rc}$$

$$R2 = \frac{Ra\,Rc}{Ra + Rb + Rc}$$

$$R3 = \frac{Ra\,Rb}{Ra + Rb + Rc}$$

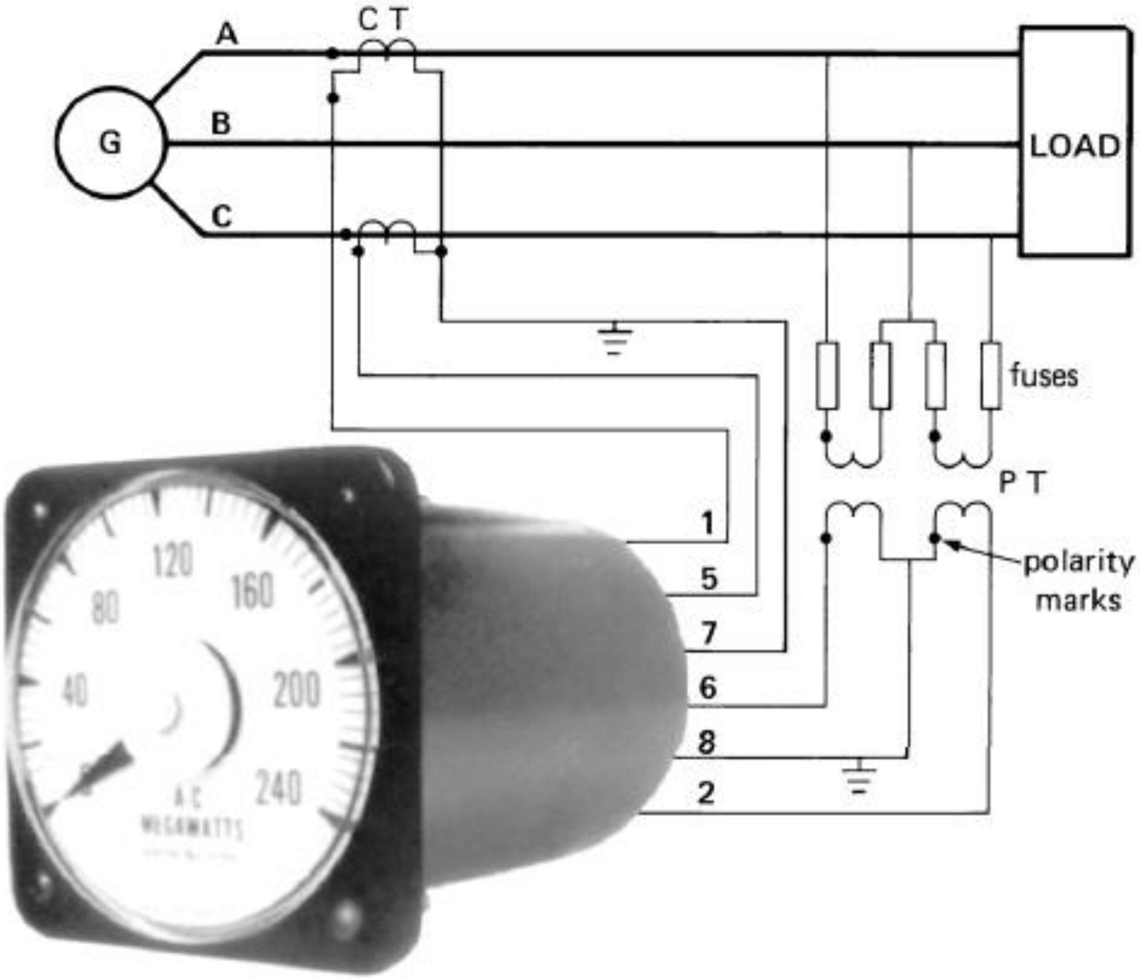
$Y \rightarrow \Delta$ transformation

$$Ra = \frac{R1\,R2 + R2\,R3 + R3\,R1}{R1}$$

$$Rb = \frac{R1\,R2 + R2\,R3 + R3\,R1}{R2}$$

$$Rc = \frac{R1\,R2 + R2\,R3 + R3\,R1}{R3}$$

Measuring active power in a high power circuit

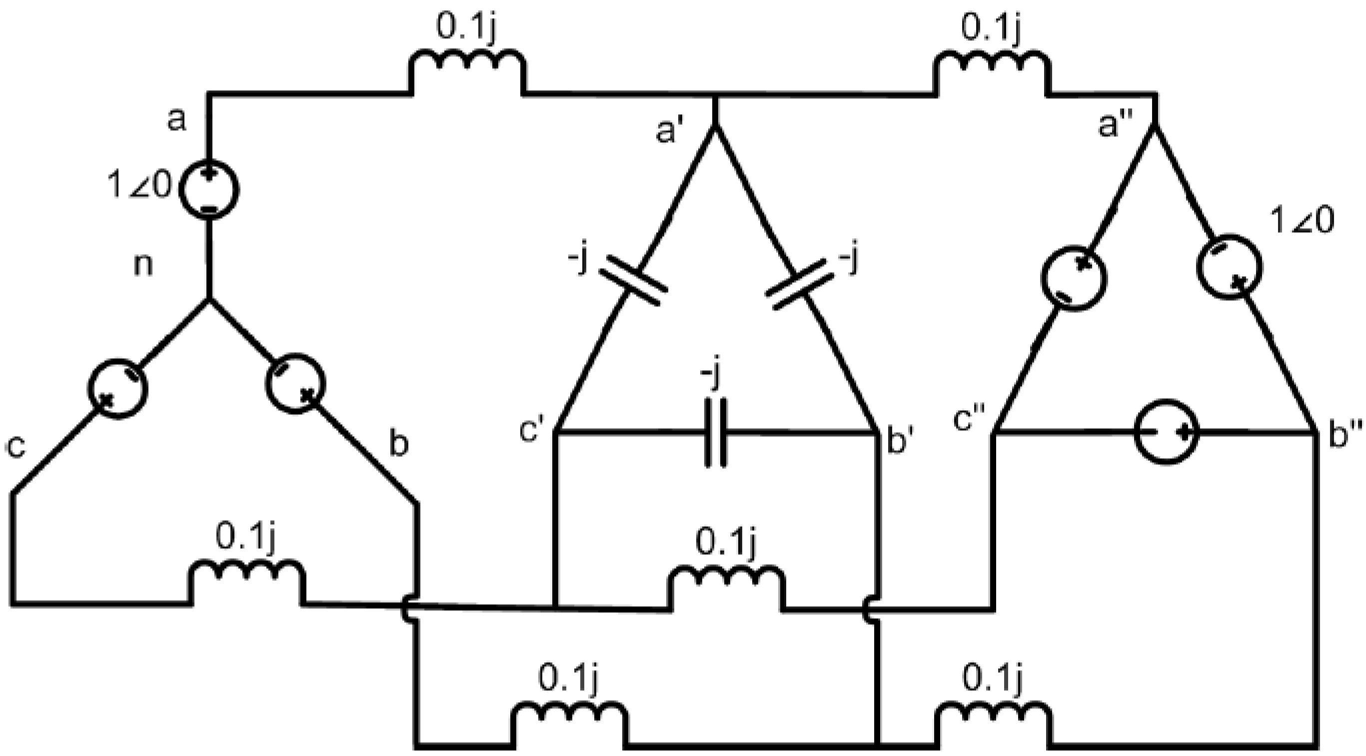


Per-Phase Analysis in Balanced 3-Phase Circuits

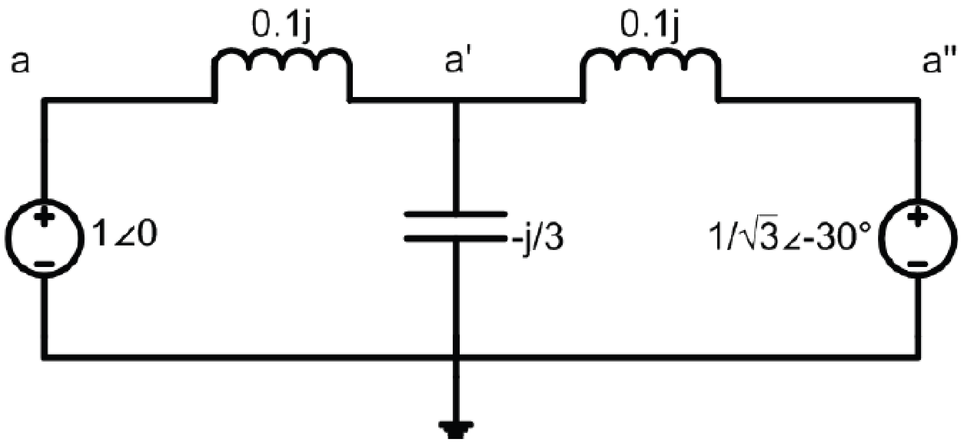
- Per phase analysis allows analysis of balanced 3ϕ systems with the same effort as for a single phase system
- **To do per phase analysis**
  1. Convert all 3ϕ load/sources to equivalent Y's
  2. Solve phase "a" independent of the other phases
  3. Total system power $S = 3\, V_a I_a^*$
  4. If desired, phase "b" and "c" values can be determined by inspection (i.e., ±120° degree phase shifts)
  5. If necessary, go back to original circuit to determine line-line values or internal 3ϕ values.

Example of per-phase analysis of balanced circuits

- Find the complex power supplied by each of the two sources.



Solution



To solve the circuit, write the KCL equation at a'

$$(V_a^{'}-1\angle 0)(-10j)+V_a^{'}(3j)+(V_a^{'}-\frac{1}{\sqrt{3}}\angle -30°)(-10j)=0$$

$$(10j+\frac{10}{\sqrt{3}}\angle 60°)=V_a^{'}(10j-3j+10j)$$

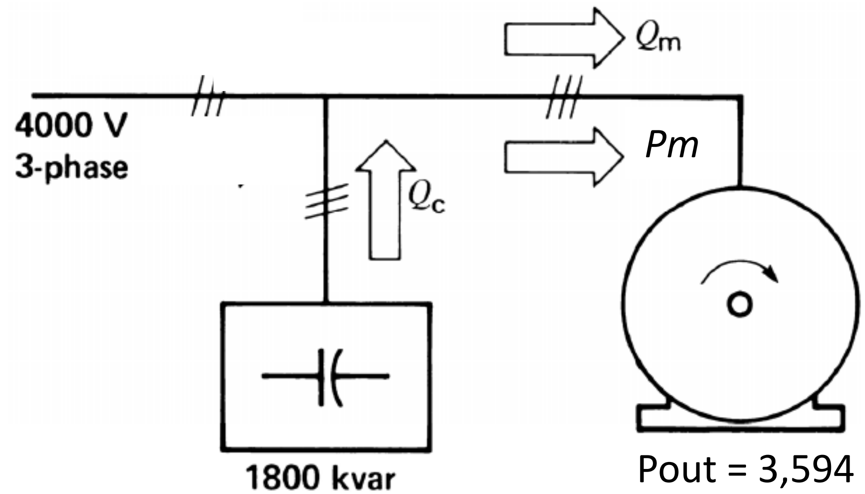
$$V_a^{'}=0.9\angle -10.9° \text{ volts} \qquad V_b^{'}=0.9\angle -130.9° \text{ volts}$$

$$V_c^{'}=0.9\angle 109.1° \text{ volts} \qquad V_{ab}^{'}=1.56\angle 19.1° \text{ volts}$$

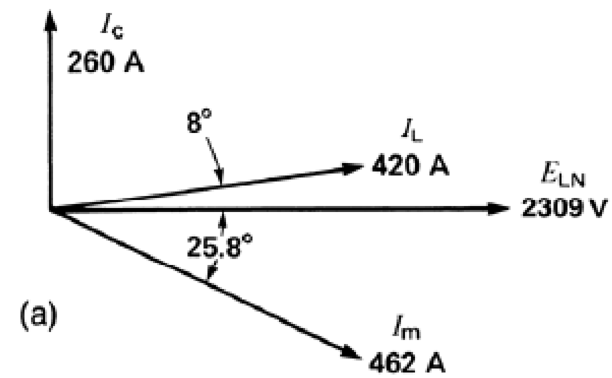
$$S_{Ygen}=3V_aI_a^*=V_a\left(\frac{V_a-V_a^{'}}{j0.1}\right)^*=5.1+j3.5 \text{ VA}$$

$$S_{\Delta gen}=3V_a^{''}\left(\frac{V_a^{''}-V_a^{'}}{j0.1}\right)^*=-5.1-j4.7 \text{ VA}$$

Example of 3-phase balanced circuit



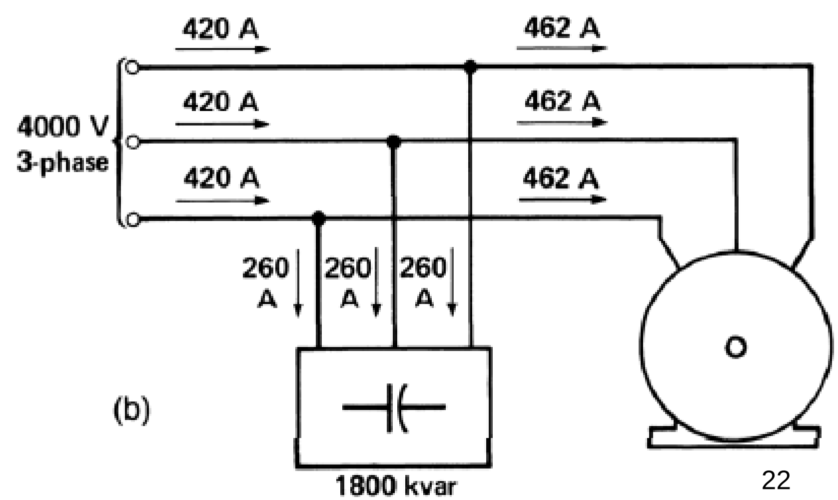
Pout = 3,594 hp
η = 93%
PF = 90 %



3-phase motor:
Pout = 3594x746 = 2,681 kW
Pm= Pout/0.93 = 2,883 kW
Sm = Pin/0.9 = 3,203 kVA
Qm= 1,395 kVAR
Im = Sm/(1.73x4000) = 462 A

3-phase capacitor bank:
Qc = 1,800 kVAR
Ic = Qc/(1.73x4000) = 260 A

Source Side:
Qs = Qm-Qc = -405 kVAR
Ss = 2,911 kVA
PFs = 99% (lead)
Is = Ss/(1,73x4000) = 420 A



Problems from Chap 2:

1, 2, 3, 4, 5, 6.

Practice problem

Consider the three-phase 3-wire load that is supplied by a balanced positive-sequence source where the rms values of phase voltages are equal to 120 V, with phase "a" voltage is taken as a reference. The load impedances across phases a-b, b-c, and c-a are purely resistive as follows: Rab = 60Ω, Rbc = 90 Ω, Rca = ∞ (i.e, open circuit).

1) Calculate the power measured by wattmeter P_1.

P1 = ……. W

2) Calculate the power measured by wattmeter P_2.

P2 = …….. W

3) Calculate the rms value of current I_b.

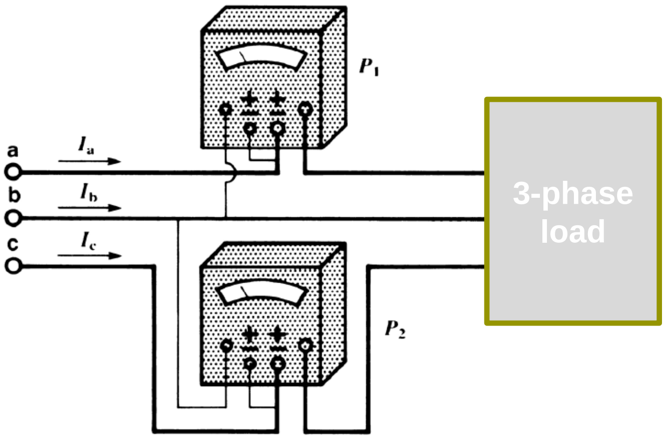
Ib = ….. A

4) Repeat the above when Rca is replaced by 120 Ω.

P1 = ………. W

P2 = ………. W

Ib = ………… A



Answer: (rounded to nearest integer)

1) P1 = 721 W, 2) P2 = 481 W, 3) Ib = 5 A, 4) P1 = 901 W, P2 = 661 W, Ib = 5 A

Homework Assignment # 2

A 3-phase power supply is feeding an unbalanced resistive load as shown below.

1) Determine the real and reactive power **supplied** by each phase.
2) It is desired to balance the load and eliminate any reactive power supplied or absorbed by the phase voltages. This can be achieved by installing a delta-connected bank of reactive elements in parallel with the load as shown (i.e., the source currents will be balanced and in-phase with their respective phase voltages. Calculate the values of such reactive elements.