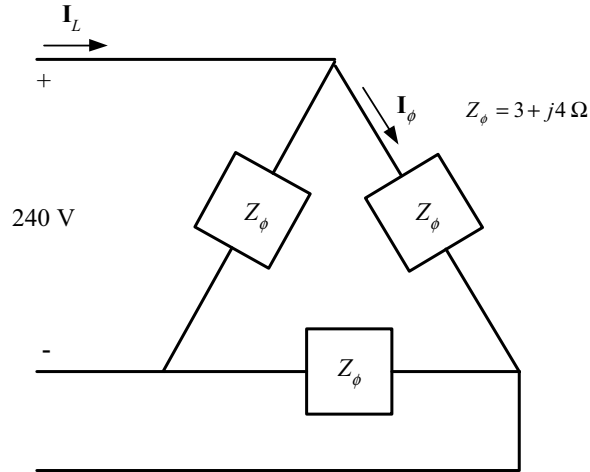


Chapter 2: Three-Phase Circuits

- 2-1. Three impedances of $4 + j3 \Omega$ are Δ -connected and tied to a three-phase 208-V power line. Find I_ϕ , I_L , P , Q , S , and the power factor of this load.

SOLUTION



Here, $V_L = V_\phi = 208 \text{ V}$, and $Z_\phi = 4 + j3 \Omega = 5 \angle 36.87^\circ \Omega$, so

$$I_\phi = \frac{V_\phi}{Z_\phi} = \frac{208 \text{ V}}{5 \Omega} = 41.6 \text{ A}$$

$$I_L = \sqrt{3} I_\phi = \sqrt{3} (41.6 \text{ A}) = 72.05 \text{ A}$$

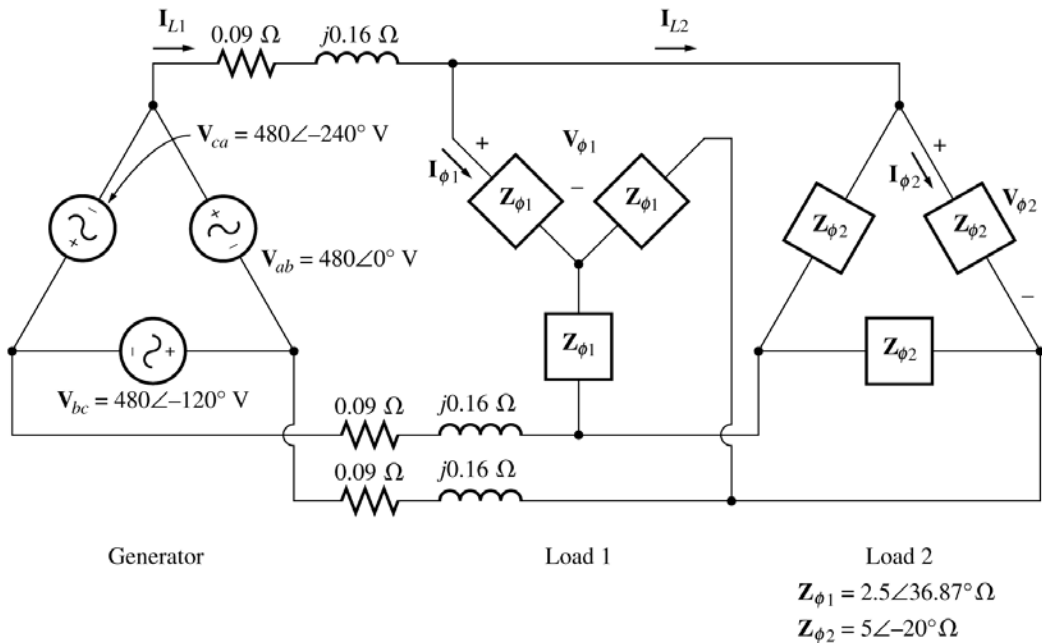
$$P = 3 \frac{V_\phi^2}{Z} \cos \theta = 3 \frac{(208 \text{ V})^2}{5 \Omega} \cos 36.87^\circ = 20.77 \text{ kW}$$

$$Q = 3 \frac{V_\phi^2}{Z} \sin \theta = 3 \frac{(208 \text{ V})^2}{5 \Omega} \sin 36.87^\circ = 15.58 \text{ kvar}$$

$$S = \sqrt{P^2 + Q^2} = 25.96 \text{ kVA}$$

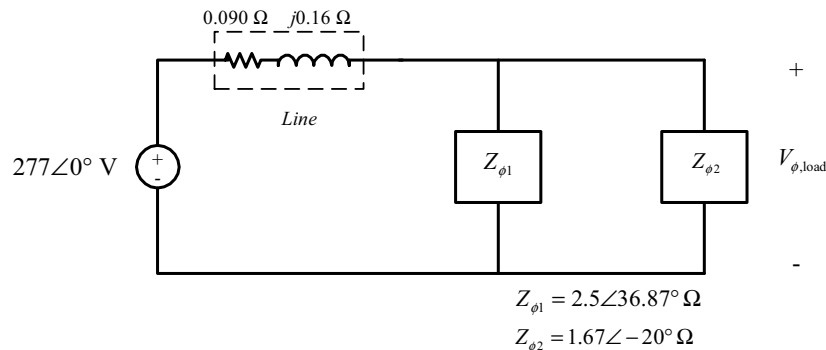
$$\text{PF} = \cos \theta = 0.8 \text{ lagging}$$

- 2-2. Figure P2-1 shows a three-phase power system with two loads. The Δ -connected generator is producing a line voltage of 480 V, and the line impedance is $0.09 + j0.16 \Omega$. Load 1 is Y-connected, with a phase impedance of $2.5 \angle 36.87^\circ \Omega$ and load 2 is Δ -connected, with a phase impedance of $5 \angle -20^\circ \Omega$.



- What is the line voltage of the two loads?
- What is the voltage drop on the transmission lines?
- Find the real and reactive powers supplied to each load.
- Find the real and reactive power losses in the transmission line.
- Find the real power, reactive power, and power factor supplied by the generator.

SOLUTION To solve this problem, first convert the two deltas to equivalent wyes, and get the per-phase equivalent circuit.



- The phase voltage of the equivalent Y-loads can be found by nodal analysis.

$$\frac{V_{\phi,load} - 277 \angle 0^\circ \text{ V}}{0.09 + j0.16 \Omega} + \frac{V_{\phi,load}}{2.5 \angle 36.87^\circ \Omega} + \frac{V_{\phi,load}}{1.67 \angle -20^\circ \Omega} = 0$$

$$(5.443 \angle -60.6^\circ)(V_{\phi,load} - 277 \angle 0^\circ \text{ V}) + (0.4 \angle -36.87^\circ)V_{\phi,load} + (0.6 \angle 20^\circ)V_{\phi,load} = 0$$

$$(5.955 \angle -53.34^\circ)V_{\phi,load} = 1508 \angle -60.6^\circ$$

$$V_{\phi,load} = 253.2 \angle -7.3^\circ \text{ V}$$

Therefore, the line voltage at the loads is $V_L \sqrt{3} V_\phi = 439 \text{ V}$.

(b) The voltage drop in the transmission lines is

$$\Delta \mathbf{V}_{\text{line}} = \mathbf{V}_{\phi, \text{gen}} - \mathbf{V}_{\phi, \text{load}} = 277 \angle 0^\circ \text{ V} - 253.2 \angle -7.3^\circ = 41.3 \angle 52^\circ \text{ V}$$

(c) The real and reactive power of each load is

$$P_1 = 3 \frac{V_\phi^2}{Z} \cos \theta = 3 \frac{(253.2 \text{ V})^2}{2.5 \Omega} \cos 36.87^\circ = 61.6 \text{ kW}$$

$$Q_1 = 3 \frac{V_\phi^2}{Z} \sin \theta = 3 \frac{(253.2 \text{ V})^2}{2.5 \Omega} \sin 36.87^\circ = 46.2 \text{ kvar}$$

$$P_2 = 3 \frac{V_\phi^2}{Z} \cos \theta = 3 \frac{(253.2 \text{ V})^2}{1.67 \Omega} \cos (-20^\circ) = 108.4 \text{ kW}$$

$$Q_2 = 3 \frac{V_\phi^2}{Z} \sin \theta = 3 \frac{(253.2 \text{ V})^2}{1.67 \Omega} \sin (-20^\circ) = -39.5 \text{ kvar}$$

(d) The line current is

$$\mathbf{I}_{\text{line}} = \frac{\Delta \mathbf{V}_{\text{line}}}{\mathbf{Z}_{\text{line}}} = \frac{41.3 \angle 52^\circ \text{ V}}{0.09 + j0.16 \Omega} = 225 \angle -8.6^\circ \text{ A}$$

Therefore, the losses in the transmission line are

$$P_{\text{line}} = 3 I_{\text{line}}^2 R_{\text{line}} = 3 (225 \text{ A})^2 (0.09 \Omega) = 13.7 \text{ kW}$$

$$Q_{\text{line}} = 3 I_{\text{line}}^2 X_{\text{line}} = 3 (225 \text{ A})^2 (0.16 \Omega) = 24.3 \text{ kvar}$$

(e) The real and reactive power supplied by the generator is

$$P_{\text{gen}} = P_{\text{line}} + P_1 + P_2 = 13.7 \text{ kW} + 61.6 \text{ kW} + 108.4 \text{ kW} = 183.7 \text{ kW}$$

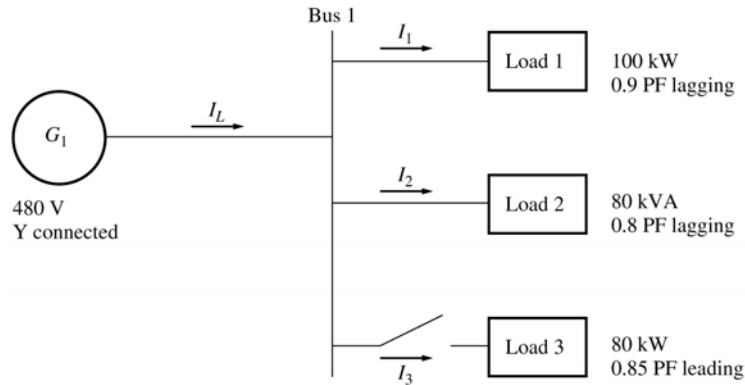
$$Q_{\text{gen}} = Q_{\text{line}} + Q_1 + Q_2 = 24.3 \text{ kvar} + 46.2 \text{ kvar} - 39.5 \text{ kvar} = 31 \text{ kvar}$$

The power factor of the generator is

$$\text{PF} = \cos \left[\tan^{-1} \frac{Q_{\text{gen}}}{P_{\text{gen}}} \right] = \cos \left[\tan^{-1} \frac{31 \text{ kvar}}{183.7 \text{ kW}} \right] = 0.986 \text{ lagging}$$

2-3. The figure shown below shows a one-line diagram of a simple power system containing a single 480 V generator and three loads. Assume that the transmission lines in this power system are lossless, and answer the following questions.

- Assume that Load 1 is Y-connected. What are the phase voltage and currents in that load?
- Assume that Load 2 is Δ -connected. What are the phase voltage and currents in that load?
- What real, reactive, and apparent power does the generator supply when the switch is open?
- What is the total line current I_L when the switch is open?
- What real, reactive, and apparent power does the generator supply when the switch is closed?
- What is the total line current I_L when the switch is closed?
- How does the total line current I_L compare to the sum of the three individual currents $I_1 + I_2 + I_3$? If they are not equal, why not?



SOLUTION Since the transmission lines are lossless in this power system, the full voltage generated by G_1 will be present at each of the loads.

(a) Since this load is Y-connected, the phase voltage is

$$V_{\phi 1} = \frac{480 \text{ V}}{\sqrt{3}} = 277 \text{ V}$$

The phase current can be derived from the equation $P = 3V_{\phi}I_{\phi} \cos \theta$ as follows:

$$I_{\phi 1} = \frac{P}{3V_{\phi} \cos \theta} = \frac{100 \text{ kW}}{3(277 \text{ V})(0.9)} = 133.7 \text{ A}$$

(b) Since this load is Δ -connected, the phase voltage is

$$V_{\phi 2} = 480 \text{ V}$$

The phase current can be derived from the equation $S = 3V_{\phi}I_{\phi}$ as follows:

$$I_{\phi 2} = \frac{S}{3V_{\phi}} = \frac{80 \text{ kVA}}{3(480 \text{ V})} = 55.56 \text{ A}$$

(c) The real and reactive power supplied by the generator when the switch is open is just the sum of the real and reactive powers of Loads 1 and 2.

$$P_1 = 100 \text{ kW}$$

$$Q_1 = P \tan \theta = P \tan(\cos^{-1} \text{PF}) = (100 \text{ kW})(\tan 25.84^\circ) = 48.4 \text{ kvar}$$

$$P_2 = S \cos \theta = (80 \text{ kVA})(0.8) = 64 \text{ kW}$$

$$Q_2 = S \sin \theta = (80 \text{ kVA})(0.6) = 48 \text{ kvar}$$

$$P_G = P_1 + P_2 = 100 \text{ kW} + 64 \text{ kW} = 164 \text{ kW}$$

$$Q_G = Q_1 + Q_2 = 48.4 \text{ kvar} + 48 \text{ kvar} = 96.4 \text{ kvar}$$

(d) The line current when the switch is open is given by $I_L = \frac{P}{\sqrt{3} V_L \cos \theta}$, where $\theta = \tan^{-1} \frac{Q_G}{P_G}$.

$$\theta = \tan^{-1} \frac{Q_G}{P_G} = \tan^{-1} \frac{96.4 \text{ kvar}}{164 \text{ kW}} = 30.45^\circ$$

$$I_L = \frac{P}{\sqrt{3} V_L \cos \theta} = \frac{164 \text{ kW}}{\sqrt{3} (480 \text{ V}) \cos(30.45^\circ)} = 228.8 \text{ A}$$

(e) The real and reactive power supplied by the generator when the switch is closed is just the sum of the real and reactive powers of Loads 1, 2, and 3. The powers of Loads 1 and 2 have already been calculated. The real and reactive power of Load 3 are:

$$P_3 = 80 \text{ kW}$$

$$Q_3 = P \tan \theta = P \tan(\cos^{-1} \text{PF}) = (80 \text{ kW})[\tan(-31.79^\circ)] = -49.6 \text{ kvar}$$

$$P_G = P_1 + P_2 + P_3 = 100 \text{ kW} + 64 \text{ kW} + 80 \text{ kW} = 244 \text{ kW}$$

$$Q_G = Q_1 + Q_2 + Q_3 = 48.4 \text{ kvar} + 48 \text{ kvar} - 49.6 \text{ kvar} = 46.8 \text{ kvar}$$

(f) The line current when the switch is closed is given by $I_L = \frac{P}{\sqrt{3} V_L \cos \theta}$, where $\theta = \tan^{-1} \frac{Q_G}{P_G}$.

$$\theta = \tan^{-1} \frac{Q_G}{P_G} = \tan^{-1} \frac{46.8 \text{ kvar}}{244 \text{ kW}} = 10.86^\circ$$

$$I_L = \frac{P}{\sqrt{3} V_L \cos \theta} = \frac{244 \text{ kW}}{\sqrt{3}(480 \text{ V}) \cos(10.86^\circ)} = 298.8 \text{ A}$$

(g) The total line current from the generator is 298.8 A. The line currents to each individual load are:

$$I_{L1} = \frac{P_1}{\sqrt{3} V_L \cos \theta_1} = \frac{100 \text{ kW}}{\sqrt{3}(480 \text{ V})(0.9)} = 133.6 \text{ A}$$

$$I_{L2} = \frac{S_2}{\sqrt{3} V_L} = \frac{80 \text{ kVA}}{\sqrt{3}(480 \text{ V})} = 96.2 \text{ A}$$

$$I_{L3} = \frac{P_3}{\sqrt{3} V_L \cos \theta_3} = \frac{80 \text{ kW}}{\sqrt{3}(480 \text{ V})(0.85)} = 113.2 \text{ A}$$

The sum of the three individual line currents is 343 A, while the current supplied by the generator is 298.8 A. These values are *not* the same, because the three loads have different impedance angles. Essentially, Load 3 is supplying some of the reactive power being consumed by Loads 1 and 2, so that it does not have to come from the generator.

2-4. Prove that the line voltage of a Y-connected generator with an *acb* phase sequence lags the corresponding phase voltage by 30° . Draw a phasor diagram showing the phase and line voltages for this generator.

SOLUTION If the generator has an *acb* phase sequence, then the three phase voltages will be

$$\mathbf{V}_{an} = V_\phi \angle 0^\circ$$

$$\mathbf{V}_{bn} = V_\phi \angle -240^\circ$$

$$\mathbf{V}_{cn} = V_\phi \angle -120^\circ$$

The relationship between line voltage and phase voltage is derived below. By Kirchhoff's voltage law, the line-to-line voltage \mathbf{V}_{ab} is given by

$$\mathbf{V}_{ab} = \mathbf{V}_a - \mathbf{V}_b$$

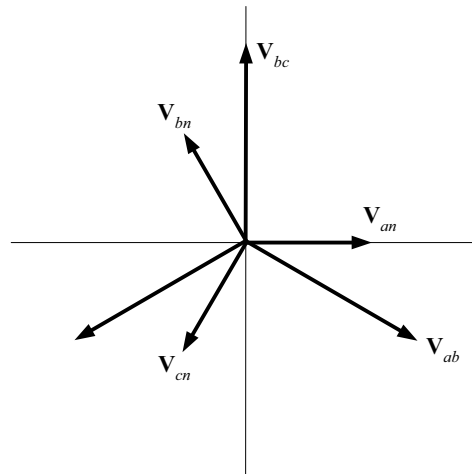
$$\mathbf{V}_{ab} = V_\phi \angle 0^\circ - V_\phi \angle -240^\circ$$

$$\mathbf{V}_{ab} = V_\phi - \left(-\frac{1}{2} V_\phi + j \frac{\sqrt{3}}{2} V_\phi \right) = \frac{3}{2} V_\phi - j \frac{\sqrt{3}}{2} V_\phi$$

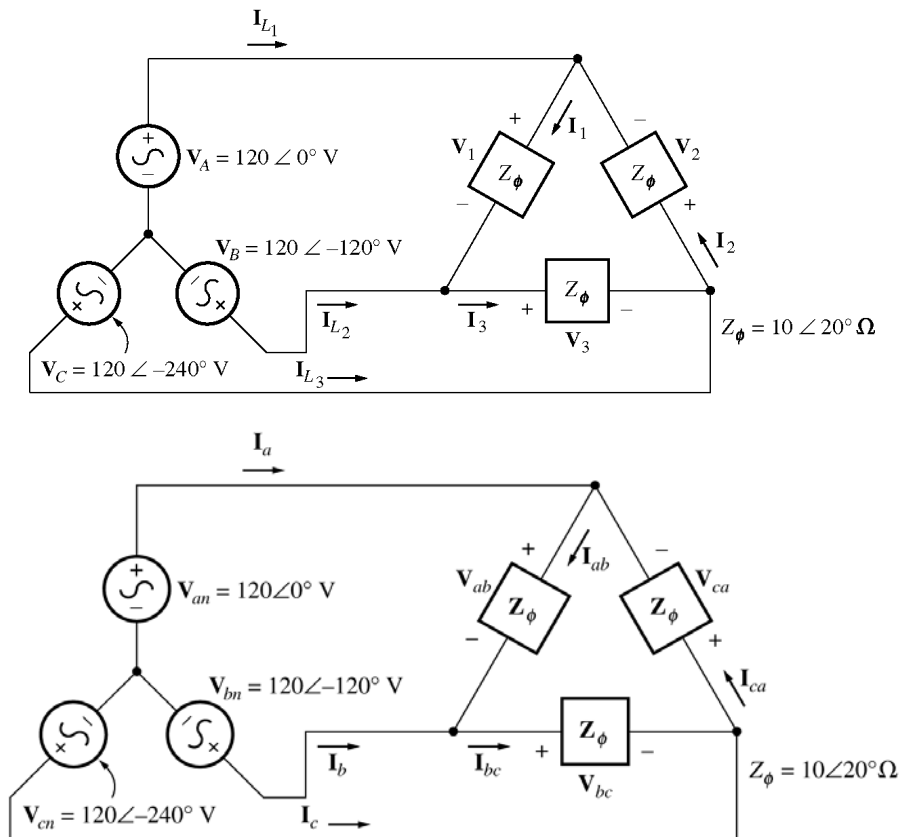
$$\mathbf{V}_{ab} = \sqrt{3} V_\phi \left(\frac{\sqrt{3}}{2} - j \frac{1}{2} \right)$$

$$\mathbf{V}_{ab} = \sqrt{3} V_\phi \angle -30^\circ$$

Thus the line voltage *lags* the corresponding phase voltage by 30° . The phasor diagram for this connection is shown below.



- 2-5. Find the magnitudes and angles of each line and phase voltage and current on the load shown in Figure P2-3.



SOLUTION Note that because this load is Δ -connected, the line and phase voltages are identical.

$$\mathbf{V}_{ab} = \mathbf{V}_{an} - \mathbf{V}_{bn} = 120\angle 0^\circ \text{ V} - 120\angle -120^\circ \text{ V} = 208\angle 30^\circ \text{ V}$$

$$\mathbf{V}_{bc} = \mathbf{V}_{bn} - \mathbf{V}_{cn} = 120\angle -120^\circ \text{ V} - 120\angle -240^\circ \text{ V} = 208\angle -90^\circ \text{ V}$$

$$\mathbf{V}_{ca} = \mathbf{V}_{cn} - \mathbf{V}_{an} = 120\angle -240^\circ \text{ V} - 120\angle 0^\circ \text{ V} = 208\angle 150^\circ \text{ V}$$

$$\mathbf{I}_{ab} = \frac{\mathbf{V}_{ab}}{Z_{\phi}} = \frac{208\angle 30^{\circ} \text{ V}}{10\angle 20^{\circ} \Omega} = 20.8\angle 10^{\circ} \text{ A}$$

$$\mathbf{I}_{bc} = \frac{\mathbf{V}_{bc}}{Z_{\phi}} = \frac{208\angle -90^{\circ} \text{ V}}{10\angle 20^{\circ} \Omega} = 20.8\angle -110^{\circ} \text{ A}$$

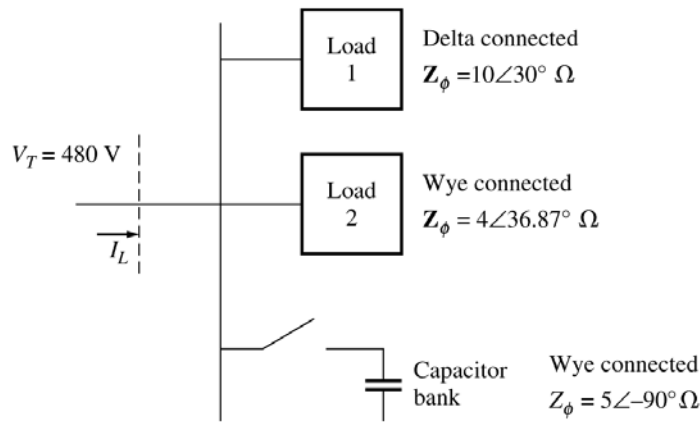
$$\mathbf{I}_{ca} = \frac{\mathbf{V}_{ca}}{Z_{\phi}} = \frac{208\angle 150^{\circ} \text{ V}}{10\angle 20^{\circ} \Omega} = 20.8\angle 130^{\circ} \text{ A}$$

$$\mathbf{I}_a = \mathbf{I}_{ab} - \mathbf{I}_{ca} = 20.8\angle 10^{\circ} \text{ A} - 20.8\angle 130^{\circ} \text{ A} = 36\angle -20^{\circ} \text{ A}$$

$$\mathbf{I}_b = \mathbf{I}_{bc} - \mathbf{I}_{ab} = 20.8\angle -110^{\circ} \text{ A} - 20.8\angle 10^{\circ} \text{ A} = 36\angle -140^{\circ} \text{ A}$$

$$\mathbf{I}_c = \mathbf{I}_{ca} - \mathbf{I}_{bc} = 20.8\angle 130^{\circ} \text{ A} - 20.8\angle -110^{\circ} \text{ A} = 36\angle 100^{\circ} \text{ A}$$

- 2-6. Figure P2-4 shows a small 480-V distribution system. Assume that the lines in the system have zero impedance.



- (a) If the switch shown is open, find the real, reactive, and apparent powers in the system. Find the total current supplied to the distribution system by the utility.
- (b) Repeat part (a) with the switch closed. What happened to the total current supplied? Why?

SOLUTION

- (a) With the switch open, the power supplied to each load is

$$P_1 = 3 \frac{V_{\phi}^2}{Z} \cos \theta = 3 \frac{(480 \text{ V})^2}{10 \Omega} \cos 30^{\circ} = 59.86 \text{ kW}$$

$$Q_1 = 3 \frac{V_{\phi}^2}{Z} \sin \theta = 3 \frac{(480 \text{ V})^2}{10 \Omega} \sin 30^{\circ} = 34.56 \text{ kvar}$$

$$P_2 = 3 \frac{V_{\phi}^2}{Z} \cos \theta = 3 \frac{(277 \text{ V})^2}{4 \Omega} \cos 36.87^{\circ} = 46.04 \text{ kW}$$

$$Q_2 = 3 \frac{V_{\phi}^2}{Z} \sin \theta = 3 \frac{(277 \text{ V})^2}{4 \Omega} \sin 36.87^{\circ} = 34.53 \text{ kvar}$$

$$P_{\text{TOT}} = P_1 + P_2 = 59.86 \text{ kW} + 46.04 \text{ kW} = 105.9 \text{ kW}$$

$$Q_{\text{TOT}} = Q_1 + Q_2 = 34.56 \text{ kvar} + 34.53 \text{ kvar} = 69.09 \text{ kvar}$$

The apparent power supplied by the utility is

$$S_{\text{TOT}} = \sqrt{P_{\text{TOT}}^2 + Q_{\text{TOT}}^2} = 126.4 \text{ kVA}$$

The power factor supplied by the utility is

$$\text{PF} = \cos \left[\tan^{-1} \frac{Q_{\text{TOT}}}{P_{\text{TOT}}} \right] = \cos \left[\tan^{-1} \frac{69.09 \text{ kvar}}{105.9 \text{ kW}} \right] = 0.838 \text{ lagging}$$

The current supplied by the utility is

$$I_L = \frac{P_{\text{TOT}}}{\sqrt{3} V_T \text{PF}} = \frac{105.9 \text{ kW}}{\sqrt{3} (480 \text{ V}) (0.838)} = 152 \text{ A}$$

(b) With the switch closed, P_3 is added to the circuit. The real and reactive power of P_3 is

$$P_3 = 3 \frac{V_\phi^2}{Z} \cos \theta = 3 \frac{(277 \text{ V})^2}{5 \Omega} \cos(-90^\circ) = 0 \text{ kW}$$

$$P_3 = 3 \frac{V_\phi^2}{Z} \sin \theta = 3 \frac{(277 \text{ V})^2}{5 \Omega} \sin(-90^\circ) = -46.06 \text{ kvar}$$

$$P_{\text{TOT}} = P_1 + P_2 + P_3 = 59.86 \text{ kW} + 46.04 \text{ kW} + 0 \text{ kW} = 105.9 \text{ kW}$$

$$Q_{\text{TOT}} = Q_1 + Q_2 + Q_3 = 34.56 \text{ kvar} + 34.53 \text{ kvar} - 46.06 \text{ kvar} = 23.03 \text{ kvar}$$

The apparent power supplied by the utility is

$$S_{\text{TOT}} = \sqrt{P_{\text{TOT}}^2 + Q_{\text{TOT}}^2} = 108.4 \text{ kVA}$$

The power factor supplied by the utility is

$$\text{PF} = \cos \left[\tan^{-1} \frac{Q_{\text{TOT}}}{P_{\text{TOT}}} \right] = \cos \left[\tan^{-1} \frac{23.03 \text{ kVAR}}{105.9 \text{ kW}} \right] = 0.977 \text{ lagging}$$

The current supplied by the utility is

$$I_L = \frac{P_{\text{TOT}}}{\sqrt{3} V_T \text{PF}} = \frac{105.9 \text{ kW}}{\sqrt{3} (480 \text{ V}) (0.977)} = 130.4 \text{ A}$$