EE 741

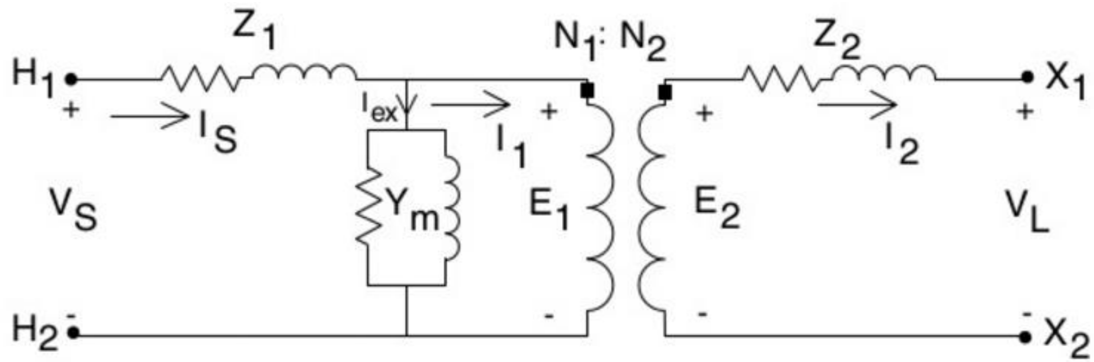
Power Transformers

Overview

- Two-winding transformer
- Autotransformers
- Three-phase transformer
  - Delta-Y-grounded
  - Ungrounded-Y-Delta
  - Y-grounded-Y-Grounded
  - Delta-Delta
  - Open-Y-Open-Delta

Two-winding transformer

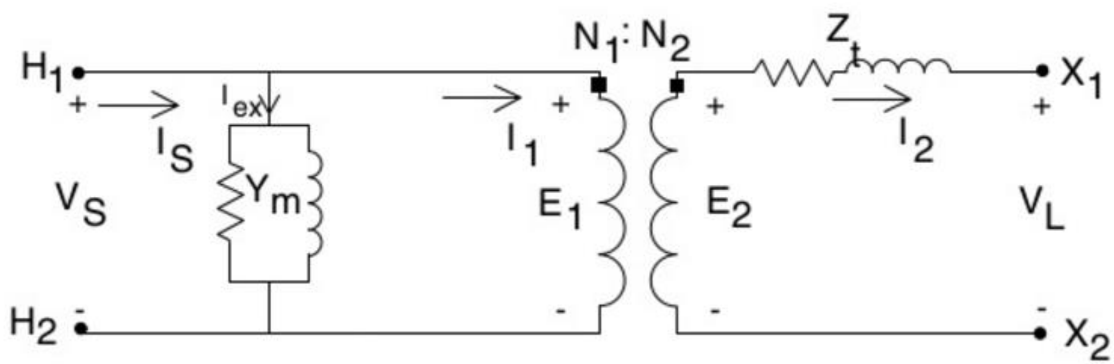
- Exact circuit



- Approximate ckt.

$$Z_t = n_t^2 \cdot Z_1 + Z_2$$

$$n_t = \frac{N_2}{N_1}$$



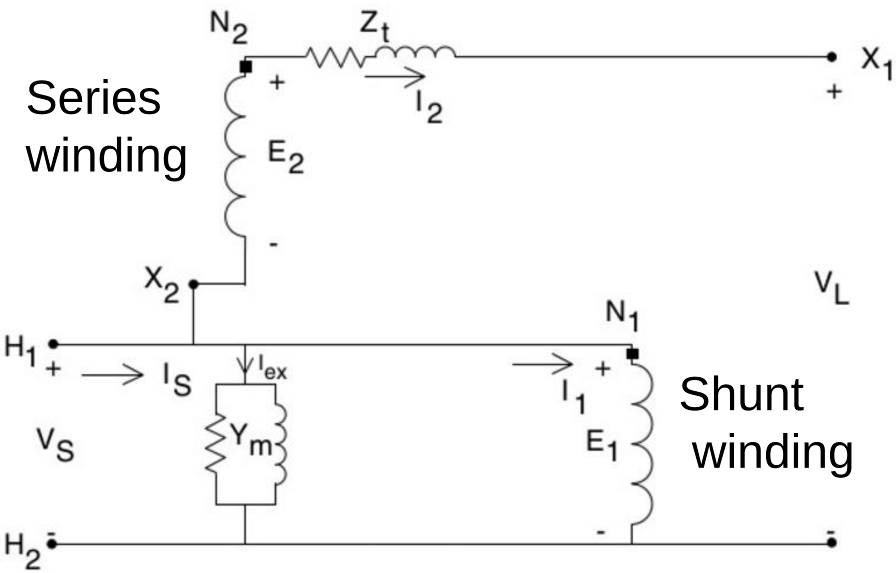
$$V_S = a \cdot V_L + b \cdot I_2$$

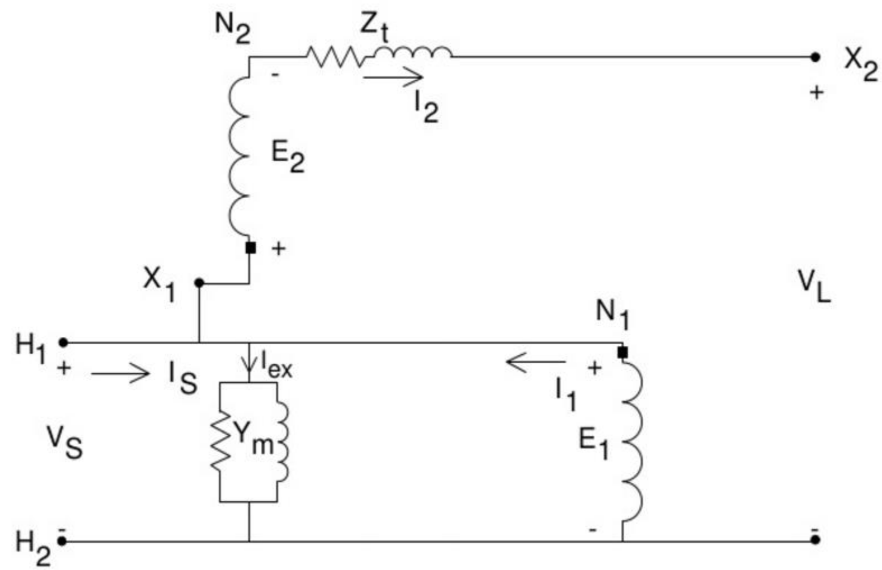
$$I_S = c \cdot V_L + d \cdot I_2$$

$$a = \frac{1}{n_t} \qquad c = \frac{Y_m}{n_t}$$

$$b = \frac{Z_t}{n_t} \qquad d = \frac{Y_m \cdot Z_t}{n_t} + n_t$$

Autotransformer





(+) sign for step-up, and (-) sign for step-down

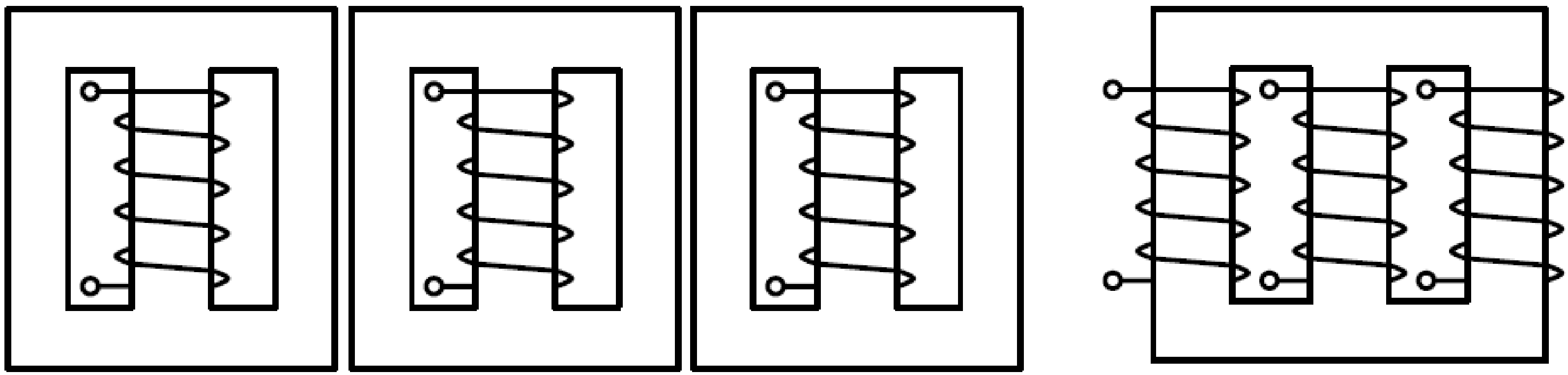
$$V_S = a \cdot V_L + b \cdot I_L$$

$$I_S = c \cdot V_L + d \cdot I_2$$

$$a = \frac{1}{1 \pm n_t} \qquad c = \frac{Y_m}{1 \pm n_t}$$

$$b = \frac{Z_t}{1 \pm n_t} \qquad d = \frac{Y_m \cdot Z_t}{1 \pm n_t} + 1 \pm n_t$$

Schematic winding structures of 3-phase transformers

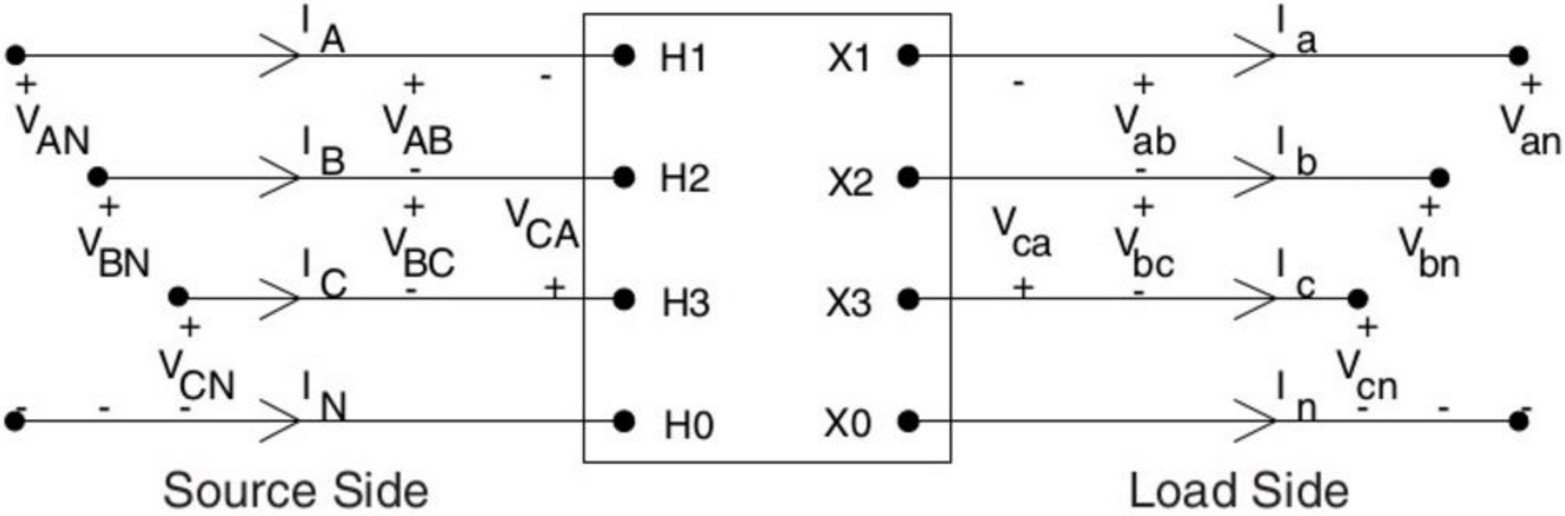


(a) (b)

Bank of 3 single-phase (triplex) core

3-legged stacked core

3-phase transformer



- Generalized matrices: $[VLN_{ABC}] = [a_t] \cdot [VLN_{abc}] + [b_t] \cdot [I_{abc}]$

$$[I_{ABC}] = [c_t] \cdot [VLN_{abc}] + [d_t] \cdot [I_{abc}]$$

- Phase shift in Y-Delta connection (American Standard),

Step-Down Connection

V_{AB} leads V_{ab} by 30 degrees

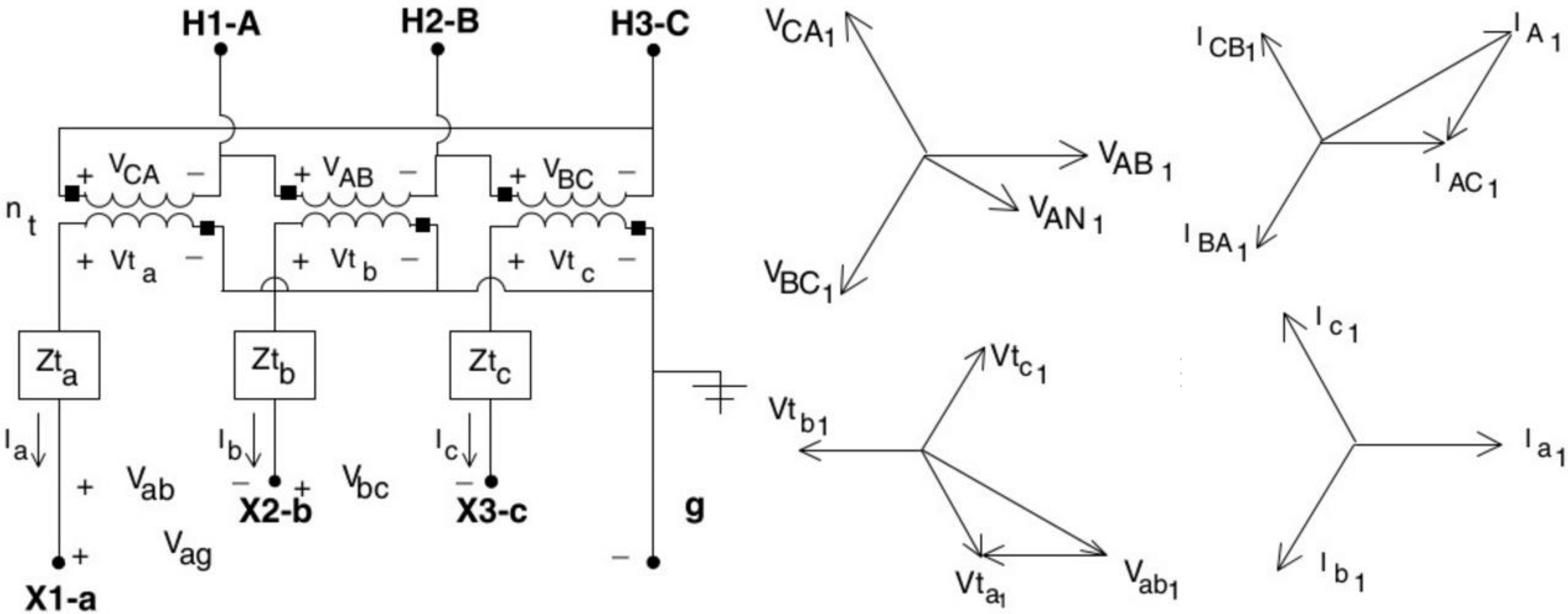
I_A leads I_a by 30 degrees

Step-Up Connection

V_{ab} leads V_{AB} by 30 degrees

I_a leads I_A by 30 degrees

Delta – Y-grounded step-down connection



Turn ratio in the following 3-phase configurations: $n_t = N_H/N_X$

Delta – Y-grounded step-down connection

- Matrices

(ignore shunt admittance)

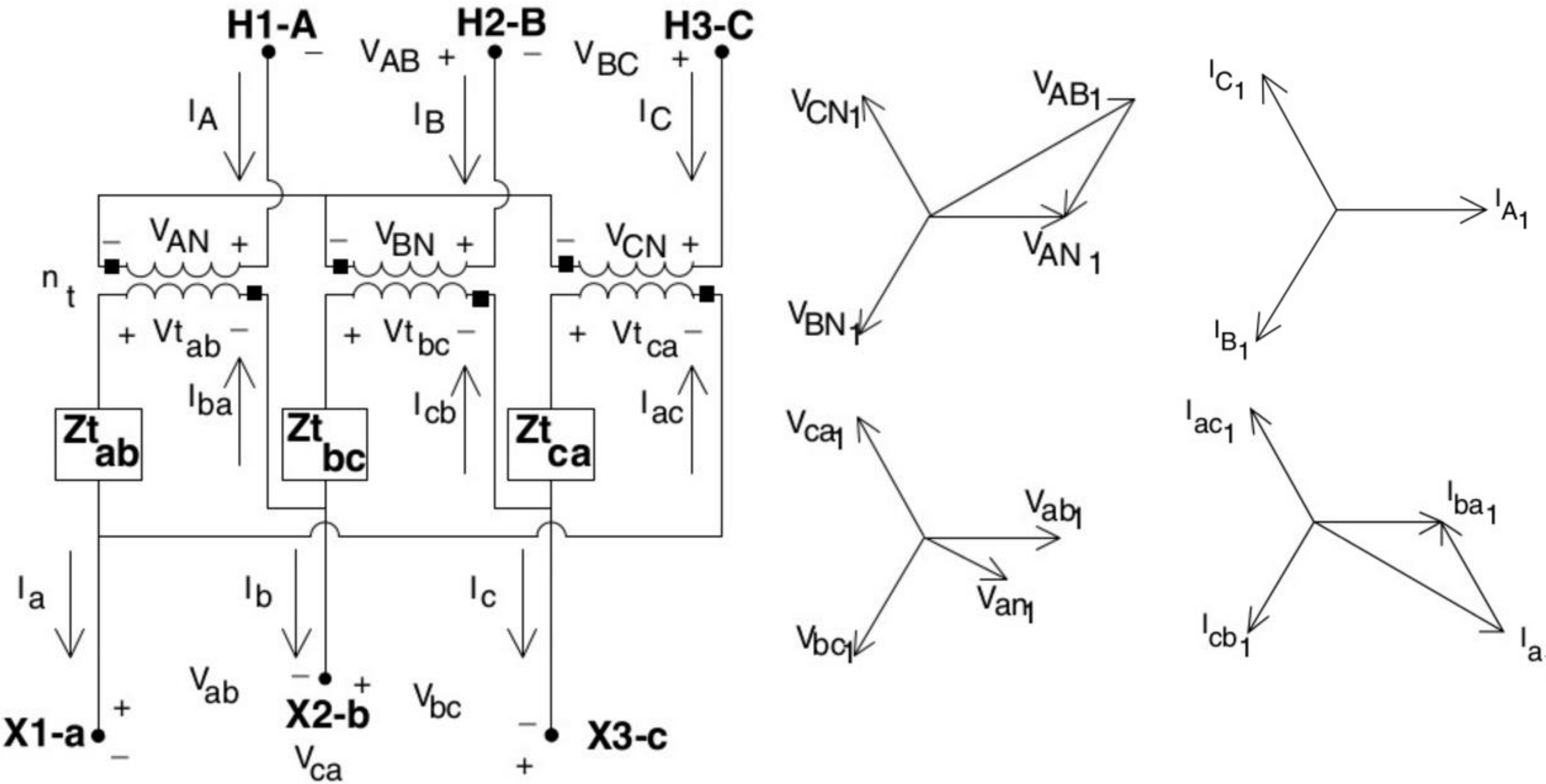
$$[a_t] = \frac{-n_t}{3} \cdot \begin{bmatrix} 0 & 2 & 1 \\ 1 & 0 & 2 \\ 2 & 1 & 0 \end{bmatrix}$$

$$[b_t] = \frac{-n_t}{3} \cdot \begin{bmatrix} 0 & 2 \cdot Zt_b & Zt_c \\ Zt_a & 0 & 2 \cdot Zt_c \\ 2 \cdot Zt_a & Zt_b & 0 \end{bmatrix}$$

$$[d_t] = \frac{1}{n_t} \cdot \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix}$$

$$[c_t] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Ungrounded-Y – Delta step-down connection



Ungrounded-Y – Delta step-down connection

- Matrices

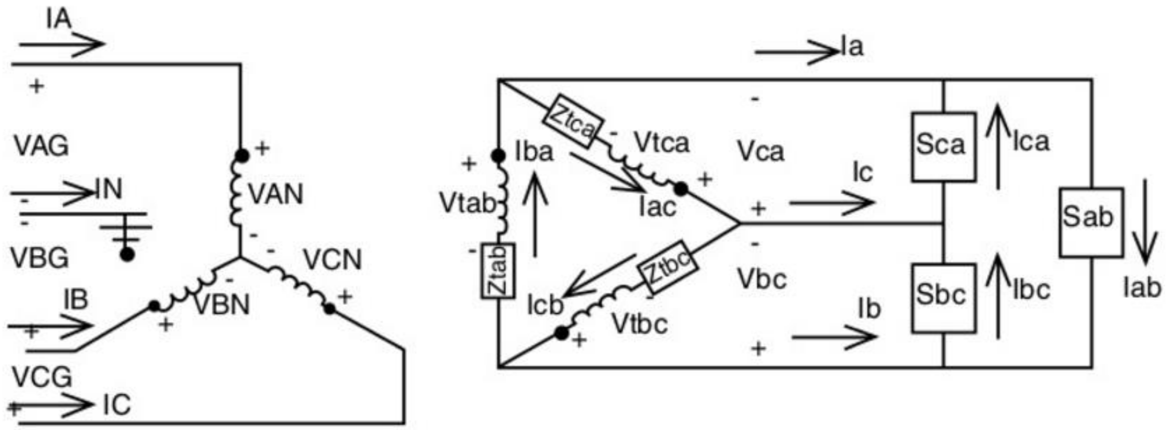
$$[a_t] = n_t \cdot \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix}$$

$$[b_t] = \frac{n_t}{3} \cdot \begin{bmatrix} Zt_{ab} & -Zt_{ab} & 0 \\ Zt_{bc} & 2 \cdot Zt_{bc} & 0 \\ -2 \cdot Zt_{ca} & -Zt_{ca} & 0 \end{bmatrix}$$

$$[c_t] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$[d_t] = \frac{1}{3 \cdot n_T} \cdot \begin{bmatrix} 1 & -1 & 0 \\ 1 & 2 & 0 \\ -2 & -1 & 0 \end{bmatrix}$$

Example 1



- Consider the following:
  - The load be unbalanced with Sab = 100 kVA @ .9 PF lag, and Sbc = Sca = 50 kVA @ 0.8 PF lag,
  - The voltage at the load is balanced at 240 V (line-to-line)
  - Transformer across a-b is rated at 100 kVA, 7200/240 V, Ztab = .01 +j.04 pu
  - Other transformers are rated at 50 kVA, 7200/240V, Ztbc=Ztca = .015+j.035 pu
- Compute a) the secondary line currents, (b) the primary line currents, c) the primary phase and line voltages, d) the kVA loading on each transformer.

Example 1 (Answer)

- a) $$[I_{abc}] = \begin{bmatrix} 522.9\,\underline{/-47.97} \\ 575.3\,\underline{/-119.06} \\ 360.8\,\underline{/53.13} \end{bmatrix} \text{ A}$$

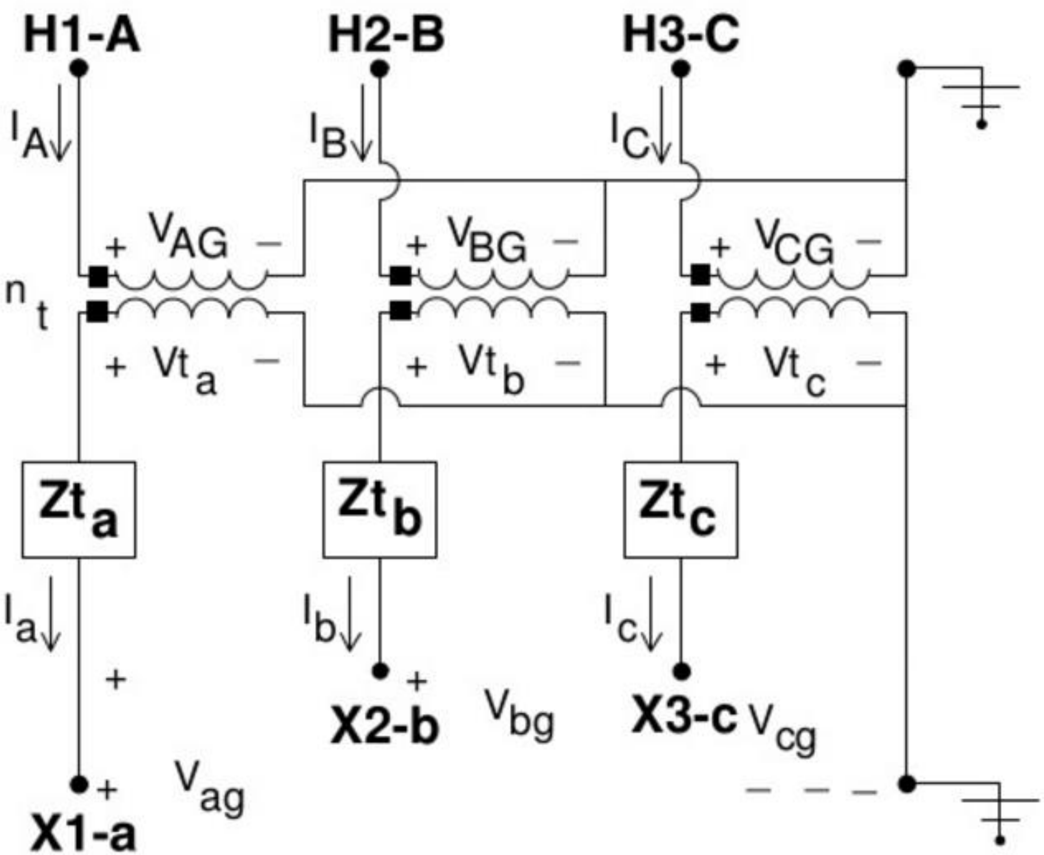
- b) $$[I_{ABC}] = [d_t]\cdot[I_{abc}] = \begin{bmatrix} 11.54\,\underline{/-28.04} \\ 8.95\,\underline{/-166.43} \\ 7.68\,\underline{/101.16} \end{bmatrix} \text{ A}$$

- c) $$[VLN_{abc}] = \begin{bmatrix} 138.56\,\underline{/-30} \\ 138.56\,\underline{/-150} \\ 138.56\,\underline{/90} \end{bmatrix} \qquad [VLN_{ABC}] = [a_t]\cdot[VLN_{abc}] + [b_t]\cdot[I_{abc}] = \begin{bmatrix} 7367.6\,\underline{/1.4} \\ 7532.3\,\underline{/-119.1} \\ 7406.2\,\underline{/121.7} \end{bmatrix} \text{ V}$$

- d) $$ST_i = \frac{VLN_{ABC_i}\cdot(I_{ABC_i})^*}{1000} = \begin{bmatrix} 85.02\,\underline{/29.46} \\ 67.42\,\underline{/47.37} \\ 56.80\,\underline{/20.58} \end{bmatrix} \text{ kVA} \qquad [VLL_{ABC}] = \begin{bmatrix} 12.94\,\underline{/31.54} \\ 12.88\,\underline{/-88.95} \\ 12.81\,\underline{/151.50} \end{bmatrix} \text{ kV}$$

$$[PF] = \begin{bmatrix} \cos(29.46) \\ \cos(47.37) \\ \cos(20.58) \end{bmatrix} = \begin{bmatrix} 87.1 \\ 67.7 \\ 93.6 \end{bmatrix} \text{ \% lagging}$$

Grounded-Y – grounded-Y step-down connection



No phase shift

Grounded-Y – grounded-Y step-down connection

- Matrices

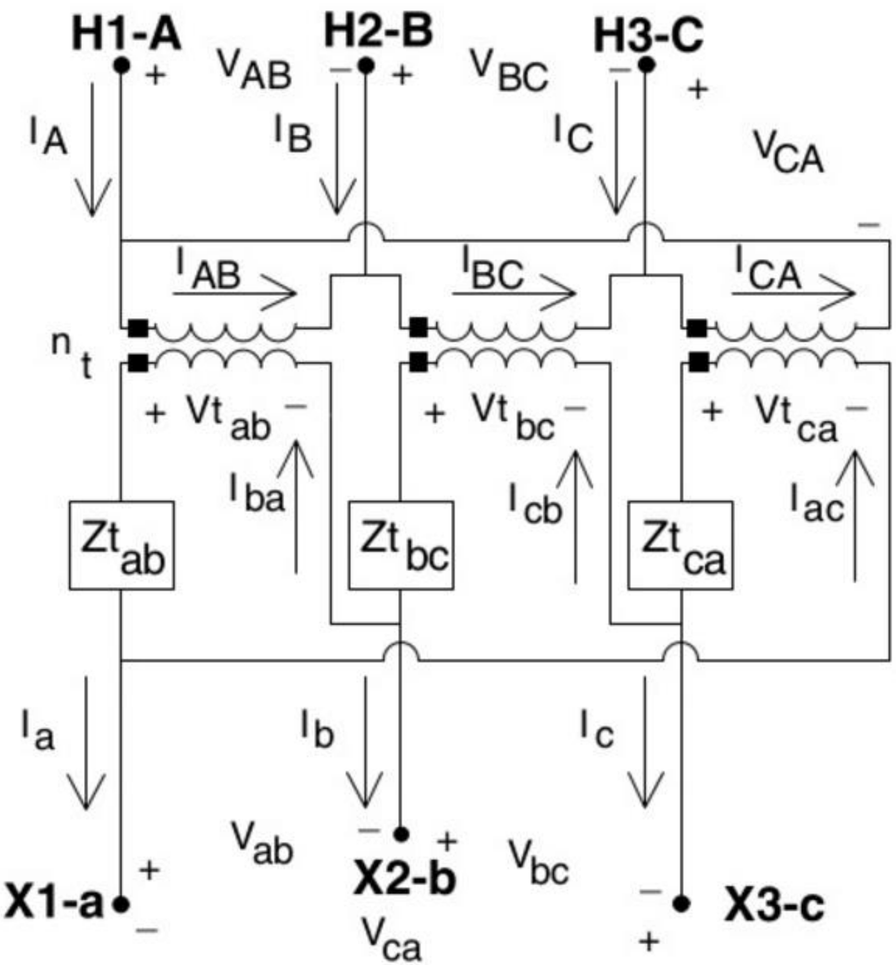
$$[a_t] = \begin{bmatrix} n_t & 0 & 0 \\ 0 & n_t & 0 \\ 0 & 0 & n_t \end{bmatrix}$$

$$[b_t] = \begin{bmatrix} n_t \cdot Zt_a & 0 & 0 \\ 0 & n_t \cdot Zt_b & 0 \\ 0 & 0 & n_t \cdot Zt_c \end{bmatrix}$$

$$[d_t] = \begin{bmatrix} \frac{1}{n_t} & 0 & 0 \\ 0 & \frac{1}{n_t} & 0 \\ 0 & 0 & \frac{1}{n_t} \end{bmatrix}$$

$$[c_t] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Delta-Delta Connection



No phase shift

Delta-Delta Connection

- Matrices

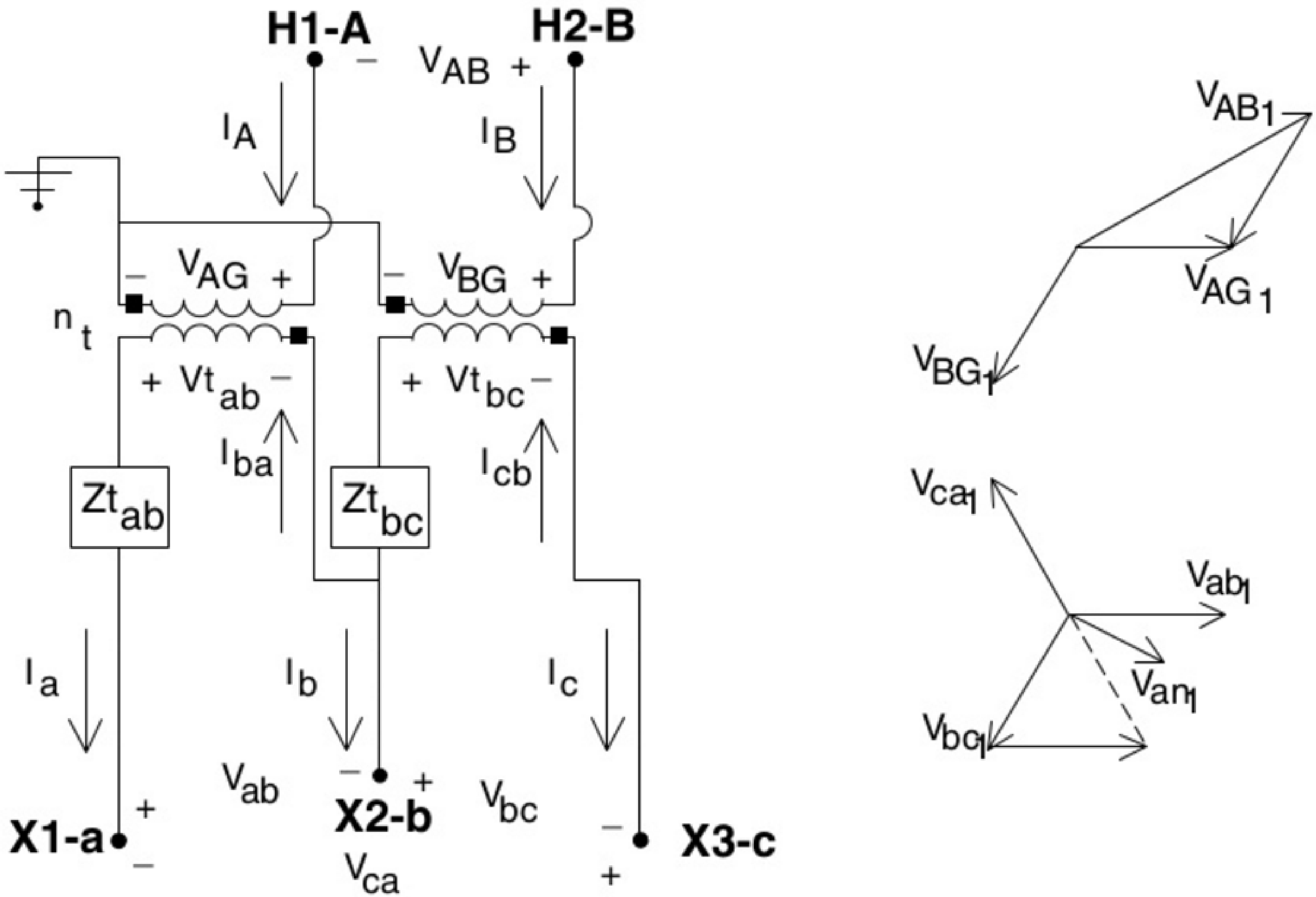
$$[a_t] = \frac{n_t}{3} \cdot \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}$$

$$[b_t] = \frac{1}{Zt_{ab} + Zt_{bc} + Zt_{ca}} \cdot \begin{bmatrix} Zt_{ca} & -Zt_{bc} & 0 \\ Zt_{ca} & Zt_{ab} + Zt_{ca} & 0 \\ -Zt_{ab} - Zt_{bc} & -Zt_{bc} & 0 \end{bmatrix} \begin{bmatrix} n_t \cdot Zt_a & 0 & 0 \\ 0 & n_t \cdot Zt_b & 0 \\ 0 & 0 & n_t \cdot Zt_c \end{bmatrix}$$

$$[d_t] = \begin{bmatrix} \frac{1}{n_t} & 0 & 0 \\ 0 & \frac{1}{n_t} & 0 \\ 0 & 0 & \frac{1}{n_t} \end{bmatrix}$$

$$[c_t] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Grounded open-Y – open Delta connection



Grounded-open-Y – open-Delta connection

- Matrices

$$[a_t] = \begin{bmatrix} n_t & -n_t & 0 \\ 0 & n_t & -n_t \\ 0 & 0 & 0 \end{bmatrix}$$

$$[b_t] = \begin{bmatrix} n_t \cdot Zt_{ab} & 0 & 0 \\ 0 & 0 & -n_t \cdot Zt_{bc} \\ 0 & 0 & 0 \end{bmatrix}$$

$$[d_t] = \begin{bmatrix} \frac{1}{n_t} & 0 & 0 \\ 0 & 0 & -\frac{1}{n_t} \\ 0 & 0 & 0 \end{bmatrix}$$

$$[c_t] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Example # 2

- Repeat Example 1 by using only phases A and B (i.e., removing one of the transformers and operating in open Y – open delta). As in Example 1, assume the voltage is balanced at the load terminals.
- Answer:

$$[VLN_{abc}] = \begin{bmatrix} 138.56\underline{/-30} \\ 138.56\underline{/-150} \\ 138.56\underline{/90} \end{bmatrix} \text{ V} \qquad [I_{abc}] = \begin{bmatrix} 522.93\underline{/-47.97} \\ 575.31\underline{/170.01} \\ 360.84\underline{/53.13} \end{bmatrix} \text{ A}$$

$$[VLG_{ABC}] = [a_t]\cdot[VLN_{abc}] + [b_t]\cdot[I_{abc}] = \begin{bmatrix} 7531\underline{/1.33} \\ 7449\underline{/-116.84} \\ 0 \end{bmatrix} \text{ V}$$

$$[I_{ABC}] = [d_t]\cdot[I_{abc}] = \begin{bmatrix} 17.43\underline{/-47.97} \\ 12.03\underline{/-126.87} \\ 0 \end{bmatrix} \text{ A}$$

$$ST_i = \left|\frac{VLG_i \cdot (I_{ABC_i})^*}{1000}\right| = \begin{bmatrix} 131.27 \\ 89.60 \\ 0 \end{bmatrix} \text{ kVA}$$

$$PF_i = \begin{bmatrix} 0.652 \\ 0.985 \\ 0 \end{bmatrix} \text{ lagging}$$

Transformer ratings: Voltage and Frequency

The **voltage** rating is used to protect the winding insulation from breakdown, and limit the magnetization current of the transformer.

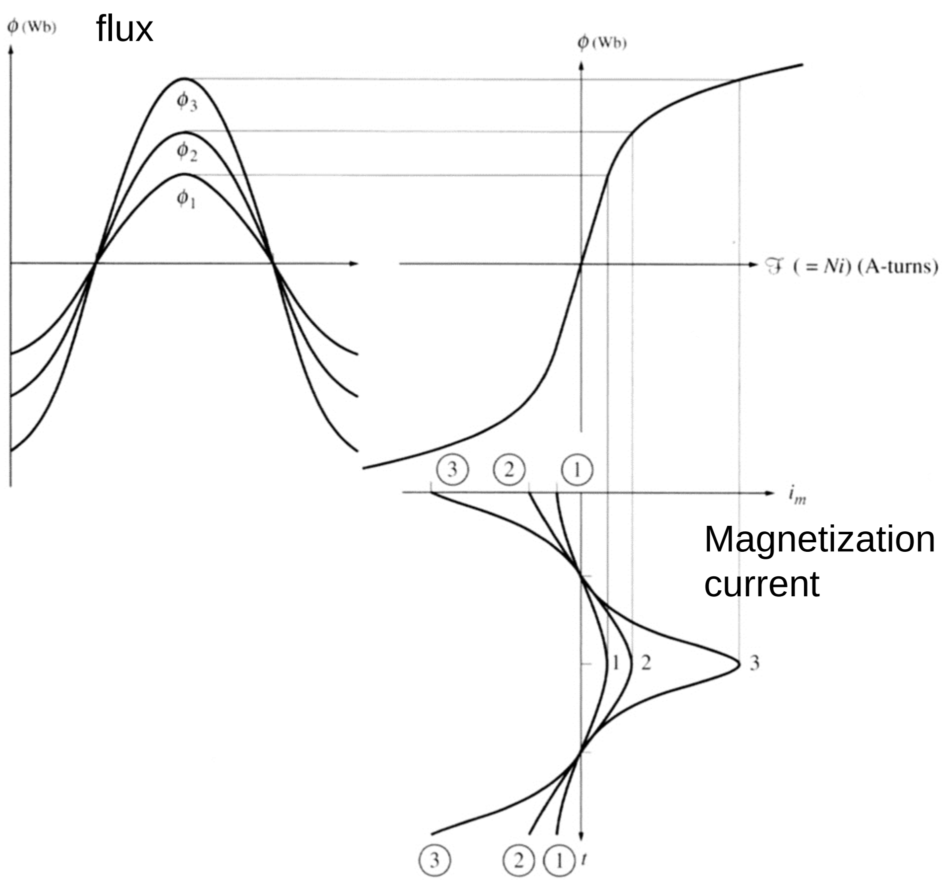
If a steady-state voltage

$$v(t) = V_M \sin \omega t$$

is applied to the transformer's primary winding, the transformer's flux will be

$$\phi(t) = \frac{1}{N_p}\int v(t)dt = -\frac{V_M}{\omega N_p}\cos \omega t$$

An increase in voltage will lead to a proportional increase in flux. However, after some point (in a saturation region). This lead to an unacceptable increase in magnetization current!



Transformer ratings: Voltage and Frequency

The maximum applied voltage (and thus the rated voltage) is set by the maximum acceptable magnetization current in the core.

The maximum flux is also related to the frequency:

$$\boxed{\phi_{\max} = \frac{V_{\max}}{\omega N_p}}$$

Therefore, to maintain the same maximum flux, a change in frequency (say, 50 Hz instead of 60 Hz) must be accompanied by the corresponding correction in the maximum allowed voltage. This reduction in applied voltage with frequency is called derating.

Transformer ratings: Apparent Power

The apparent power rating sets (together with the voltage rating) the current through the windings. The current determines the i^2R losses and, therefore, the heating of the coils. Remember, overheating shortens the life of transformer’s insulation!

In addition to apparent power rating for the transformer itself, additional higher rating(s) may be specified if a forced cooling is used. Under any circumstances, the temperature of the windings must be limited.

Transformer ratings: Current inrush

Assuming that the following voltage is applied to the transformer at the moment it is connected to the line:

$$v(t) = V_M \sin\left(\omega t + \theta\right)$$

The maximum flux reached on the first half-cycle depends on the phase of the voltage at the instant the voltage is applied. If the initial voltage is

$$v(t) = V_M \sin\left(\omega t + 90°\right) = V_M \cos \omega t$$

and the initial flux in the core is zero, the maximum flux during the first half-cycle is equals to the maximum steady-state flux (which is ok):

$$\phi_{\max} = \frac{V_M}{\omega N_p}$$

However, if the voltage's initial phase is zero, i.e.

$$v(t) = V_M \sin\left(\omega t\right)$$

Transformer ratings: Current inrush

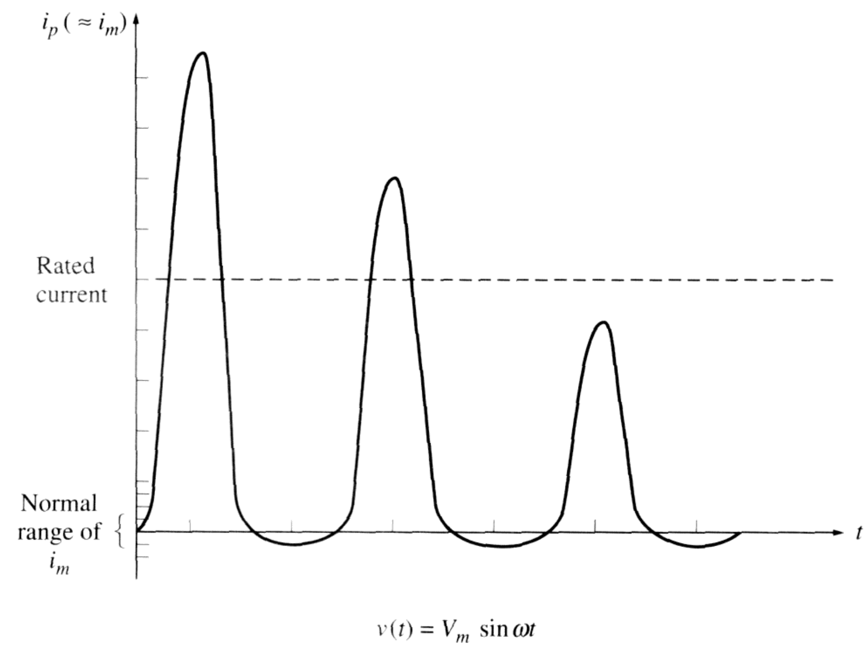
the maximum flux during the first half-cycle will be

$$\phi_{\max} = \frac{1}{N_p}\int_0^{\pi/\omega} V_M \sin(\omega t)\,dt = -\frac{V_M}{\omega N_p}\cos(\omega t)\Big|_0^{\pi/\omega} = \frac{2V_M}{\omega N_p}$$

Which is **twice** higher than a normal steady-state flux!

Doubling the maximum flux in the core can lead to saturation, thus may result in a huge magnetization current!

Normally, the voltage phase angle cannot be controlled, thus a large inrush current is possible during the first several cycles after turn ON.



Typical Transformer Data Sheet

45kVA HIGH EFFICIENCY K-RATED COPPER WOUND TRANSFORMER, LOW VOLTAGE DRY TYPE

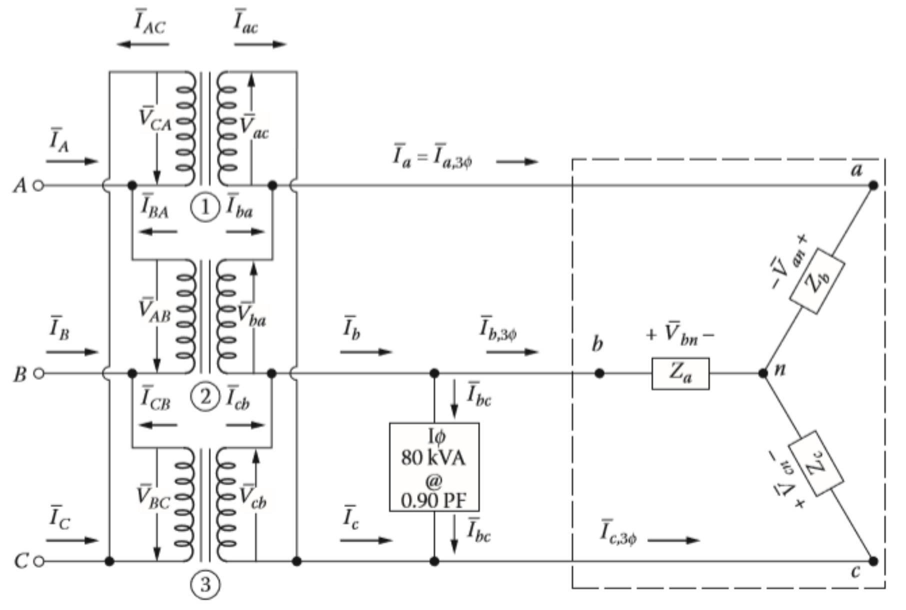
PARAMETER		DATA	UNITS
Electrical Rating	Power	45	kVA
	Frequency	60	Hz
	Primary	480 (Nominal)	Volts
	Primary Taps	2 x 2 ½ % FCAN, 4x 2 ½ % FCBN	%
	Secondary	208/120 (no load)	Volts
	Phase Shift	30 (Primary/Secondary)	Degrees
	K-Rating	7	K-Factor
Construction	Connection	Primary: 3 Ph 3-Wire, Secondary: 3 Ph. 4 -Wire	
	Neutral Bus Rating	200% of Line Current	
	Coils	Copper, Braised Internal Connections	
	Core	3-Leg, Very Low Loss Grain Oriented Steel	
	Insulation Class	220 (UL Listed)	
	Insulation	Nomex with Epoxy Co-Polymer impregnant	
	Impregnation / Properties	Epoxy Copolymer: Build > 2 mils @ 3.2 kV/mil (dielectric); H_2O absorption < 0.05%; Curing VOC < 1.65 lbs/gal.	

PARAMETER									UNITS
Losses	Linear Loading	no load	15%:	25%:	35%:	50%:	75%:	100%:	%
	Watt Loss	139	160	199	259	398	794	1,503	Watts
	Heat	474	546	679	884	1,358	2,709	5,128	BTU/hr

PARAMETER		DATA					UNITS
Efficiency	35% load per NEMA TP-2	98.3	(NEMA TP-1/CSA C802.3-01; requirement 97.7%) (DOE 10 CFR Part 430 CSL 3 requirement 98.3 %)				%
	Linear Loading	1/6: 97.6	¼: 98.2	½: 98.2	¾: 97.7	1/1: 96.7	
	Non-Linear Loading (K7)	1/6: 97.6	¼: 98.1	½: 98.0	¾: 97.2	1/1: 96.0	

PARAMETER			DATA	UNITS
Operation	Temperature Rise		< 130 (full linear load at nominal conditions)	°C
	Excitation		0.620	Amps
	Audible Noise		45 (per NEMA ST-20)	dBA
	Ambient		40 °C (per ANSI C57.96-01.100)	°C
	B.I.L.		10	kV
Abnormal	Short Circuit	Primary	1,380 (symmetrical output short circuit)	Amps
		Secondary	3,185 (asymmetrical L-N/G short circuit)	
		Inrush	300 (typical 3 cycle recovery)	
Impedance	Z: (+/- Sequence)		Z: 4.5 X: 3.1 R: 3.1	%

Assignment # 1



Consider 3 single-phase transformers that are connected in Δ-Δ as shown above.

- The 3-phase load is balanced with a rating of 200 kVA @ .8 PF lag, while the single-phase load is rated at 80 kVA @ 0.9 PF lag,
- The voltage at the load is balanced at 240 V (line-to-line)
- Transformer across b-c is rated at 100 kVA, 7620/240 V, and the other transformers are rated at 75 kVA, 7620/240V. The impedance of each transformer is .03+j.04 pu.

Compute the following:

- The secondary currents [I_{abc}] and primary currents [I_{ABC}],
- The primary phase voltages [VLN_{ABC}], and line voltages [VLL_{ABC}].
- The kVA loading on each transformer (hint: Vba + Vac + Vcb =0)