EE 741

Primary & Secondary Distribution Systems
Radial-Type Primary Feeder

- Most common, simplest and lowest cost
Example of Overhead Primary Feeder Layout
Example of Underground Primary Feeder Layout
Radial-Type Feeder with Tie and Sectionalizing Switches

- Higher service reliability
- Fast restoration of service by switching un-faulted sections of the feeder to adjacent primary feeders
Radial-Type Primary with Express Feeder
Radial-Type Phase Area Feeder
Loop Type Primary Feeder

- Loop tie disconnect switch normally open or normally closed
Primary Network

- Supplied by multiple substations
- Load supplied from multiple directions
- Highest reliability but more difficult to operate
Primary Voltage Levels: Factors Affecting Selection

- Voltage drops
- Feeder lengths
- Subtransmission voltage
- Load projection
- Power losses
- Equipment availability costs
- Adjacent substation and feeder voltages
- Company policies
Voltage Square Rule

\[ V_{L-N} = 1 \quad Z = 1 \quad V_{L-N} = \frac{IZ}{V_{L-N}} = \frac{(1)(1)}{1} = 1 \text{ pu} \]

(a) Base case

\[ V_{L-N} = 2 \quad Z = 4 \]

\[ I = \frac{1}{2} \quad \text{Voltage drop} = \frac{(\frac{1}{2})^4}{2} = 1 \text{ pu} \]

(b) Same kVA load but \( \left(\frac{V_2}{V_1}\right)^2 \) times the distance

\[ V_{L-N} = 2 \quad Z = 2 \]

\[ I = 1 \quad \text{Voltage drop} = \frac{(1)(2)}{2} = 1 \text{ pu} \]

(c) Double kVA and \( \frac{1}{2} \left(\frac{V_2}{V_1}\right)^2 \) times the distance
Factor Affecting Conductor Size Selection

- Load forecast
- Voltage drops
- Transformer rating
- Conductor rating
- Load growth rate
- Power losses
- Total cost
Tie Lines

• Provides emergency service from an adjacent feeder to customers thereby reducing outage time.
Voltage drop and Power Loss in Radial Feeder with Uniformly Distributed Load

\[ VD = \frac{1}{2} z l I_s \]
\[ P_{loss} = \frac{1}{3} r l I_s^2 \]
Derivation

\[ \frac{dI}{dx} = c = \frac{I_s}{l} \]

\[ I_x = I_s \left(1 - \frac{x}{l}\right) \]

\[ dV = I_x z \, dx = I_s z \left(1 - \frac{x}{l}\right) \, dx \]

\[ dP_{LS} = I_x^2 \, r \, dx = \left[I_s \left(1 - \frac{x}{l}\right)\right]^2 \, r \, dx \]

\[ VD_x = \int_{0}^{x} dV = I_s z x \left(1 - \frac{x}{2l}\right) \]

\[ VD_{x=l} = \frac{1}{2} z l I_s \]

\[ P_{LS,x} = \int_{0}^{x} dP_{LS} = r I_s \left(x + \frac{x^3}{3l^2} - \frac{x^2}{l}\right) \]

\[ P_{LS,x=l} = \frac{1}{3} r l I_s^2 \]
VD and Power Loss in Radial Feeder with Uniformly Increasing Load

\[ VD = \frac{2}{3} z l I_s \]

\[ P_{loss} = \frac{8}{15} r l I_s^2 \]
Radial Secondary Distribution System

- Common secondary main feeding a group of customers,
- In rural areas, a distribution transformer serves one customer.
Secondary Banking

- Secondary main served by multiple transformers (in parallel) that are fed from the same primary feeder.
- Improved voltage regulation and service reliability, reduced voltage dip
Secondary Network

- Meshed network that is powered by multiple feeders, through network-type transformers
- Fault current limiters, network protectors (i.e., air circuit breakers) with back-up fuses are used for secondary network protection.
Economic Design of Secondary Circuits

- Total Annual Cost (TAC) = annual installed cost of transformer, secondary cable, pole and hardware + annual operating cost of transformer (excitation current, core and copper loss) and cable.
- TAC is a function of the transformer and cable sizes.

\[ TAC = \sum IC_T + \sum IC_{SL} + \sum IC_{SD} + \sum IC_{PHI} + \sum OC_{exc} + \sum OC_{T, Fe} + \sum OC_{T, Cu} + \sum OC_{SL, Cu} + \sum OC_{SD, Cu}. \]
Physical Characteristics – Overhead lines

• An overhead line usually consists of three conductors or bundles of conductors containing the three phases of the power system.

• In overhead lines, the bare conductors are suspended from a pole or a tower via insulators. The conductors are usually made of aluminum cable steel reinforced (ACSR).
Physical Characteristics – underground cables

• Cable lines are designed to be placed underground or under water. The conductors are insulated from one another and surrounded by protective sheath.

• Cable lines are more expensive and harder to maintain. They also have capacitance problem – not suitable for long distance.
Electrical Characteristics

• Transmission/distribution lines are characterized by a series resistance, inductance, and shunt capacitance per unit length.

• These values determine the power-carrying capacity of the line and the voltage drop across it at full load.

\[ R_{DC} = \frac{\rho l}{A} \]
Cable resistance

- The resistivity increases linearly with temperature over normal range of temperatures.
- If the resistivity at one temperature and material temperature constant are known, the resistivity at another temperature can be found by

\[ \rho_{T_2} = \frac{M + T_2}{M + T_1} \rho_{T_1} \]

<table>
<thead>
<tr>
<th>Material</th>
<th>Resistivity at 20°C [Ω·m]</th>
<th>Temperature constant [°C]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Annealed copper</td>
<td>1.72·10⁻⁸</td>
<td>234.5</td>
</tr>
<tr>
<td>Hard-drawn copper</td>
<td>1.77·10⁻⁸</td>
<td>241.5</td>
</tr>
<tr>
<td>Aluminum</td>
<td>2.83·10⁻⁸</td>
<td>228.1</td>
</tr>
<tr>
<td>Iron</td>
<td>10.00·10⁻⁸</td>
<td>180.0</td>
</tr>
<tr>
<td>Silver</td>
<td>1.59·10⁻⁸</td>
<td>243.0</td>
</tr>
</tbody>
</table>
Cable Resistance

• AC resistance of a conductor is always higher than its DC resistance due to the skin effect forcing more current flow near the outer surface of the conductor. The higher the frequency of current, the more noticeable skin effect would be.

• Wire manufacturers usually supply tables of resistance per unit length at common frequencies (50 and 60 Hz). Therefore, the resistance can be determined from such tables.
Line inductance

- The series inductance of a transmission line consists of two components: internal and external inductances, which are due the magnetic flux inside and outside the conductor respectively.
- The inductance of a transmission line is defined as the number of flux linkages [Wb-turns] produced per ampere of current flowing through the line:

\[ L = \frac{\lambda}{I} \]

- The inductance of a single-phase transmission line is given by (see derivation in the book): \((r : \text{conductor radius} - \text{assumed solid}, D: \text{distance between cables}, \mu = 4\pi \times 10^{-7} \text{ H/m}, r' = r e^{-1/4} = .7788 r)\)

\[
\frac{I}{\pi} = \frac{\mu}{4} \left(1 + \ln \frac{D}{r} \right) \quad [H/m] \\
L = 4 \times 10^{-7} \ln \left(\frac{D}{r'}\right) \quad \text{H/m}
\]
Remarks on line inductance

• The greater the spacing between the phases of a transmission line, the greater the inductance of the line.
  – Since the phases of a high-voltage overhead line must be spaced further apart to ensure proper insulation, a high-voltage line will have a higher inductance than a low-voltage line.
  – Since the spacing between lines in buried cables is very small, series inductance of cables is much smaller than the inductance of overhead lines.

• The greater the radius of the conductors, the lower the inductance of the line. In practical transmission lines, instead of using heavy and inflexible conductors of large radii, two and more conductors are bundled together to approximate a large diameter conductor, and reduce corona loss.
Per-Phase Inductance of 3-phase power line

\[ L = 2 \times 10^{-7} \ln \left( \frac{GMD}{GMR} \right) \text{ H/m} \]

where the Geometric Mean Distance (GMD) is defined by

\[ GMD = \sqrt[3]{D_1 D_2 D_3} \]

where D1, D2, and D3 are the distances between the 3 conductors. The Geometric Mean Radius (GMR) is supplied by the manufacturer (takes into account the cable strands). For a solid conductor, GMR = 0.7788 r.

For a 60 Hz system, the reactance of the line is

\[ X_L = 0.754 \times 10^{-4} \ln \left( \frac{GMD}{GMR} \right) \text{ Ohms/m} \]

\[ X_L = 0.1213 \ln \left( \frac{GMD}{GMR} \right) \text{ Ohms/mi} \]
Shunt capacitance

- Since a voltage $V$ is applied to a pair of conductors separated by a dielectric (air), charges $q$ of equal magnitude but opposite sign will accumulate on the conductors. Capacitance $C$ between the two conductors is defined by

$$C = \frac{q}{V}$$

- The capacitance of a single-phase transmission line is given by (see derivation in the book): ($\varepsilon = 8.85 \times 10^{-12} \text{ F/m}$)

$$C = \frac{2\pi \varepsilon}{\ln\left(\frac{D}{r}\right)} \text{ F/m}$$
Capacitance of 3-phase power line

- The capacitance per phase is computed by

\[ C = \frac{2\pi \epsilon}{\ln\left(\frac{GMD}{r}\right)} \text{ F/m} \]

- The shunt admittance per phase at 60 Hz is given by

\[ y = 2\pi f C = \frac{0.35 \times 10^{-8}}{\ln\left(\frac{GMD}{r}\right)} \text{ S.m} \]

- The shunt capacitive reactance per phase at 60 Hz is given by

\[ X_c = 47.7 \times 10^6 \ln \left(\frac{GMD}{r}\right) \text{ Ohm.m} \]
1. The greater the spacing between the phases of a line, the lower the capacitance of the line.
   - Since the phases of a high-voltage overhead line must be spaced further apart to ensure proper insulation, a high-voltage line will have a lower capacitance than a low-voltage line.
   - Since the spacing between lines in buried cables is very small, shunt capacitance of cables is much larger than the capacitance of overhead lines.

2. The greater the radius of the conductors, the higher the capacitance of the line. Therefore, bundling increases the capacitance.
Short line model

- Overhead lines shorter than 50 miles can be modeled as a series resistance and inductance, since the shunt capacitance can be neglected over short distances.

\[ R = rd \]
\[ X = xd \]

- The total series resistance and series reactance can be calculated as

where \( r \), \( x \) are resistance and reactance per unit length and \( d \) is the length of the transmission line.
Short line model

- Two-port network model:

\[ V_S = AV_R + BI_R \]
\[ I_S = CV_R + DI_R \]
\[ I_S = I_R \]

\[ V_R = V_S - RI - jX_L I \]

- The equation is similar to that of a synchronous generator and transformer (w/o shunt impedance)
Voltage Regulation:

1. If unity-PF (resistive) loads are added at the end of a line, the voltage at the end of the transmission line decreases slightly – small positive VR.

2. If leading (capacitive) loads are added at the end of a line, the voltage at the end of the transmission line increases – negative VR.

3. If lagging (inductive) loads are added at the end of a line, the voltage at the end of the transmission line decreases significantly – large positive VR.
If the resistance of the line is ignored, then

\[ I \cos \theta = \frac{V_S \sin \delta}{X_L} \]

\[ P = \frac{3V_S V_R \sin \delta}{X_L} \]

Therefore, for fixed voltages, the power flow through a transmission line depends on the angle between the input and output voltages.

Maximum power flow occurs when \( \delta = 90^\circ \).

Notes:

- The maximum power handling capability of a transmission line is a function of the square of its voltage.

- The maximum power handling capability of a transmission line is inversely proportional to its series reactance (some very long lines include series capacitors to reduce the total series reactance).

- The angle \( \delta \) controls the power flow through the line. Hence, it is possible to control power flow by placing a phase-shifting transformer.
Example

• A line with reactance $X$ and negligible resistance supplies a pure resistive load from a fixed source $V_S$. Determine the maximum power transfer, and the load voltage $V_R$ at which this occurs. *(Hint: recall the maximum power transfer theorem from your basic circuits course)*

• Ans: $P_{\text{max}} = \frac{V_S^2}{2X}$, $V_R = \frac{V_S}{\sqrt{2}}$
To prevent excessive voltage variations in a power system, the ratio of the magnitude of the receiving end voltage to the magnitude of the ending end voltage is generally within

$$0.95 \leq \frac{V_S}{V_R} \leq 1.05$$

The angle $\delta$ in a transmission line should typically be $\leq 30^\circ$ to ensure that the power flow in the transmission line is well below the static stability limit.

Any of these limits can be more or less important in different circumstances.

- In short lines, where series reactance $X$ is relatively small, the **resistive heating** usually limits the power that the line can supply.
- In longer lines operating at lagging power factors, the **voltage drop** across the line is usually the limiting factor.
- In longer lines operating at leading power factors, the **maximum angle** $\delta$ can be the limiting factor.
Medium Line (50-150 mi)

- the shunt admittance must be included in calculations. However, the total admittance is usually modeled (π model) as two capacitors of equal values (each corresponding to a half of total admittance) placed at the sending and receiving ends.

- The total series resistance and series reactance are calculated as before. Similarly, the total shunt admittance is given by

\[
Y = yd
\]

- where \( y \) is the shunt admittance per unit length and \( d \) is the length of the transmission line.
Two-port network:

\[ V_S = AV_R + BI_I \]
\[ I_S = CV_R + DI_I \]

\[ A = \frac{ZY}{2} + 1 \]
\[ B = Z \]
\[ C = Y \left( \frac{ZY}{4} + 1 \right) \]
\[ D = \frac{ZY}{2} + 1 \]