EE 742
Chap. 5: Electromechanical Dynamics

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introduction

- In this chapter, a longer time scale is considered during which the rotor speed will vary (order of seconds – 10s' of seconds - transient period).
- The change in rotor speed interacts with the electro-magnetic changes to produce electro-mechanical dynamic effects.
- Some important stability concepts will be introduced mathematically with physical implications.

Swing equation

- Rotor dynamic equation (Newton's Law on motion):

$$J\frac{d\omega_m}{dt} + D_d\,\omega_m = \tau_t - \tau_e, \tag{5.1}$$

where J is the total moment of inertia of the turbine and generator rotor (kg m^2), ω_m is the rotor shaft velocity (mechanical rad/s), τ_t is the torque produced by the turbine (N m), τ_e is the counteracting electromagnetic torque and D_d is the damping-torque coefficient (N m s) and accounts for the mechanical rotational loss due to windage and friction.

- At steady state, $\omega_m = \omega_{sm}$, and

$$\tau_t = \tau_e + D_d\omega_{sm} \quad \text{or} \quad \tau_m = \tau_t - D_d\omega_{sm} = \tau_e, \tag{5.2}$$

where τ_m is the net mechanical shaft torque, that is the turbine torque less the rotational losses at $\omega_m = \omega_{sm}$. It is this torque that is converted into electromagnetic torque. If, due to some disturbance, $\tau_m > \tau_e$ then the rotor accelerates; if $\tau_m < \tau_e$ then it decelerates.

- Rotor speed:

$$\omega_m = \omega_{sm} + \Delta\omega_m = \omega_{sm} + \frac{d\delta_m}{dt}, \tag{5.3}$$

where δ_m is the rotor angle expressed in mechanical radians and $\Delta\omega_m = d\delta_m/dt$ is the *speed deviation* in mechanical radians per second.

Swing Equation

- After substitution,

$$J\omega_{sm}\frac{d^2\delta_m}{dt^2}+\omega_{sm}D_d\frac{d\delta_m}{dt}=\frac{\omega_{sm}}{\omega_m}P_m-\frac{\omega_{sm}}{\omega_m}P_e, \tag{5.6}$$

where P_m is the net shaft power input to the generator and P_e is the electrical air-gap power, both expressed in watts. During a disturbance the speed of a synchronous machine is normally quite close to synchronous speed so that $\omega_m \approx \omega_{sm}$ and Equation (5.6) becomes

$$J\omega_{sm}\frac{d^2\delta_m}{dt^2}+\omega_{sm}D_d\frac{d\delta_m}{dt}=P_m-P_e. \tag{5.7}$$

The coefficient $J\omega_{sm}$ is the *angular momentum* of the rotor at synchronous speed and, when given the symbol M_m, allows Equation (5.7) to be written as

$$M_m\frac{d^2\delta_m}{dt^2}=P_m-P_e-D_m\frac{d\delta_m}{dt}, \tag{5.8}$$

where $D_m=\omega_{sm}D_d$ is the damping coefficient. Equation (5.8) is called the *swing equation* and is the fundamental equation governing the rotor dynamics.

Swing Equation

- Inertia constant:

The inertia constant is given the symbol H defined as the stored kinetic energy in megajoules at synchronous speed divided by the machine rating S_n in megavolt-amperes so that

$$H = \frac{0.5 J \omega_{sm}^2}{S_n} \quad \text{and} \quad M_m = \frac{2 H S_n}{\omega_{sm}}. \tag{5.9}$$

The units of H are seconds.

the power angle and angular speed can be expressed in electrical radians and electrical radians per second respectively, rather than their mechanical equivalent, by substituting

$$\delta = \frac{\delta_m}{p/2} \quad \text{and} \quad \omega_s = \frac{\omega_{sm}}{p/2}, \tag{5.11}$$

where p is the number of poles. Introducing the inertia constant and substituting Equations (5.11) into Equation (5.8) allows the swing equation to be written as

$$\frac{2 H S_n}{\omega_s} \frac{d^2\delta}{dt^2} + D \frac{d\delta}{dt} = P_m - P_e$$

where D, the damping coefficient, is $D = 2D_m/p$.

Swing Equation

The equations in (5.12) can be rationalized by defining an *inertia coefficient* M and *damping power* P_D such that

$$M = \frac{2HS_n}{\omega_s} = \frac{T_m S_n}{\omega_s}, \quad P_D = D\frac{d\delta}{dt}, \tag{5.13}$$

when the swing equation takes the common form

$$M\frac{d^2\delta}{dt^2} = P_m - P_e - P_D = P_{acc}, \tag{5.14}$$

where P_{acc} is the net accelerating power. The time derivative of the rotor angle $d\delta/dt = \Delta\omega = \omega - \omega_s$ is the *rotor speed deviation* in electrical radians per second. Often it is more convenient to replace the second-order differential equation (5.14) by two first-order equations:

$$\begin{aligned} M\frac{d\Delta\omega}{dt} &= P_m - P_e - P_D = P_{acc} \\ \frac{d\delta}{dt} &= \Delta\omega. \end{aligned} \tag{5.15}$$

Damping Power

- Assumptions:

(i) the resistances of both the armature and the field winding are neglected;
(ii) damping is produced only by the damper windings;
(iii) the leakage reactance of the armature winding can be neglected;
(iv) excitation does not affect the damping torque.

- Generator equivalent circuit resembles that of an induction motor:
  - When ignoring rotor saliency, $s = \Delta\omega/\omega_s$

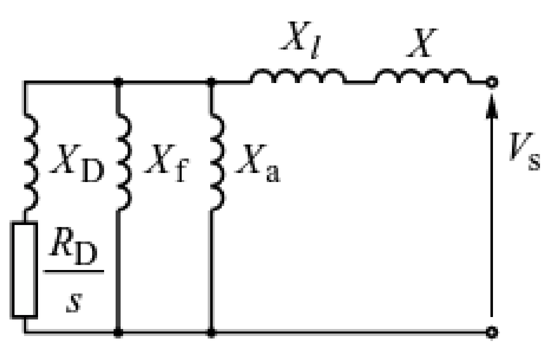
$$P_D = I_D^2 \frac{R_D}{s} \cong V_s^2 \frac{X'_d - X''_d}{(X + X'_d)^2} \frac{X'_d}{X''_d} \frac{T''_d \, \Delta\omega}{1 + (T''_d \, \Delta\omega)^2}.$$

where $X'_d \cong \dfrac{1}{\frac{1}{X_f} + \frac{1}{X_a}}$, $\quad X''_d \cong \dfrac{1}{\frac{1}{X_f} + \frac{1}{X_a} + \frac{1}{X_D}}$ $\qquad T''_d = \dfrac{X_D}{\omega_s R_D} \cong \dfrac{X'_d X''_d}{\omega_s R_D (X'_d - X''_d)}$,

and

$$X_D \cong \frac{X'_d X''_d}{X'_d - X''_d}.$$

(a)



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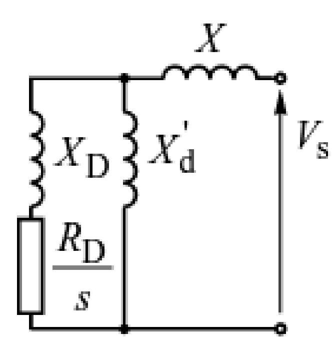


Damping power

- With rotor saliency, the following formula is derived when using d-q axis decomposition – note the dependency on rotor angle δ.

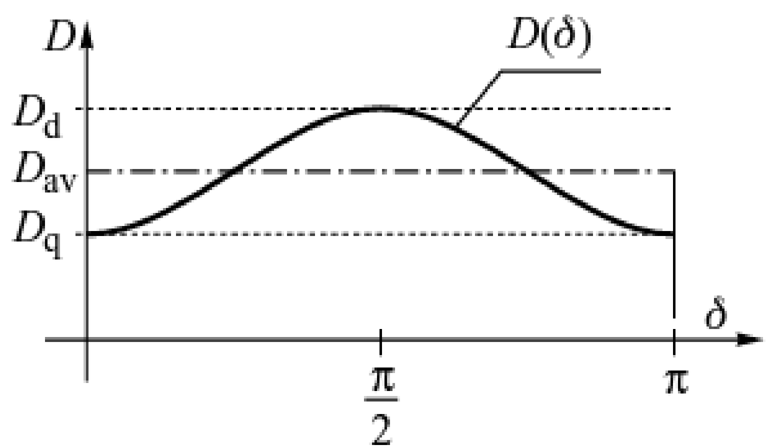
$$P_D = V_s^2 \left[\frac{X'_d - X''_d}{(X + X'_d)^2} \frac{X'_d}{X''_d} \frac{T''_d \Delta\omega}{1 + (T''_d \Delta\omega)^2} \sin^2\delta + \frac{X'_q - X''_q}{(X + X'_q)^2} \frac{X'_q}{X''_q} \frac{T''_q \Delta\omega}{1 + (T''_q \Delta\omega)^2} \cos^2\delta \right].$$

- For small speed deviations, the above expression can be approximated by

$$P_D = V_s^2 \left[\frac{X'_d - X''_d}{(X + X'_d)^2} \frac{X'_d}{X''_d} T''_d \sin^2\delta + \frac{X'_q - X''_q}{(X + X'_q)^2} \frac{X'_q}{X''_q} T''_q \cos^2\delta \right] \Delta\omega.$$

$$P_D = [D_d \sin^2\delta + D_q \cos^2\delta]\Delta\omega = D(\delta)\Delta\omega,$$

where $D(\delta) = D_d \sin^2\delta + D_q \cos^2\delta$ and D_d, D_q are damping coefficients in both axes.



Damping power

- For large speed deviation values, it is convenient to rewrite P_D as

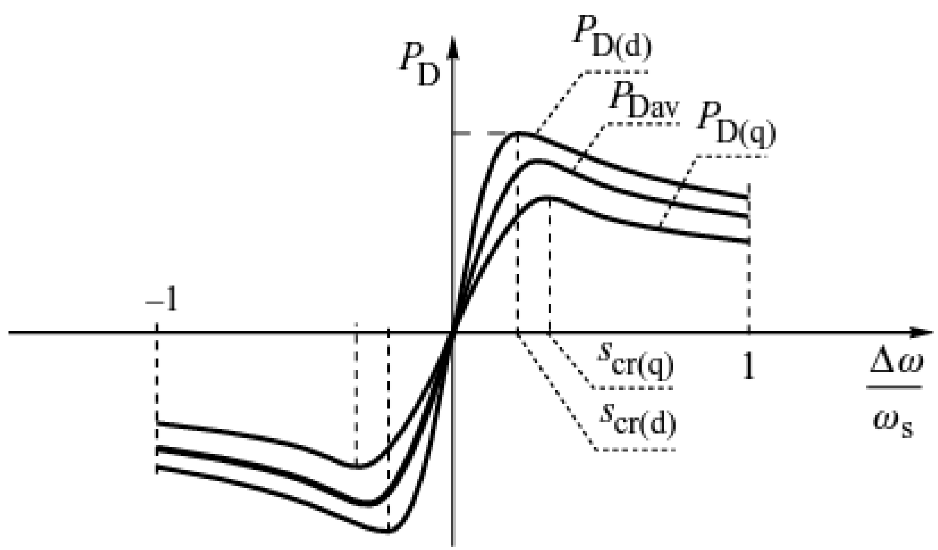
$$P_{\mathrm{D}} = P_{\mathrm{D(d)}} \sin^2 \delta + P_{\mathrm{D(q)}} \cos^2 \delta,$$

- Critical speed deviation in each axis:

$$s_{\mathrm{cr(d)}} = \frac{\Delta \omega_{\mathrm{cr(d)}}}{\omega_{\mathrm{s}}} = \frac{1}{T''_{\mathrm{d}}\, \omega_{\mathrm{s}}}, \quad s_{\mathrm{cr(q)}} = \frac{\Delta \omega_{\mathrm{cr(q)}}}{\omega_{\mathrm{s}}} = \frac{1}{T''_{\mathrm{q}}\, \omega_{\mathrm{s}}},$$

- Critical damping power in each axis:

$$P_{\mathrm{D(d)cr}} = \frac{V_{\mathrm{s}}^2}{2} \frac{X'_{\mathrm{d}} - X''_{\mathrm{d}}}{(X + X'_{\mathrm{d}})^2} \frac{X'_{\mathrm{d}}}{X''_{\mathrm{d}}}, \quad P_{\mathrm{D(q)cr}} = \frac{V_{\mathrm{s}}^2}{2} \frac{X'_{\mathrm{d}} - X''_{\mathrm{d}}}{(X + X'_{\mathrm{d}})^2} \frac{X'_{\mathrm{d}}}{X''_{\mathrm{d}}}.$$

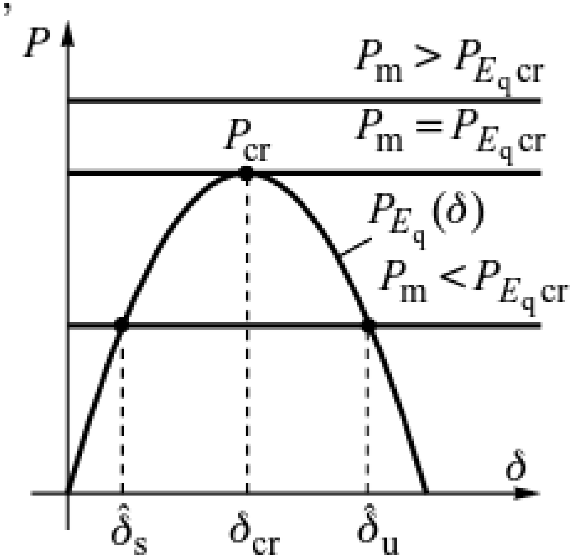


Equilibrium points

- Recall the generator power-angle characteristic at steady-state (chap. 3):
  - For salient pole: $$P_e = P_{E_q} = \frac{E_q V_s}{x_d} \sin\delta + \frac{V_s^2}{2}\frac{x_d - x_q}{x_q x_d}\sin 2\delta,$$
  - For round rotor: $$P_e(\delta) = P_{E_q}(\delta) = \frac{E_q V_s}{x_d}\sin\delta.$$
- The swing equation can be rewritten as:

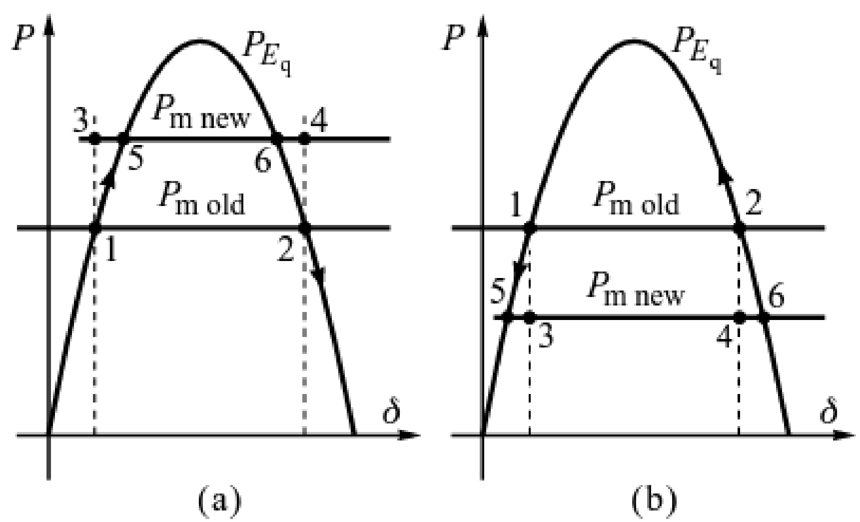
$$M\frac{d^2\delta}{dt^2} = P_m - P_e(\delta) - D\frac{d\delta}{dt},$$

- At equilibrium, $\left.\frac{d\delta}{dt}\right|_{\delta=\hat{\delta}} = 0 \quad \text{and} \quad \left.\frac{d^2\delta}{dt^2}\right|_{\delta=\hat{\delta}} = 0,$
- Hence, $P_m = P_e(\hat{\delta}).$
- Using power-angle curve for simplicity:
  - No equilibrium point when P_m > critical power
  - one equilibrium point when P_m = critical power
  - two equilibrium points when P_m < critical power



Steady-state stability of unregulated generator

- We first ignore the controls of the generator and turbine (i.e., the mechanical power and excitation voltage are constant).
- Small-disturbance or small-signal stability: is system is said to be steady-state stable for a specific operating condition if, following a small disturbance, it reaches a steady-state operating point at or close to the pre-disturbance condition.
- Herein, the power system may be linearized near the operating point for analytical purposes.
- The generator-infinite bus bar system is stable only in the left-hand side of the power-angle curve



Steady-state stability of unregulated generator

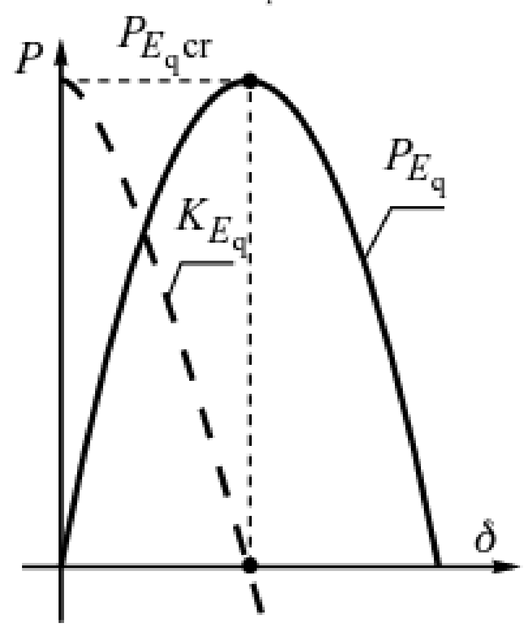
that is, when the slope K_{E_q} of the characteristic is positive

$$K_{E_q} = \left.\frac{\partial P_{E_q}}{\partial \delta}\right|_{\delta=\hat{\delta}_s} > 0. \tag{5.33}$$

K_{E_q} is referred to as the *steady-state synchronizing power coefficient* and the critical power $P_{E_q\,\mathrm{cr}}$ is often referred to as the *pull-out power*

. The value of $P_{E_q\,\mathrm{cr}}$ is also referred to as the *steady-state stability limit* and can be used to determine the *steady-state stability margin* as

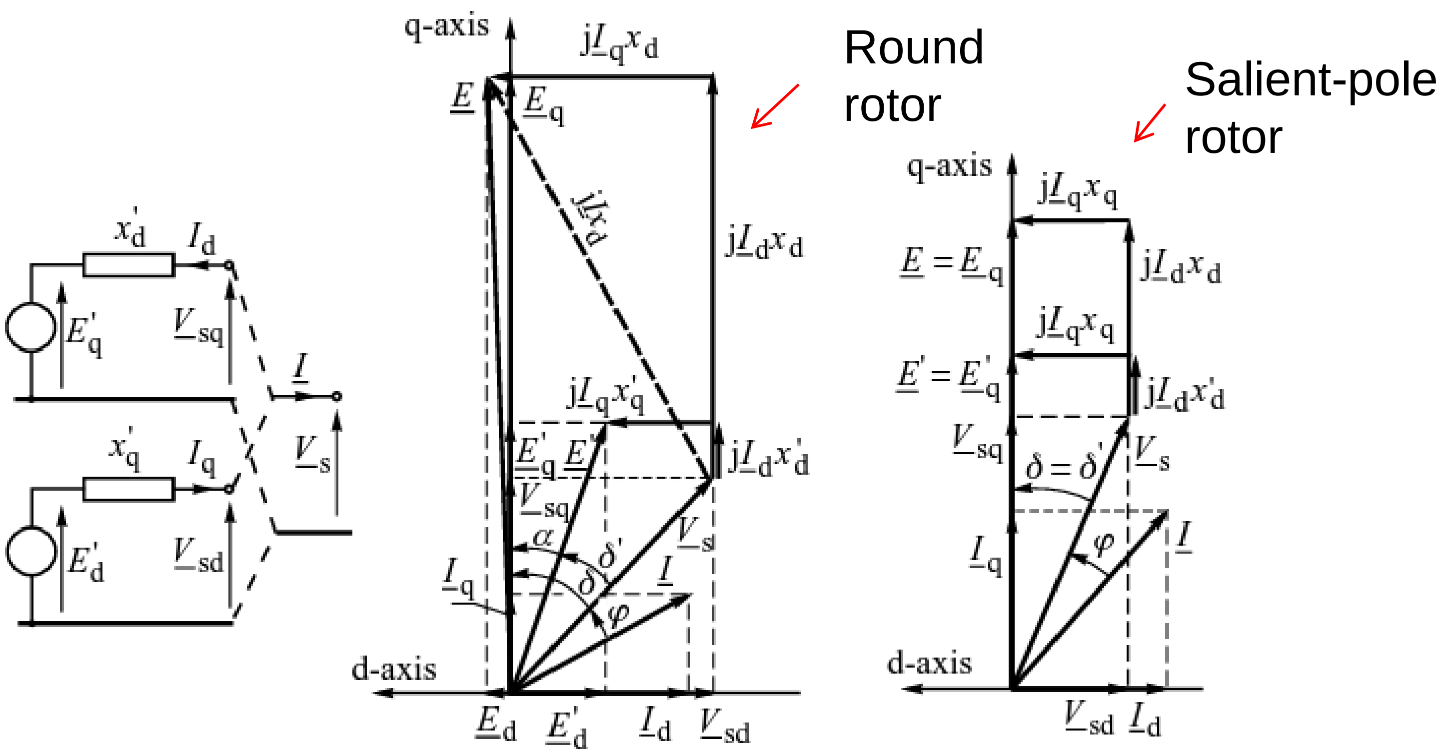
$$c_{E_q} = \frac{P_{E_q\,\mathrm{cr}} - P_m}{P_{E_q\,\mathrm{cr}}}, \tag{5.34}$$



Transient power-angle characteristic

It should be emphasized that the pull-out power is determined by the steady-state characteristic $P_{E_q}(\delta)$ and the dynamic response of the generator to a disturbance is determined by the transient power–angle characteristic

- Rotor oscillations occur in the same time scale as the transient period → generator model during transient state:
  - For generator model with constant flux linkage, see Fig. below (Chap. 4)



(a) (b) (c)

Transient power-angle characteristic

– Constant flux linkage model:

$$P_e = P_s = V_{sd} I_d + V_{sq} I_q = -\frac{E'_q V_{sd}}{x'_d} + \frac{V_{sd} V_{sq}}{x'_d} + \frac{E'_d V_{sq}}{x'_q} - \frac{V_{sd} V_{sq}}{x'_q}$$

$$P_e = P_{E'}(\delta) = \frac{E'_q V_s}{x'_d} \sin\delta + \frac{E'_d V_s}{x'_q} \cos\delta - \frac{V_s^2}{2} \frac{x'_q - x'_d}{x'_q x'_d} \sin 2\delta.$$

$$P_e = P_{E'}(\delta') = \frac{E' V_s}{x'_d} \left[\sin\delta' \left(\cos^2\alpha + \frac{x'_d}{x'_q} \sin^2\alpha \right) + \frac{1}{2} \left(\frac{x'_q - x'_d}{x'_q} \right) \cos\delta' \sin 2\alpha \right]$$
$$- \frac{V_s^2}{2} \frac{x'_q - x'_d}{x'_d x'_q} \sin 2(\delta' + \alpha).$$

$$\delta = \delta' + \alpha,$$

For a generator with a salient-pole rotor, $x'_q = x_q$ $\alpha = 0$ and $\delta' = \delta$.

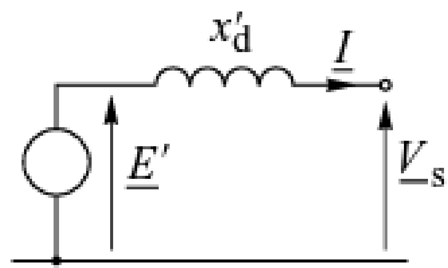
the above transient power expression simplifies to

$$P_e = P_{E'_q}(\delta')\Big|_{x'_q = x_q} = \frac{E'_q V_s}{x'_d} \sin\delta' - \frac{V_s^2}{2} \frac{x_q - x'_d}{x_q x'_d} \sin 2\delta'.$$

Transient power-angle characteristic

- *Classical generator model*: the constant flux linkage model can be simplified further by ignoring the transient saliency, i.e., assuming that $x'_d \approx x'_q$. The transient power equation becomes equal to

$$P_e = P_{E'}(\delta')\big|_{x'_d \approx x'_q} \approx \frac{E' V_s}{x'_d} \sin \delta'.$$



- Note that

$$\delta = \delta' + \alpha, \quad \frac{d\delta}{dt} = \frac{d\delta'}{dt} \quad \text{and} \quad \frac{d^2\delta}{dt^2} = \frac{d^2\delta'}{dt^2}.$$

This allows δ' to be used in the swing equation instead of δ when Equation (5.14) becomes

$$M\frac{d^2\delta'}{dt^2} = P_m - \frac{E' V_s}{x'_d} \sin \delta' - D\frac{d\delta'}{dt}.$$

Examples 5.1

The round-rotor generator considered in Example 4.1 is connected to the power system (infinite busbar) via a transformer with series reactance $X_T = 0.13$ pu and a transmission line with series reactance $X_L = 0.17$ pu. Find, and plot, the steady-state and the transient characteristics using both the constant flux linkage and the classical generator model. As in Example 4.1, the generator real power output is 1 pu, the reactive power output is 0.5 pu and the terminal voltage is 1.1 pu.

$\underline{I}_0 = 1.016\angle -26.6^\circ,$

$E_{q0} = 2.336\angle 38.5^\circ$

$I_{d0} = -0.922$

$I_{q0} = 0.428$

$E'_{d0} = -0.522$

$E'_{q0} = 1.073$

$E' = 1.193 \angle 12.5^o$

$\alpha = \text{atan}(E'_d/E'_q) = 26^o$

$x_d = x_q = 1.9$

$x'_d = 0.53$

$x'_q = 0.68$

$V_s = V_g - j(X_T + X_L)I = 1.0 \angle -15.8^o$

$\delta_o = 38.5 + 15.8 = 54.3^o,$

$\delta'_o = 12.5 + 15.8 = 28.3^o$

$\Phi_o = 26.6 - 15.8 = 10.8^o$

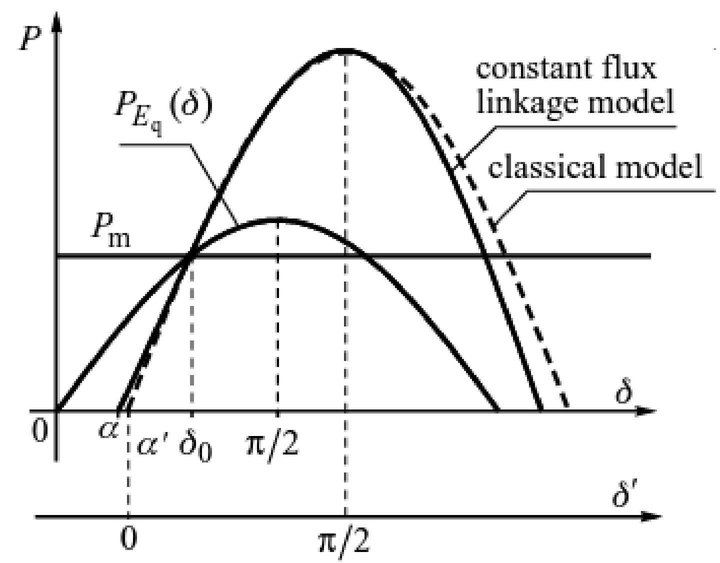
$V_{sd} = -1\sin 54.3^o = -.814$

$V_{sq} = 1\cos 54.3^o = 0.584$

$$P_{E_q}(\delta) = \frac{E_d V_s}{x_d}\sin\delta = \frac{2.336\times 1}{1.9}\sin\delta = 1.23\sin\delta.$$

$$P_{E'}(\delta) = \frac{1.07\times 1}{0.53}\sin\delta + \frac{-0.5224\times 1}{0.68}\cos\delta - \frac{1^2}{2}\frac{0.68-0.53}{0.68\times 0.53}\sin 2\delta$$
$$= 2.02\sin\delta - 0.768\cos\delta - 0.208\sin 2\delta.$$

$$P_{E'}(\delta') \approx \frac{1.223\times 1}{0.53}\sin\delta' = 2.31\sin\delta'.$$



(a)

Examples 5.2

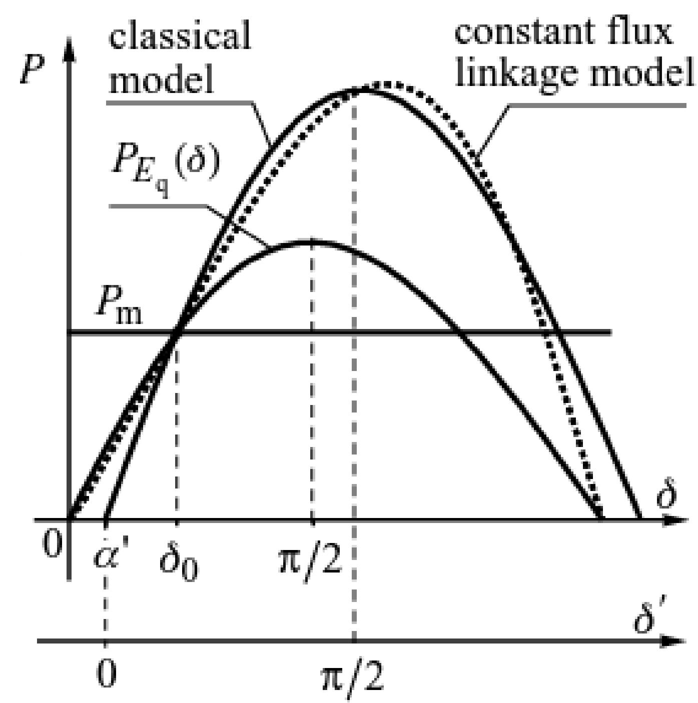
Recalculate all the characteristics from Example 5.1 for the salient-pole generator considered previously in Example 4.2.

$$P_{E_q}(\delta) = \frac{E_q V_s}{x_d}\sin\delta + \frac{V_s^2}{2}\frac{x_d - x_q}{x_d x_q}\sin 2\delta = \frac{1.735 \times 1}{1.23}\sin\delta + \frac{1}{2}\frac{1.23 - 0.99}{1.23 \times 0.99}\sin 2\delta$$

$$= 1.41\sin\delta + 0.099\sin 2\delta \cong 1.41\sin\delta.$$

$$P_{E'}(\delta) = \frac{1.241 \times 1}{0.6}\sin\delta - \frac{1^2}{2}\frac{0.99 - 0.6}{0.99 \times 0.6}\sin 2\delta = 2.07\sin\delta - 0.322\sin 2\delta$$

$$P_{E'}(\delta') \approx \frac{1.265 \times 1}{0.6}\sin\delta' = 2.108\sin\delta'.$$

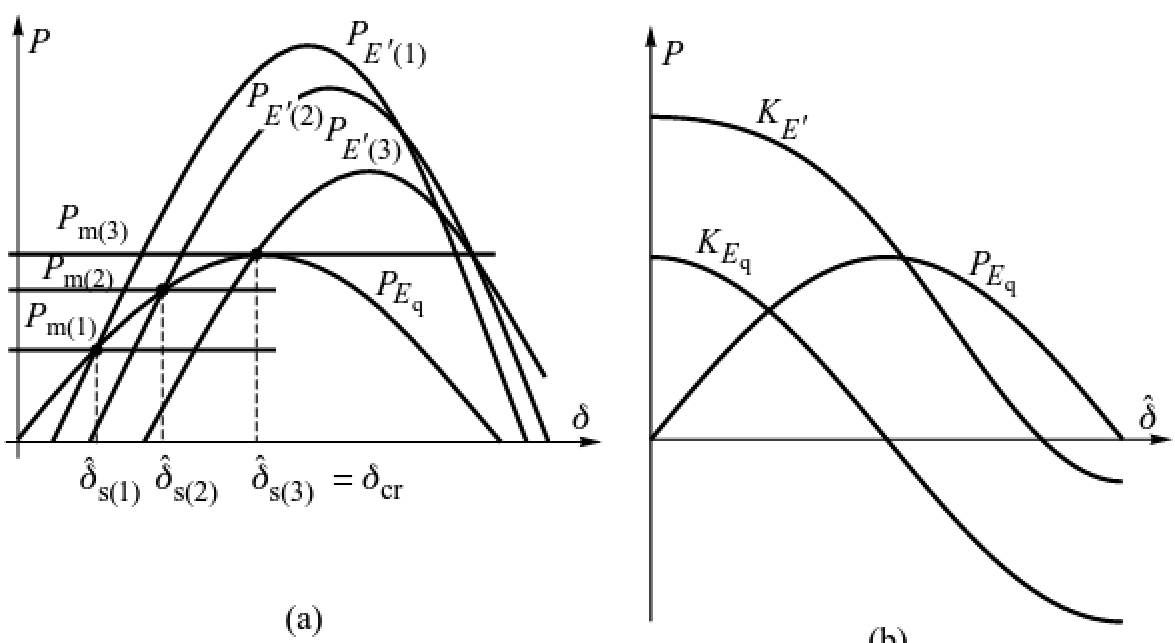


Impact of increase in load

- The increase in load modifies the transient characteristic as shown below (results in smaller transient emf E’, hence smaller peak value of the transient power curve).
- The *transient synchronizing power coefficient*

$$K_{E'} = \left. \frac{\partial P_{E'}}{\partial \delta'} \right|_{\delta' = \hat{\delta}'_s},$$

is steeper than its steady-state counterpart

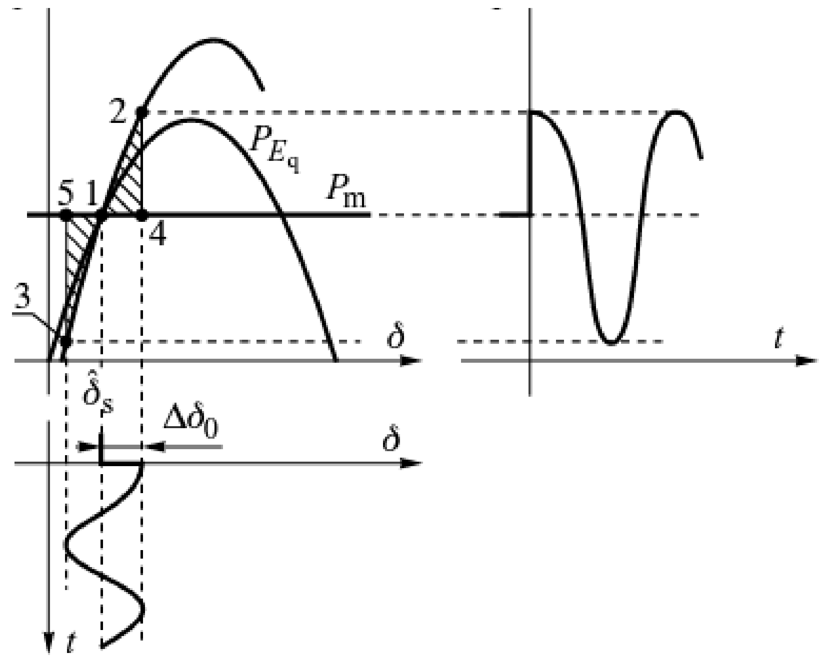


Rotor Swing and equal area criterion

- Consider the effect of disturbing the rotor angle δ from its equilibrium point $\hat{\delta}_s$ to a new value $(\hat{\delta}_s + \Delta\delta_0)$.
- The disturbance performs work on the rotor. It increases the system potential energy by

$$W_{1-2} = \int_{\hat{\delta}_s}^{\hat{\delta}_s+\Delta\delta_0} [P_{E'}(\delta) - P_m]\,d\delta = \text{area } 1-2-4.$$

- At point 2, $P_m < P_{E'}$
  → the machine decelerates.
- At point 1, the above potential energy is converted to kinetic energy, pushing the rotor past the equilibrium point, but starts to accelerate.
- At point 3, area 1-2-4 = area 1-3-5.
- The process repeats indefinitely when no damping is present.

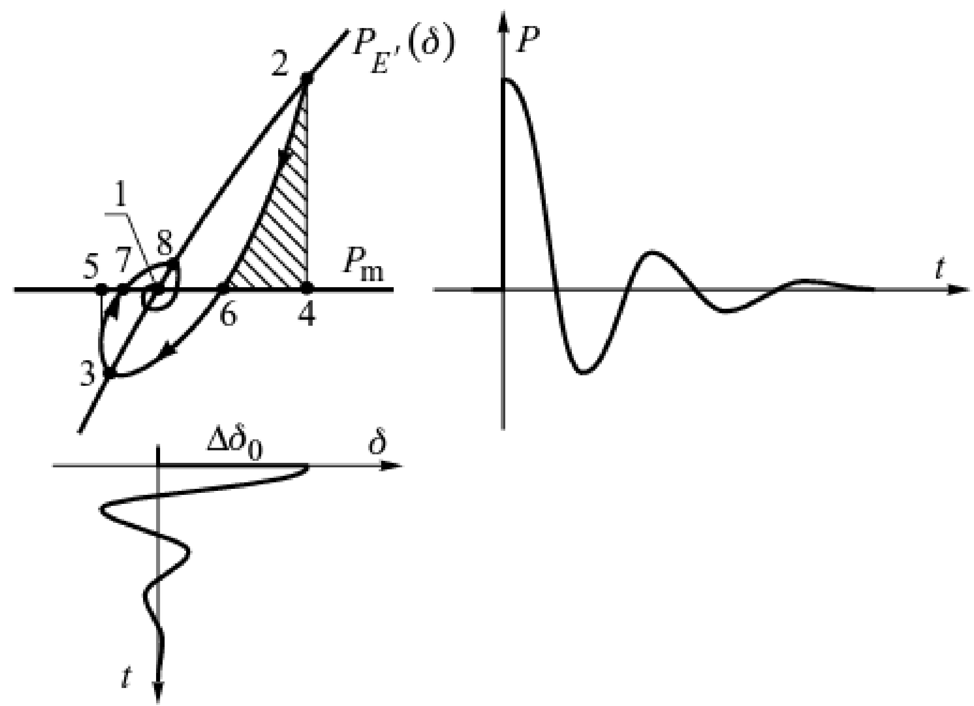


Effect of damper winding

- For small deviations in speed, the damper winding produces damping power that is proportional to speed deviation and adds up to the air-gap power. The sign of P_D depends on the sign of $\Delta\omega$.

$$M\frac{d^2\delta}{dt^2} = P_m - [P_e(\delta) + P_D],$$

- The rotor will therefore move along a modified power-angle trajectory.
- Starting from point 2, the rotor begins to decelerate and reaches minimum speed at point 6.
- Past point 6, the rotor accelerates and reaches sychronous speed when area 2-4-6 = area 6-3-5
- The rotor oscillations are damped and the system quickly reaches the equilibrium point 1.



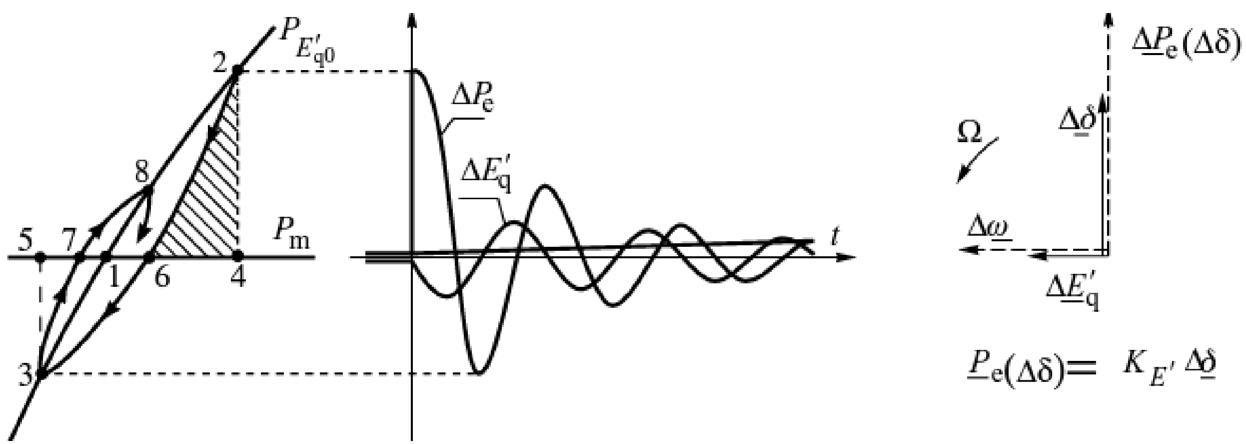
Effect of rotor flux linkage variation

- In reality, E’ is not constant (as the armature flux enters the rotor winding , the rotor flux linkage changes with time).
- To simplify the analysis, only the salient pole machine is considered (i.e., $E' = E'_q$ and $E'_d = 0$;)
- Linearize the transient power equation at the initial operating point:

$$M\frac{d^2\Delta\delta}{dt^2} + D\Delta\omega + K_{E'_q}\,\Delta\delta + D_{\delta'}\Delta E'_q = 0,$$

where $K_{E'_q} = \partial P_e/\partial\delta'$ and $D_{\delta'} = \partial P_e/\partial E'_q$

- $\Delta E_{q'}$ can be in phase with the speed deviation $\Delta\omega$ → in this case, it induces an additional damping torque, (see first figure below).

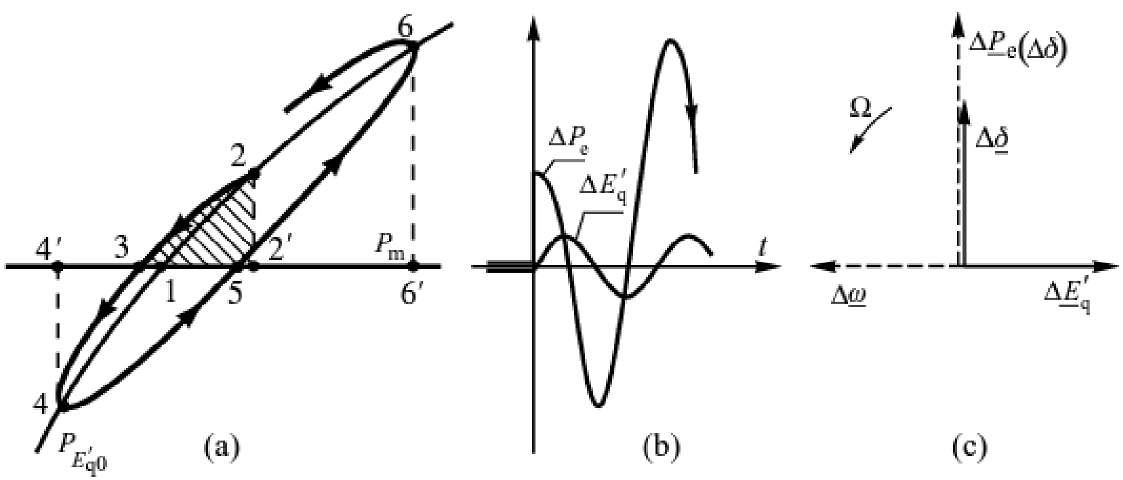


Effect of rotor flux linkage variation

- Updated necessary condition for steady state stability (i.e., $\Delta E_{q'}$ to be in phase with the speed deviation $\Delta\omega$):

$$K_{E_q} = \frac{\partial P_{E_q}}{\partial \delta} > 0 \quad \text{and} \quad K_{E'} = \frac{\partial P_{E'}}{\partial \delta} > K_{E_q}.$$

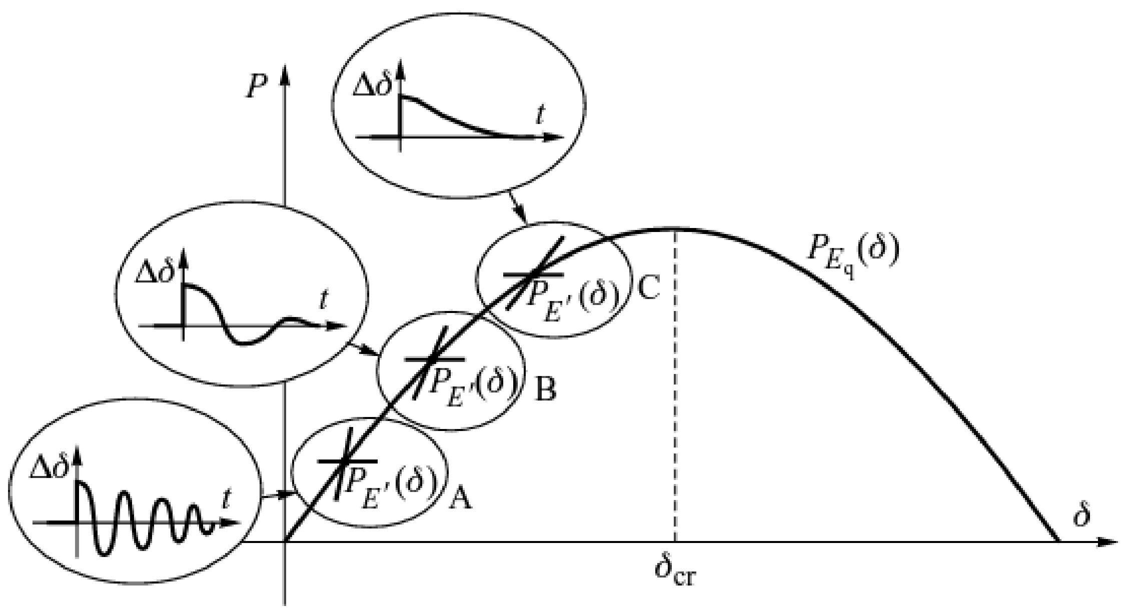
- In case of large network resistance, $\Delta E_{q'}$ can be 180^o out of phase with the speed deviation, → generator instability can occur if this negative damping is larger that the positive damping produced by the damper winding (see second figure below).



Rotor swings around the equilibrium point

- Linearized swing equation: $$M\frac{d^2\Delta\delta}{dt^2} + D\frac{d\Delta\delta}{dt} + K_{E'}\ \Delta\delta = 0,$$
- Roots of characteristic equation: $$\lambda_{1,2} = -\frac{D}{2M} \pm \sqrt{\left(\frac{D}{2M}\right)^2 - \frac{K_{E'}}{M}}.$$
- Three possible solutions: under-damped, critically damped and over-damped.

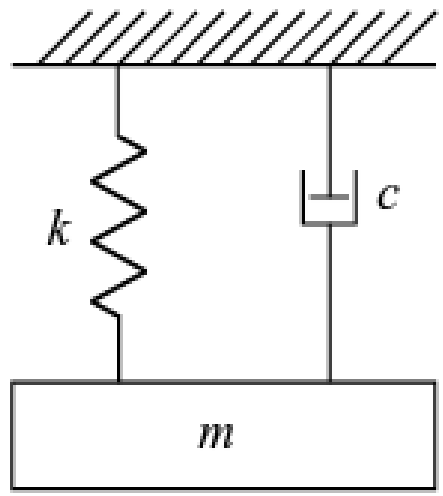
As the values of the roots $\lambda_{1,2}$ depends on the actual values of $K_{E'}$, D and M, so too does the type of response. The inertia coefficient M is constant while both D and $K_{E'}$ depend on the generator loading. Figure 5.3 shows that the damping coefficient D increases with load and Figure 5.10b shows that the transient synchronizing power coefficient $K_{E'}$ decreases with load.



Mechanical analogues of generator-infinite bus-bar system

- Dynamic equation of mass-spring-damper system

$$m\frac{\mathrm{d}^2\Delta x}{\mathrm{d}t^2}+c\frac{\mathrm{d}\Delta x}{\mathrm{d}t}+k\Delta x=0.$$

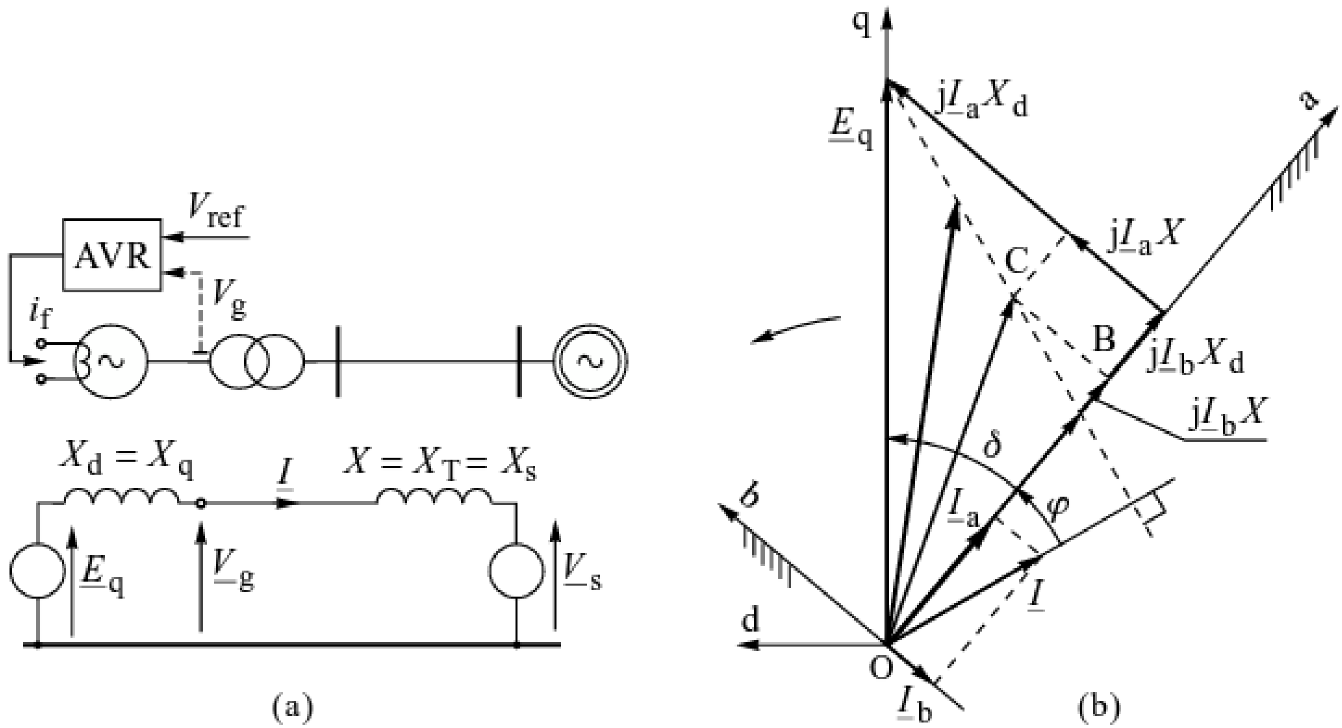


Steady-state stability of regulated system (when action of AVR is included)

- Study restricted to generator with round rotor and negligible resistance.

$$\left(E_{qa} + \frac{X_d}{X} V_s\right)^2 + E_{qb}^2 = \left[\frac{X_d + X}{X} V_g\right]^2. \tag{5.76}$$

This equation describes a circle of radius $\rho = (X_d / X + 1) V_g$ with centre lying on the a-axis at a distance $A = -X_d V_s / X$ from the origin. This means that with V_g = constant and V_s = constant, the tip of E_q moves on this circle.



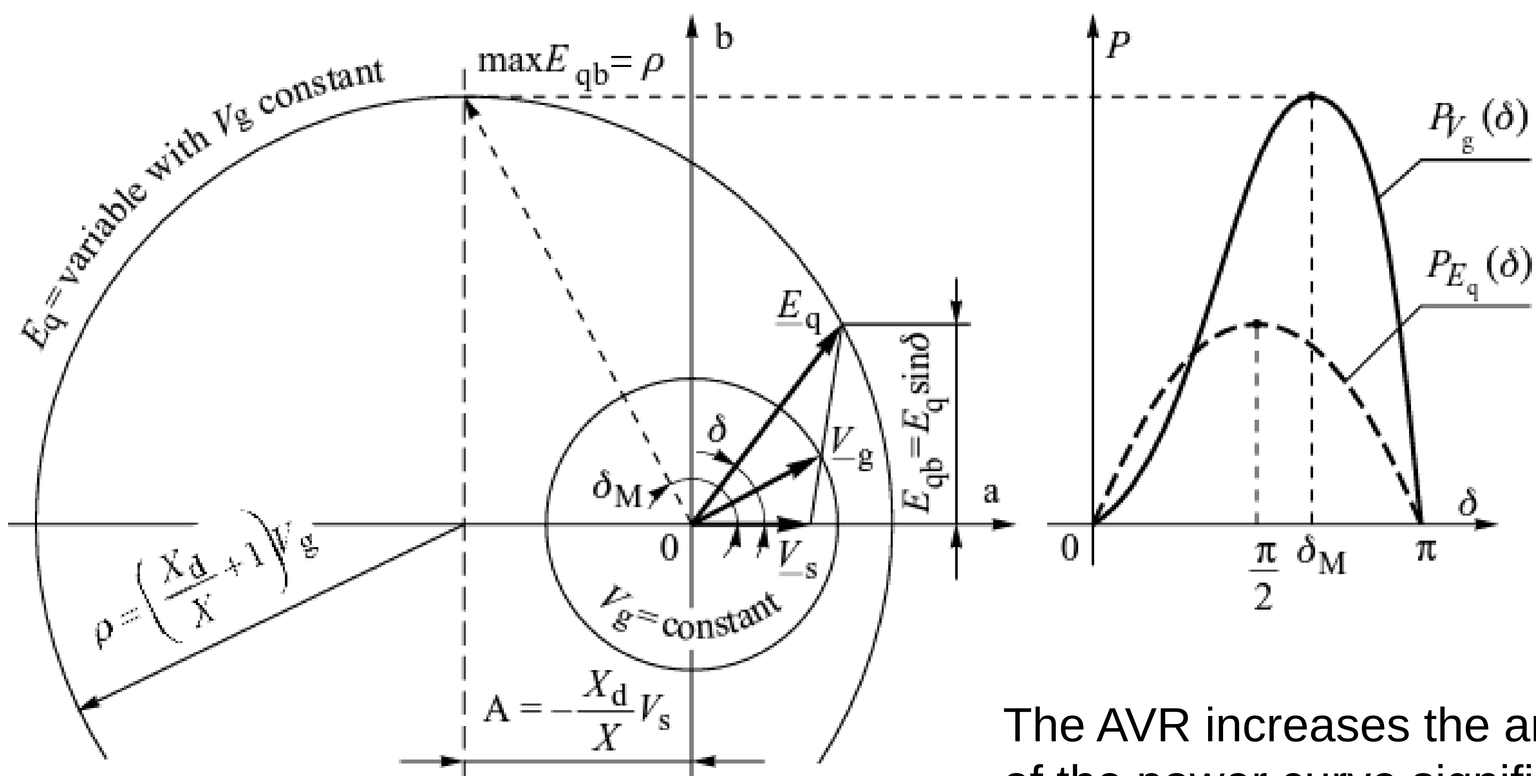
(a) (b)

- Solve for Eq in terms of Vs, Vg, and δ:

$$E_\mathrm{q} = \sqrt{\left(\frac{X_\mathrm{d}+X}{X}V_\mathrm{g}\right)^2 - \left(\frac{X_\mathrm{d}}{X}V_\mathrm{s}\sin\delta\right)^2} - \frac{X_\mathrm{d}}{X}V_\mathrm{s}\cos\delta,$$

- Substitute in the power equation:

$$P_{V_\mathrm{g}}(\delta) = \frac{V_\mathrm{s}}{X_\mathrm{d}+X}\sin\delta\sqrt{\left(\frac{X_\mathrm{d}+X}{X}V_\mathrm{g}\right)^2 - \left(\frac{X_\mathrm{d}}{X}V_\mathrm{s}\sin\delta\right)^2} - \frac{1}{2}\frac{X_\mathrm{d}}{X}\frac{V_\mathrm{s}^2}{X_\mathrm{d}+X}\sin 2\delta.$$



The AVR increases the amplitude of the power curve significantly

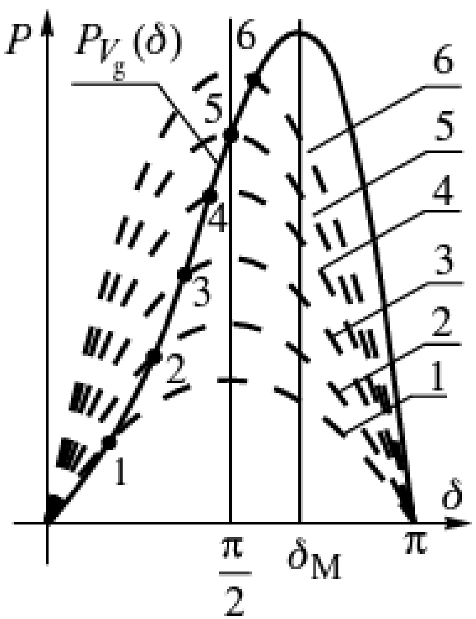
- Generator power is proportional to E_{qb} (= $E_q \sin\delta$)

$$P_{V_g}(\delta) = \frac{V_s}{X_d + X} E_{qb},$$

- Maximum power occurs when

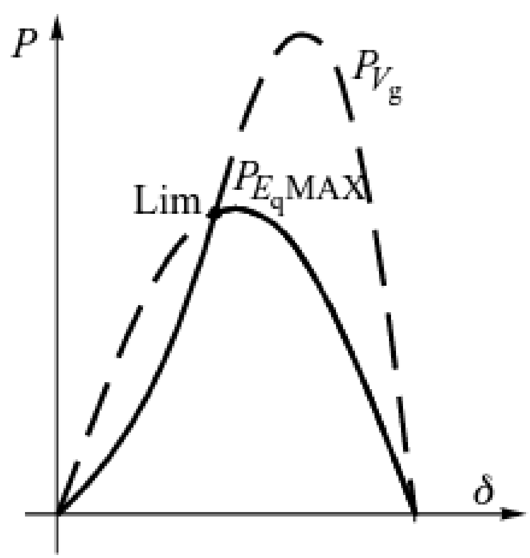
$$E_{qb} = \rho = \left(\frac{X_d}{X} + 1\right) V_g \quad \text{at} \quad \delta_M = \arctan\left(\frac{\rho}{A}\right) = \arctan\left(-\frac{X_d + X}{X_d} \frac{V_g}{V_s}\right).$$

$$P_{V_g\,M} = P_{V_g}(\delta)\Big|_{\delta=\delta_M} = \frac{V_g V_s}{X},$$



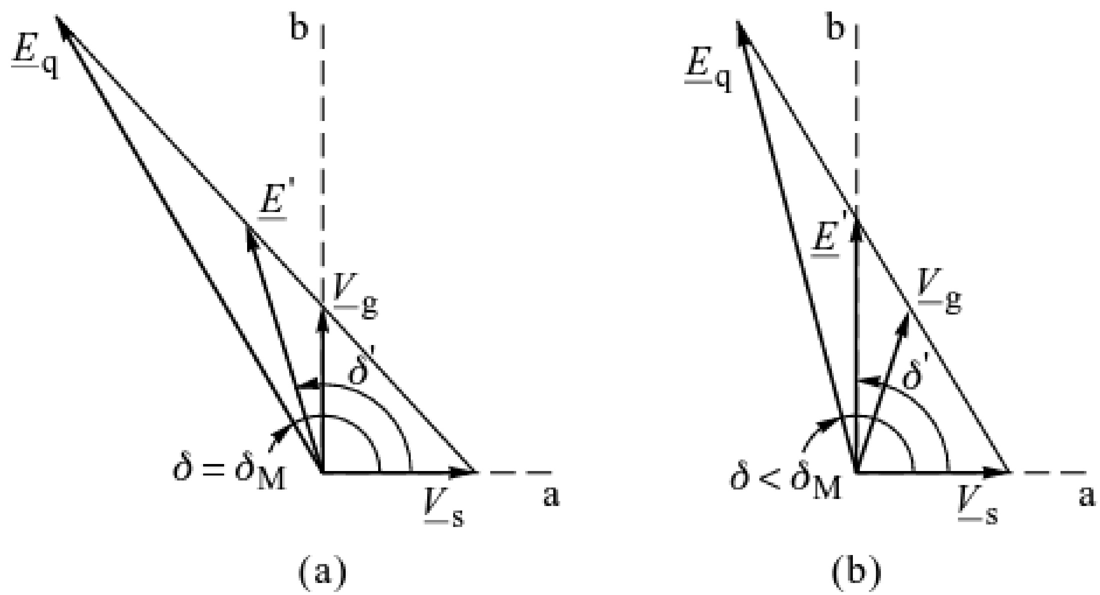
Dashed curves 1-6 represent $P_{Eq}(\delta)$ for higher values of E_q.

- The slow-acting AVR (one with a large time constant) will not be able to respond during the transient period, hence the stability limit corresponds to $\delta = \pi/2$.
- For a fast-acting AVR (with short time constant), the stability limit corresponds to $\delta > \pi/2$. This value depends on the system and AVR parameters (i.e., *conditional stability*).
- The influence of field current limiter is illustrated in the figure below. The field current limit is reached before $P_{Vg\,MAX}$ if X is small.
  - Below the limiting point, the generator steady-state characteristic follows P_{Vg}.
  - Above the limiting point, the generator steady-state characteristic follows P_{Eq}.



Transient power-angle characteristics of regulated generator

- We consider AVR with large time constant. In here, the transient characteristics is the same as the unregulated system except,
  - the value of E_q' is higher (hence higher amplitude of $P_{E'}(\delta')$) since the increased loading in the regulated system causes an increase in the steady-state field current.
  - In addition, the angle δ' reaches $\pi/2$ before δ reached its critical value.
- Note that when the δ reached its critical value, $\delta' > \pi/2$, hence $K_{E'}$ is negative. So at which point on the $P_{Vg}(\delta)$ curve where $K_{E'} = 0$?



- Using classical model and phasor diagram in Fig. 20b,

$$\left(E'_{\mathrm{a}} + \frac{X'_{\mathrm{d}}}{X} V_{\mathrm{s}}\right)^2 + E'^{\,2}_{\mathrm{b}} = \left[\frac{X'_{\mathrm{d}} + X}{X} V_{\mathrm{g}}\right]^2.$$

$$\Rightarrow \quad E' = \sqrt{\left(\frac{X'_{\mathrm{d}} + X}{X} V_{\mathrm{g}}\right)^2 - \left(\frac{X'_{\mathrm{d}}}{X} V_{\mathrm{s}} \sin\delta\right)^2} - \frac{X'_{\mathrm{d}}}{X} V_{\mathrm{s}} \cos\delta.$$

$$\Rightarrow \quad E'\big|_{\delta'=\pi/2} = \frac{V_{\mathrm{g}}}{X} \sqrt{(X'_{\mathrm{d}} + X)^2 - \left(X'_{\mathrm{d}} \frac{V_{\mathrm{s}}}{V_{\mathrm{g}}}\right)^2}.$$

$$\Rightarrow \quad P_{V_{\mathrm{g}}\,\mathrm{cr}} = P_{V_{\mathrm{g}}}(\delta')\big|_{\delta'=\pi/2} = \frac{V_{\mathrm{s}} V_{\mathrm{g}}}{X} \sqrt{1 - \left(\frac{X'_{\mathrm{d}}}{X'_{\mathrm{d}} + X}\right)^2 \left(\frac{V_{\mathrm{s}}}{V_{\mathrm{g}}}\right)^2}.$$

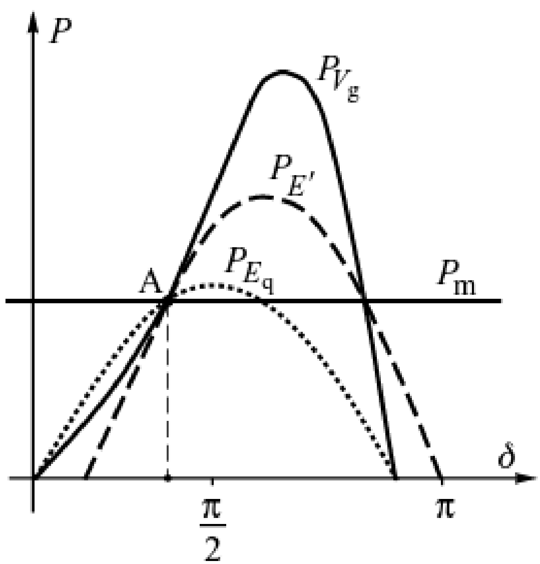
- Ratio of power at which $K_{E'} = 0$ to the power at which $K_{Vg} = 0$:

$$\alpha = \frac{P_{V_{\mathrm{g}}}(\delta' = \pi/2)}{P_{V_{\mathrm{g}}}(\delta = \delta_{\mathrm{M}})} = \frac{P_{V_{\mathrm{g}}\,\mathrm{cr}}}{P_{V_{\mathrm{g}}\,\mathrm{M}}} = \sqrt{1 - \left(\frac{X'_{\mathrm{d}}}{X'_{\mathrm{d}} + X}\right)^2 \left(\frac{V_{\mathrm{s}}}{V_{\mathrm{g}}}\right)^2}.$$

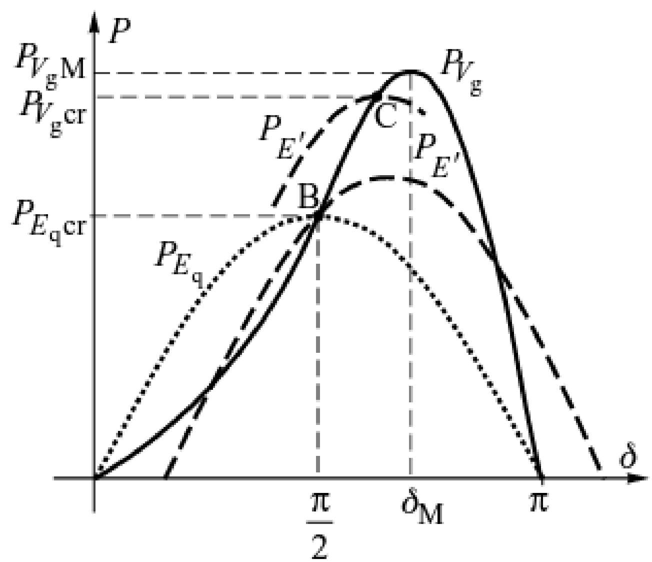
- α is strongly dependent on the system reactance X.
- Refer to the figures below:
  - At operating point A (light load) , $0 < K_{Eq} < K_{E'} < K_{Vg}$
  - At operating point B (medium load), $0 = K_{Eq}$ while $0 < K_{E'} < K_{Vg}$
  - At operating point C (heavy load) , $0 > K_{Eq}$, $K_{E'} = 0$, and $0 < K_{Vg}$

After a disturbance the rotor swings follow the transient power–angle characteristic $P_{E'}(\delta')$. The system is unstable above the point $P_{V_g\,\mathrm{cr}}$ where $K_{E'} = \partial P_{E'}/\partial\delta < 0$ and no deceleration area is available. Therefore the necessary stability condition is

$$K_{E'} = \frac{\partial P_{E'}}{\partial \delta} > 0. \tag{5.89}$$



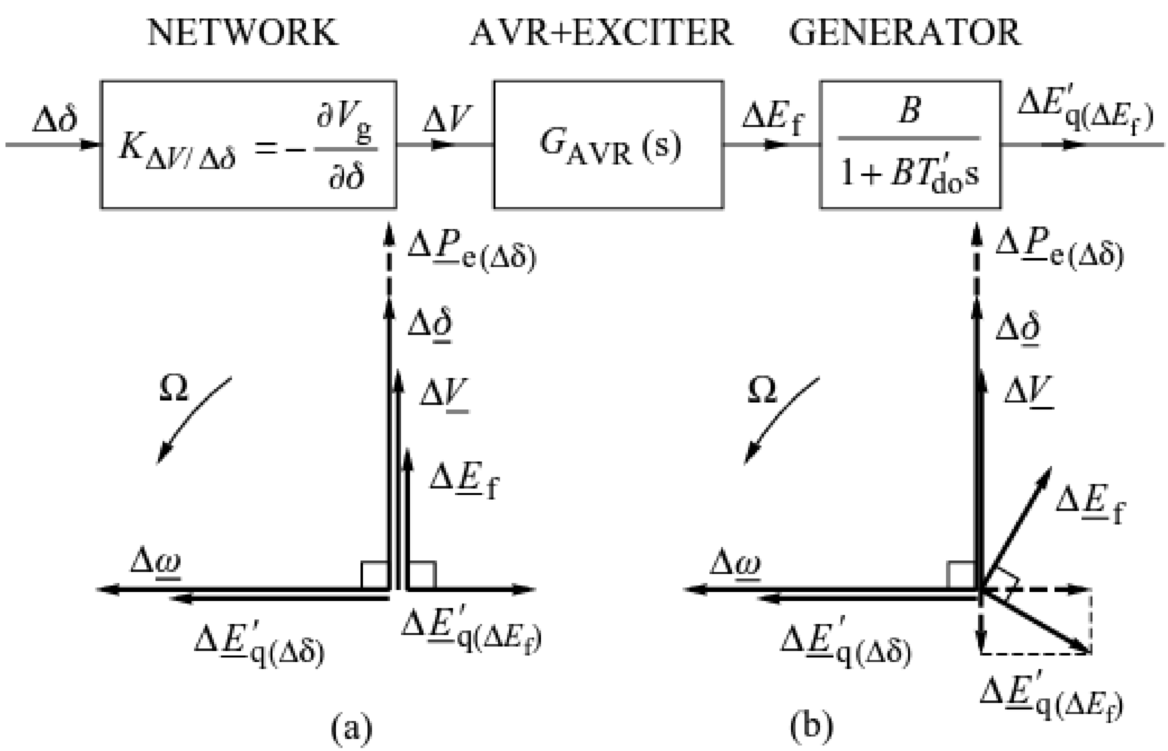
(a)



(b)

Effect or rotor flux variation

- Determine the phase shift between speed change and $\Delta E'_{q(\Delta E_f)}$ (see Section 5.5.3 for details)
- The phasors below represent two types of AVRs:
  - For static exciter ΔE_f is in phase with ΔV
  - For rotating machine exciter, ΔE_f lags ΔV by 10s' of degrees.
- the out-of-phase component $\Delta E'_{q(\Delta E_f)}$ relative to Δω reduces damping (i.e., increases chances of instability).



Effect of AVR action on damper winding

- Change in speed induces a voltage $e_{D(\Delta\omega)}$ in damper winding proportional to Δω. The induced current $i_{D(\Delta\omega)}$ lags by and angle due to high winding inductance.
  - In-phase component of current with Δω gives rise to the natural damping torque
  - The quadrature component of current enhances the synchronizing power coefficient.
- A change in field winding voltage ΔE_f, induces an additional current $i_{D(\Delta Ef)}$ in damper winding (transformer action).
  - The horizontal component of this current is 180^o of-of-phase with respect to Δω.
  - → the voltage regulator weakens the natural damping above.
  - This detrimental effect can be compensated by a supplementary control loop (PSS).