EE 742
Chap. 5: Electromechanical Dynamics
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introduction

• In this chapter, a longer time scale is considered during which the rotor speed will vary (order of seconds – 10s’ of seconds - transient period).

• The change in rotor speed interacts with the electro-magnetic changes to produce electro-mechanical dynamic effects.

• Some important stability concepts will be introduced mathematically with physical implications.
Swing equation

- Rotor dynamic equation (Newton’s Law on motion):

\[ J \frac{d\omega_m}{dt} + D_d \omega_m = \tau_t - \tau_e, \quad (5.1) \]

where \( J \) is the total moment of inertia of the turbine and generator rotor (kg m\(^2\)), \( \omega_m \) is the rotor shaft velocity (mechanical rad/s), \( \tau_t \) is the torque produced by the turbine (N m), \( \tau_e \) is the counteracting electromagnetic torque and \( D_d \) is the damping-torque coefficient (N m s) and accounts for the mechanical rotational loss due to windage and friction.

- At steady state, \( \omega_m = \omega_{sm} \), and

\[ \tau_t = \tau_e + D_d \omega_{sm} \quad \text{or} \quad \tau_m = \tau_t - D_d \omega_{sm} = \tau_e, \quad (5.2) \]

where \( \tau_m \) is the net mechanical shaft torque, that is the turbine torque less the rotational losses at \( \omega_m = \omega_{sm} \). It is this torque that is converted into electromagnetic torque. If, due to some disturbance, \( \tau_m > \tau_e \) then the rotor accelerates; if \( \tau_m < \tau_e \) then it decelerates.

- Rotor speed:

\[ \omega_m = \omega_{sm} + \Delta \omega_m = \omega_{sm} + \frac{d\delta_m}{dt}, \quad (5.3) \]

where \( \delta_m \) is the rotor angle expressed in mechanical radians and \( \Delta \omega_m = \frac{d\delta_m}{dt} \) is the speed deviation in mechanical radians per second.
Swing Equation

• After substitution,

$$J \omega_{sm} \frac{d^2 \delta_m}{dt^2} + \omega_{sm} D_d \frac{d \delta_m}{dt} = \frac{\omega_{sm}}{\omega_m} P_m - \frac{\omega_{sm}}{\omega_m} P_e,$$

where $P_m$ is the net shaft power input to the generator and $P_e$ is the electrical air-gap power, both expressed in watts. During a disturbance the speed of a synchronous machine is normally quite close to synchronous speed so that $\omega_m \approx \omega_{sm}$ and Equation (5.6) becomes

$$J \omega_{sm} \frac{d^2 \delta_m}{dt^2} + \omega_{sm} D_d \frac{d \delta_m}{dt} = P_m - P_e.$$

The coefficient $J \omega_{sm}$ is the angular momentum of the rotor at synchronous speed and, when given the symbol $M_m$, allows Equation (5.7) to be written as

$$M_m \frac{d^2 \delta_m}{dt^2} = P_m - P_e - D_m \frac{d \delta_m}{dt},$$

where $D_m = \omega_{sm} D_d$ is the damping coefficient. Equation (5.8) is called the swing equation and is the fundamental equation governing the rotor dynamics.
Swing Equation

- Inertia constant:

The inertia constant is given the symbol $H$ defined as the stored kinetic energy in megajoules at synchronous speed divided by the machine rating $S_n$ in megavolt-amperes so that

$$H = \frac{0.5J\omega_{sm}^2}{S_n} \quad \text{and} \quad M_m = \frac{2HS_n}{\omega_{sm}}. \quad (5.9)$$

The units of $H$ are seconds.

the power angle and angular speed can be expressed in electrical radians and electrical radians per second respectively, rather than their mechanical equivalent, by substituting

$$\delta = \frac{\delta_m}{p/2} \quad \text{and} \quad \omega_s = \frac{\omega_{sm}}{p/2}, \quad (5.11)$$

where $p$ is the number of poles. Introducing the inertia constant and substituting Equations (5.11) into Equation (5.8) allows the swing equation to be written as

$$\frac{2HS_n}{\omega_s} \frac{d^2\delta}{dt^2} + D \frac{d\delta}{dt} = P_m - P_e$$

where $D$, the damping coefficient, is $D = 2D_m/p$. 
The equations in (5.12) can be rationalized by defining an *inertia coefficient* $M$ and *damping power* $P_D$ such that

$$M = \frac{2HS_n}{\omega_s} = \frac{T_m S_n}{\omega_s}, \quad P_D = D \frac{d\delta}{dt},$$

(5.13)

when the swing equation takes the common form

$$M \frac{d^2 \delta}{dt^2} = P_m - P_e - P_D = P_{acc},$$

(5.14)

where $P_{acc}$ is the net accelerating power. The time derivative of the rotor angle $d\delta/dt = \Delta \omega = \omega - \omega_s$ is the *rotor speed deviation* in electrical radians per second. Often it is more convenient to replace the second-order differential equation (5.14) by two first-order equations:

$$M \frac{d \Delta \omega}{dt} = P_m - P_e - P_D = P_{acc}$$

$$\frac{d\delta}{dt} = \Delta \omega.$$
Damping Power

• Assumptions:
  
  (i) the resistances of both the armature and the field winding are neglected;
  (ii) damping is produced only by the damper windings;
  (iii) the leakage reactance of the armature winding can be neglected;
  (iv) excitation does not affect the damping torque.

• Generator equivalent circuit resembles that of an induction motor:
  
  When ignoring rotor saliency, $s = \Delta \omega / \omega_s$

$$P_D = I_D^2 \frac{R_D}{s} \approx V_s^2 \frac{X'_d - X''_d}{(X + X'_d)^2} \frac{X'_d}{X''_d} \frac{T''_d \Delta \omega}{1 + (T''_d \Delta \omega)^2}.$$ 

where $X'_d \equiv \frac{1}{\frac{1}{X_f} + \frac{1}{X_a}}$, $X''_d \equiv \frac{1}{\frac{1}{X_f} + \frac{1}{X_a} + \frac{1}{X_D}}$, $T''_d = \frac{X_D}{\omega_s R_D} \approx \frac{X'_d X''_d}{\omega_s R_D (X'_d - X''_d)}$,

and

$$X_D \approx \frac{X'_d X''_d}{X'_d - X''_d}.$$
Damping power

- With rotor saliency, the following formula is derived when using d-q axis decomposition – note the dependency on rotor angle $\delta$.

\[
P_D = V_s^2 \left[ \frac{X_d' - X_d''}{X_d'} \frac{X_d'}{X_d''} \frac{T_d'' \Delta \omega}{1 + (T_d'' \Delta \omega)^2} \sin^2 \delta + \frac{X_q' - X_q''}{X_q'} \frac{X_q'}{X_q''} \frac{T_q'' \Delta \omega}{1 + (T_q'' \Delta \omega)^2} \cos^2 \delta \right].
\]

- For small speed deviations, the above expression can be approximated by

\[
P_D = V_s^2 \left[ \frac{X_d' - X_d''}{X_d'} \frac{X_d'}{X_d''} T_d'' \sin^2 \delta + \frac{X_q' - X_q''}{X_q'} \frac{X_q'}{X_q''} T_q'' \cos^2 \delta \right] \Delta \omega.
\]

\[
P_D = [D_d \sin^2 \delta + D_q \cos^2 \delta] \Delta \omega = D(\delta) \Delta \omega,
\]

where $D(\delta) = D_d \sin^2 \delta + D_a \cos^2 \delta$ and $D_d$, $D_a$ are damping coefficients in both axes.
Damping power

- For large speed deviation values, it is convenient to rewrite \( P_D \) as
  \[
  P_D = P_{D(d)} \sin^2 \delta + P_{D(q)} \cos^2 \delta,
  \]

- Critical speed deviation in each axis:
  \[
  s_{cr(d)} = \frac{\Delta \omega_{cr(d)}}{\omega_s} = \frac{1}{T_d'' \omega_s}, \quad s_{cr(q)} = \frac{\Delta \omega_{cr(q)}}{\omega_s} = \frac{1}{T_q'' \omega_s},
  \]

- Critical damping power in each axis:
  \[
  P_{D(d)cr} = \frac{V_s^2}{2} \frac{X_d' - X_d''}{(X + X_d')^2} \frac{X_d'}{X_d''}, \quad P_{D(q)cr} = \frac{V_s^2}{2} \frac{X_d' - X_d''}{(X + X_d')^2} \frac{X_d'}{X_d''}.
  \]
Equilibrium points

Recall the generator power-angle characteristic at steady-state (chap. 3):

- For salient pole:
  \[ P_e = P_{E_q} = \frac{E_q V_s}{x_d} \sin \delta + \frac{V_s^2}{2} \frac{x_d - x_q}{x_q x_d} \sin 2\delta, \]

- For round rotor:
  \[ P_e(\delta) = P_{E_q}(\delta) = \frac{E_q V_s}{x_d} \sin \delta. \]

The swing equation can be rewritten as:

\[ M \frac{d^2\delta}{dt^2} = P_m - P_e(\delta) - D \frac{d\delta}{dt}, \]

At equilibrium,

\[ \frac{d\delta}{dt} \bigg|_{\delta = \delta} = 0 \quad \text{and} \quad \frac{d^2\delta}{dt^2} \bigg|_{\delta = \delta} = 0, \]

Hence, \( P_m = P_e(\hat{\delta}). \)

Using power-angle curve for simplicity:

- No equilibrium point when \( P_m > \text{critical power} \)
- One equilibrium point when \( P_m = \text{critical power} \)
- Two equilibrium points when \( P_m < \text{critical power} \)
Steady-state stability of unregulated generator

• We first ignore the controls of the generator and turbine (i.e., the mechanical power and excitation voltage are constant).

• Small-disturbance or small-signal stability: is system is said to be steady-state stable for a specific operating condition if, following a small disturbance, it reaches a steady-state operating point at or close to the pre-disturbance condition.

• Herein, the power system may be linearized near the operating point for analytical purposes.

• The generator-infinite bus bar system is stable only in the left-hand side of the power-angle curve
Steady-state stability of unregulated generator

that is, when the slope $K_{E_q}$ of the characteristic is positive

$$K_{E_q} = \left. \frac{\partial P_{E_q}}{\partial \delta} \right|_{\delta = \delta_s} > 0.$$ (5.33)

$K_{E_q}$ is referred to as the *steady-state synchronizing power coefficient* and the critical power $P_{E_q \text{ cr}}$ is often referred to as the *pull-out power*. The value of $P_{E_q \text{ cr}}$ is also referred to as the *steady-state stability limit* and can be used to determine the *steady-state stability margin* as

$$c_{E_q} = \frac{P_{E_q \text{ cr}} - P_m}{P_{E_q \text{ cr}}},$$ (5.34)
Transient power-angle characteristic

It should be emphasized that the pull-out power is determined by the steady-state characteristic $P_{Eq}(\delta)$ and the dynamic response of the generator to a disturbance is determined by the transient power–angle characteristic.

- Rotor oscillations occur in the same time scale as the transient period $\rightarrow$ generator model during transient state:
  - For generator model with constant flux linkage, see Fig. below (Chap. 4)
Transient power-angle characteristic

– Constant flux linkage model:

\[ P_e = P_s = V_{sd} I_d + V_{sq} I_q = \frac{E_q' V_{sd}}{x_d'} + \frac{V_{sd} V_{sq}}{x_d'} + \frac{E_d' V_{sq}}{x_q'} - \frac{V_{sd} V_{sq}}{x_q'} \]

\[ P_e = P_{E_e}(\delta) = \frac{E_q' V_s}{x_d'} \sin \delta + \frac{E_d' V_s}{x_q'} \cos \delta - \frac{V_s^2}{2} \frac{x_q' - x_d'}{x_q' x_d'} \sin 2\delta. \]

\[ P_e = P_{E_e}(\delta') = \frac{E_q' V_s}{x_d'} \left[ \sin \delta' \left( \cos^2 \alpha + \frac{x_d'}{x_q'} \sin^2 \alpha \right) + \frac{1}{2} \left( \frac{x_q' - x_d'}{x_q'} \right) \cos \delta' \sin 2\alpha \right] \]

\[ - \frac{V_s^2}{2} \frac{x_q' - x_d'}{x_d' x_q'} \sin 2(\delta' + \alpha). \]

\[ \delta = \delta' + \alpha, \quad x_q' = x_q, \quad \alpha = 0 \text{ and } \delta' = \delta. \]

For a generator with a salient-pole rotor, the above transient power expression simplifies to

\[ P_e = P_{E_{eq}}(\delta') \bigg|_{x_q' = x_q} = \frac{E_q' V_s}{x_d'} \sin \delta' - \frac{V_s^2}{2} \frac{x_q' - x_d'}{x_q x_d'} \sin 2\delta'. \]
Transient power-angle characteristic

- **Classical generator model**: the constant flux linkage model can be simplified further by ignoring the transient saliency, i.e., assuming that $x_d' \approx x_q'$. The transient power equation becomes equal to

\[ P_e = P_{E'}(\delta') \bigg|_{x_d \approx x_q' \approx \frac{E'V_s}{x_d'}} \approx \frac{E'V_s}{x_d'} \sin \delta'. \]

- Note that

\[ \delta = \delta' + \alpha, \quad \frac{d\delta}{dt} = \frac{d\delta'}{dt} \quad \text{and} \quad \frac{d^2\delta}{dt^2} = \frac{d^2\delta'}{dt^2}. \]

This allows $\delta'$ to be used in the swing equation instead of $\delta$ when Equation (5.14) becomes

\[ M \frac{d^2\delta'}{dt^2} = P_m - \frac{E'V_s}{x_d'} \sin \delta' - D \frac{d\delta'}{dt}. \]
Examples 5.1

The round-rotor generator considered in Example 4.1 is connected to the power system (infinite busbar) via a transformer with series reactance \(X_T = 0.13\) pu and a transmission line with series reactance \(X_L = 0.17\) pu. Find, and plot, the steady-state and the transient characteristics using both the constant flux linkage and the classical generator model. As in Example 4.1, the generator real power output is 1 pu, the reactive power output is 0.5 pu and the terminal voltage is 1.1 pu.

\[
I_n = 1.016\angle -26.6^\circ, \quad x_d = x_q = 1.9
\]
\[
E_{q0} = 2.336\angle 38.5^\circ
\]
\[
I_{d0} = -0.922 \quad x_d' = 0.53
\]
\[
I_{q0} = 0.428 \quad x_q' = 0.68
\]
\[
E_{d0}' = -0.522
\]
\[
E_{q0}' = 1.073
\]
\[
E' = 1.193\angle 12.5^\circ\]
\[
\alpha = \text{atan}(E'_d/E'_q) = 26^\circ
\]
\[
\delta_0 = 38.5 + 15.8 = 54.3^\circ,
\]
\[
\delta'_0 = 12.5 + 15.8 = 28.3^\circ
\]
\[
\Phi_0 = 26.6 - 15.8 = 10.8^\circ
\]
\[
V_{sd} = -1\sin 54.3^\circ = -0.814
\]
\[
V_{sq} = 1\cos 54.3^\circ = 0.584
\]

\[
\begin{align*}
P_{E_q}(\delta) &= \frac{E_d V_s}{x_d} \sin \delta = \frac{2.336 \times 1}{1.9} \sin \delta = 1.23 \sin \delta. \\
P_{E_q}'(\delta) &= \frac{1.07 \times 1}{0.53} \sin \delta + \frac{-0.5224 \times 1}{0.68} \cos \delta - \frac{1^2}{2} \frac{0.68 - 0.53}{0.68 \times 0.53} \sin 2\delta \\
&= 2.02 \sin \delta - 0.768 \cos \delta - 0.208 \sin 2\delta. \\
P_E(\delta') &\approx \frac{1.223 \times 1}{0.53} \sin \delta' = 2.31 \sin \delta'.
\end{align*}
\]
Examples 5.2

Recalculate all the characteristics from Example 5.1 for the salient-pole generator considered previously in Example 4.2.

\[
P_{Eq}(\delta) = \frac{E_q V_s}{x_d} \sin \delta + \frac{V_s^2}{2} \frac{x_d - x_q}{x_d x_q} \sin 2\delta = \frac{1.735 \times 1}{1.23} \sin \delta + \frac{11.23 - 0.99}{2 \times 1.23 \times 0.99} \sin 2\delta
\]

\[
= 1.41 \sin \delta + 0.099 \sin 2\delta \cong 1.41 \sin \delta.
\]

\[
P_{E'}(\delta) = \frac{1.241 \times 1}{0.6} \sin \delta - \frac{1^2 0.99 - 0.6}{2 \times 0.99 \times 0.6} \sin 2\delta = 2.07 \sin \delta - 0.322 \sin 2\delta
\]

\[
P_{E'}(\delta') \approx \frac{1.265 \times 1}{0.6} \sin \delta' = 2.108 \sin \delta'.
\]
Impact of increase in load

- The increase in load modifies the transient characteristic as shown below (results in smaller transient emf $E'$, hence smaller peak value of the transient power curve).
- The *transient synchronizing power coefficient* 

$$K_{E'} = \frac{\partial P_{E'}}{\partial \delta'} \bigg|_{\delta' = \hat{\delta}_s},$$

is steeper than its steady-state counterpart.
**Rotor Swing and equal area criterion**

- Consider the effect of disturbing the rotor angle $\delta$ from its equilibrium point $\hat{\delta}_s$ to a new value $(\hat{\delta}_s + \Delta\delta_0)$.

- The disturbance performs work on the rotor. It increases the system potential energy by

\[
W_{1-2} = \int_{\hat{\delta}_s}^{\hat{\delta}_s+\Delta\delta_0} [P_E'(\delta) - P_m] \, d\delta = \text{area 1} - \text{2} - \text{4}.
\]

- At point 2, $P_m < P_{E'}$ → the machine decelerates.

- At point 1, the above potential energy is converted to kinetic energy, pushing the rotor past the equilibrium point, but starts to accelerate.

- At point 3, area 1-2-4 = area 1-3-5.

- The process repeats indefinitely when no damping is present.
Effect of damper winding

- For small deviations in speed, the damper winding produces damping power that is proportional to speed deviation and adds up to the air-gap power. The sign of $P_D$ depends on the sign of $\Delta \omega$.

$$M \frac{d^2 \delta}{dt^2} = P_m - [P_e(\delta) + P_D],$$

- The rotor will therefore move along a modified power-angle trajectory.
- Starting from point 2, the rotor begins to decelerate and reaches minimum speed at point 6.
- Past point 6, the rotor accelerates and reaches synchronous speed when area 2-4-6 = area 6-3-5.
- The rotor oscillations are damped and the system quickly reaches the equilibrium point 1.
Effect of rotor flux linkage variation

• In reality, $E'$ is not constant (as the armature flux enters the rotor winding, the rotor flux linkage changes with time).
• To simplify the analysis, only the salient pole machine is considered (i.e., $E' = E'_q$ and $E'_d = 0$).
• Linearize the transient power equation at the initial operating point:

$$M \frac{d^2 \Delta \delta}{dt^2} + D \Delta \omega + K_{E'_q} \Delta \delta + D_{\delta'} \Delta E'_q = 0,$$

where $K_{E'_q} = \frac{\partial P_e}{\partial \delta'}$ and $D_{\delta'} = \frac{\partial P_e}{\partial E'_q}$

• $\Delta E'_q$ can be in phase with the speed deviation $\Delta \omega$ → in this case, it induces an additional damping torque, (see first figure below).
Effect of rotor flux linkage variation

- Updated necessary condition for steady state stability (i.e., $\Delta E_{q'}$ to be in phase with the speed deviation $\Delta \omega$):

$$K_{Eq} = \frac{\partial P_{Eq}}{\partial \delta} > 0 \quad \text{and} \quad K_{E'} = \frac{\partial P_{E'}}{\partial \delta} > K_{Eq}.$$  

- In case of large network resistance, $\Delta E_{q'}$ can be 180$^\circ$ out of phase with the speed deviation, $\rightarrow$ generator instability can occur if this negative damping is larger than the positive damping produced by the damper winding (see second figure below).
Rotor swings around the equilibrium point

- Linearized swing equation:
  \[ M \frac{d^2 \Delta \delta}{dt^2} + D \frac{d \Delta \delta}{dt} + K_{E'} \Delta \delta = 0, \]

- Roots of characteristic equation:
  \[ \lambda_{1,2} = -\frac{D}{2M} \pm \sqrt{\left(\frac{D}{2M}\right)^2 - \frac{K_{E'}}{M}}. \]

- Three possible solutions: under-damped, critically damped and over-damped.

As the values of the roots \( \lambda_{1,2} \) depends on the actual values of \( K_{E'} \), \( D \) and \( M \), so too does the type of response. The inertia coefficient \( M \) is constant while both \( D \) and \( K_{E'} \) depend on the generator loading. Figure 5.3 shows that the damping coefficient \( D \) increases with load and Figure 5.10b shows that the transient synchronizing power coefficient \( K_{E'} \) decreases with load.
Mechanical analogues of generator-infinite bus-bar system

- Dynamic equation of mass-spring-damper system

\[ m \frac{d^2 \Delta x}{dt^2} + c \frac{d\Delta x}{dt} + k \Delta x = 0. \]
Steady-state stability of regulated system (when action of AVR is included)

- Study restricted to generator with round rotor and negligible resistance.

\[
\left( E_{qa} + \frac{X_d}{X} V_s \right)^2 + E_{qb}^2 = \left[ \frac{X_d + X}{X} V_g \right]^2.
\]  \hspace{1cm} (5.76)

This equation describes a circle of radius \( \rho = (X_d / X + 1) V_g \) with centre lying on the a-axis at a distance \( A = -X_d V_s / X \) from the origin. This means that with \( V_g \) constant and \( V_s \) constant, the tip of \( E_q \) moves on this circle.
• Solve for Eq in terms of Vs, Vg, and δ:

\[ E_q = \sqrt{\left( \frac{X_d + X}{X} V_g \right)^2 - \left( \frac{X_d}{X} V_s \sin \delta \right)^2 - \frac{X_d}{X} V_s \cos \delta}, \]

• Substitute in the power equation:

\[ P_{V_g}(\delta) = \frac{V_s}{X_d + X} \sin \delta \sqrt{\left( \frac{X_d + X}{X} V_g \right)^2 - \left( \frac{X_d}{X} V_s \sin \delta \right)^2 - \frac{1}{2} \frac{X_d}{X} \frac{V_s^2}{X_d + X} \sin 2\delta}. \]

The AVR increases the amplitude of the power curve significantly.
• Generator power is proportional to $E_{qb} (= E_q \sin \delta)$

\[ P_{V_g} (\delta) = \frac{V_s}{X_d + X} E_{qb}, \]

• Maximum power occurs when

\[ E_{qb} = \rho = \left( \frac{X_d}{X} + 1 \right) V_g \quad \text{at} \quad \delta_M = \arctan \left( \frac{\rho}{A} \right) = \arctan \left( -\frac{X_d + X}{V_s} \frac{V_g}{V_s} \right). \]

\[ P_{V_g M} = P_{V_g} (\delta) \bigg|_{\delta=\delta_M} = \frac{V_g V_s}{X}, \]

Dashed curves 1-6 represent $P_{Eq}(\delta)$ for higher values of $E_q$. 
• The slow-acting AVR (one with a large time constant) will not be able to respond during the transient period, hence the stability limit corresponds to $\delta = \pi/2$.
• For a fast-acting AVR (with short time constant), the stability limit corresponds to $\delta > \pi/2$. This value depends on the system and AVR parameters (i.e., conditional stability).
• The influence of field current limiter is illustrated in the figure below. The field current limit is reached before $P_{Vg\, MAX}$ if $X$ is small:
  – Below the limiting point, the generator steady-state characteristic follows $P_{Vg}$.
  – Above the limiting point, the generator steady-state characteristic follows $P_{Eq}$.
Transient power-angle characteristics of regulated generator

• We consider AVR with large time constant. In here, the transient characteristics is the same as the unregulated system except,
  – the value of $E_q'$ is higher (hence higher amplitude of $P_{E'}(\delta')$) since the increased loading in the regulated system causes an increase in the steady-state field current.
  – In addition, the angle $\delta'$ reaches $\pi/2$ before $\delta$ reached its critical value.

• Note that when the $\delta$ reached its critical value, $\delta' > \pi/2$, hence $K_{E'}$ is negative. So at which point on the $P_{Vg}(\delta)$ curve where $K_{E'} = 0$?
• Using classical model and phasor diagram in Fig. 20b,

\[
\left( E_a' + \frac{X_d'}{X} V_s \right)^2 + E_b'^2 = \left[ \frac{X_d' + X}{X} V_g \right]^2.
\]

\[
E' = \sqrt{\left( \frac{X_d' + X}{X} V_g \right)^2 - \left( \frac{X_d'}{X} V_s \sin \delta \right)^2} - \frac{X_d'}{X} V_s \cos \delta.
\]

\[
E'\big|_{\delta' = \pi/2} = \frac{V_g}{X} \sqrt{\left( X_d' + X \right)^2 - \left( \frac{X_d'}{V_g} \right)^2}.
\]

\[
P_{V_g, cr} = P_{V_g}(\delta')\big|_{\delta' = \pi/2} = \frac{V_s V_g}{X} \sqrt{1 - \left( \frac{X_d'}{X_d' + X} \right)^2 \left( \frac{V_s}{V_g} \right)^2}.
\]

• Ratio of power at which \( K_{E'} = 0 \) to the power at which \( K_{Vg} = 0 \):

\[
\alpha = \frac{P_{V_g}(\delta' = \pi/2)}{P_{V_g}(\delta = \delta_M)} = \frac{P_{V_g, cr}}{P_{V_g, M}} = \sqrt{1 - \left( \frac{X_d'}{X_d' + X} \right)^2 \left( \frac{V_s}{V_g} \right)^2}.
\]
• $\alpha$ is strongly dependent on the system reactance $X$.

• Refer to the figures below:
  
  - At operating point A (light load), $0 < K_{\text{Eq}} < K_{E'} < K_{Vg}$
  - At operating point B (medium load), $0 = K_{\text{Eq}}$ while $0 < K_{E'} < K_{Vg}$
  - At operating point C (heavy load), $0 > K_{\text{Eq}}$, $K_{E'} = 0$, and $0 < K_{Vg}$

After a disturbance the rotor swings follow the transient power–angle characteristic $P_{E'}(\delta')$. The system is unstable above the point $P_{Vg_{\text{cr}}}$ where $K_{E'} = \partial P_{E'}/\partial \delta < 0$ and no deceleration area is available. Therefore the necessary stability condition is

$$K_{E'} = \frac{\partial P_{E'}}{\partial \delta} > 0.$$  \hspace{1cm} (5.89)
Effect or rotor flux variation

• Determine the phase shift between speed change and $\Delta E'_{q(\Delta E_f)}$ (see Section 5.5.3 for details)

• The phasors below represent two types of AVRs:
  – For static exciter $\Delta E_f$ is in phase with $\Delta V$
  – For rotating machine exciter, $\Delta E_f$ lags $\Delta V$ by 10s’ of degrees.

• the out-of-phase component $\Delta E'_{q(\Delta E_f)}$ relative to $\Delta \omega$ reduces damping (i.e., increases chances of instability).
Effect of AVR action on damper winding

• Change in speed induces a voltage $e_{D(\Delta \omega)}$ in damper winding proportional to $\Delta \omega$. The induced current $i_{D(\Delta \omega)}$ lags by an angle due to high winding inductance.
  – In-phase component of current with $\Delta \omega$ gives rise to the natural damping torque
  – The quadrature component of current enhances the synchronizing power coefficient.

• A change in field winding voltage $\Delta E_f$, induces an additional current $i_{D(\Delta E_f)}$ in damper winding (transformer action).
  – The horizontal component of this current is $180^\circ$ of-of-phase with respect to $\Delta \omega$.
  – $\rightarrow$ the voltage regulator weakens the natural damping above.
  – This detrimental effect can be compensated by a supplementary control loop (PSS).