EE 742
Chapter 3: Power System in the Steady State

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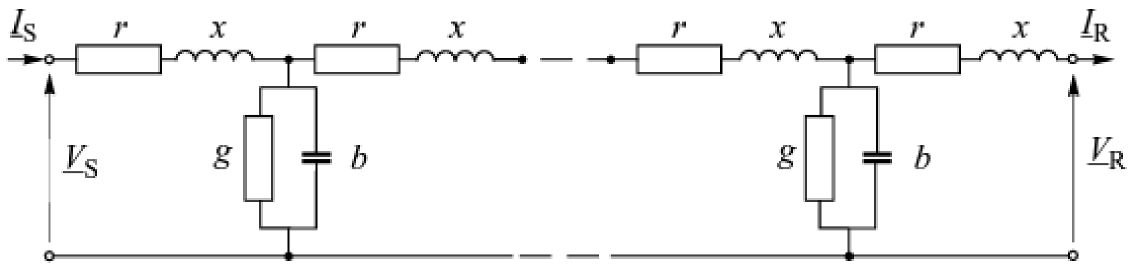
Transmission Line Model

Distributed Parameter Model:

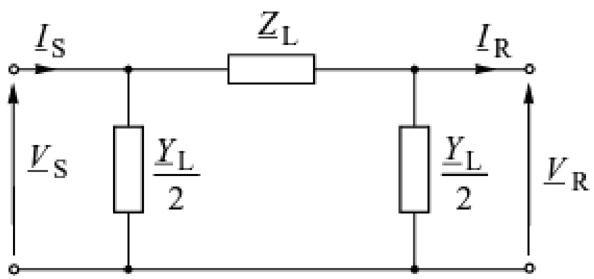


Terminal Voltage/Current Relations:

$$\begin{bmatrix} \underline{V}_S \\ \underline{I}_S \end{bmatrix} = \begin{bmatrix} \cosh \underline{\gamma} l & \underline{Z}_C \sinh \underline{\gamma} l \\ \sinh \underline{\gamma} l / \underline{Z}_C & \cosh \underline{\gamma} l \end{bmatrix} \begin{bmatrix} \underline{V}_R \\ \underline{I}_R \end{bmatrix},$$

Characteristic impedance: $\underline{Z}_C = \sqrt{\underline{z}/\underline{y}}$ Propagation constant: $\underline{\gamma} = \sqrt{\underline{z}\underline{y}}$

π equivalent Circuit:



π equivalent Circuit parameters:

$$\underline{Z}_L = \underline{Z}\frac{\sinh \underline{\gamma} l}{\underline{\gamma} l}, \quad \underline{Y}_L = \underline{Y}\frac{\tanh\left(\underline{\gamma} l/2\right)}{\underline{\gamma} l/2},$$

where $\underline{Z} = \underline{z}l$ is the total series impedance and $\underline{Y} = \underline{y}l$ is the total shunt admittance:

Transmission Line Model

- Simplified models for medium and short lines:
  - Medium line (50 mi < *l* < 150 mi): $\underline{Z}_L = \underline{Z}, \quad \underline{Y}_L = \underline{Y}.$
  - Short line (*l* < 50 mi): $\underline{Z}_L = \underline{Z}, \quad \underline{Y}_L = 0.$
- In a typical line, g can be neglected. *r* can also be neglected (since *r* << *x*), then the line becomes lossless and the characteristic impedance becomes a pure real number ($Z_c = (L/C)^{1/2}$), while propagation constant become pure imaginary number ($\gamma = j\beta = j\omega(LC)^{1/2}$). The voltage/current equations become

$$\underline{V}_S = \underline{V}_R \cos\beta l + jZ_C \underline{I}_R \sin\beta l$$

$$\underline{I}_S = \underline{I}_R \cos\beta l + j\left(\underline{V}_R / Z_C\right) \sin\beta l,$$

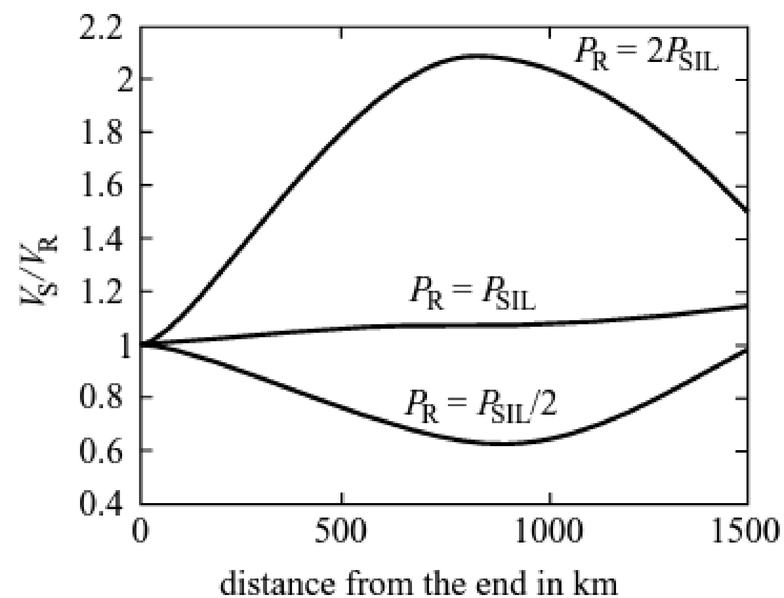
- Note that the voltage and current vary sinusoidally along the line length. The wavelength of the full cycle is $\lambda = 2\pi/\beta$.

Surge Impedance Loading (SIL)

- Power delivered at rated voltage to a load whose impedance is equal to the surge impedance:

$$P_{\mathrm{SIL}} = \frac{V_n^2}{Z_{\mathrm{C}}}.$$

- Under such condition
  - The voltage and current profiles are flat
  - The reactive power loss in the line is zero.
- In reality, the loading is rarely equal to this “natural load”.
- Voltage variation with loading (see fig.)



Real Power Transmission

- Complex power at receiving end (assuming lossless line):

$$\underline{S}_{\mathrm{R}} = \underline{V}_{\mathrm{R}} \underline{I}_{\mathrm{R}}^{*} = \frac{V_{\mathrm{R}} V_{\mathrm{S}}}{Z_{\mathrm{C}} \sin \beta l} \mathrm{e}^{\mathrm{j}(\pi/2 - \delta_{\mathrm{SR}})} - \frac{V_{\mathrm{R}}^{2} \cos \beta l}{Z_{\mathrm{C}} \sin \beta l} \mathrm{e}^{\mathrm{j}\pi/2}.$$

- Real power at receiving end:

$$P_{\mathrm{R}} = \mathrm{Re}\,[\underline{S}] = \frac{V_{\mathrm{S}} V_{\mathrm{R}}}{Z_{\mathrm{C}} \sin \beta l} \sin \delta_{\mathrm{SR}},$$

- Maximum power at receiving end

$$P_{\mathrm{R,max}} = \frac{V_{\mathrm{S}} V_{\mathrm{R}}}{Z_{\mathrm{C}} \sin \beta l} \approx \frac{P_{\mathrm{SIL}}}{\sin \beta l}.$$

- For lines less than 150 miles,

$$\sin \beta l \cong \beta l, \quad \cos \beta l \approx 1, \quad Z_{\mathrm{C}} \sin \beta l \cong \sqrt{\frac{L}{C}} \omega \sqrt{LC}\, l = \omega L l = X,$$

$$P_{\mathrm{R}} \cong \frac{V_{\mathrm{S}} V_{\mathrm{R}}}{X} \sin \delta_{\mathrm{SR}},$$

Reactive Power Considerations

- Complex power at receiving end (assuming lossless line):

$$\underline{S}_\mathrm{R} = \underline{V}_\mathrm{R} \underline{I}_\mathrm{R}^* = \frac{V_\mathrm{R} V_\mathrm{S}}{Z_\mathrm{C} \sin \beta l} \mathrm{e}^{\mathrm{j}(\pi/2 - \delta_\mathrm{SR})} - \frac{V_\mathrm{R}^2 \cos \beta l}{Z_\mathrm{C} \sin \beta l} \mathrm{e}^{\mathrm{j}\pi/2}.$$

- Reactive power at receiving end:

$$Q_\mathrm{R} = \mathrm{Im}\left[\underline{V}_\mathrm{R} \underline{I}_\mathrm{R}^*\right] = \frac{V_\mathrm{S} V_\mathrm{R}}{Z_\mathrm{C} \sin \beta l} \cos \delta_\mathrm{SR} - \frac{V_\mathrm{R}^2 \cos \beta l}{Z_\mathrm{C} \sin \beta l} = \frac{V_\mathrm{R}}{Z_\mathrm{C} \sin \beta l} \left(V_\mathrm{S} \cos \delta_\mathrm{SR} - V_\mathrm{R} \cos \beta l\right).$$

- Approximate expression of lines less than 150 mi long:

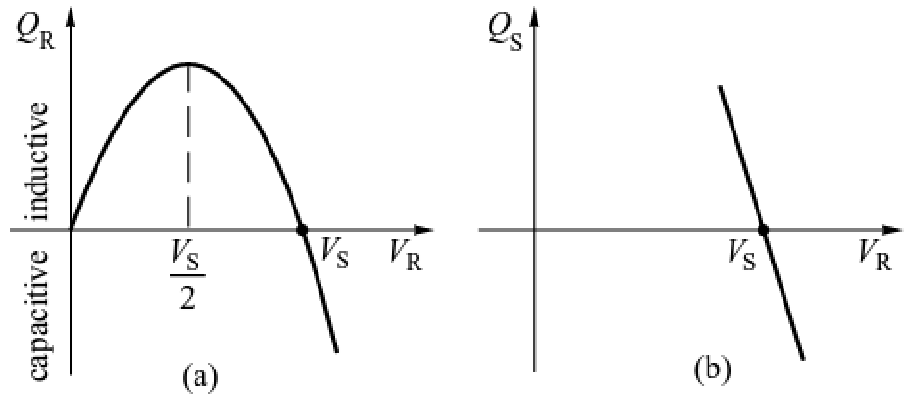
$$Q_\mathrm{R} \cong \frac{V_\mathrm{R}}{X} \left(V_\mathrm{S} \cos \delta_\mathrm{SR} - V_\mathrm{R}\right). \qquad Q_\mathrm{S} \cong \frac{V_\mathrm{S}}{X} \left(V_\mathrm{S} - V_\mathrm{R} \cos \delta_\mathrm{SR}\right).$$

- And for small transmission angles,

$$Q_\mathrm{R} \approx \frac{V_\mathrm{R}(V_\mathrm{S} - V_\mathrm{R})}{X}, \qquad Q_\mathrm{S} \approx \frac{V_\mathrm{S}}{X} \left(V_\mathrm{S} - V_\mathrm{R}\right).$$

Reactive Power Considerations

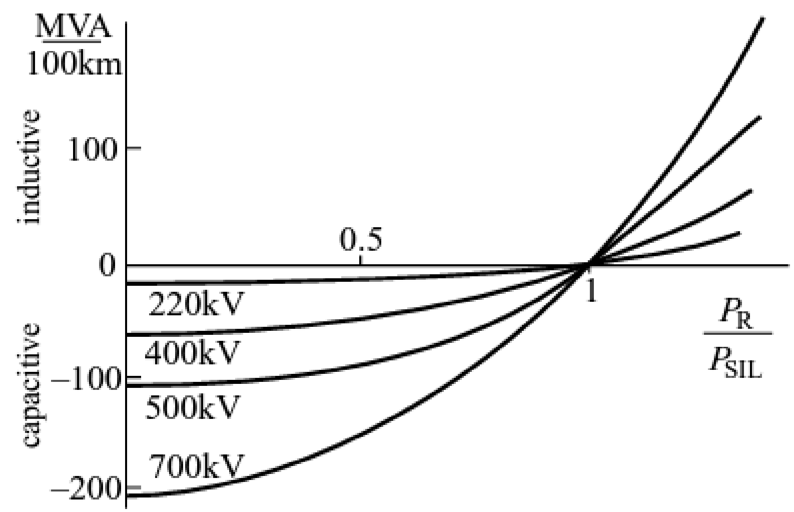
- Change in Q_R and in Q_S (when the influence of real power flow is neglected)



- Reactive power loss under the presence of real power flow:

$$\Delta Q = Q_S - Q_R = \frac{V_S^2 \cos\beta l - 2 V_S V_R \cos\delta_{SR} + V_R^2 \cos\beta l}{Z_C \sin\beta l}.$$

$$\Delta Q(P_R) \approx \frac{2 P_{SIL}}{\sin\beta l}\left[\cos\beta l - \sqrt{1 - \left(\frac{P_R \sin\beta l}{P_{SIL}}\right)^2}\right].$$



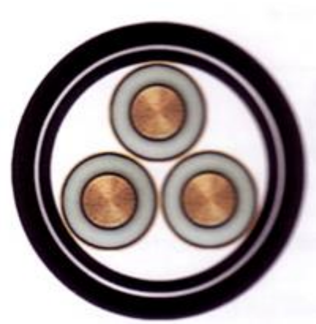
Homework # 2

Consider a 500 kV, 60 Hz transmission line that is 250 mi long. Assume the line parameters are as follows: resistance r=0.018 Ω/mi, reactance x= 0.47 Ω/mi, suceptance b=7.5 μS/mi, receiving end voltage is fixed at rated value (i.e., 288.7 kV/phase), receiving end power = 300 MW/phase @ unity power factor. Calculate the following:

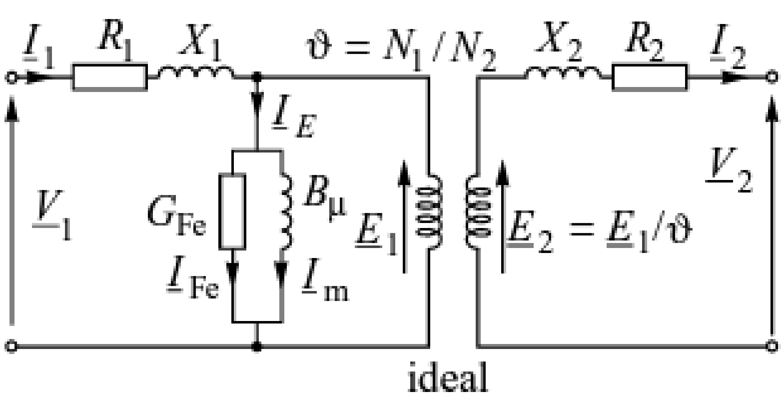
a) Voltage at the sending end.
b) Real and reactive power supplied at the sending end.
c) Recalculate the above using the approximate (lossless) line model.

Underground cables

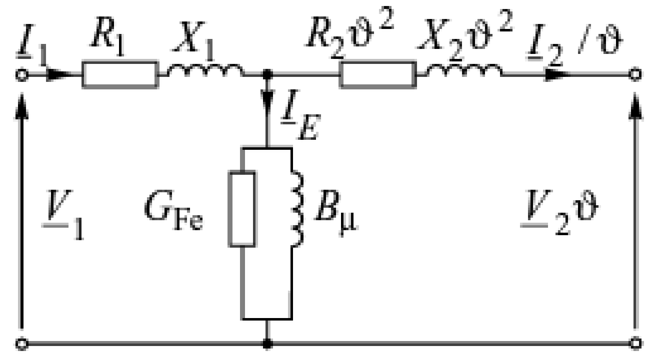
- Modeled the same way as overhead lines
- The typical per-unit series reactance of a cable is about half of that of an overhead line of the same rating.
- The per-unit charging current of a cable is about 30 times larger that that of an overhead line of the same rating – thus severely limiting transmission capacity.



Transformers



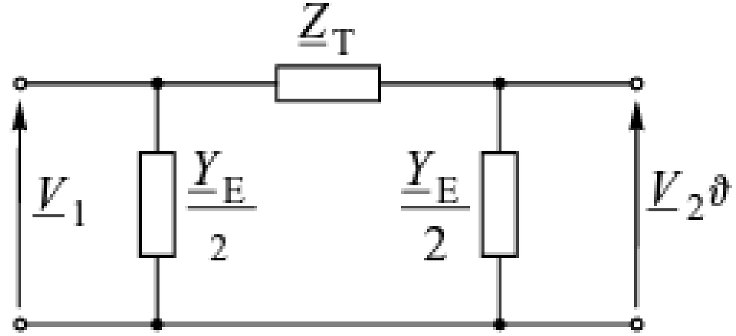
(a)



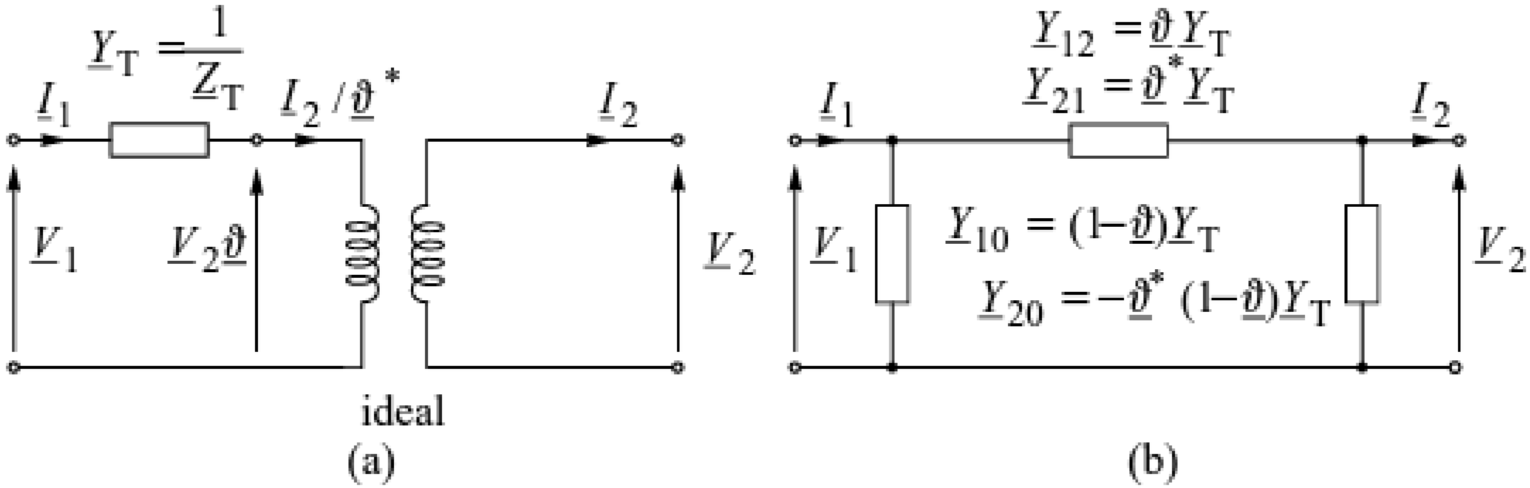
(b)

- Equivalent circuit (see above figure)
- Approximate equivalent π circuit (see figure below)

$$\underline{Z}_{\mathrm{T}} = \underline{Z}_1 + \underline{Z}_2 = R + \mathrm{j}X, \quad \mathrm{Y}_{\mathrm{E}} = G_{\mathrm{FE}} + \mathrm{j}B_{\mu}.$$



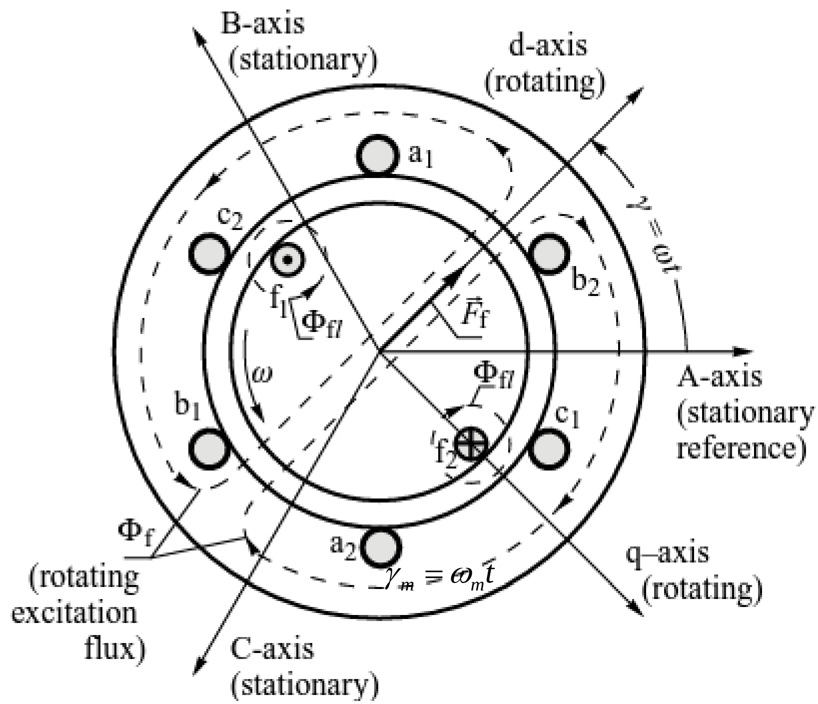
Transformers



- Equivalent circuit with off-nominal turn ratio (including phase shift) → turn ratio is represented by a complex number. Herein, the shunt branch is ignored.

$$\begin{bmatrix} \underline{I}_1 \\ -\underline{I}_2/\underline{\vartheta}^* \end{bmatrix} = \begin{bmatrix} \underline{Y}_T & -\underline{Y}_T \\ -\underline{Y}_T & \underline{Y}_T \end{bmatrix} \begin{bmatrix} \underline{V}_1 \\ \underline{\vartheta}\,\underline{V}_2 \end{bmatrix} \rightarrow \begin{bmatrix} \underline{I}_1 \\ -\underline{I}_2 \end{bmatrix} = \begin{bmatrix} \underline{Y}_T & -\underline{\vartheta}\,\underline{Y}_T \\ -\underline{\vartheta}^*\underline{Y}_T & \underline{\vartheta}^*\underline{\vartheta}\,\underline{Y}_T \end{bmatrix} \begin{bmatrix} \underline{V}_1 \\ \underline{V}_2 \end{bmatrix}.$$

Synchronous Generators: 2-pole round rotor



- For a 2-pole machine: velocity $\omega_m = \omega_e$, and angle $\gamma_m = \gamma_e$, $\gamma = \omega t$
- Rotor flux produced by field current : $\Phi_f = \dfrac{F_f}{\Re} = \dfrac{N_f i_f}{\Re}$,
- Flux linkage induced in phase a (ref).): $\Psi_{fA}(t) = N_\phi \Phi_f \cos(\omega t) = \Psi_{fA} \cos(\omega t)$
- Induced emfs (internal voltages) in stator windings:

$$e_{fA} = -\frac{d\Psi_{fA}(t)}{dt} = \omega M_f i_f \sin \omega t, \qquad e_{fB} = -\frac{d\Psi_{fB}(t)}{dt} = \omega M_f i_f \sin\left(\omega t - \frac{2\pi}{3}\right)$$

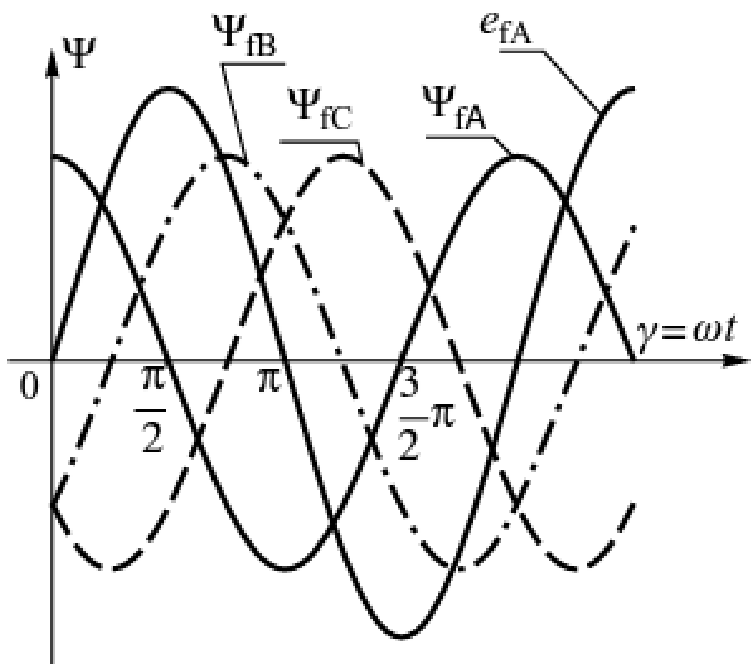
$$e_{fC} = -\frac{d\Psi_{fC}(t)}{dt} = \omega M_f i_f \sin\left(\omega t - \frac{4\pi}{3}\right).$$

Mutual inductance: $M_f = N_\phi N_f / \Re$

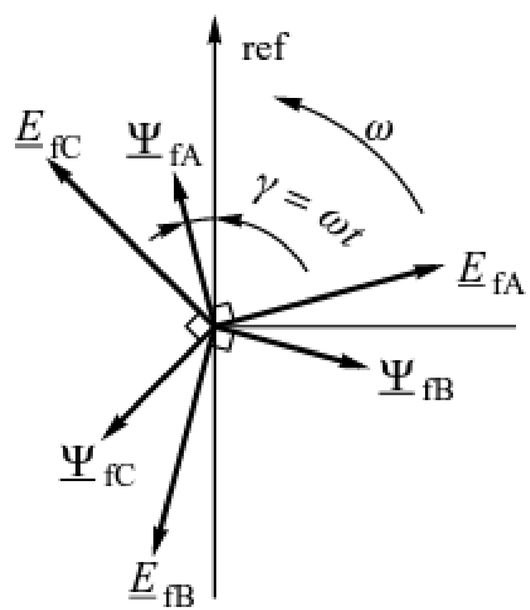
Synchronous Generator under no-load

- Time variation of flux linkages and induced emfs (fig. a)
- Phasor representation of flux linkages and induced emfs (fig. b)
- RMS value of each emf:

$$E_f = \frac{1}{\sqrt{2}}\omega\Psi_{fa} = \frac{1}{\sqrt{2}}\omega N_\phi \Phi_f = \frac{1}{\sqrt{2}}\omega M_f i_f \cong 4.44 f M_f i_f .$$



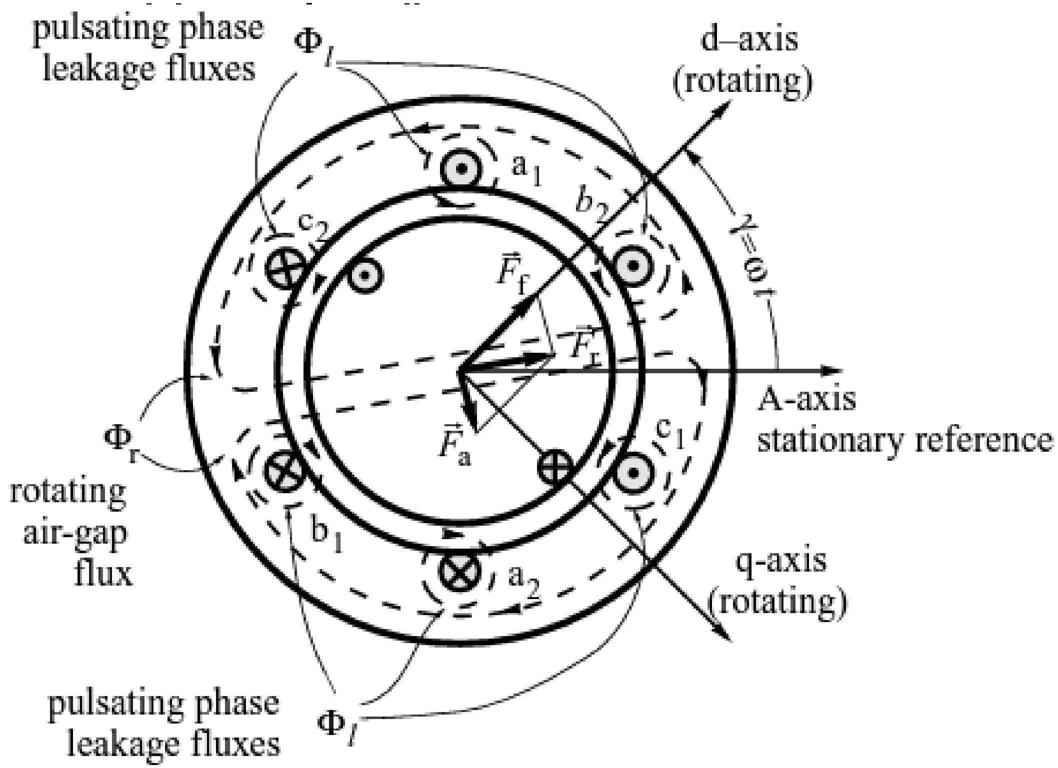
(a)



(b)

Synchronous generator – armature reaction

- The stator phase currents produce pulsating phase mmfs that are shifted in both space and time.
- The resultant stator mmf F_a is constant ($=1.5\ N_a I_m$) and rotates with angular velocity ω (i.e., stationary w.r.t. the field mmf F_f). N_a is the effective number of turns per phase per pole ($4N_\varphi/4p$)
- The new air-gap flux mmf F_r is equal to the vector sum of F_a and F_f. (weaker



Equivalent Circuit

- Total mmf in phase "a":

$$F_{rA}(t) = F_f \cos\omega t + F_a \cos(\omega t - \lambda) = N_f i_f \cos\omega t + 1.5 N_a I_m \cos(\omega t - \lambda).$$

- Flux linkage in phase "a":

$$\Psi_{rA}(t) = N_\phi \frac{F_{rA}(t)}{\Re} = M_f i_f \cos\omega t + L_a I_m \cos(\omega t - \lambda),$$

$$M_f = N_\phi N_f / \Re, \quad L_a = 1.5 N_a N_\phi / \Re$$

- Induced emf in phase "a":

$$e_{rA} = -\frac{d\Psi_{rA}}{dt} = \omega M_f i_f \sin\omega t + \omega L_a I_m \sin(\omega t - \lambda) = e_{fA}(t) + e_{aA}(t),$$

- Phasor form:

$$\underline{E}_r = \underline{E}_f + \underline{E}_a = \underline{E}_f - jX_a \underline{I},$$

- Terminal voltage (after taking stator winding resistance and leakage flux into account):

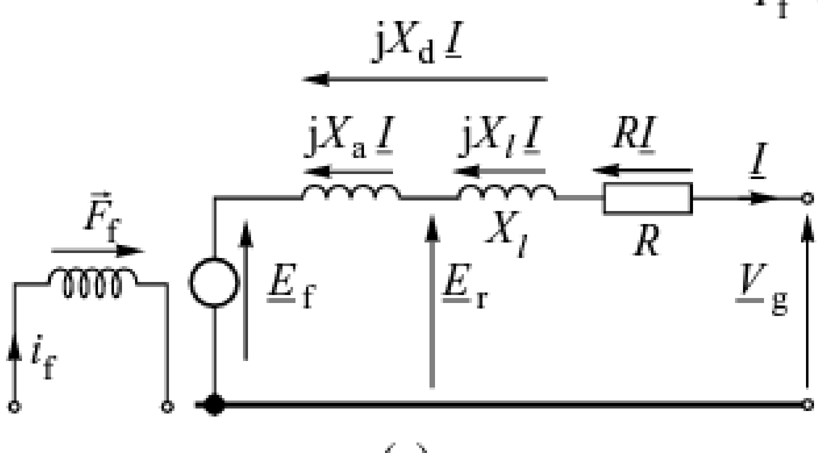
$$\underline{V}_g = \underline{E}_f - jX_a \underline{I} - jX_l \underline{I} - R\underline{I} = \underline{E}_f - jX_d \underline{I} - R\underline{I},$$

Equivalent circuit

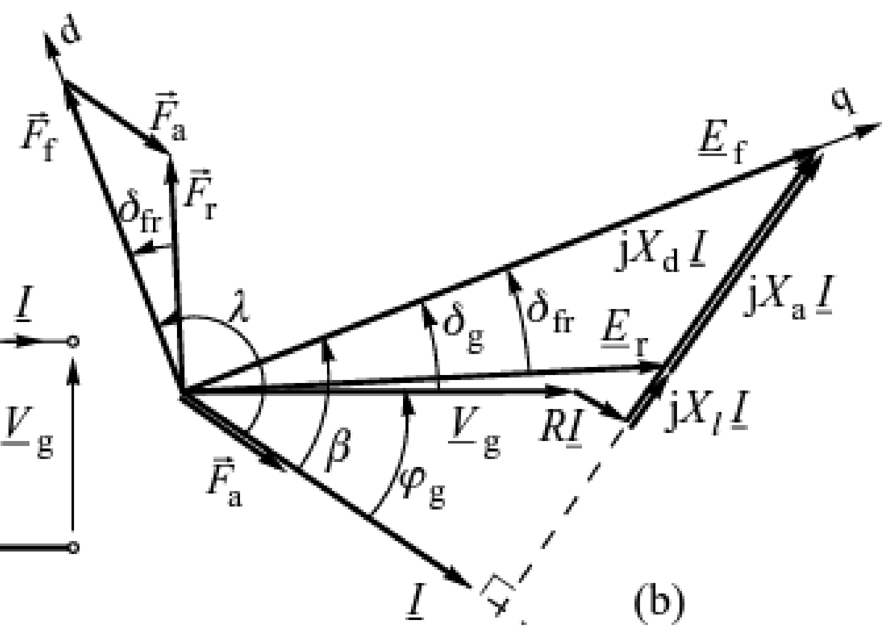
- Armature reaction (or magnetizing) reactance: X_a
- Leakage reactance: X_l
- Synchronous reactance (or d-axis synchronous reactance):

$$X_d = X_a + X_l$$

- Internal emf (or voltage behind the synchronous reactance): E_f



(a)



(b)

Electromagnetic Torque

- For the rotor speed to be constant, the two opposing mechanical torque τ_m and electromagnetic torque τ must be equal.
- Neglecting the mechanical losses, the air gap power must equal the mechanical power: $\tau_m \omega_m = 3E_f I \cos\beta$ (where β is the angle of *I* with respect to E_f)
- For a machine with p-poles, the electromagnetic torque can be written in a number of forms (note: $\lambda=\pi/2+\beta$, and the torque angle δ_{fr} is the angle between F_r and F_f.)

$$\tau = \frac{3}{4} p \Phi_f N_\phi I_m \cos\beta.$$

$$\tau = \frac{\pi}{8} p^2 \Phi_f F_a \sin\lambda = \frac{\pi}{8} p^2 \frac{F_f F_a}{\Re} \sin\lambda,$$

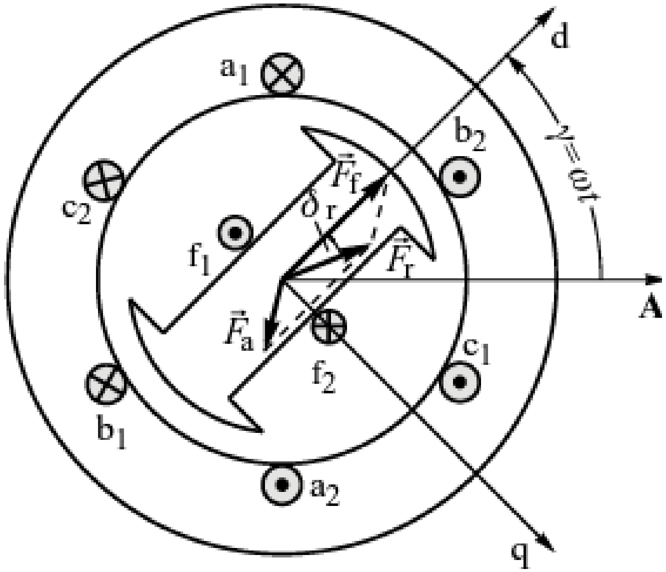
$$\tau = \frac{\pi}{8} p^2 F_r \frac{F_f}{\Re} \sin\delta_{fr} = \frac{\pi}{8} p^2 F_r \Phi_f \sin\delta_{fr},$$

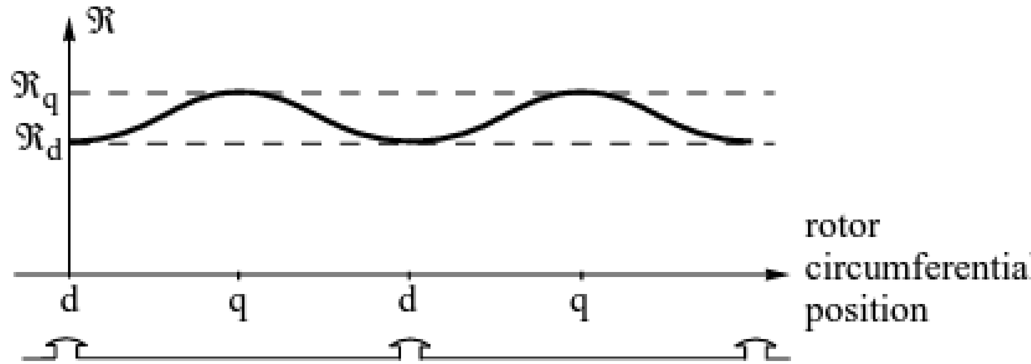
$$\tau = \frac{\pi}{2} F_r \Phi_f \sin\delta_{fr}.$$ ← for a 2-pole machine

Homework # 3

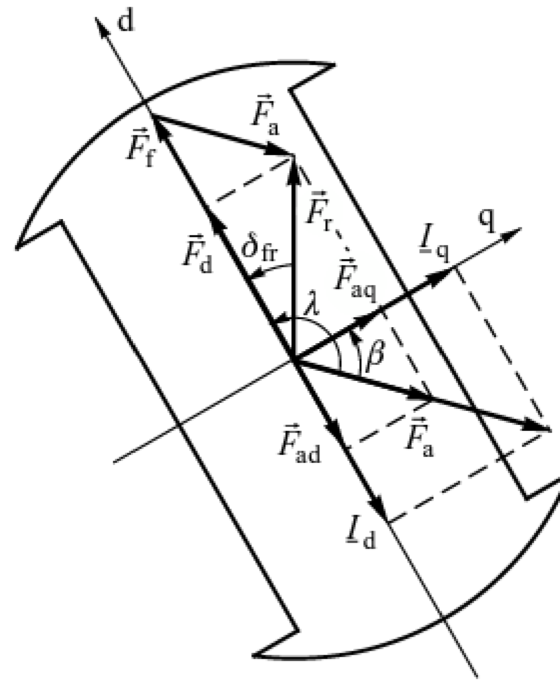
- Consider a cylindrical 2-pole, 60 Hz, 25 kV, 200 MVA generator . The per-unit values of R, X_a and X_l are 0. 05, 0.8 and 0.1, respectively. The generator is delivering 0.8 + j0.5 pu of complex power to the network and the terminal voltage is at 1 pu. Calculate the following:
  - E_f and E_r
  - δ_g and torque angle δ_{fr}
  - λ and internal power factor angle β
  - Real and reactive power losses in the generator
  - Airgap power and mechanical torque (ignore mech. losses).

Salient pole machines – 2 poles





- Non-uniform air gap, hence variable reluctance.
- → Decomposition of mmfs and current into d-component and q-component.
- Let X_{ad} and X_{aq} represent the direct-axis and quadrature-axis armature reaction reactances.



Salient pole machines – 2 poles

- Resultant air gap emf:

$$\underline{E}_r = \underline{E}_f + \underline{E}_{aq} + \underline{E}_{ad} = \underline{E}_f - jX_{ad}\underline{I}_d - jX_{aq}\underline{I}_q.$$

- Resultant terminal voltage:

$$\underline{V}_g = \underline{E}_r - jX_l\underline{I} - R\underline{I} = \underline{E}_f - jX_{ad}\underline{I}_d - jX_{aq}\underline{I}_q - jX_l(\underline{I}_d + \underline{I}_q) - R\underline{I}$$

$$= \underline{E}_f - j(X_{ad} + X_l)\underline{I}_d - j(X_{aq} + X_l)\underline{I}_q - R\underline{I},$$

$$\underline{E}_f = \underline{V}_g + jX_d\underline{I}_d + jX_q\underline{I}_q + R\underline{I},$$

$$\underline{E}_f = \underline{V}_g + R\underline{I} + jX_q\underline{I} + j(X_d - X_q)\underline{I}_d = \underline{E}_Q + j(X_d - X_q)\underline{I}_d,$$

Herein, $\underline{E}_Q = \underline{V}_g + (R + jX_q)\underline{I}$. and X_d and X_q are the direct- and quadrature-axis synchronous reactances.

- Note: $X_d > X_q$ because the reluctance along the q-axis is highest.
- Note: for a round rotor, $X_d = X_q$, and $E_Q = E_f$.

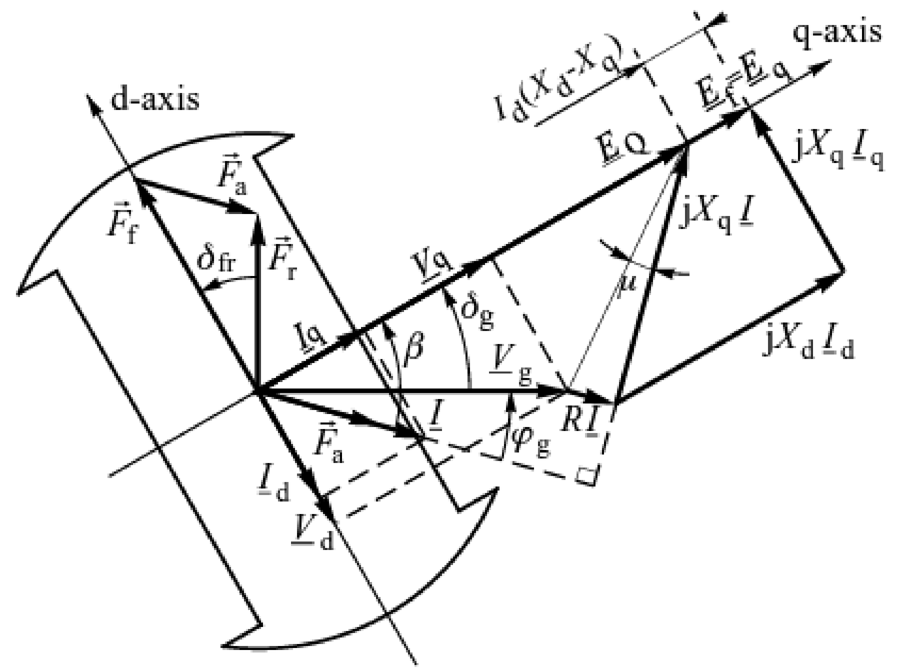
Phasor Diagram and Equivalent Circuit

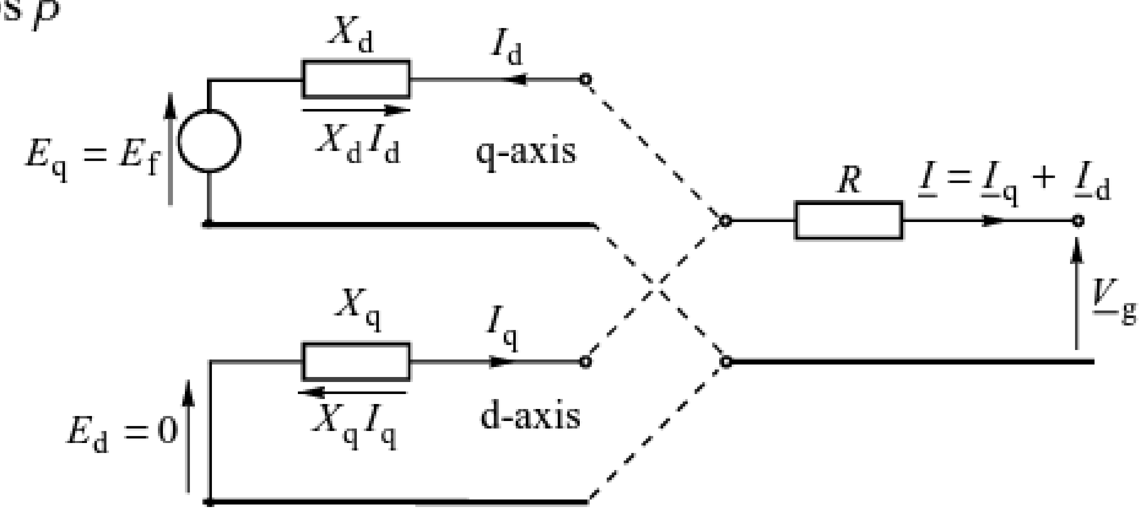
d-axis: $E_{\mathrm{d}} = V_{\mathrm{d}} + RI_{\mathrm{d}} + X_{\mathrm{q}} I_{\mathrm{q}} = 0$

q-axis: $E_{\mathrm{q}} = V_{\mathrm{q}} + RI_{\mathrm{q}} - X_{\mathrm{d}} I_{\mathrm{d}} = E_{\mathrm{f}}.$

$V_{\mathrm{d}} = -V_{\mathrm{g}} \sin \delta_{\mathrm{g}}, \qquad V_{\mathrm{q}} = V_{\mathrm{g}} \cos \delta_{\mathrm{g}}$

$I_{\mathrm{d}} = -I \sin \beta, \qquad I_{\mathrm{q}} = I \cos \beta$





Torque

- Electromagnetic torque of a generator with 2-pole salient rotor (see derivation on pp. 88)

$$\tau = \frac{\pi}{2}\Phi_{\mathrm{f}} F_{\mathrm{r}} \sin\delta_{\mathrm{fr}} + \frac{\pi}{4} F_{\mathrm{r}}^2 \frac{\mathfrak{R}_{\mathrm{q}} - \mathfrak{R}_{\mathrm{d}}}{\mathfrak{R}_{\mathrm{q}}\mathfrak{R}_{\mathrm{d}}} \sin 2\delta_{\mathrm{fr}}.$$

↑ Synchronous torque

↑ Reluctance torque

Generator-transformer unit

- The generator impedance is increase by the transformer impedance. For the round rotor machine;

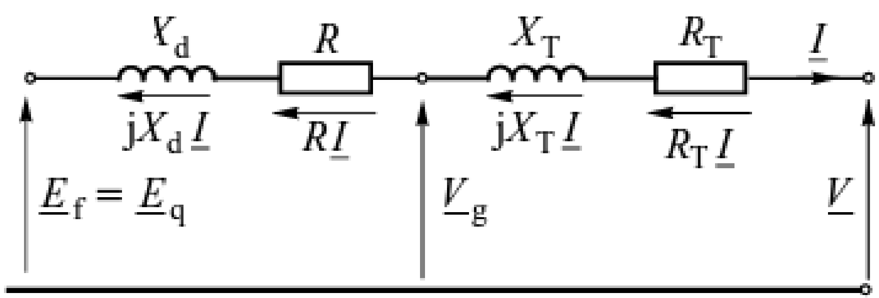
$$\underline{V} = \underline{E}_{\mathrm{f}} - \mathrm{j}X_{\mathrm{d}}\underline{I} - R\underline{I} - \mathrm{j}X_{\mathrm{T}}\underline{I} - R_{\mathrm{T}}\underline{I} = \underline{E}_{\mathrm{f}} - \mathrm{j}\left(X_{\mathrm{d}} + X_{\mathrm{T}}\right)\underline{I} - \left(R + R_{\mathrm{T}}\right)\underline{I},$$

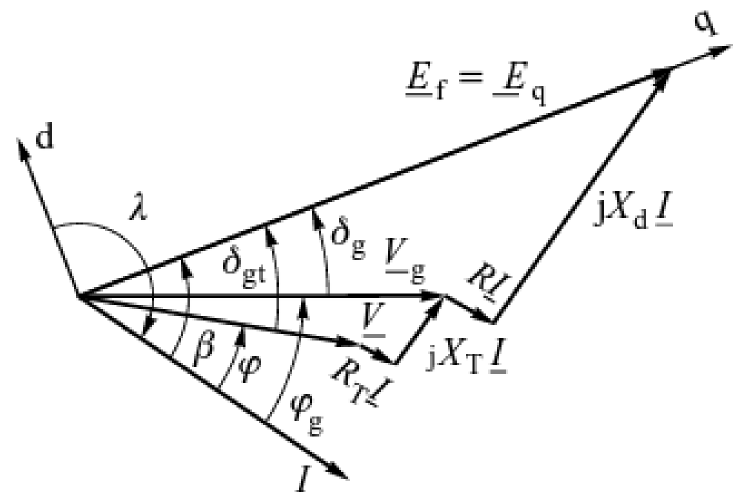
- For the salient rotor machine

$$\underline{V}_{\mathrm{g}} = \underline{E}_{\mathrm{f}} - \mathrm{j}X_{\mathrm{d}}\underline{I}_{\mathrm{d}} - \mathrm{j}X_{\mathrm{q}}\underline{I}_{\mathrm{q}} - R\underline{I} - X_{\mathrm{T}}\underline{I} - R_{\mathrm{T}}\underline{I}$$

$$= \underline{E}_{\mathrm{f}} - \mathrm{j}X_{\mathrm{d}}\underline{I}_{\mathrm{d}} - \mathrm{j}X_{\mathrm{q}}\underline{I}_{\mathrm{q}} - R\underline{I} - X_{\mathrm{T}}\left(\underline{I}_{\mathrm{d}} + \underline{I}_{\mathrm{q}}\right) - R_{\mathrm{T}}\underline{I}.$$

Including network impedance is achieved by adding X_s and R_s in series.





Real and reactive power of generator-transformer unit

- Real power (see pp. 91): $P = V_d I_d + V_q I_q$ $\quad z^2 = r^2 + x_d x_q.$

$$P = \frac{E_q V}{z}\frac{x_q}{z}\sin\delta_{gt} + \frac{1}{2}\frac{V^2}{z}\frac{x_d - x_q}{z}\sin 2\delta_{gt} + \frac{E_q V}{z}\frac{r}{z}\cos\delta_{gt} - \frac{V^2}{z}\frac{r}{z}.$$

- For a round rotor machine and when neglecting the resistance:

$$P = \frac{E_q V}{x_d}\sin\delta_{gt}.$$

- Reactive power (see pp. 92): Q= $V_d I_q - V_q I_d$

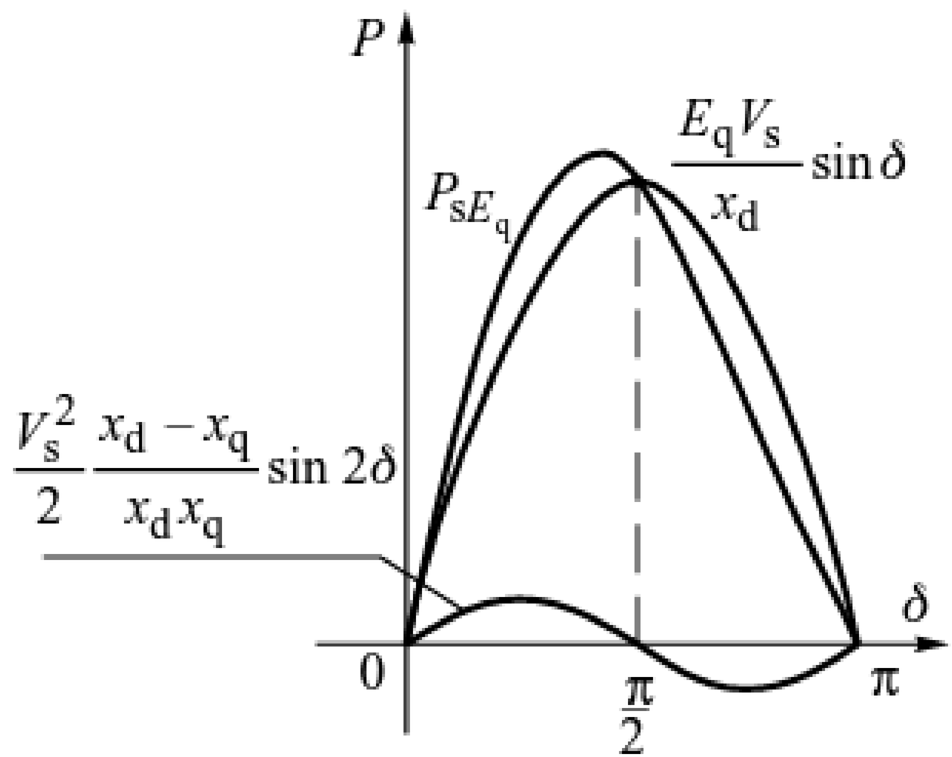
$$Q = \frac{E_q V}{z}\frac{x_q}{z}\cos\delta_{gt} - \frac{V^2}{z}\frac{x_d\sin^2\delta_{gt} + x_q\cos^2\delta_{gt}}{z} - \frac{E_q V}{z}\frac{r}{z}\sin\delta_{gt}.$$

- For a round rotor machine and when neglecting the resistance:

$$Q = \frac{E_q V}{x_d}\cos\delta_{gt} - \frac{V^2}{x_d}.$$

Power-angle characteristic of generator with salient rotor

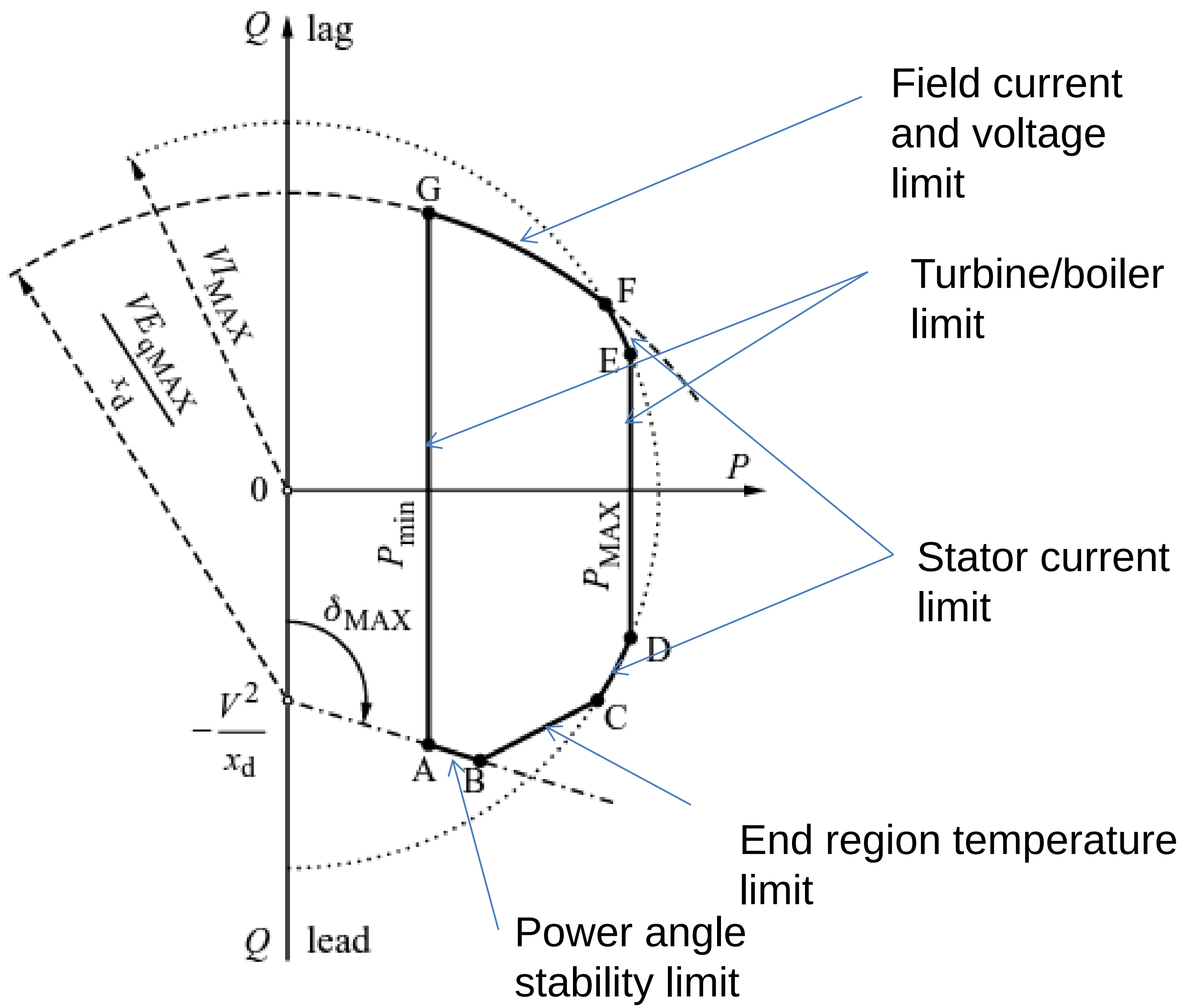
- When *r* is ignored and E_q is held constant, then the power contains two terms. The latter (i.e., reluctance term) deforms the sinusoid. Hence peak power occurs at an angle that is less than 90°.



Homework # 4

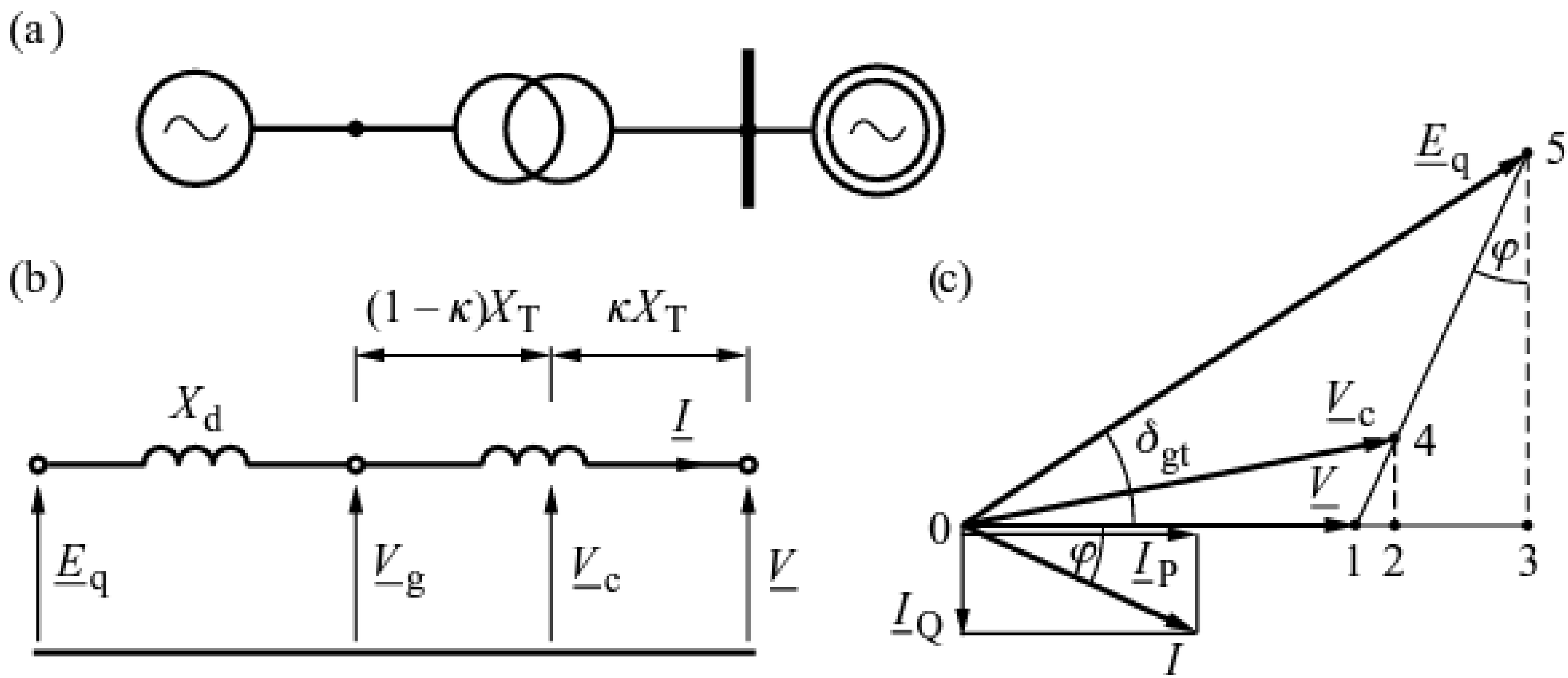
- Consider a 25 kV, 230 MVA generator with salient-pole rotor. The per-unit values of R, X_d and X_q are 0.05, 0.93 and 0.69, respectively. The generator is delivering 0.6 + j0.3 pu of complex power to the network and the terminal voltage is at 1 pu.
  - Determine the d-axis and q-axis components of the generator current, terminal voltage, and internal emf.
  - Calculate the active and reactive powers supplied by the generator using the expressions in the previous slides and compare to the above values.
  - Finally, determine the maximum power and corresponding angle δ if the E and V are held constant.

Real and reactive power capability curves



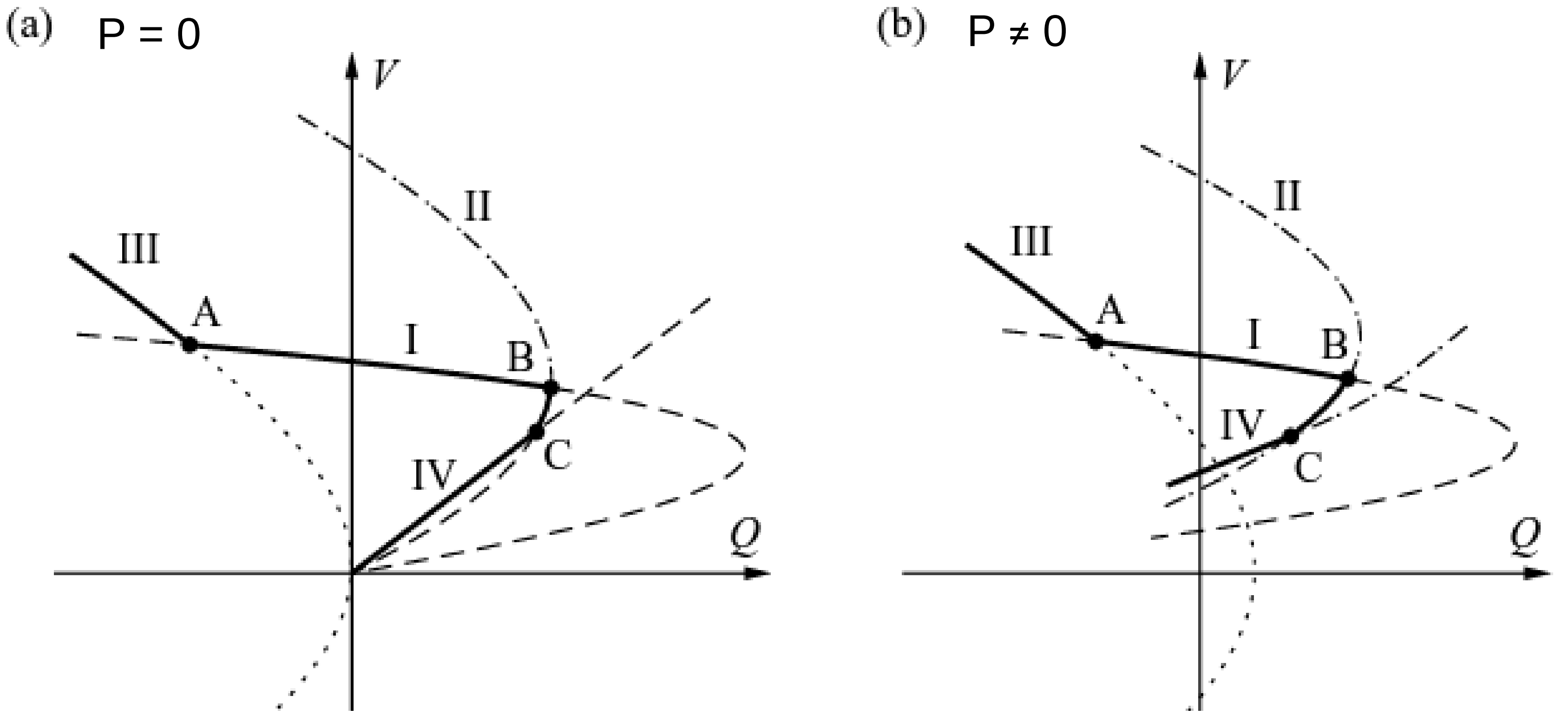
Voltage-reactive power capability curve

- Consider a round rotor generator connected to a power system through a transformer.
- Voltage is to be regulated at a fictitious point (V_c) within the transformer.



V(Q) Generator Curves

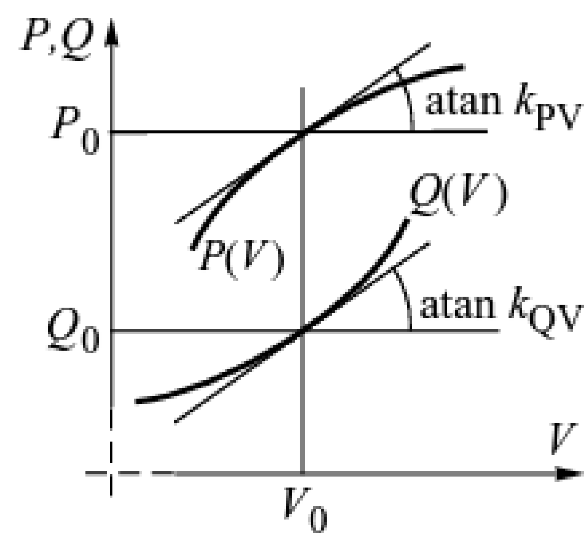
- Curve I: Field current less than its maximum value (eqn. 3.105)
- Curve II: Maximum field current (eqn. 3.106)
- Curve III: Maximum power angle (eqn. 3.107)
- Curve IV: Maximum stator current (eqn. 3.108)



Power System Loads

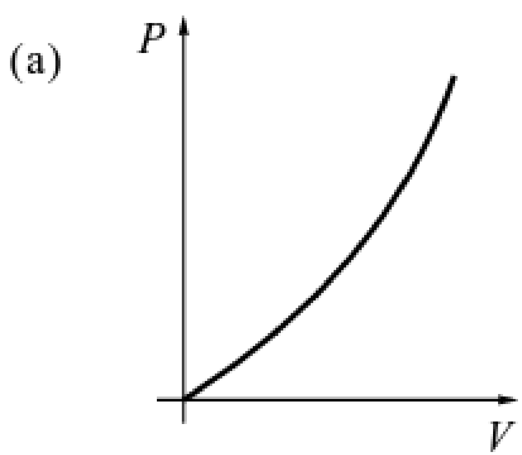
- Only simple static composite load models are described (dynamic models will be seen in later chapters).
- The active and reactive power demand of a static composite load depends on the voltage and frequency.
- *Voltage and frequency sensitivity*: slope of load-voltage or load-frequency characteristics (see fig. below)

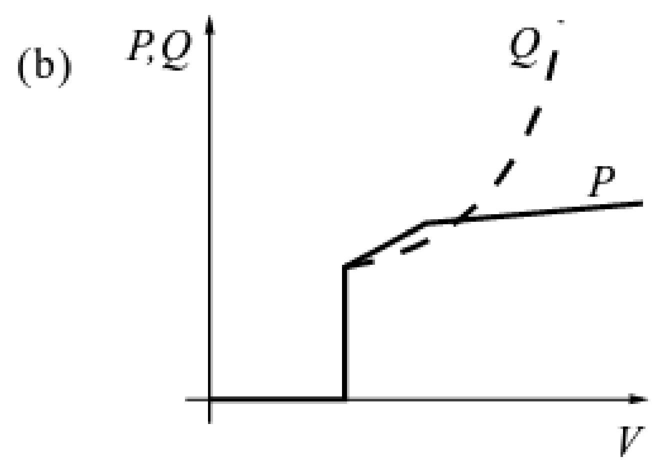
$$k_{PV} = \frac{\Delta P / P_0}{\Delta V / V_0}, \quad k_{QV} = \frac{\Delta Q / Q_0}{\Delta V / V_0}, \quad k_{Pf} = \frac{\Delta P / P_0}{\Delta f / f_0}, \quad k_{Qf} = \frac{\Delta Q / Q_0}{\Delta f / f_0},$$



Lighting and heating load characteristics

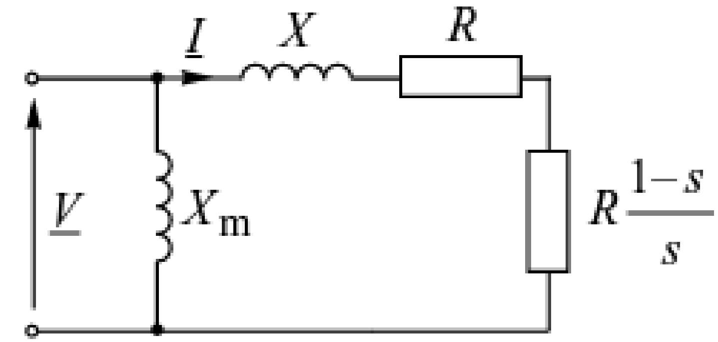
- Voltage characteristics of incandescent and fluorescent bulbs (see fig. below)
- Heating load equipped with thermostat is considered a constant power load. If not, its is considered a constant resistance load.



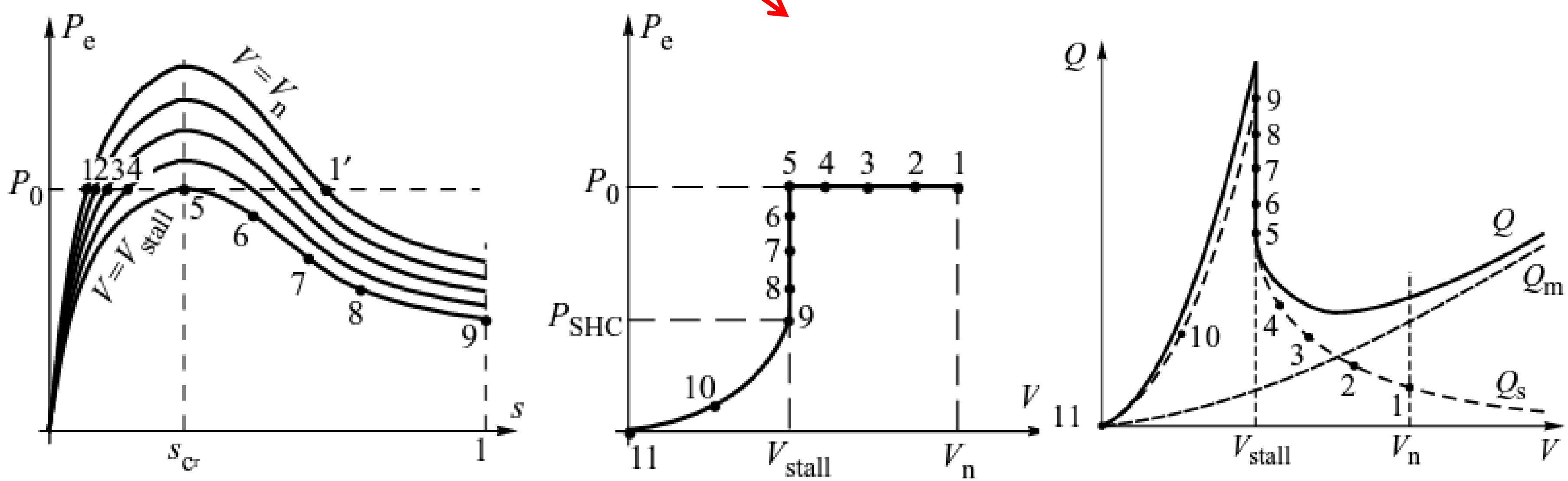


Induction Motors

- Equivalent circuit with stator impedance neglected.

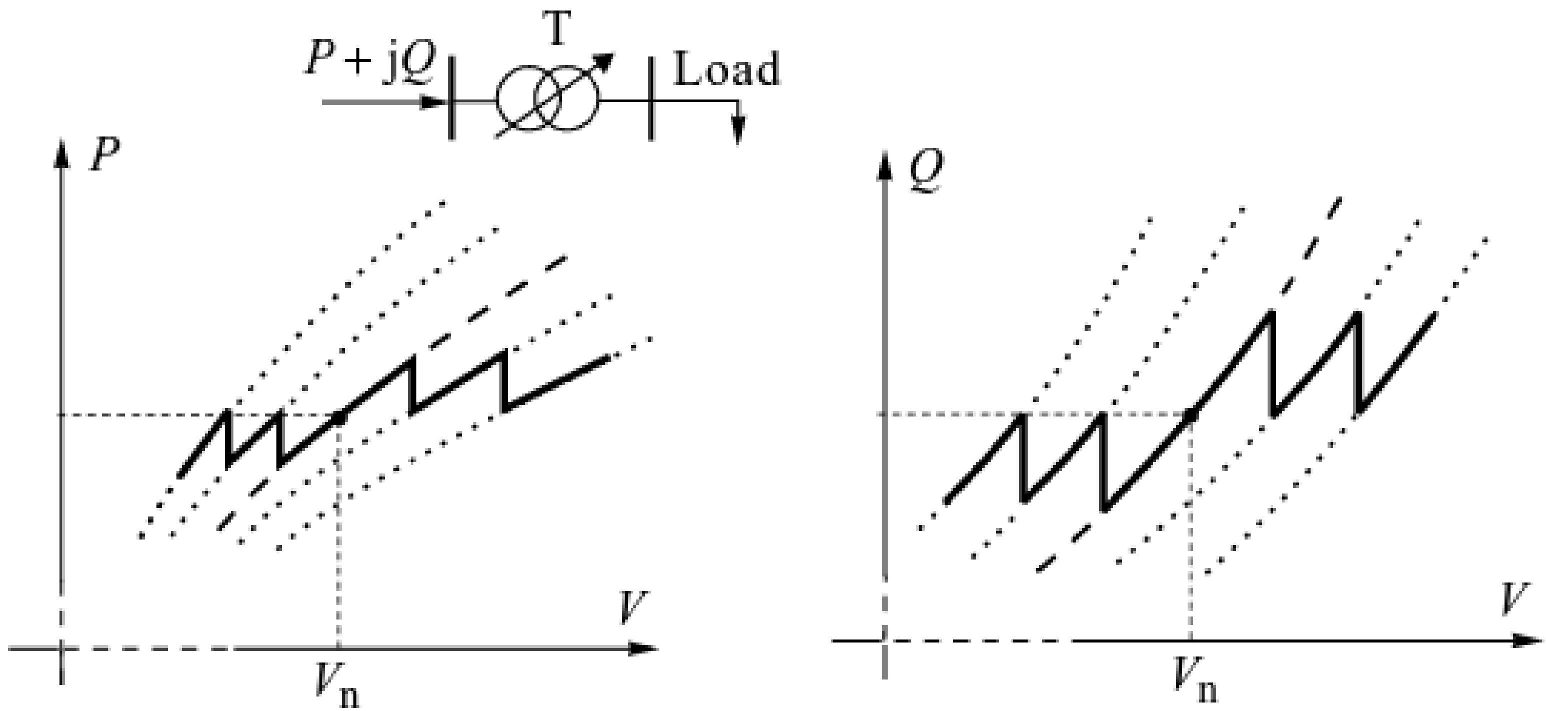


- Voltage characteristic under constant load torque.



(a) (b)

Influence of tap-changing transformer on composite load voltage characteristics



ZIP and Exponential and Frequency-Dependent Load Models

- ZIP model:

$$P = P_0\left[a_1\left(\frac{V}{V_0}\right)^2 + a_2\left(\frac{V}{V_0}\right) + a_3\right]$$

$$Q = Q_0\left[a_4\left(\frac{V}{V_0}\right)^2 + a_5\left(\frac{V}{V_0}\right) + a_6\right],$$

- Exponential model:

$$P = P_0\left(\frac{V}{V_0}\right)^{n_\mathrm{p}} \quad \text{and} \quad Q = Q_0\left(\frac{V}{V_0}\right)^{n_\mathrm{q}},$$

- Frequency dependent model:

$$P = P(V)\left[1 + k_{\mathrm{Pf}}\frac{\Delta f}{f_0}\right]$$

$$Q = Q(V)\left[1 + k_{\mathrm{Qf}}\frac{\Delta f}{f_0})\right],$$

Network Equations

- Bus admittance matrix

$$\begin{bmatrix} \underline{I}_1 \\ \vdots \\ \underline{I}_i \\ \vdots \\ \underline{I}_N \end{bmatrix} = \begin{bmatrix} \underline{Y}_{11} & \cdots & \underline{Y}_{1i} & \cdots & \underline{Y}_{1N} \\ \vdots & \ddots & \vdots & & \vdots \\ \underline{Y}_{i1} & \cdots & \underline{Y}_{ii} & \cdots & \underline{Y}_{iN} \\ \vdots & & \vdots & & \vdots \\ \underline{Y}_{N1} & \cdots & \underline{Y}_{Ni} & \cdots & \underline{Y}_{NN} \end{bmatrix} \begin{bmatrix} \underline{V}_1 \\ \vdots \\ \underline{V}_i \\ \vdots \\ \underline{V}_N \end{bmatrix} \quad \text{or} \quad \underline{\boldsymbol{I}} = \underline{\boldsymbol{Y}}\underline{\boldsymbol{V}}.$$

- Bus impedance matrix

$$\begin{bmatrix} \underline{V}_1 \\ \vdots \\ \underline{V}_i \\ \vdots \\ \underline{V}_N \end{bmatrix} = \begin{bmatrix} \underline{Z}_{11} & \cdots & \underline{Z}_{1i} & \cdots & \underline{Z}_{1N} \\ \vdots & \ddots & \vdots & & \vdots \\ \underline{Z}_{i1} & \cdots & \overline{Z_{ii}} & \cdots & \underline{Z}_{iN} \\ \vdots & & \vdots & & \vdots \\ \underline{Z}_{N1} & \cdots & \underline{Z}_{Ni} & \cdots & \underline{Z}_{NN} \end{bmatrix} \begin{bmatrix} \underline{I}_1 \\ \vdots \\ \underline{I}_i \\ \vdots \\ \underline{I}_N \end{bmatrix} \quad \text{or} \quad \underline{\boldsymbol{V}} = \underline{\boldsymbol{Z}\boldsymbol{I}},$$

Power flow equations

$$\underline{I}_i = \underline{Y}_{ii}\underline{V}_i + \sum_{j=1;j\neq i}^{N} \underline{Y}_{ij}\underline{V}_j,$$

$$\underline{S}_i = P_i + \mathrm{j}\,Q_i = \underline{V}_i \underline{I}_i^* = V_i \mathrm{e}^{\mathrm{j}\delta_i}\left[Y_{ii}\,V_i \mathrm{e}^{-\mathrm{j}(\delta_i+\theta_{ii})} + \sum_{j=1;j\neq i}^{N} V_j\,Y_{ij}\mathrm{e}^{-\mathrm{j}(\delta_j+\theta_{ij})}\right]$$

$$= V_i^2\,Y_{ii}\,\mathrm{e}^{-\mathrm{j}\theta_{ii}} + V_i \sum_{j=1;j\neq i}^{N} V_j\,Y_{ij}\mathrm{e}^{\mathrm{j}(\delta_i-\delta_j-\theta_{ij})}.$$

$$P_i = V_i^2\,Y_{ii}\cos\theta_{ii} + \sum_{j=1;j\neq i}^{N} V_i\,V_j\,Y_{ij}\cos(\delta_i-\delta_j-\theta_{ij})$$

$$Q_i = -V_i^2\,Y_{ii}\sin\theta_{ii} + \sum_{j=1;j\neq i}^{N} V_i\,V_j\,Y_{ij}\sin(\delta_i-\delta_j-\theta_{ij}).$$