EE 742 – Chap 4
Electromagnetic Phenomenon

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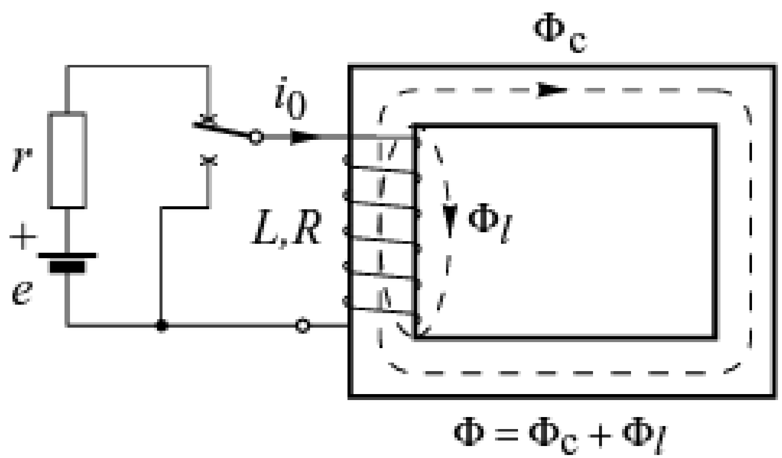
Introduction

- This chapter addresses the electromagnetic interactions that occur within a generator immediately following a disturbance.
- These dynamics have a time scale of several milliseconds – a time period where the inertia of the turbine and generator is sufficient to prevent any change in rotor speed.
- Will consider various types of faults, round and salient pole generators, and when a generator is synchronized with the grid.

Fundamentals

- *Law of constant flux linkage*: The magnetic flux linking in a closed winding cannot change instantly, → the current through an inductor and the energy stored in it cannot change instantly.
- In the simple DC circuit below:
  - The circuit was at steady-state, when the switch (make-before-break) is activated.
  - The current through the inductor starts from i_0 then decays exponentially.



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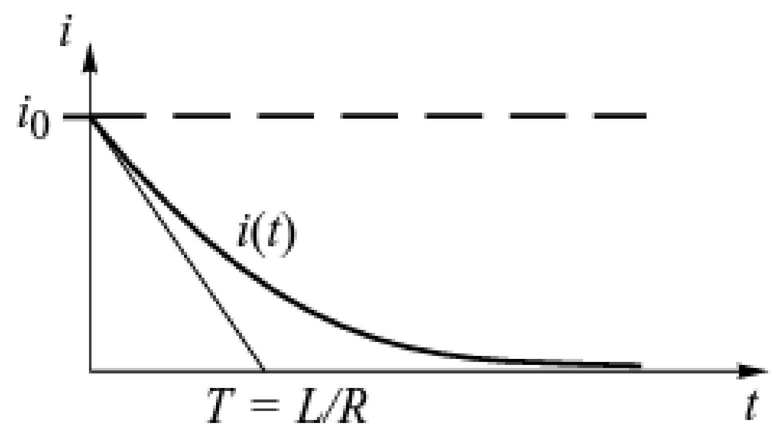

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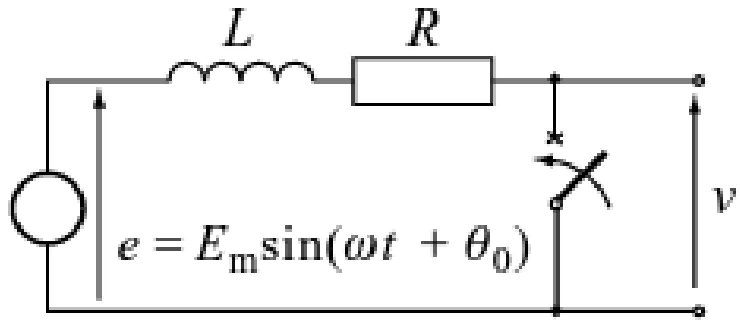
Simple R-L Circuit

- Circuit Equation:

$$E_{\mathrm{m}} \sin(\omega t + \theta_0) = L\frac{\mathrm{d}i}{\mathrm{d}t} + Ri,$$



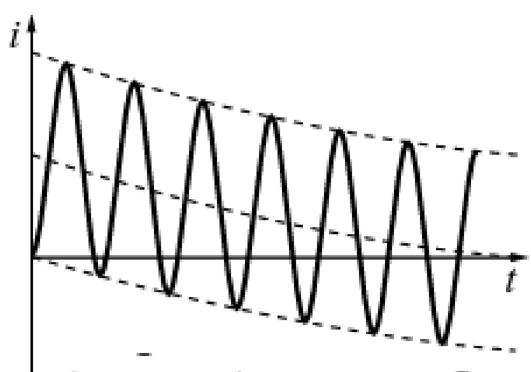
- Solution:

Forced response

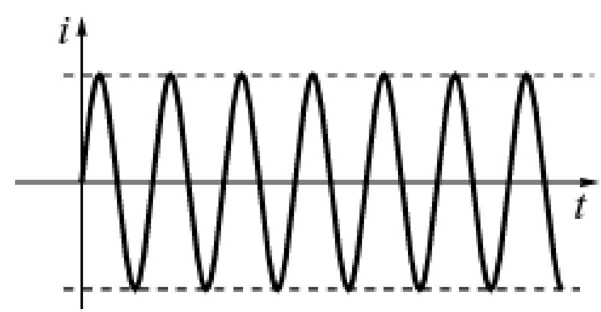
Natural response (DC offset)

$$i(t) = \frac{E_{\mathrm{m}}}{Z}\sin(\omega t + \theta_0 - \phi) - \frac{E_{\mathrm{m}}}{Z}\sin(\theta_0 - \phi)\mathrm{e}^{-Rt/L},$$

where $Z = \sqrt{\omega^2 L^2 + R^2}.$ $\phi = \arctan(\omega L / R)$



$(\theta_0 - \phi) = -\pi/2.$



$(\theta_0 - \phi) = 0;$

3-phase short circuit at generator terminals

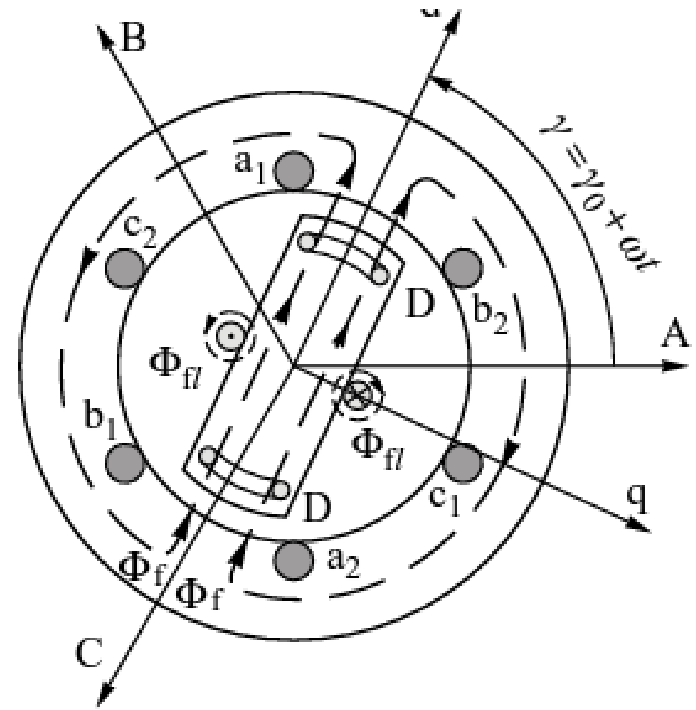
(no load and neglect resistance)

- Fault occurs at t = 0, the rotor position is at $\gamma = \gamma_0$
- The damper winding imbedded in the rotor pole surfaces is denoted by D.
- Initial flux linkages in armature (stator) windings:

$$\Psi_{\mathrm{fA0}} = \Psi_{\mathrm{fa}} \cos \gamma_0, \quad \Psi_{\mathrm{fB0}} = \Psi_{\mathrm{fa}} \cos(\gamma_0 - 2\pi/3), \quad \Psi_{\mathrm{fC0}} = \Psi_{\mathrm{fa}} \cos(\gamma_0 - 4\pi/3),$$

where $\Psi_{\mathrm{fa}} = N_\phi \Phi_{\mathrm{f0}}$

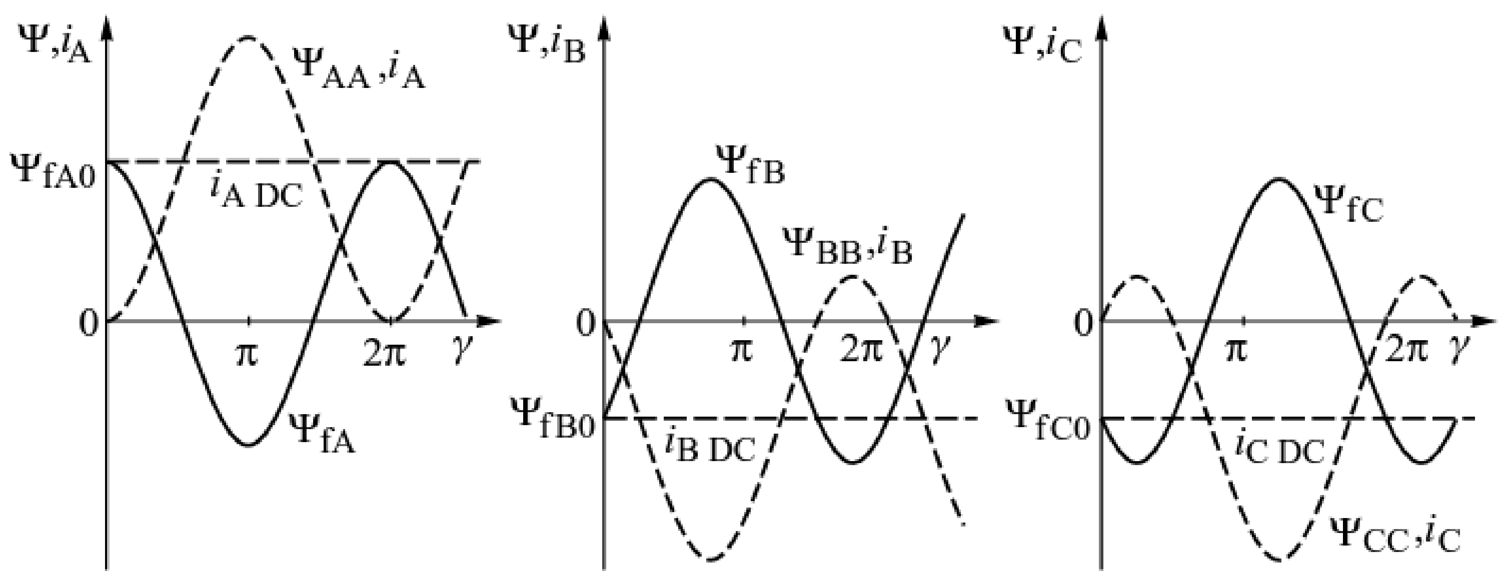
Pre-fault excitation
Flux per pole



3-phase short circuit at generator terminals
(no load and neglect resistance)

- After the fault, the rotor continues to rotate and flux linkages continue to change in a sinusoidal fashion: Ψ_{fA}, Ψ_{fB} and Ψ_{fC}
- In order to keep the total flux linkage in each phase constant, additional currents must be induced in the stator windings to generate the difference between the initial flux linkage and change above

$$\Psi_{AA} = \Psi_{fA0} - \Psi_{fA}, \quad \Psi_{BB} = \Psi_{fB0} - \Psi_{fB}, \quad \Psi_{CC} = \Psi_{fC0} - \Psi_{fC}.$$

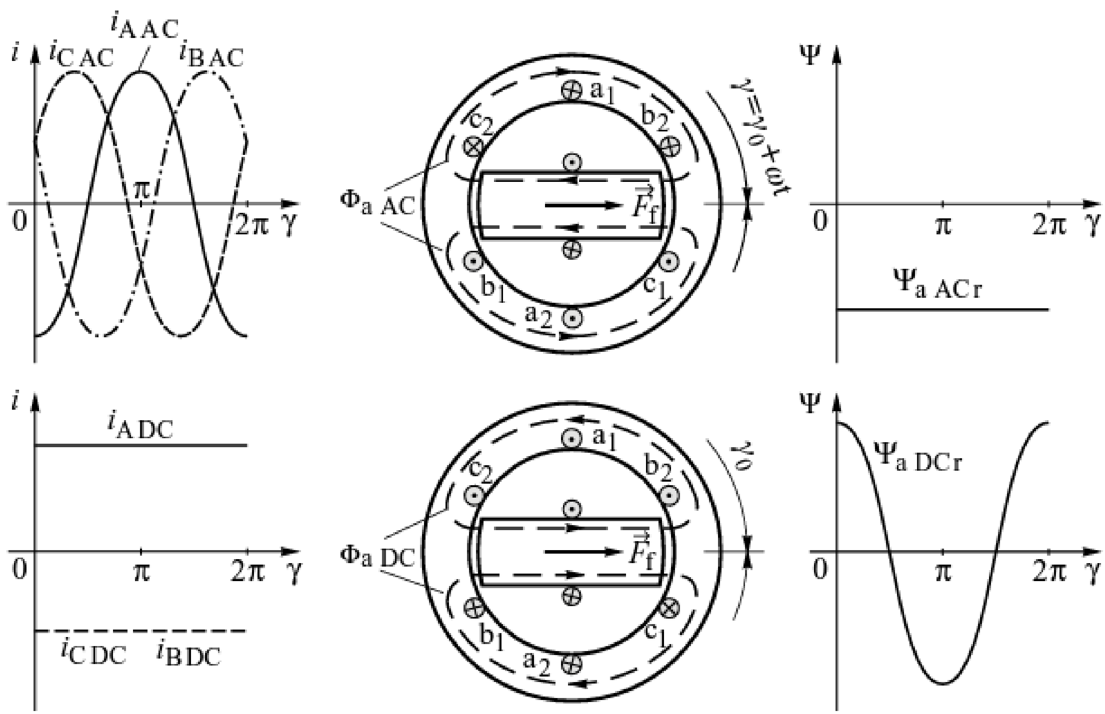


Cont.

- Stator currents (contain both AC and DC components):

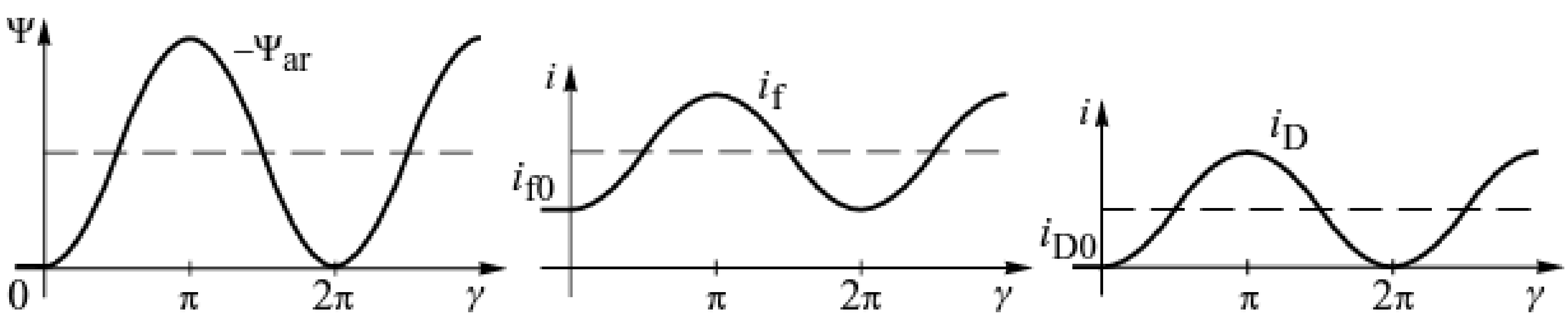
$$
\begin{aligned}
i_A &= -i_m(t)\cos(\omega t + \gamma_0) + i_m(0)\cos\gamma_0 \\
i_B &= -i_m(t)\cos(\omega t + \gamma_0 - 2\pi/3) + i_m(0)\cos(\gamma_0 - 2\pi/3) \\
i_C &= -i_m(t)\cos(\omega t + \gamma_0 - 4\pi/3) + i_m(0)\cos(\gamma_0 - 4\pi/3),
\end{aligned}
$$

- When the winding resistance is neglected, $i_m(t) = i_m(0) = \text{constant}$
- These currents produce an armature reaction mmf, hence armature reaction flux across the air gap. Link with the rotor windings.



Cont.

- As the field and damper windings are closed, the total flux linkage must remain unchanged immediately after the fault.
- Therefore, additional current must flow in these windings to compensate for the armature reaction fluxes that link with the rotor.
- The figure below shows the necessary rotor flux linkage for this and the field and damper winding currents to set up such linkages.
- Both currents contain AC and DC components.



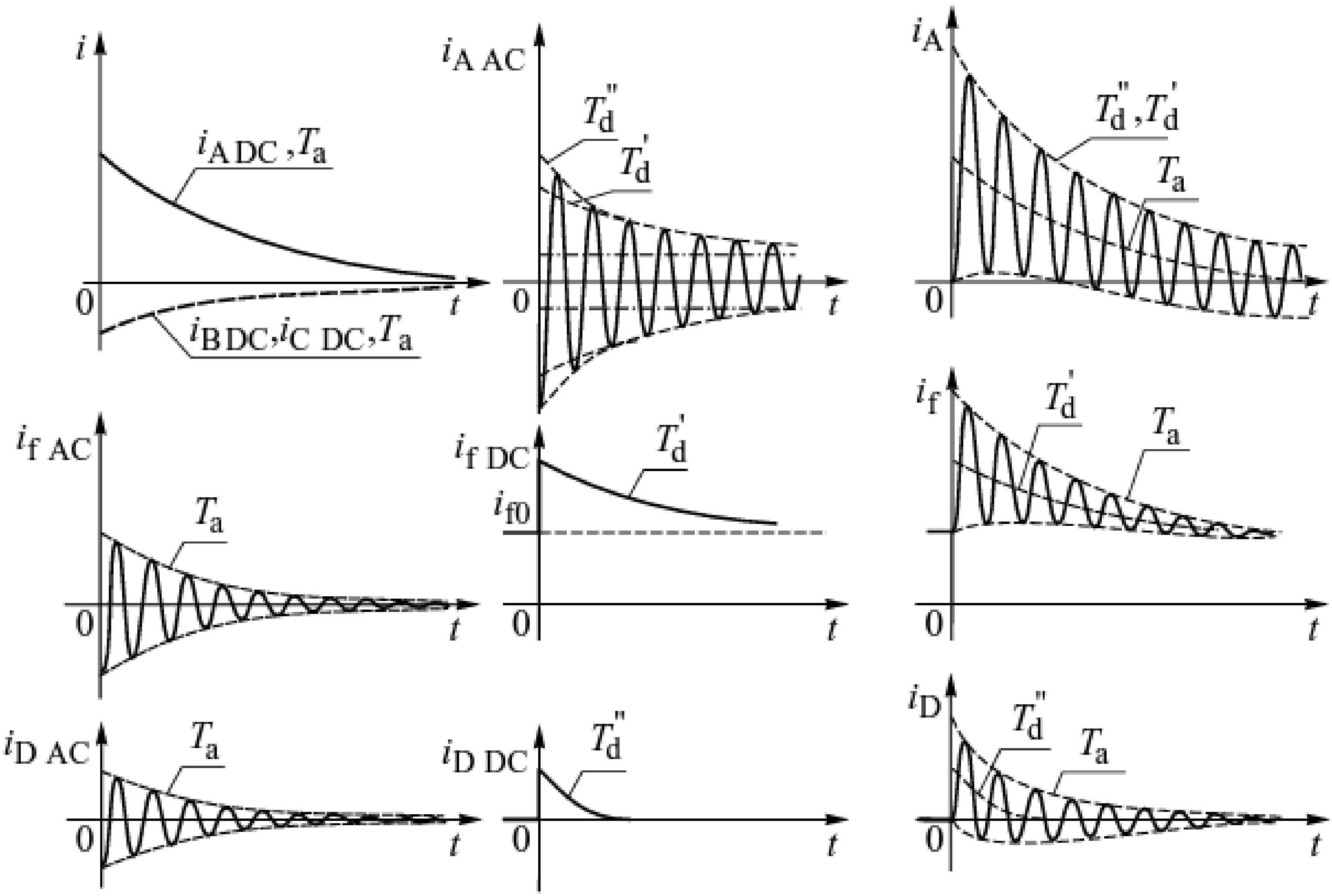
Cont.

- *Under the presence of winding resistance*, the DC component of stator windings will decay exponentially to zero with a time constant T_a = L/R.

$$i_A = -i_m(t)\cos(\omega t + \gamma_0) + i_m(0)e^{-t/T_a}\cos\gamma_0$$
$$i_B = -i_m(t)\cos(\omega t + \gamma_0 - 2\pi/3) + i_m(0)e^{-t/T_a}\cos(\gamma_0 - 2\pi/3)$$
$$i_C = -i_m(t)\cos(\omega t + \gamma_0 - 4\pi/3) + i_m(0)e^{-t/T_a}\cos(\gamma_0 - 4\pi/3).$$

- Why the peak value of the AC component varies?
  - The induced DC currents in the rotor windings by the AC component of the stator currents will decay with time constants T_a'' (for damper winding) and T_a' (for field winding).
  - As the DC rotor currents induce AC stator currents, $i_m(t)$ will decay to its steady-state value with the two time constants.
- the damper winding resistance is much larger than that of the field winding → $T_a'' \ll T_a'$.

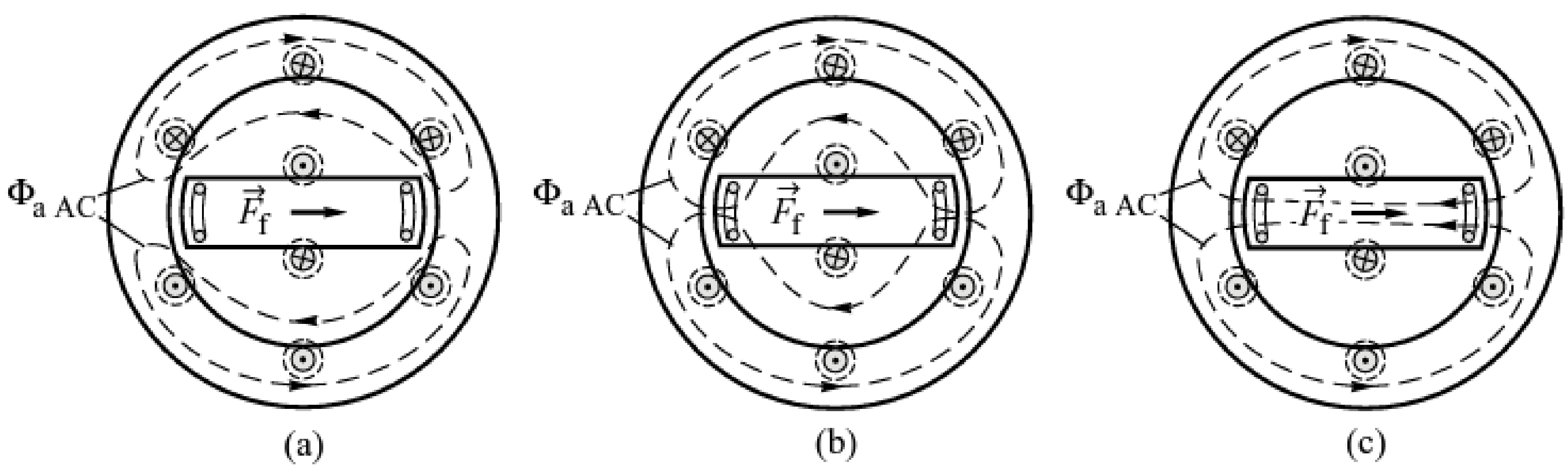
Short Circuit Currents in Generator Windings



Armature flux paths

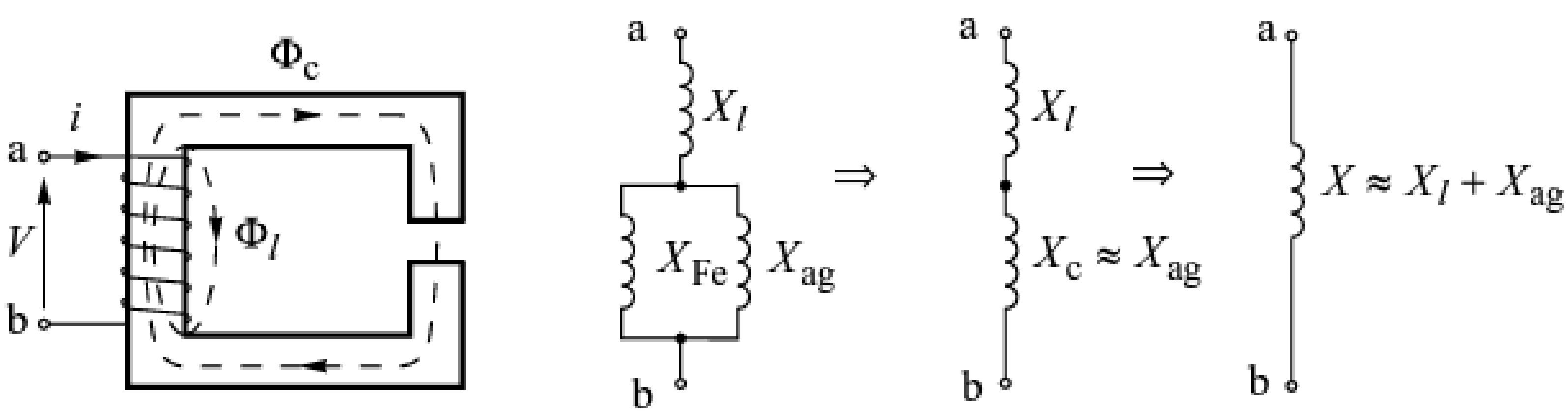
(Quest for quantifying $i_m(t)$)

- The value of the armature reactance will be different as the induced rotor currents force the armature flux to take a different path.
  - Sub-transient state (a): induced rotor currents keep armature reaction flux out of rotor core – to maintain rotor flux linkages constant.
  - Transient state (b): Once energy is dissipated in damper winding resistance, the armature flux enters the damper winding, but not the field winding.
  - Steady state (c): Once energy is dissipated in the field winding resistance, the armature flux enters the field winding and crosses the rotor core.

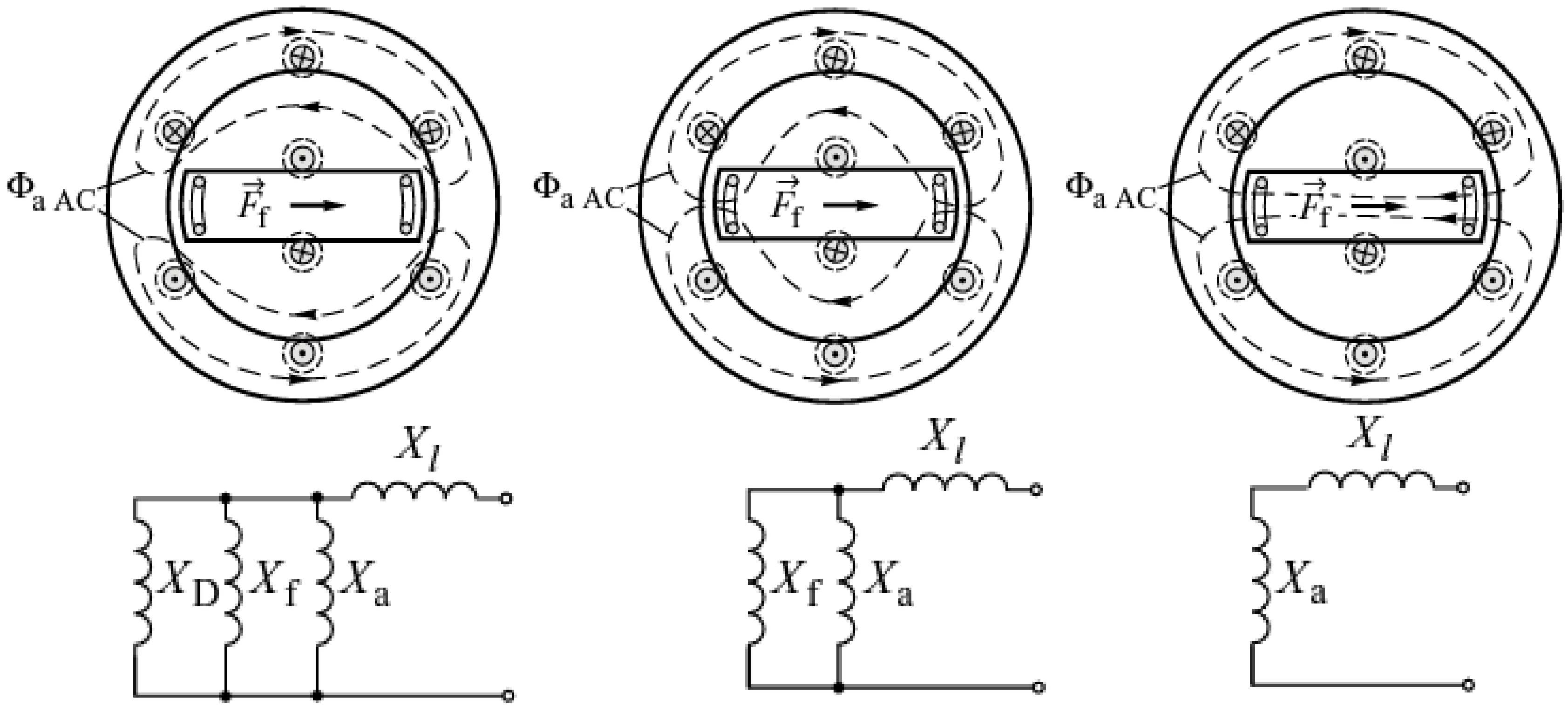


Equivalent reactances at different states

- It is convenient to a analyze the generator dynamics at different states separately → Assign different reactances for each state.
- *Inductance:* ratio of flux linkages to the current producing the flux → Low reluctance path results in large flux and large inductance or reactance and vice versa.
- Reminder: Parallel (series) flux paths correspond to series (parallel) connection of reactances – see illustration below. This principal is applied to the synchronous machine at different states.



Generator reactance at different states



$$X_d'' = X_l + \frac{1}{\frac{1}{X_a} + \frac{1}{X_D} + \frac{1}{X_f}}$$

$$X_d' = X_l + \frac{1}{\frac{1}{X_a} + \frac{1}{X_f}}$$

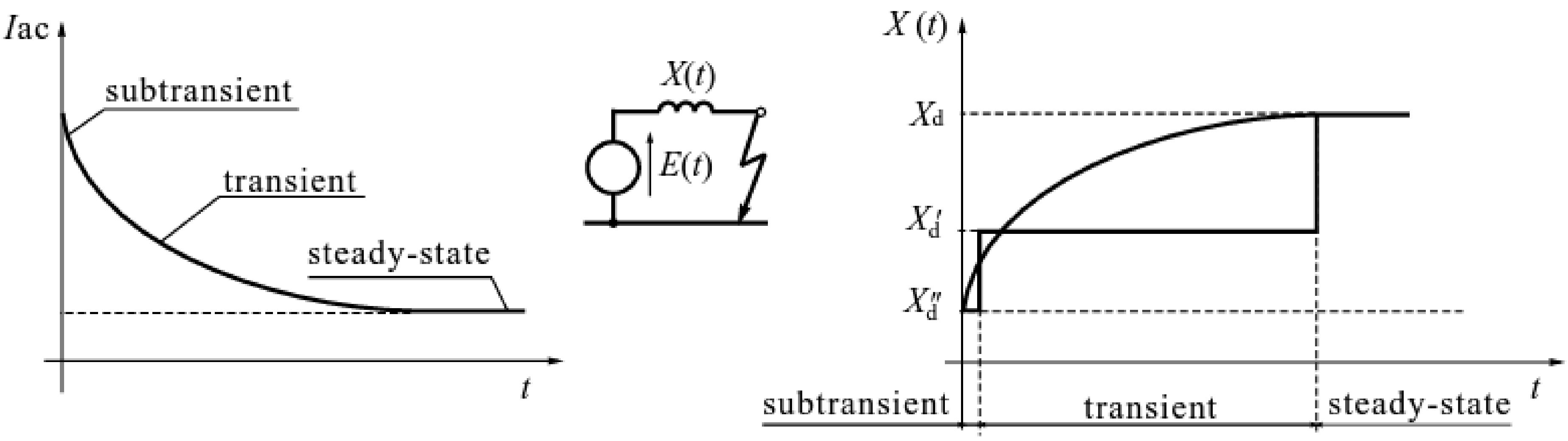
$$X_d = X_l + X_a$$

X_d''	direct-axis subtransient reactance
X_d'	direct-axis transient reactance
X_d	direct-axis synchronous reactance.

$X_d'' < X_d' < X_d$

3-Step approximation of generator model

- Following a fault, the generator becomes a dynamic source with time-varying reactance X(t) and internal voltage E(t).
- It is convenient to consider the three states separately using conventional AC circuit analysis.
- In each state, the generator is represent by a constant emf behind a constant reactance.



(a)

(b)

q-axis reactances

- When the generator is operating under load, or for other types of disturbances, the armature mmf is no-longer directed on the d-axis, and will have both d- and q- components → Analyze the two components separately using the two reaction method presented in Chap. 3.
- *Subtransient State*: for the armature mmf component directed on the q-axis, the currents forcing the armature reaction flux out of the rotor are the rotor eddy-currents and q-axis damper winding (if any).
  - In generators with q-axis damper windings, $X_q'' \approx X_d''$
  - In generators with no q-axis damper windings, $X_q'' > X_d''$. The difference between these reactances is called *subtransient saliency.*
- *Transient State:* the currents forcing the armature reaction flux out of the rotor are the rotor eddy-currents, hence $X_q' > X_d'$.

Saliency in different states

State	Generator type	Reactance	Saliency: Yes/no	Saliency: Reason
Subtransient	Any type but with a d-axis damper only	$X''_q > X''_d$	Yes	Weaker screening in the q-axis because of the lack of damper winding
	Any type but with both d- and q-axis damper windings	$X''_q \approx X''_d$	No	Similar screening in both axes
Transient	Round rotor	$X'_q > X'_d$	Yes	Strong screening in the d-axis due to the field winding but weak screening in the q-axis due to the rotor body currents
	Salient pole	$X'_q = X_q$	Yes	No screening in the q-axis because of the laminated rotor core
Steady state	Round rotor	$X_q \approx X_d$	No	Symmetrical air gap in both axes
	Salient pole	$X_q < X_d$	Yes	Larger air gap on the q-axis

Typical generator parameter values

	Round rotor			Salient-pole rotor	
Parameter	200 MVA	600 MVA	1500 MVA	150 MVA	230 MVA
X_d	1.65	2.00	2.20	0.91	0.93
X_q	1.59	1.85	2.10	0.66	0.69
X'_d	0.23	0.39	0.44	0.3	0.3
X'_q	0.38	0.52	0.64	—	—
X''_d	0.17	0.28	0.28	0.24	0.25
X''_q	0.17	0.32	0.32	0.27	0.27
T'_d	0.83	0.85	1.21	1.10	3.30
T'_q	0.42	0.58	0.47	—	—
T''_d	0.023	0.028	0.030	0.05	0.02
T''_q	0.023	0.058	0.049	0.06	0.02

Relation between open- and short-circuit time constants

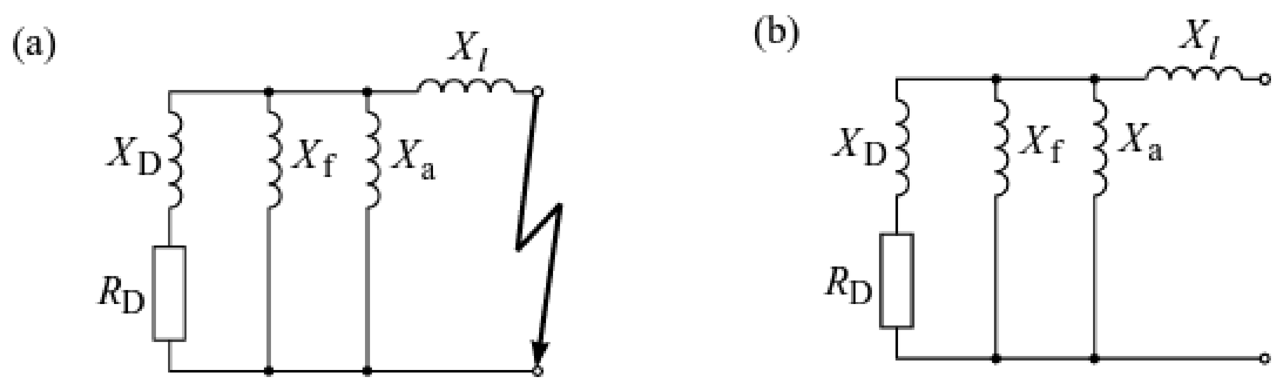
- First approximation - neglect X_l:

$$X'_d \cong \frac{X_a X_f}{X_a + X_f}, \quad \frac{1}{X'_d} \cong \frac{1}{X_a} + \frac{1}{X_f} \qquad X''_d \cong \frac{1}{\frac{1}{X_D} + \frac{1}{X'_d}}, \quad X_D \cong \frac{X'_d X''_d}{X'_d - X''_d}.$$

- Insert damper winding resistance is series with X_D, then compute the time constant of each circuit.

$$T''_d = \frac{X_D}{\omega R_D}. \qquad T''_{do} = \frac{X_D + \frac{X_a X_f}{X_a + X_f}}{\omega R_D} \cong \frac{X_D + X'_d}{\omega R_D}.$$

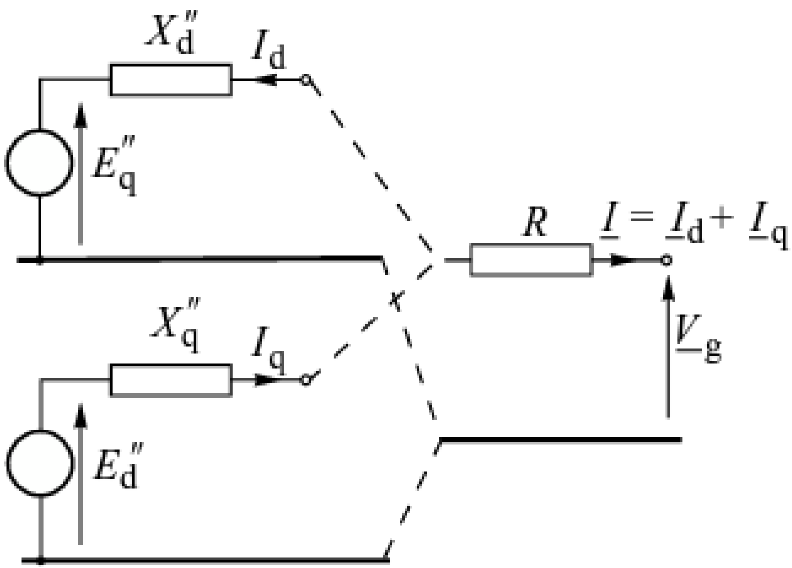
$$\Longrightarrow \quad T''_d \cong T''_{do} \frac{X''_d}{X'_d}, \quad T''_q \cong T''_{qo} \frac{X''_q}{X'_q}, \quad T'_d \cong T'_{do} \frac{X'_d}{X_d}, \quad T'_q \cong T'_{qo} \frac{X'_q}{X_q}.$$

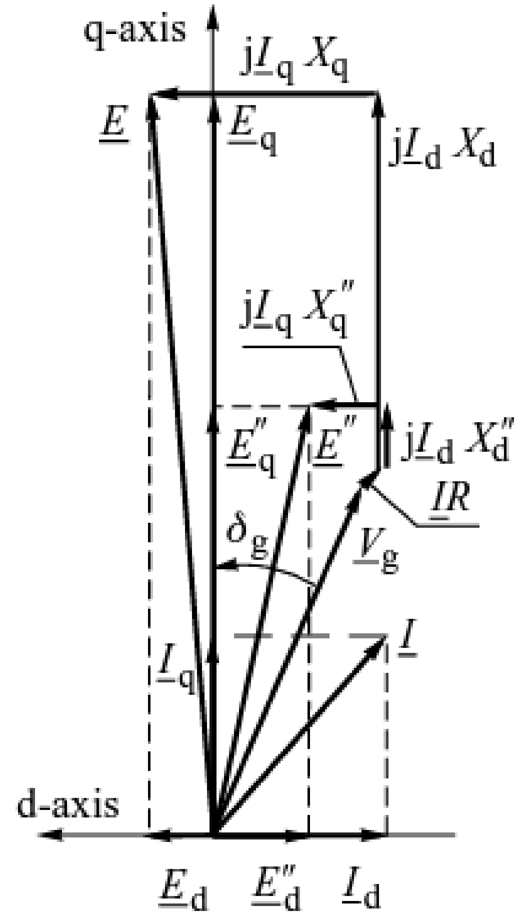


Equivalent Circuit in sub-transient State

$$
\begin{aligned}
\underline{E}'' &= \underline{V}_{\mathrm{g}} + R\underline{I} + \mathrm{j}\underline{I}_{\mathrm{d}}X''_{\mathrm{d}} + \mathrm{j}\underline{I}_{\mathrm{q}}X''_{\mathrm{q}}, \\
\underline{E} &= \underline{V}_{\mathrm{g}} + R\underline{I} + \mathrm{j}\underline{I}_{\mathrm{d}}X_{\mathrm{d}} + \mathrm{j}\underline{I}_{\mathrm{q}}X_{\mathrm{q}} \\
&= \underline{V}_{\mathrm{g}} + R\underline{I} + \mathrm{j}\underline{I}_{\mathrm{d}}\left(X_{\mathrm{d}} - X''_{\mathrm{d}}\right) + \mathrm{j}\underline{I}_{\mathrm{d}}X''_{\mathrm{d}} + \mathrm{j}\underline{I}_{\mathrm{q}}\left(X_{\mathrm{q}} - X''_{\mathrm{q}}\right) + \mathrm{j}\underline{I}_{\mathrm{q}}X''_{\mathrm{q}} \\
&= \underline{E}'' + \mathrm{j}\underline{I}_{\mathrm{d}}\left(X_{\mathrm{d}} - X''_{\mathrm{d}}\right) + \mathrm{j}\underline{I}_{\mathrm{q}}\left(X_{\mathrm{q}} - X''_{\mathrm{q}}\right).
\end{aligned}
$$

I: sub-transient current



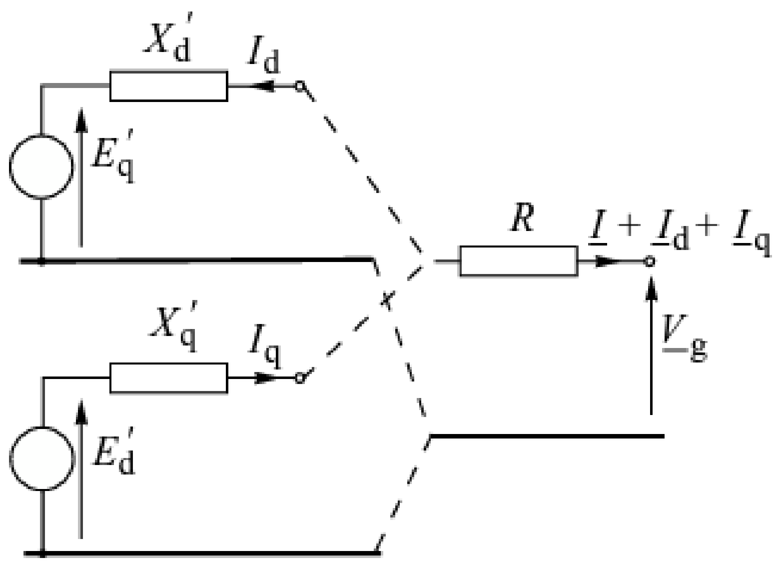


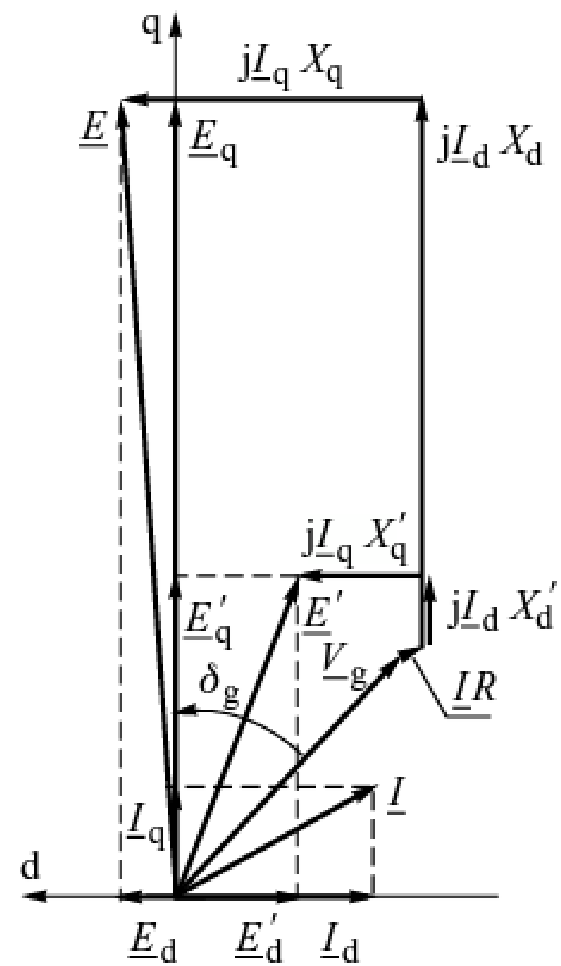
Equivalent circuit in transient state

$$\underline{E}' = \underline{E}'_{\mathrm{q}} + \underline{E}'_{\mathrm{d}} = \underline{V}_{\mathrm{g}} + R\underline{I} + \mathrm{j}\underline{I}_{\mathrm{d}} X'_{\mathrm{d}} + \mathrm{j}\underline{I}_{\mathrm{q}} X'_{\mathrm{q}},$$

$$\underline{E} = \underline{V}_{\mathrm{g}} + R\underline{I} + \mathrm{j}\underline{I}_{\mathrm{d}} X_{\mathrm{d}} + \mathrm{j}\underline{I}_{\mathrm{q}} X_{\mathrm{q}} = \underline{E}' + \mathrm{j}\underline{I}_{\mathrm{d}}\left(X_{\mathrm{d}} - X'_{\mathrm{d}}\right) + \mathrm{j}\underline{I}_{\mathrm{q}}\left(X_{\mathrm{q}} - X'_{\mathrm{q}}\right)$$

I: transient current





Equivalent circuit in steady state

$$\underline{E}_{\mathrm{f}} = \underline{V}_g + \mathrm{j}X_{\mathrm{d}}\underline{I}_{\mathrm{d}} + \mathrm{j}X_{\mathrm{q}}\underline{I}_{\mathrm{q}} + R\underline{I},$$

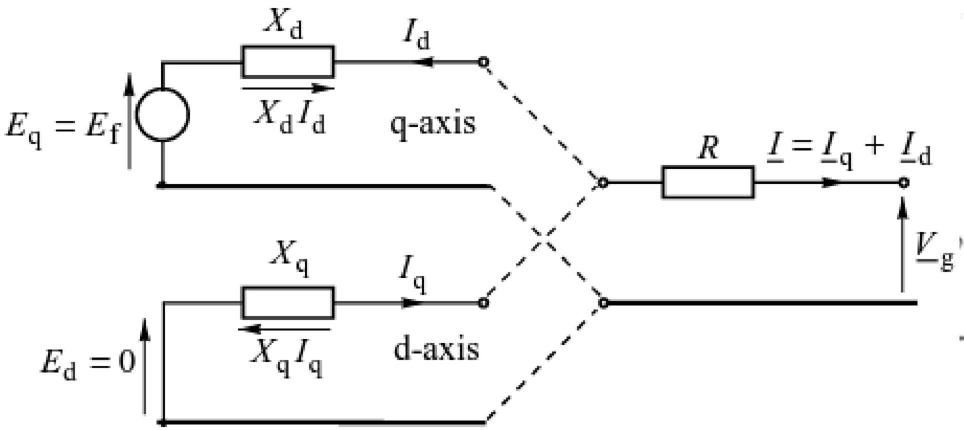
$$\underline{E}_{\mathrm{f}} = \underline{V}_{\mathrm{g}} + R\underline{I} + \mathrm{j}X_{\mathrm{q}}\underline{I} + \mathrm{j}(X_{\mathrm{d}} - X_{\mathrm{q}})\underline{I}_{\mathrm{d}} = \underline{E}_{\mathrm{Q}} + \mathrm{j}(X_{\mathrm{d}} - X_{\mathrm{q}})\underline{I}_{\mathrm{d}},$$

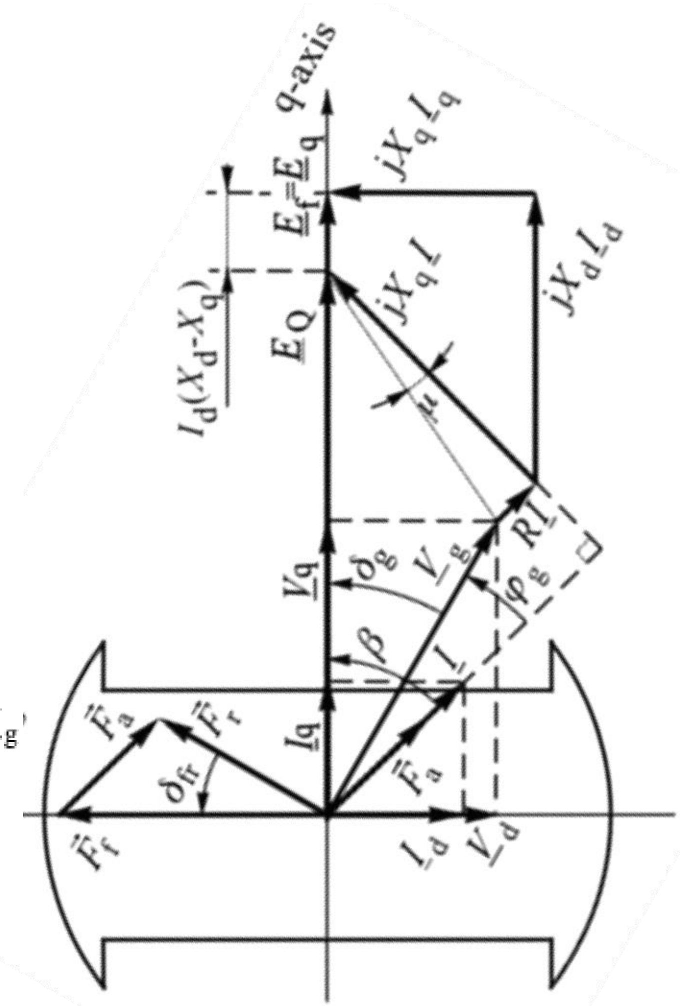
I: steady-state current

$E_d = 0$

$E_q = E_f$

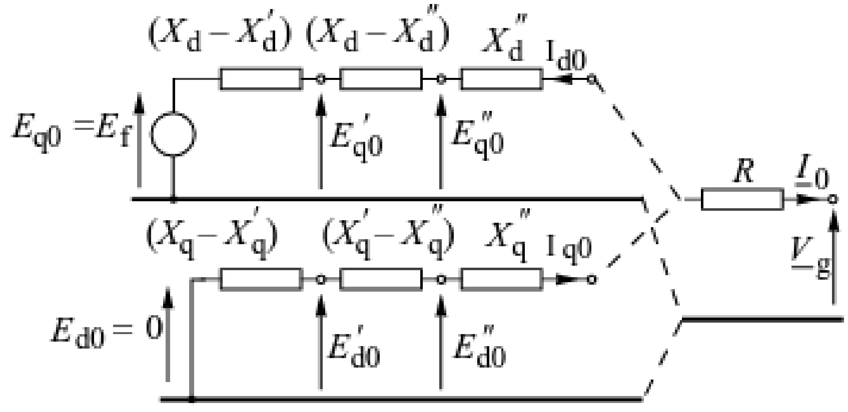
For round rotor machine, $X_d = X_q$





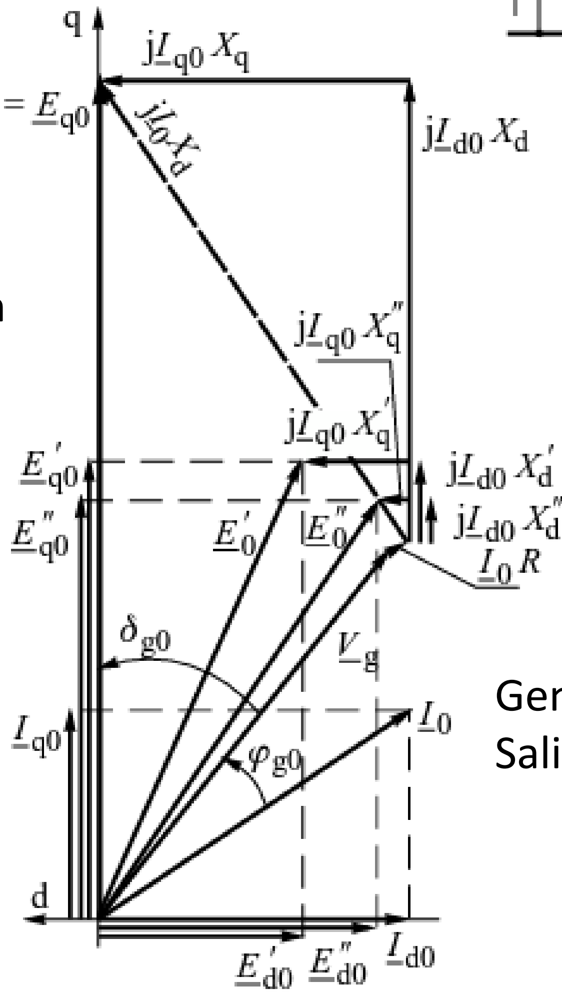
Determining the initial Values of the emfs.

I_0: initial current



(a)

Generator with round rotor



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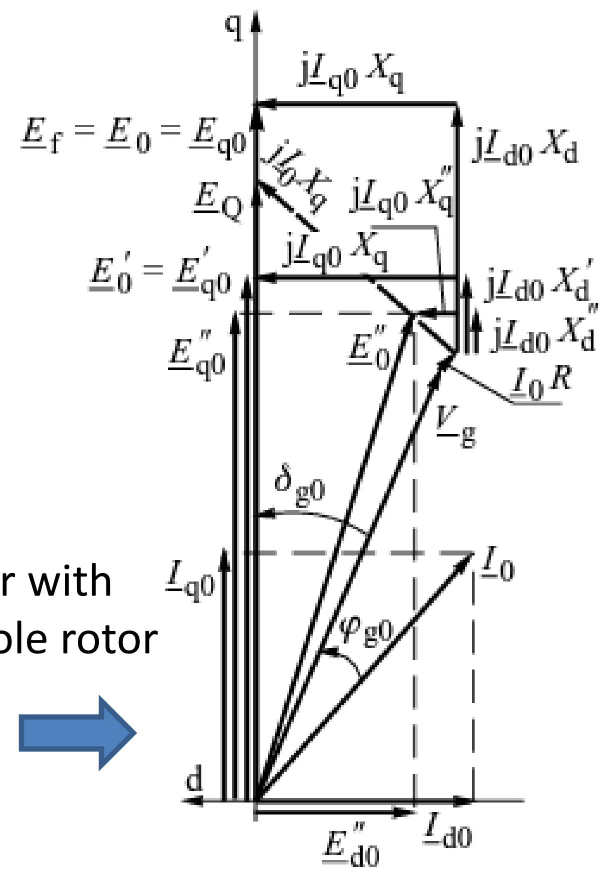
Generator with Salient-pole rotor



(c)

Example 4.1

A 200 MVA round-rotor generator with the parameters given in Table 4.3 is loaded with 1 pu of real power and 0.5 pu of reactive power (lagging). The voltage at the generator terminals is 1.1 pu. Find the prefault values of the steady-state, transient and subtransient emfs. Assume $X_d = X_q = 1.6$ and neglect the armature resistance.

Assuming the generator voltage to be the reference, the load current is

$$\underline{I}_0 = \left(\frac{S}{\underline{V}_g}\right)^* = \frac{P - jQ}{V_g} = \frac{1 - j0.5}{1.1} = 1.016\angle - 26.6^\circ,$$

so that $\varphi_{g0} = 26.6^\circ$. The steady-state internal emf is

$$E_{q0} = \underline{V}_g + jX_d\underline{I}_0 = 1.1 + j1.6 \times 1.016\angle - 26.6^\circ = 2.336\angle 38.5^\circ.$$

Thus $E_{q0} = 2.336$ and $\delta_{g0} = 38.5^\circ$. The d- and q-components of the current and voltage are

$$\begin{aligned}
I_{d0} &= -I_0 \sin(\varphi_{g0} + \delta_{g0}) = -1.016 \sin(26.6^\circ + 38.5^\circ) = -0.922 \\
I_{q0} &= I_0 \cos(\varphi_{g0} + \delta_{g0}) = 0.428 \\
V_{gd} &= -V_g \sin \delta_{g0} = -1.1 \sin 38.5^\circ = -0.685, \qquad V_{gq} = V_g \cos \delta_{g0} = 0.861.
\end{aligned}$$

Now the d- and q-components of the transient and subtransient emfs can be calculated from the phasor diagram in Figure 4.15 as

$$\begin{aligned}
E'_{d0} &= V_{gd} + X'_q I_{q0} = -0.685 + 0.38 \times 0.428 = -0.522 \\
E'_{q0} &= V_{gq} - X'_d I_{d0} = 0.861 - 0.23 \times (-0.922) = 1.073 \\
E''_{d0} &= V_{gd} + X''_q I_{q0} = -0.612 \\
E''_{q0} &= V_{gq} - X''_d I_{d0} = 1.018.
\end{aligned}$$

Example 4.2

Solve a similar problem to that in Example 4.1 but for the 230 MVA salient-pole generator in Table 4.3.

The main problem with the salient-pole generator is in finding the direction of the q-axis. Equation (3.64) gives $\underline{E}_{\mathrm{Q}} = \underline{V}_{\mathrm{g}} + \mathrm{j}X_{\mathrm{q}}\underline{I}_0 = 1.1 + \mathrm{j}0.69 \times 1.016\angle - 26.6^\circ = 1.546\angle 23.9^\circ$. Thus $\delta_{\mathrm{g}0} = 23.9^\circ$ and

$$
\begin{aligned}
I_{\mathrm{d}0} &= -1.016\sin(26.6^\circ + 23.9^\circ) = -0.784, \quad I_{\mathrm{q}0} = -1.016\cos(26.6^\circ + 23.9^\circ) = 0.647 \\
V_{\mathrm{gd}} &= -1.1\sin 23.9^\circ = -0.446, \quad V_{\mathrm{gq}} = 1.1\cos 23.9^\circ = 1.006 \\
E_{\mathrm{q}0} &= V_{\mathrm{gq}} - I_{\mathrm{d}0}X_{\mathrm{d}} = 1.006 - (-0.784) \times 0.93 = 1.735 \\
E'_{\mathrm{d}0} &= -0.446 + 0.69 \times 0.647 = 0, \quad E'_{\mathrm{q}0} = 1.006 - 0.3 \times (-0.784) = 1.241, \\
E''_{\mathrm{d}0} &= -0.446 + 0.27 \times 0.647 = -0.271, \quad E''_{\mathrm{q}0} = 1.006 - 0.25 \times (-0.784) = 1.202.
\end{aligned}
$$

Formula of Short-circuit current under no load

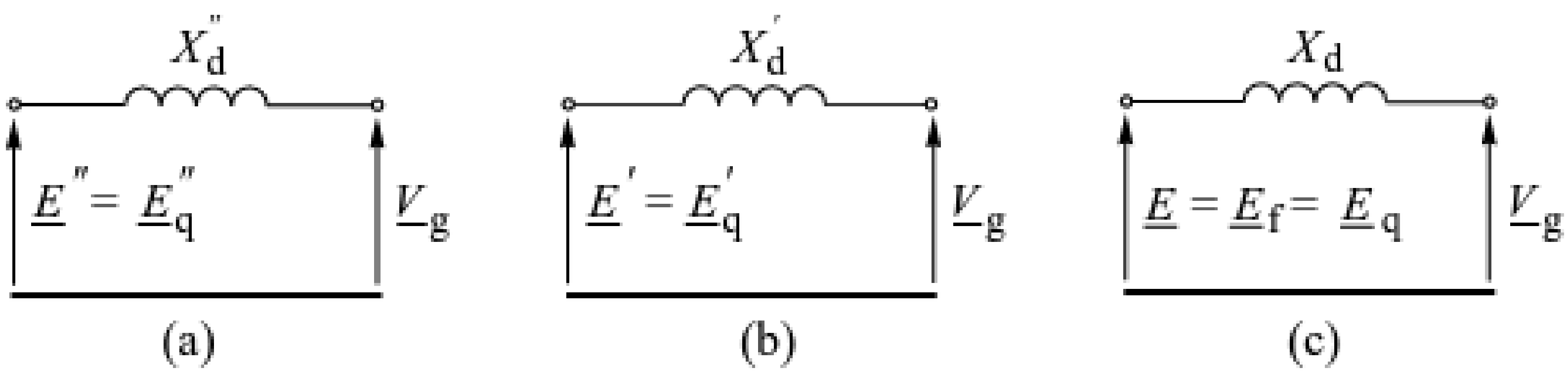
- Under no load, the armature current is zero. Hence,

$$E'' = E' = E = E_f = V_g,$$

- Since the d-axis components are zero, the generator equivalent circuit can be simplified as shown below.
- The amplitude of the AC fault current components are given by

$$i''_m = \frac{E_{fm}}{X_{d''}}, \qquad i'_m = \frac{E_{fm}}{X'_d}, \qquad i^{\infty}_m = \frac{E_{fm}}{X_d},$$

where $E_{fm} = \sqrt{2}E_f$.

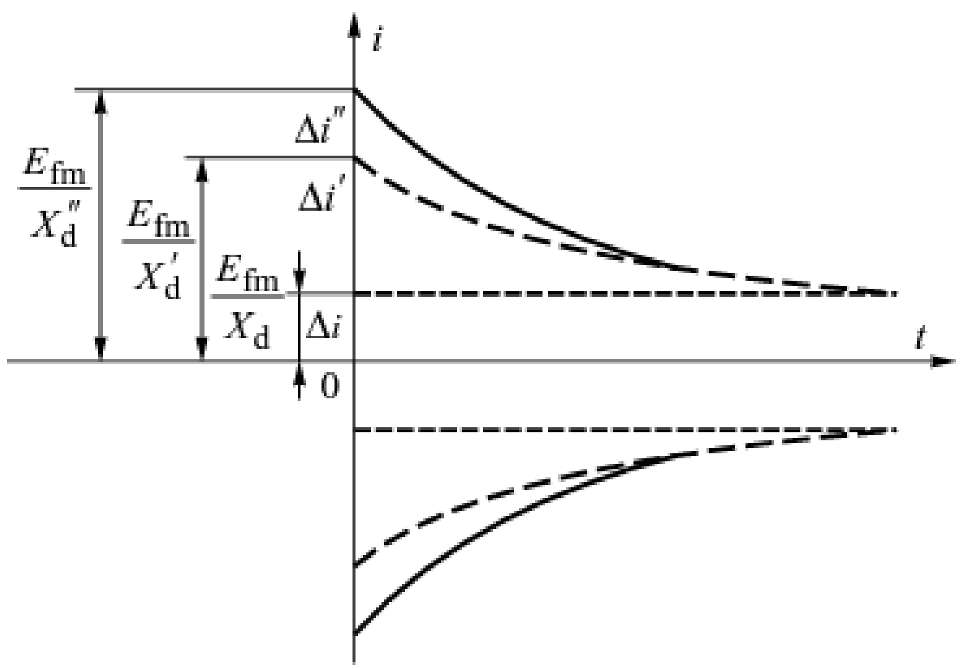


Formula of Short-circuit current under no load

$$i_{\mathrm{m}}(t) = \Delta i'' \mathrm{e}^{-t/T_{\mathrm{d}}''} + \Delta i' \mathrm{e}^{-t/T_{\mathrm{d}}'} + \Delta i,$$

$$\Delta i = E_{\mathrm{fm}} \frac{1}{X_{\mathrm{d}}}, \quad \Delta i' = E_{\mathrm{fm}} \left(\frac{1}{X_{\mathrm{d}}'} - \frac{1}{X_{\mathrm{d}}} \right), \quad \Delta i'' = E_{\mathrm{fm}} \left(\frac{1}{X_{\mathrm{d}}''} - \frac{1}{X_{\mathrm{d}}'} \right).$$

$$i_{\mathrm{m}}(t) = E_{\mathrm{fm}} \left[\left(\frac{1}{X_{\mathrm{d}}''} - \frac{1}{X_{\mathrm{d}}'} \right) \mathrm{e}^{-t/T_{\mathrm{d}}''} + \left(\frac{1}{X_{\mathrm{d}}'} - \frac{1}{X_{\mathrm{d}}} \right) \mathrm{e}^{-t/T_{\mathrm{d}}'} + \frac{1}{X_{\mathrm{d}}} \right].$$



Formula of Short-circuit current under no load

$$i_{\mathrm{m}}(0) = \frac{E_{\mathrm{fm}}}{X''_{\mathrm{d}}}.$$

$$i_{\mathrm{A}} = -\frac{E_{\mathrm{fm}}}{X''_{\mathrm{d}}}\left[g_3(t)\cos(\omega t + \gamma_0) - \mathrm{e}^{-t/T_{\mathrm{a}}}\cos\gamma_0\right]$$

$$i_{\mathrm{B}} = -\frac{E_{\mathrm{fm}}}{X''_{\mathrm{d}}}\left[g_3(t)\cos(\omega t + \gamma_0 - 2\pi/3) - \mathrm{e}^{-t/T_{\mathrm{a}}}\cos(\gamma_0 - 2\pi/3)\right]$$

$$i_{\mathrm{C}} = -\frac{E_{\mathrm{fm}}}{X''_{\mathrm{d}}}\left[g_3(t)\cos(\omega t + \gamma_0 - 4\pi/3) - \mathrm{e}^{-t/T_{\mathrm{a}}}\cos(\gamma_0 - 4\pi/3)\right].$$

$$g_3(t) = X''_{\mathrm{d}}\left[\left(\frac{1}{X''_{\mathrm{d}}} - \frac{1}{X'_{\mathrm{d}}}\right)\mathrm{e}^{-t/T''_{\mathrm{d}}} + \left(\frac{1}{X'_{\mathrm{d}}} - \frac{1}{X_{\mathrm{d}}}\right)\mathrm{e}^{-t/T'_{\mathrm{d}}} + \frac{1}{X_{\mathrm{d}}}\right],$$

With subtransient saliency (i.e., $X_d'' \neq X_q''$), the current expression becomes

$$i_{\mathrm{A}} = -\frac{E_{\mathrm{fm}}}{X''_{\mathrm{d}}}\left[g_3(t)\cos(\omega t + \gamma_0)\right] + \frac{E_{\mathrm{fm}}}{2}\mathrm{e}^{-t/T_{\mathrm{a}}}\left[\left(\frac{1}{X''_{\mathrm{d}}} + \frac{1}{X''_{\mathrm{q}}}\right)\cos\gamma_0 + \left(\frac{1}{X''_{\mathrm{d}}} - \frac{1}{X''_{\mathrm{q}}}\right)\cos(2\omega t + \gamma_0)\right]$$

Short-circuit current under load

- Under load condition, the initial values of the subtransient and transient emfs are functions of the initial load current.
- These values can be computed as illustrated in the previous example.
- The d-axis emf forces the flow of q- axis AC currents (sine), while the q-axis emf forces the flow of d- axis AC currents (cosine).
- The resulting current expressions becomes

$$
\begin{aligned}
i_{\mathrm{A}} = & -\left[\left(\frac{E''_{\mathrm{qm0}}}{X''_{\mathrm{d}}} - \frac{E'_{\mathrm{qm0}}}{X'_{\mathrm{d}}}\right)\mathrm{e}^{-t/T''_{\mathrm{d}}} + \left(\frac{E'_{\mathrm{qm0}}}{X'_{\mathrm{d}}} - \frac{E_{\mathrm{qm0}}}{X_{\mathrm{d}}}\right)\mathrm{e}^{-t/T'_{\mathrm{d}}} + \frac{E_{\mathrm{qm0}}}{X_{\mathrm{d}}}\right]\cos(\omega t + \gamma_0) \\
& + \left[\left(\frac{E''_{\mathrm{dm0}}}{X''_{\mathrm{q}}} - \frac{E'_{\mathrm{dm0}}}{X'_{\mathrm{q}}}\right)\mathrm{e}^{-t/T''_{\mathrm{q}}} + \frac{E'_{\mathrm{dm0}}}{X'_{\mathrm{q}}}\mathrm{e}^{-t/T'_{\mathrm{q}}}\right]\sin(\omega t + \gamma_0) \\
& + \frac{V_{\mathrm{gm0}}}{2}\mathrm{e}^{-t/T_{\mathrm{a}}}\left[\left(\frac{1}{X''_{\mathrm{d}}} + \frac{1}{X''_{\mathrm{q}}}\right)\cos(\gamma_0 + \delta_{\mathrm{g}}) + \left(\frac{1}{X''_{\mathrm{d}}} - \frac{1}{X''_{\mathrm{q}}}\right)\cos(2\omega t + \gamma_0 + \delta_{\mathrm{g}})\right]
\end{aligned}
$$

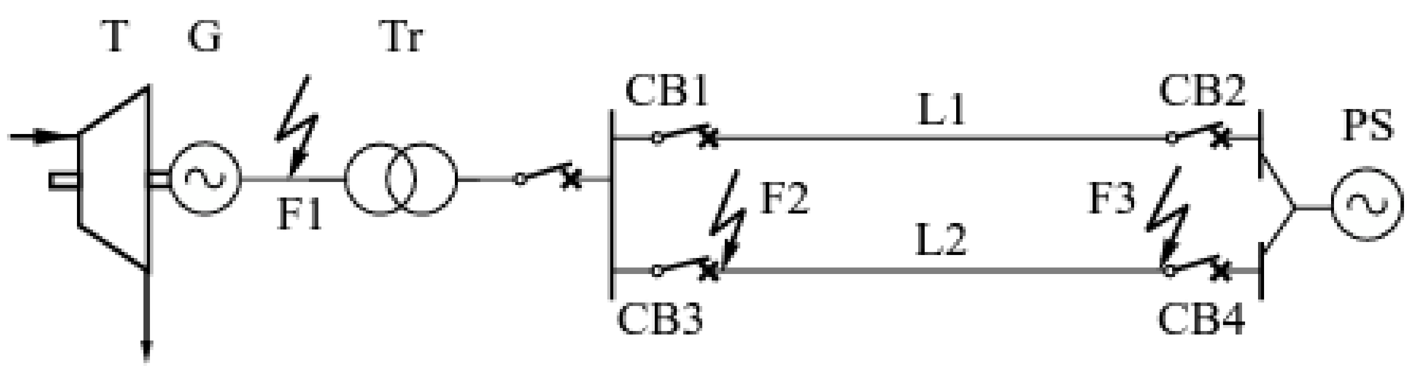
Short circuit in a network

- Most faults occur at some distance away from the generator terminals. When a fault occurs at F2, the transformer reactance must be added to the generator reactances:

$$x''_d = X''_d + X_T, \quad x'_d = X'_d + X_T, \quad x_d = X_d + X_T.$$

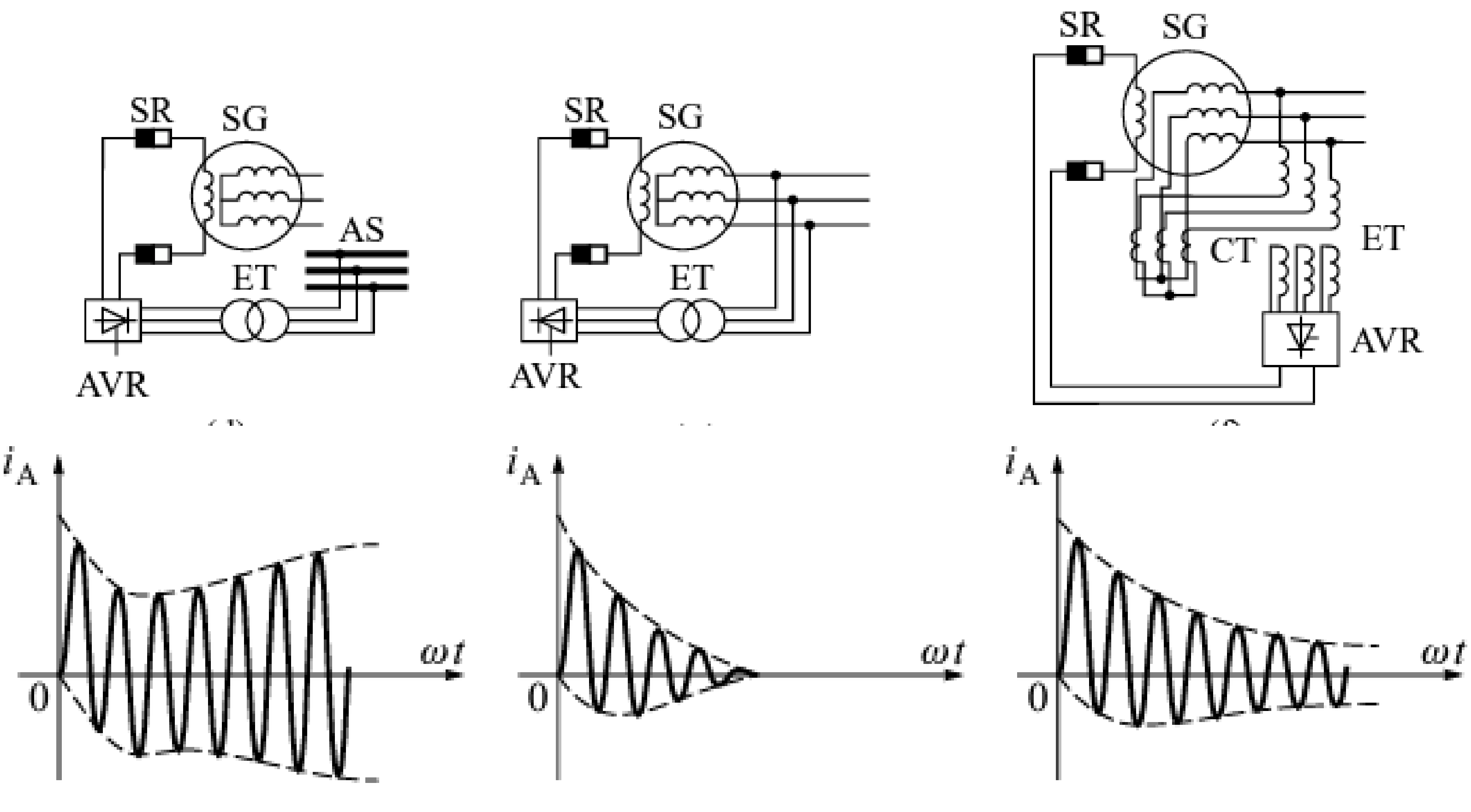
- Secondly, the transformer resistance must be taken into account (i.e., the DC component will decay more rapidly – affects the value of T_a.). In addition,

$$T''_{d(\text{network})} = T''_d \left(\frac{X'_d}{X''_d}\right)\left(\frac{X''_d + X_T}{X'_d + X_T}\right), \quad T'_{d(\text{network})} = T'_d \left(\frac{X_d}{X'_d}\right)\left(\frac{X'_d + X_T}{X_d + X_T}\right).$$



Influence of the AVR on fault current

- Rotating exciter or static exciter fed by auxiliary source: current increases
- Static exciter fed solely from generator terminal voltage: current decays to 0.
- Compound static exciter: current settle to non-zero value.



(a) (b) (c)

Sub-transient torque - 4 components

- Torque due to DC component of short-circuit current (see fig. below)

$$\tau_\omega(t) = \frac{3}{\omega}\frac{E_f^2}{X_d''} g_3(t) e^{-t/T_a} \sin \omega t \quad \mathrm{N\,m},$$

- Additional component if sub-transient saliency is taken into count.

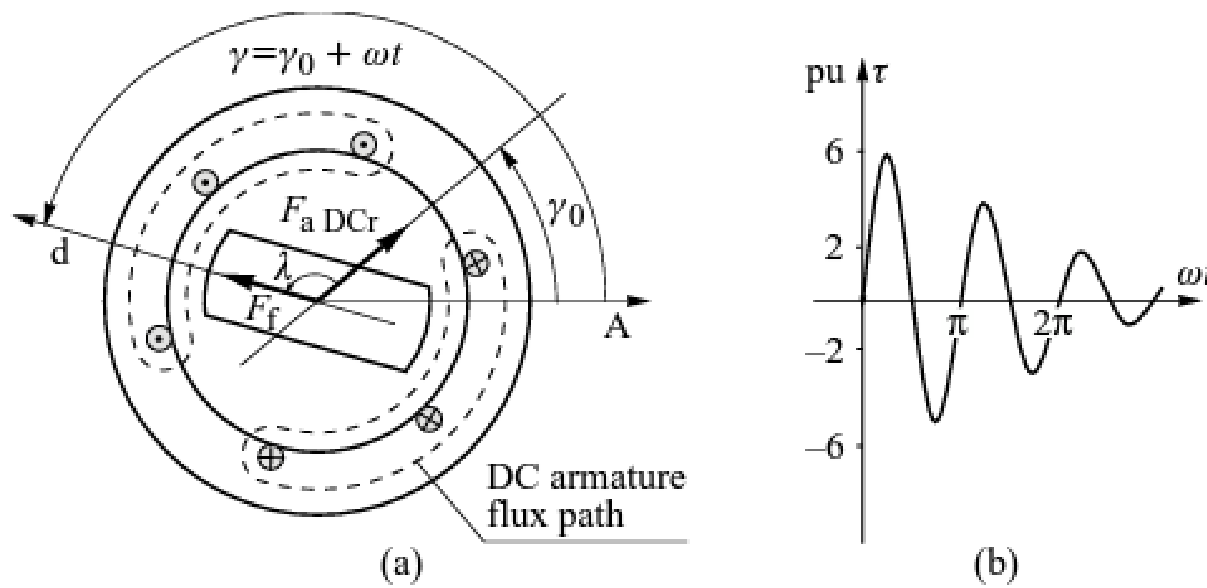
$$\tau_{2\omega}(t) = -\frac{3}{2}\frac{E_f^2}{\omega}\left(\frac{1}{X_d''} - \frac{1}{X_q''}\right) e^{-2t/T_a} \sin 2\omega t \quad \mathrm{N\,m}.$$

- Torque corresponding to AC current decay in stator windings.

$$\tau_R(t) = \frac{3}{\omega}\left[\frac{E_f}{X_d''} g_3(t)\right]^2 R \quad \mathrm{N\,m}.$$

- Torque due to current decay in ro windings.

$$\tau_r(t) = \frac{3}{\omega}\left(\frac{i_m(0)}{\sqrt{2}} e^{-t/T_a}\right)^2 r = \frac{3}{\omega}\left(\frac{E_f}{X_d''}\right)^2 r e^{-2t/T_a} \quad \mathrm{N\,m},$$

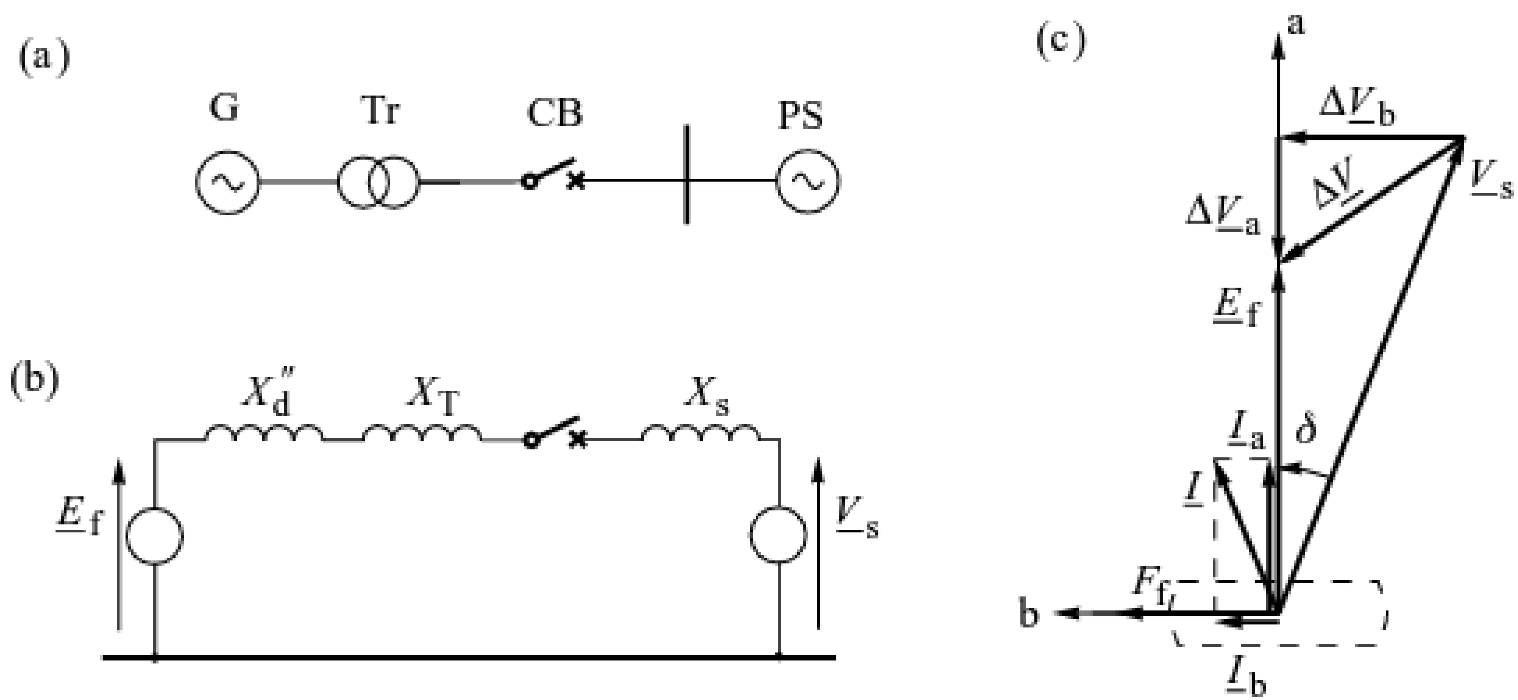


Synchronization

- Ideal synchronization occurs when

$$\omega = \omega_s,\ \underline{E}_f = \underline{V}_s \text{ and } \delta = 0$$

- In practice, these conditions and not exactly satisfied. Hence, there will be a circulating current that contains both AC and DC components.
- For simplicity, assume a non zero angle and a difference in voltage ΔV and ignore stator winding resistance and rotor subtransient saliency.



Torque during synchronization: sub-transient period

- Maximum values of b-axis current component and resulting torque due to its DC component (mmf produced by AC component is in phase with the rotor mmf – hence produces no torque):

$$i_{\mathrm{bm}} = \frac{\Delta V_{\mathrm{am}}}{x_{\mathrm{d}}''} = \frac{V_{\mathrm{sm}} \cos\delta - E_{\mathrm{fm}}}{x_{\mathrm{d}}''},$$

where $x_{\mathrm{d}}'' = X_{\mathrm{d}}'' + X_{\mathrm{T}} + X_{\mathrm{s}}$, $E_{\mathrm{fm}} = \sqrt{2}E_{\mathrm{f}}$ and $V_{\mathrm{sm}} = \sqrt{2}V_{\mathrm{s}}$.

$$\tau_{\mathrm{I}} = -\frac{3}{2}\frac{1}{\omega}E_{\mathrm{fm}} i_{\mathrm{bm}} \sin\omega t = -\frac{3}{\omega}\frac{E_{\mathrm{f}}}{x_{\mathrm{d}}''}(V_{\mathrm{s}}\cos\delta - E_{\mathrm{f}})\sin\omega t \quad \mathrm{N\,m}.$$

- Maximum values of a-axis current component and resulting torque (AC & DC components produce constant and sinusoidal torque components) :

$$i_{\mathrm{am}} = \frac{\Delta V_{\mathrm{bm}}}{x_{\mathrm{d}}''} = \frac{V_{\mathrm{sm}}\sin\delta}{x_{\mathrm{d}}''}.$$

$$\tau_{\mathrm{II}} = \frac{3}{2}\frac{1}{\omega}E_{\mathrm{fm}} i_{\mathrm{am}} = \frac{3}{\omega}\frac{E_{\mathrm{f}} V_{\mathrm{s}}}{x_{\mathrm{d}}''}\sin\delta \quad \mathrm{N\,m}.$$

$$\tau_{\mathrm{III}} = \frac{3}{2}\frac{1}{\omega}E_{\mathrm{fm}} i_{\mathrm{am}} \sin\left(\omega t - \frac{\pi}{2}\right) = -\frac{3}{\omega}\frac{E_{\mathrm{f}} V_{\mathrm{s}}}{x_{\mathrm{d}}''}\sin\delta\cos\omega t \quad \mathrm{N\,m}.$$

Torque during synchronization

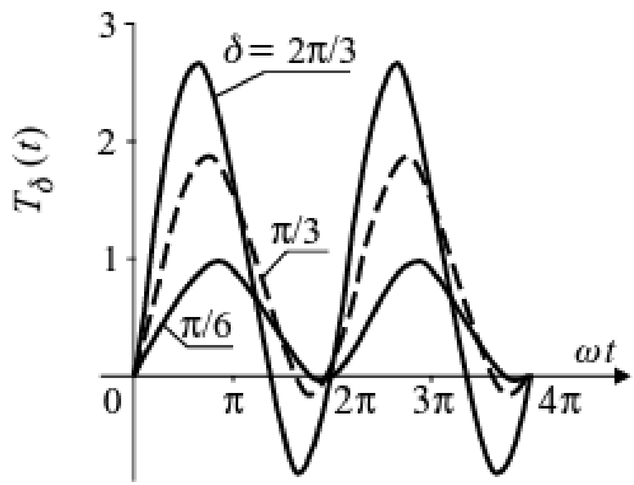
- Resultant torque (resistance ignored)

$$\tau = \frac{3}{\omega}\left[\frac{E_f V_s}{x_d''}\sin\delta\left(1-\cos\omega t+\tan\frac{\delta}{2}\sin\omega t\right)+\frac{E_f(E_f-V_s)}{x_d''}\sin\omega t\right] \quad \mathrm{N\,m}.$$

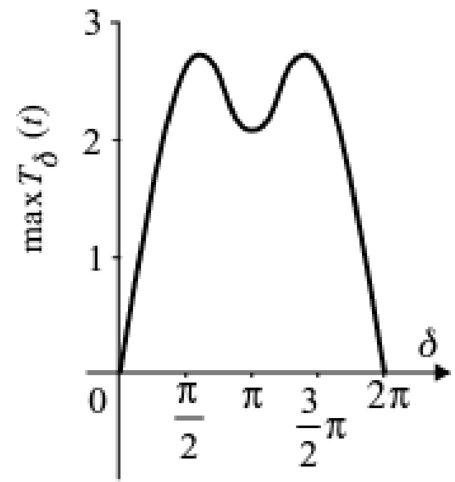
- When $E_f \approx V_s$, then the above expression simplifies to

$$T_\delta(t) = \sin\delta\left(1-\cos\omega t+\tan\frac{\delta}{2}\sin\omega t\right).$$

- Fig (a): torque variation and Fig (b): maximum torque for different values of δ.

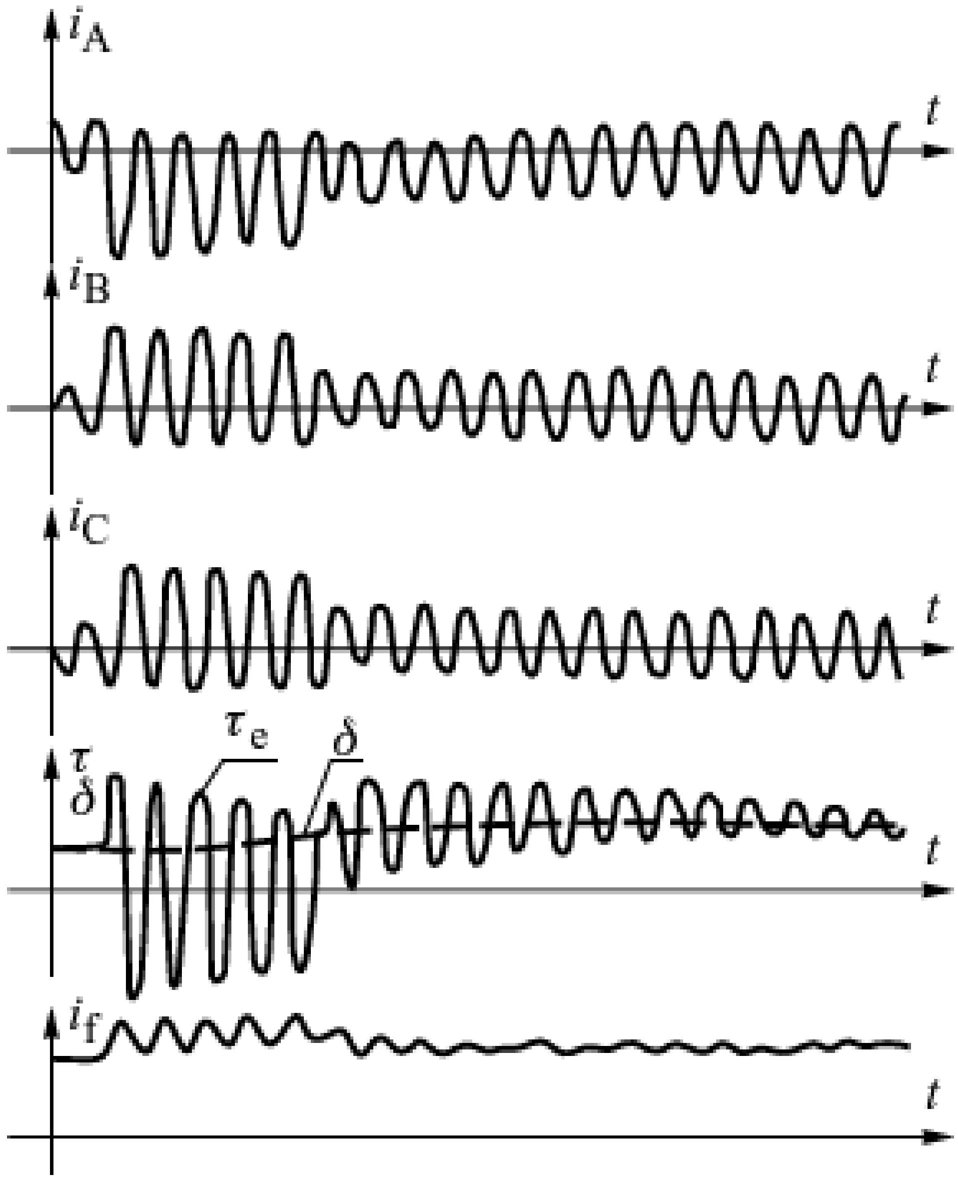


(a)



(b)

Sample of 3-phase fault and its clearing



Homework # 5

1) Derive the analytical expressions of phase "a" currents when a three-phase fault occurs at the terminals of the generators under load described in Example 4.1 & 4.2. Then plot these wave using Excel. Assume the fault occurs when $\Upsilon_o = 0$ and the armature winding time constant $T_a = 1$ sec.
2) Repeat 1) above when the generators are under no load with $E_f = 1$ p.u.