EE 742
Chap 8: Voltage Stability

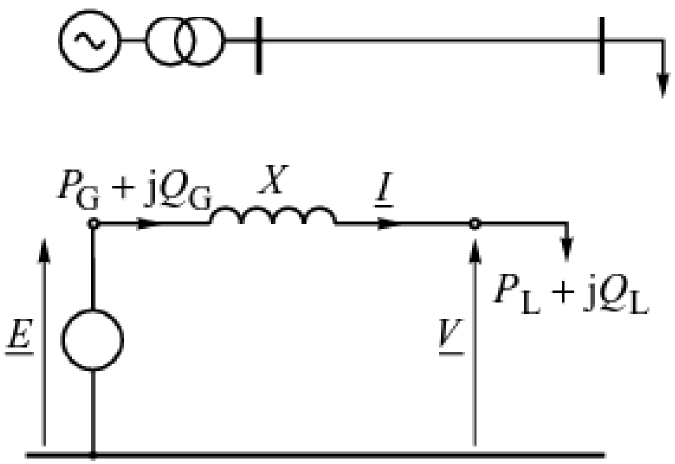
Loadability of a simple network

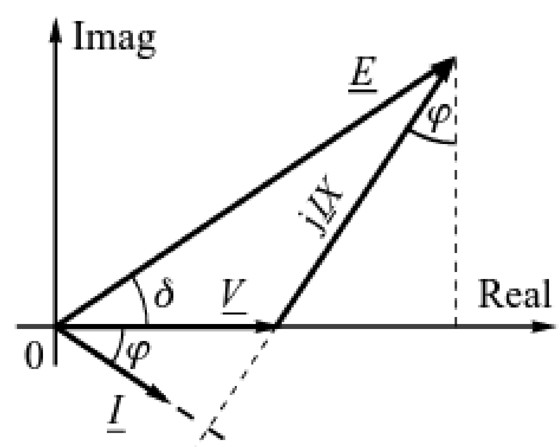
- What are the possible solutions for the network below? What are the limits (if any)?
- Static power-voltage equations:

$$P_L(V) = VI\cos\varphi = V\frac{IX\cos\varphi}{X} = \frac{EV}{X}\sin\delta \quad (1)$$

$$Q_L(V) = VI\sin\varphi = V\frac{IX\sin\varphi}{X} = \frac{EV}{X}\cos\delta - \frac{V^2}{X}. \quad (2)$$

$$\left(\frac{EV}{X}\right)^2 = [P_L(V)]^2 + \left[Q_L(V) + \frac{V^2}{X}\right]^2. \quad (3)$$





Case of ideally stiff load

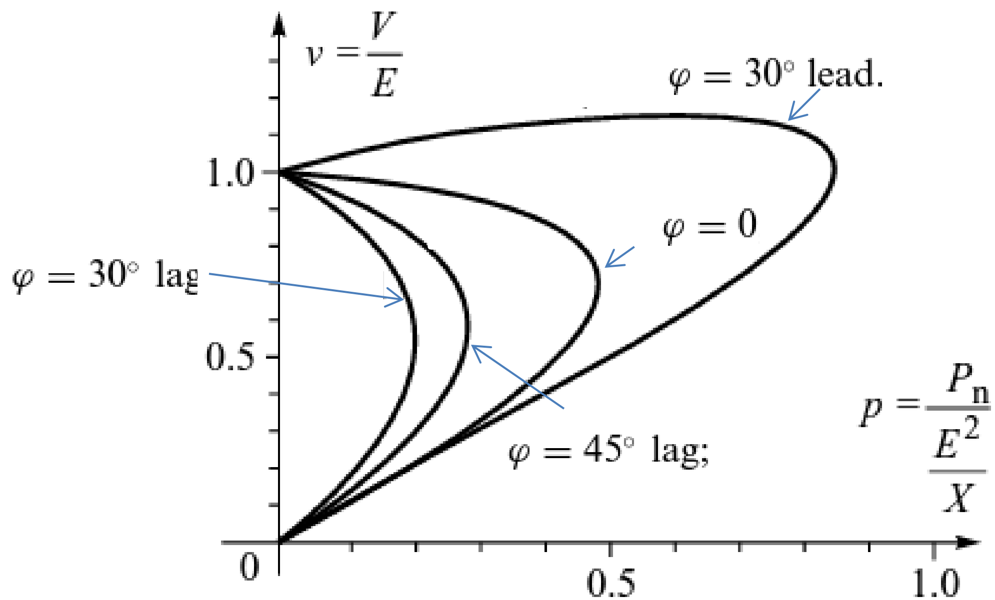
$$P_L(V) = P_n \quad \text{and} \quad Q_L(V) = Q_n,$$

- Substitution in (3) yields real power in terms of power factor angle:

$$p = -v^2 \sin\varphi \cos\varphi + v \cos\varphi \sqrt{1 - v^2 \cos^2\varphi},$$

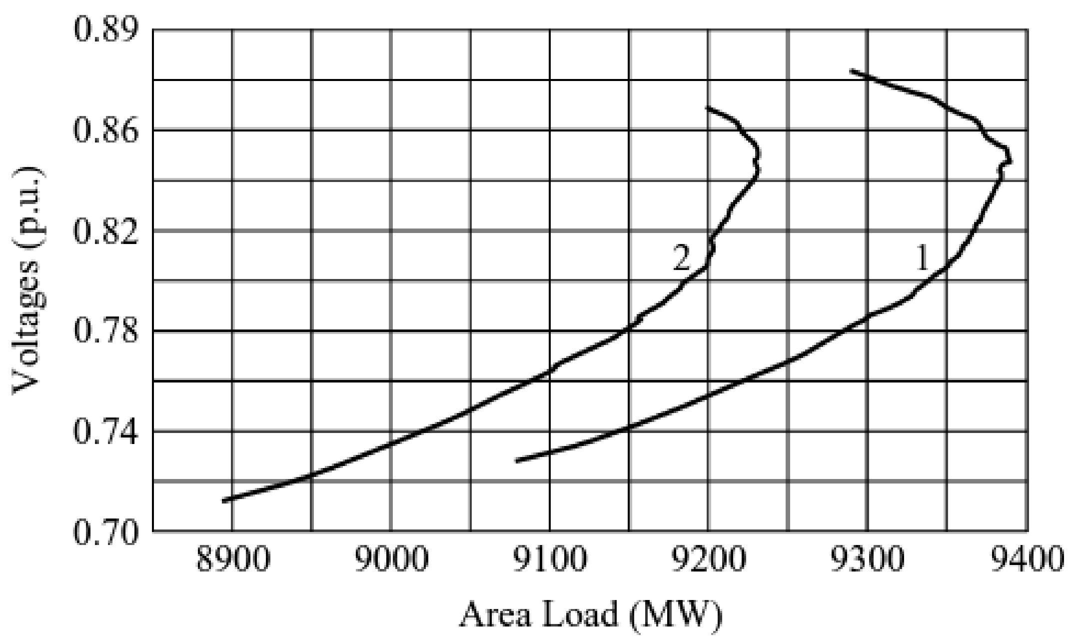
- Where $$v = \frac{V}{E}, \quad p = \frac{P_n}{\frac{E^2}{X}}.$$

- The nose curves V(P) below show the voltage dependency on real power



Examples of system blackout due to voltage problems Athens, Greece, 2004

- The load was on the increase (9.39 GW) in mid-day of July 2004.
- A loss of a generator brought the system into an emergency state
- Load shedding was initiated when another generator was lost.
- See the PV nose curves below - inevitable voltage collapse
  - 1 loss of first generator
  - 2 loss of second generator



Assignment # 7

Consider a 500 kV transmission line with total series impedance = 8+j100Ω and total shunt admittance = 0.0008 S. The line is connected to a stiff source that is fixed at the sending end (V_s = 1 pu). Assume the load is stiff.

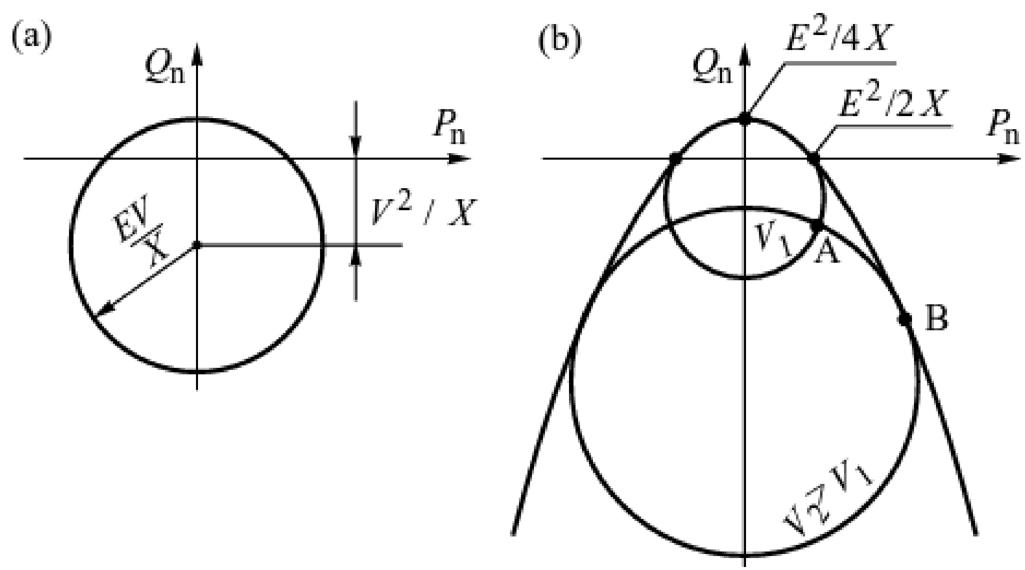
1) Plot the nose curves (i.e., receiving end voltage as a function of load real power for a) 0.9 power factor (lag), b) unity power factor, and c) 0.9 power factor (lead).
2) Plot the source reactive power Qs as a function of receiving end voltage for a) P = 0 MW, b) P = 500 MW, c) P = 1000 MW and d) P = 1500 MW, e) 2000 MW.

Case of ideally stiff load

$$P_L(V) = P_n \quad \text{and} \quad Q_L(V) = Q_n,$$

- Different values of voltage V correspond to different circles in the (P_n, Q_n) plane. The analytical expression of the envelope which encloses all solutions of the network equation(3) is given by:
  - Inside envelope: two solutions
  - On the envelope: one solution
  - Outside the envelope: no solution

$$Q_n = \frac{E^2}{4X} - \frac{P_n^2}{\frac{E^2}{X}},$$



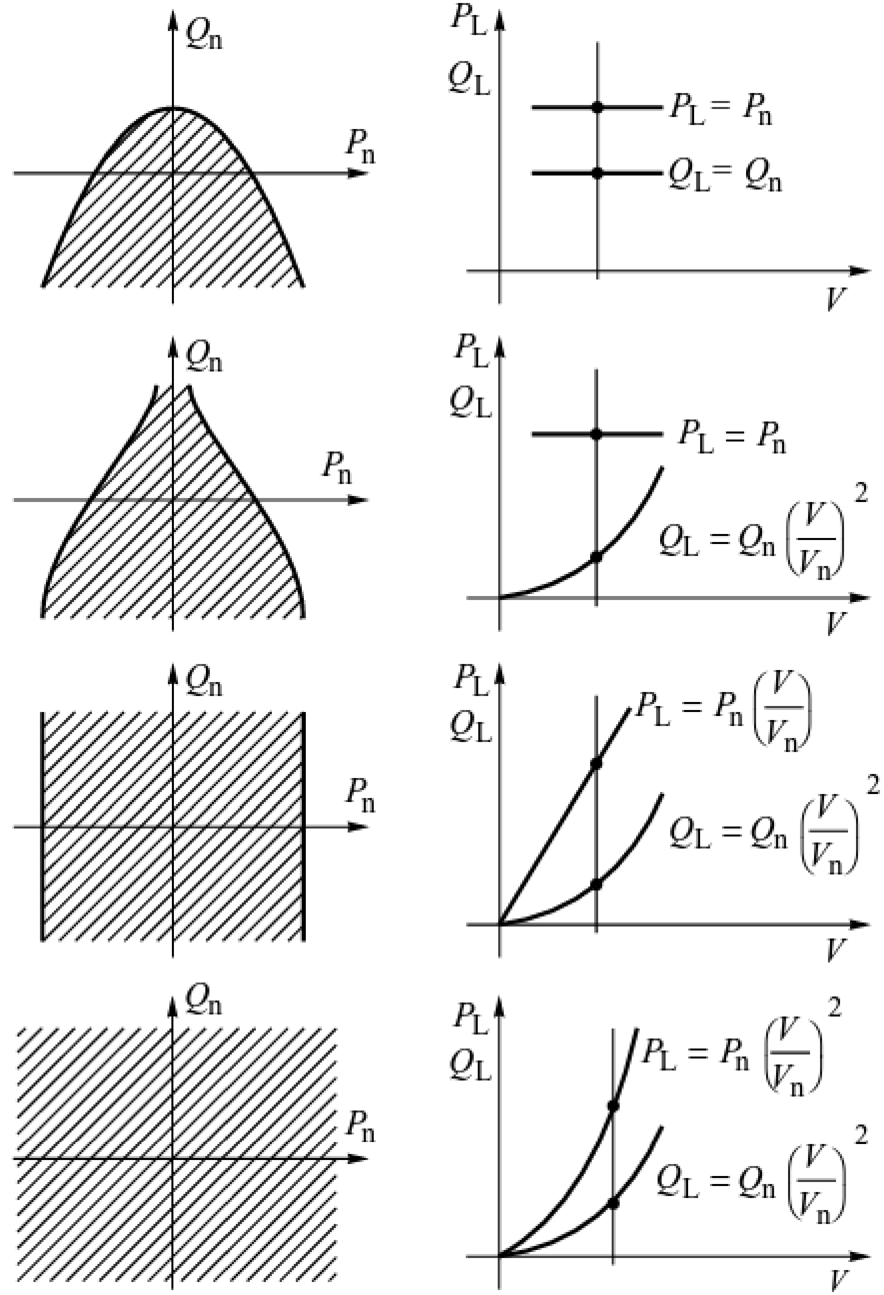
Influence of load characteristics

- Note the infinite solution space for the case where both P and Q vary quadratically with voltage. Herein,

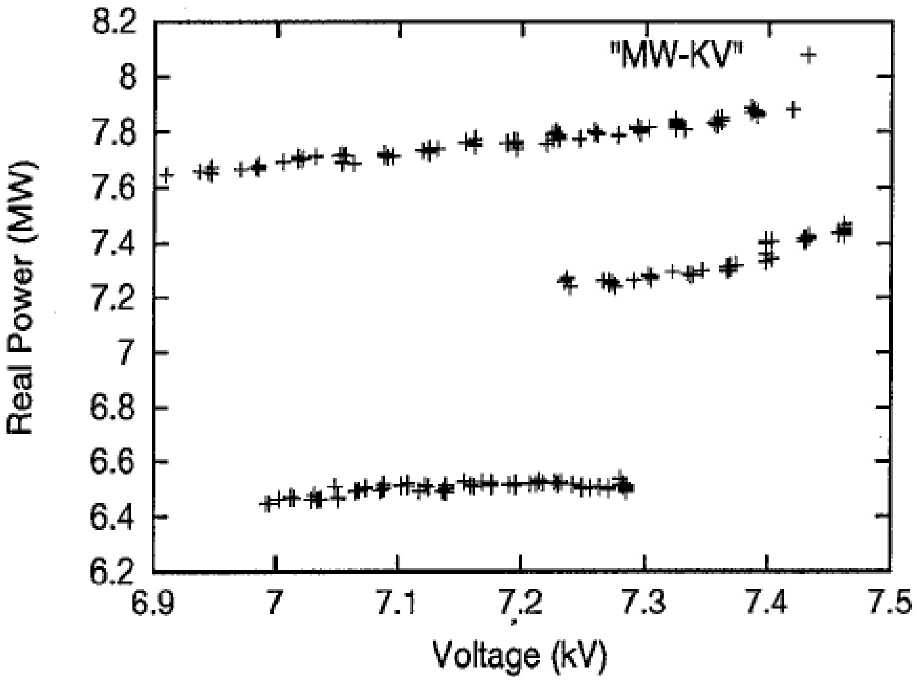
$$P_L(V) = P_n \left(\frac{V}{V_n}\right)^2 = \frac{P_n}{V_n^2} V^2 = G_n V^2,$$

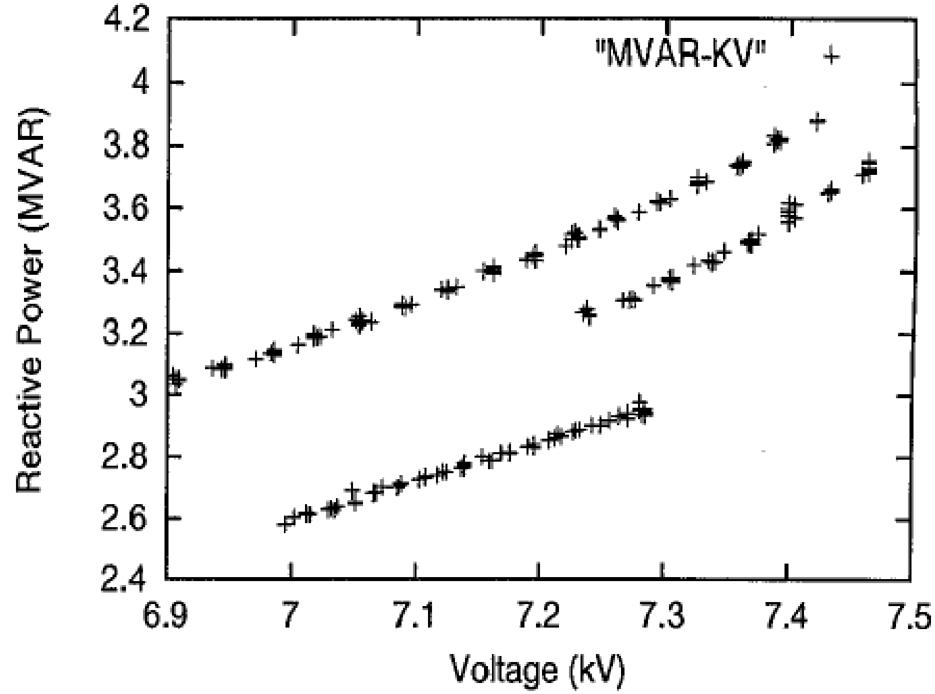
$$Q_L(V) = Q_n \left(\frac{V}{V_n}\right)^2 = \frac{Q_n}{V_n^2} V^2 = B_n V^2,$$

- This is the case of a constant admittance (or impedance) load where there is always a solution (obtained by voltage division).



Sample of measured P(V) and Q(V) curves on a distribution feeder

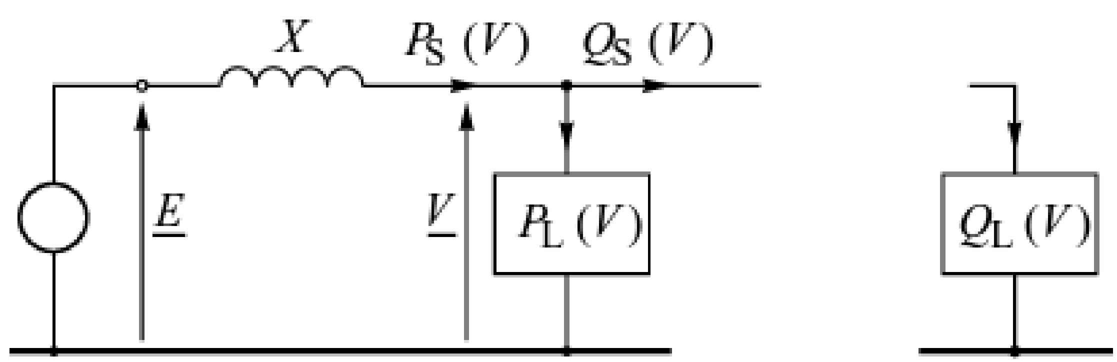




Stability Criteria

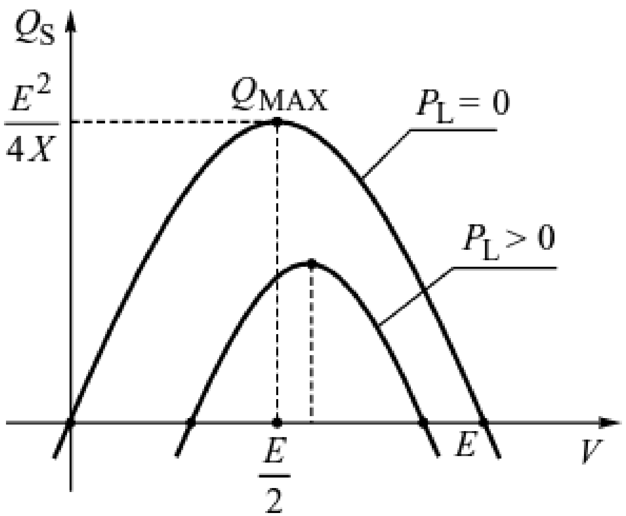
- *Voltage stability problem:* In case of two solutions with respect to the voltage, which one corresponds to a stable equilibrium point? What are the conditions for stability? Several types of criterion are considered.
- *dΔQ/dV Criterion* (idea of separating the load and source reactive powers)
  - Reactive power expression in terms of voltage and real power from the network equation (3)

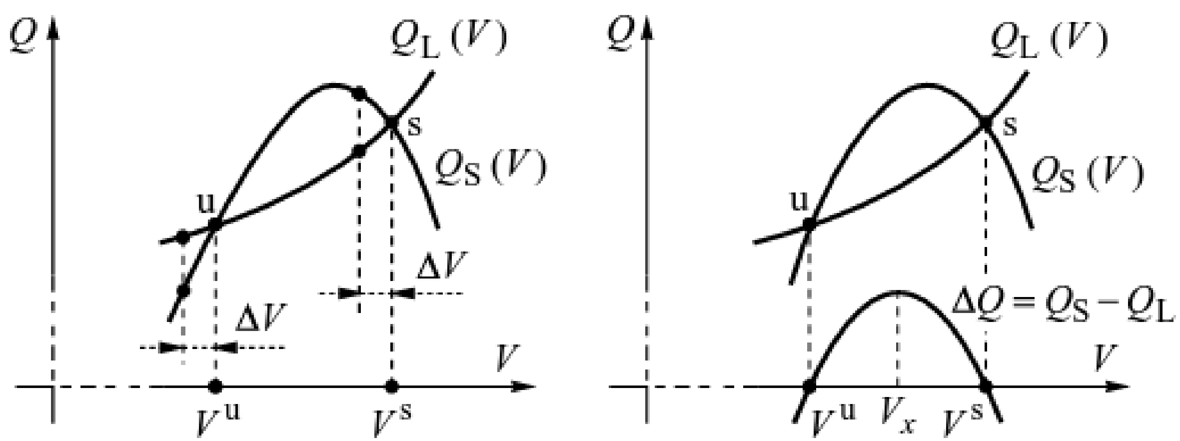
$$Q_S(V) = \sqrt{\left[\frac{EV}{X}\right]^2 - [P_L(V)]^2} - \frac{V^2}{X}.$$



Voltage Stability – *dΔQ/dV* criterion

- The left figure below shows how the source reactive power varies with voltage at different levels or real power.
- Now the load reactive power curve can be drown on the same figure as shown below. The two reactive powers must be equal to each other at equilibrium.
  - Recall that access reactive power tends to raise the voltage (Chap. 3).
  - A small voltage disturbance shows that point *s* is stable while *u* is unstable.
  - Condition for stability: $$\frac{\mathrm{d}(Q_{\mathrm{S}} - Q_{\mathrm{L}})}{\mathrm{d}V} < 0 \quad \text{or} \quad \frac{\mathrm{d}Q_{\mathrm{S}}}{\mathrm{d}V} < \frac{\mathrm{d}Q_{\mathrm{L}}}{\mathrm{d}V}.$$
  - Convinent with load flow.





(a) (b)

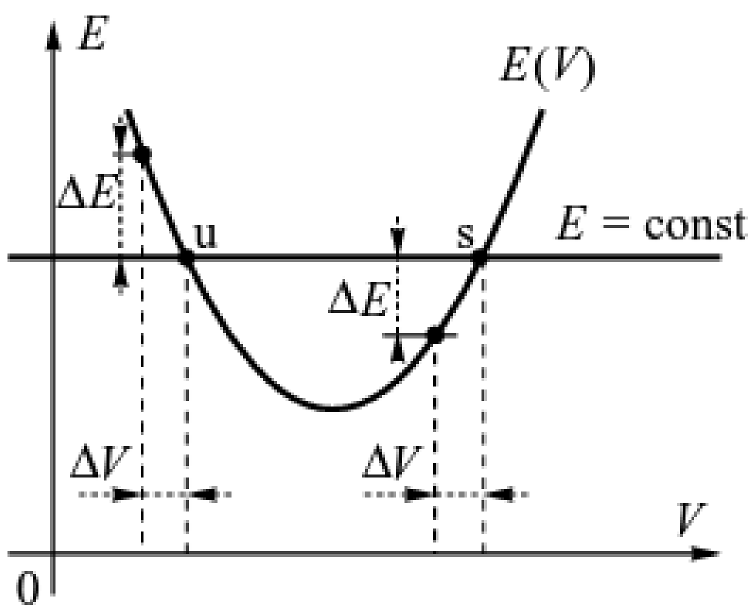
Voltage Stability – *dE/dV* criterion

- In here, the system equivalent emf E is expressed in terms of V:

$$E(V) = \sqrt{\left(V + \frac{Q_L(V)X}{V}\right)^2 + \left(\frac{P_L(V)X}{V}\right)^2},$$

- Stability criterion: $\frac{dE}{dV} > 0.$

- Not convenient when using load flow.

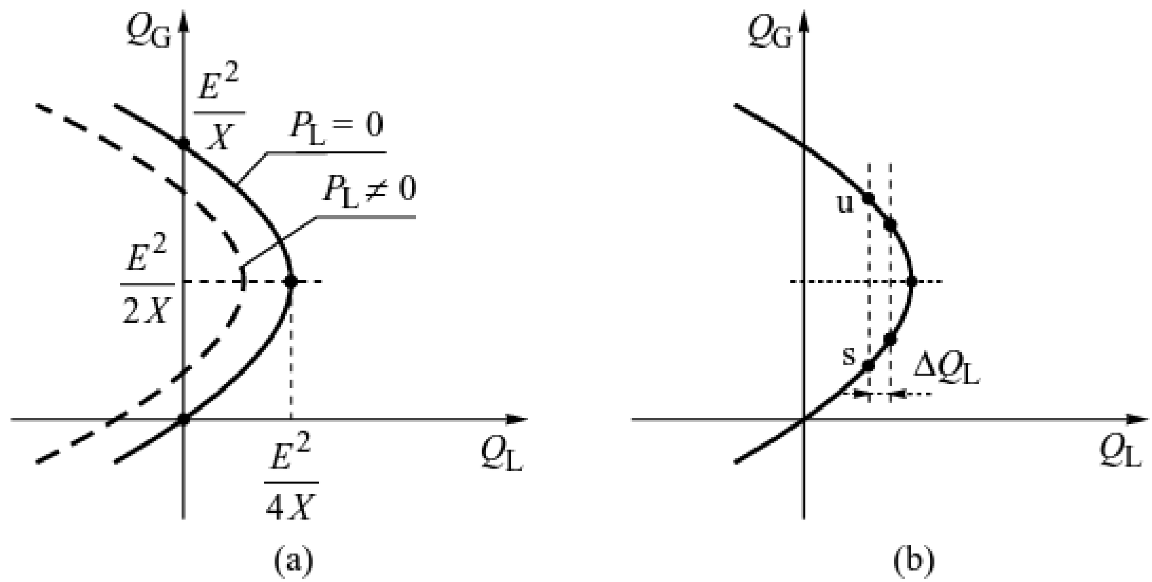


Voltage Stability – dQ_G/dQ_L criterion

- The generated reactive power Q_G includes the network reactive power demand (I^2X):

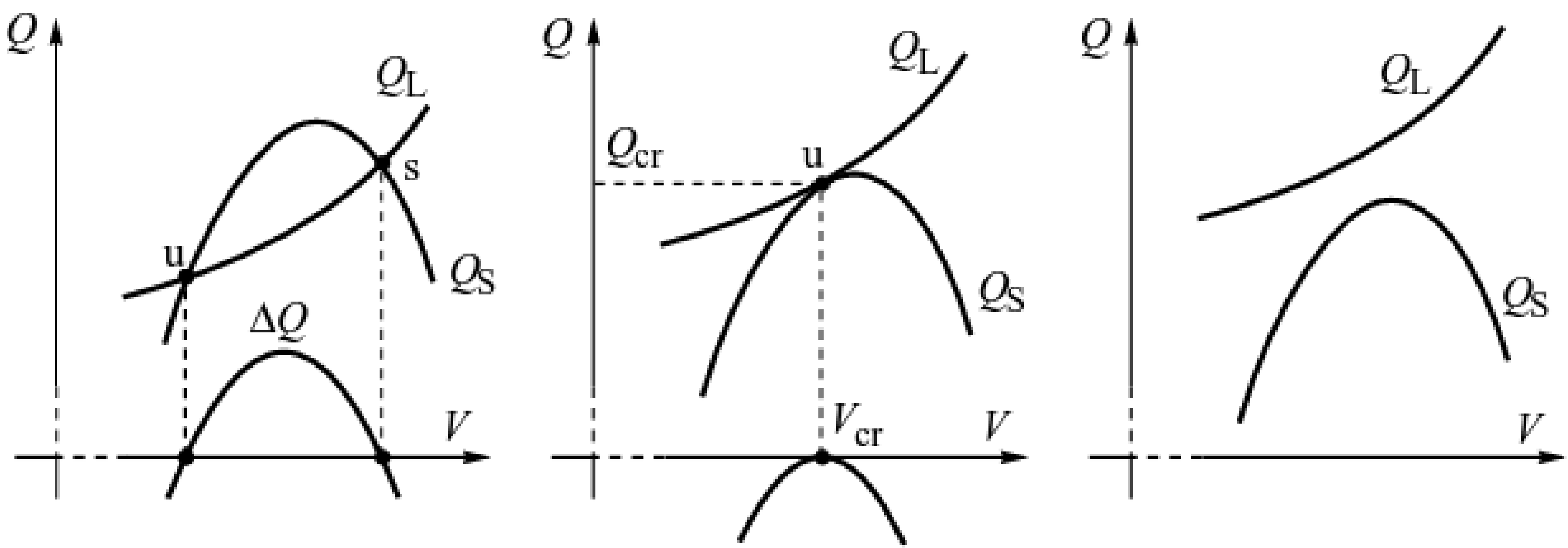
$$Q_L(V) = -\frac{Q_G^2(V)}{\frac{E^2}{X}} + Q_G(V) - \frac{P_L^2(V)}{\frac{E^2}{X}}.$$

- For constant real power, the above equation is a parabola (see fig.)
- Point *s* is a stable equilibrium point (as generation follows load), while point *u* is and unstable point.
- Stability criterion: $\frac{dQ_G}{dQ_L} > 0.$
- Convenient with power flow.



Critical load demand and voltage collapse

- The figure below shows a stable state (left), a critical state (center) and a state that does not have an equilibrium point (left).
- An increase in load (both P & Q), lowers the curve Q_S and raises curve Q_L. This brings the stable equilibrium point *s* closer to the critical point *u*. Increasing the load beyond this point results in voltage collapse (where the reactive power demand is larger than the supply).



Avoiding voltage instability

- It is important to stay away from the critical value of the voltage as far as possible. However, such a point is difficult to determine.
- An iterative formula can be used under the following assumptions:
  - (a) The power factor is maintained constant $$\frac{P_n(t)}{P_0} = \frac{Q_n(t)}{Q_0} = \xi,$$
  - (b) The active and reactive powers vary with the voltage as follows:

$$\frac{Q_L}{Q_n} = a_2\left(\frac{V}{V_n}\right)^2 - a_1\left(\frac{V}{V_n}\right) + a_0, \quad \frac{P_L}{P_n} = b_1\left(\frac{V}{V_n}\right),$$

- Then the critical voltage and critical load can be found iteratively by

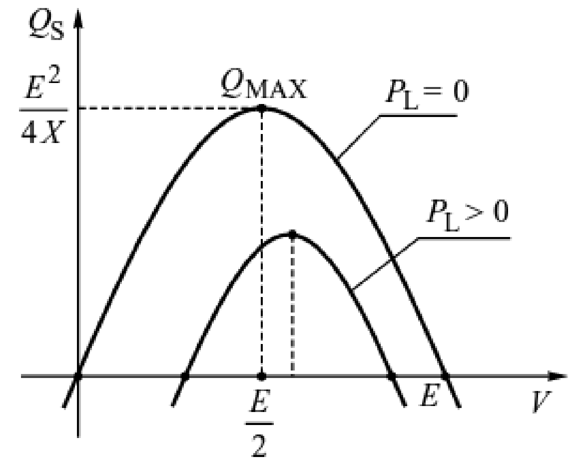
$$\xi_{cr} = \frac{1}{\frac{\alpha_0 X}{V_{cr}^2} - \alpha_2 X}. \qquad V_{cr} = \frac{\sqrt{\left(\frac{E}{\beta_1 X}\right)^2 - \xi_{cr}^2 + \frac{\alpha_1}{\beta_1}\xi_{cr}}}{2\frac{\alpha_2}{\beta_1}\xi_{cr} + \frac{2}{\beta_1 X}}.$$

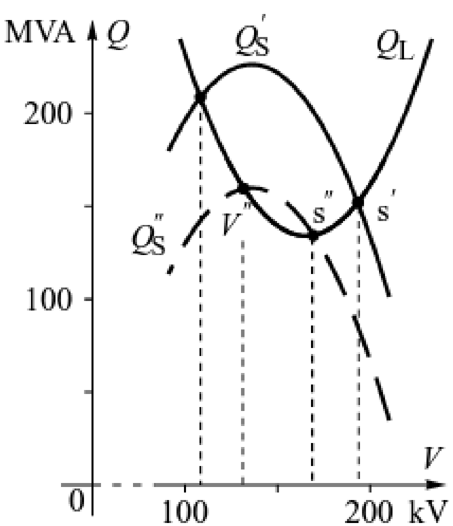
- See Example 1 (pp. 313)

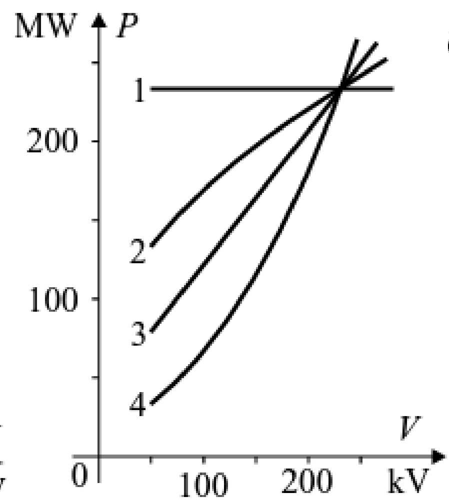
Effect of line outage and load characteristic

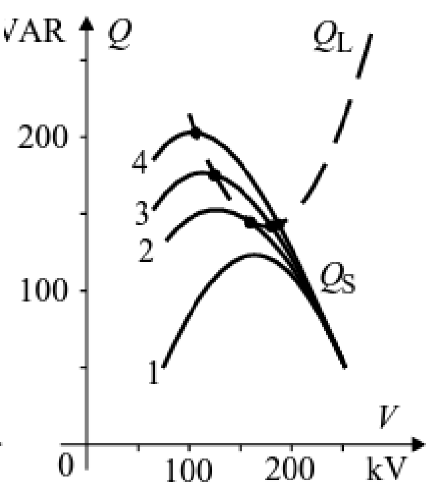
- Recall that the reactive power curve depends on the system impedance and real power characteristic.
- A network outage may cause voltage stability problems as this lowers Qs (see Ex. 2, p. 315)
- The variation of real power with voltage may improve voltage stability (see Ex. 3, p. 315)
- Constant impedance loads are always stable.

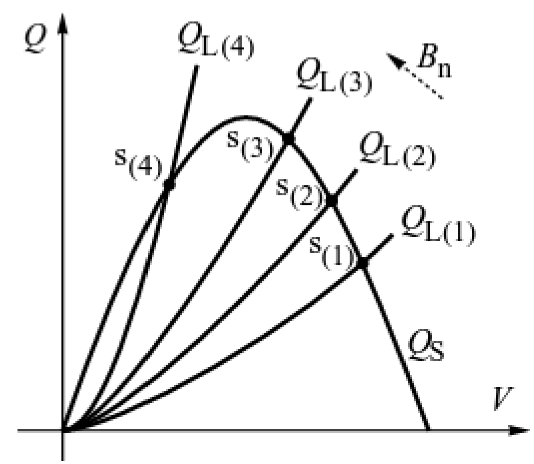
$$Q_S(V) = \sqrt{\left[\frac{EV}{X}\right]^2 - [P_L(V)]^2} - \frac{V^2}{X}.$$











Voltage stability assessment through load flow

- Voltage stability of a network with multiple generators and many composite loads is more complicated.
- Recall the power flow equations (Chap. 3)

$$P_i = V_i^2 Y_{ii} \cos\theta_{ii} + \sum_{j=1;j\neq i}^{N} V_i V_j Y_{ij} \cos(\delta_i - \delta_j - \theta_{ij})$$

$$Q_i = -V_i^2 Y_{ii} \sin\theta_{ii} + \sum_{j=1;j\neq i}^{N} V_i V_j Y_{ij} \sin(\delta_i - \delta_j - \theta_{ij}).$$

- Linearization results in

$$\begin{bmatrix} \Delta \boldsymbol{P} \\ \hdashline \Delta \boldsymbol{Q} \end{bmatrix} = \begin{bmatrix} \boldsymbol{H} & \boldsymbol{M} \\ \boldsymbol{N} & \boldsymbol{K} \end{bmatrix} \begin{bmatrix} \Delta \boldsymbol{\delta} \\ \hdashline \Delta \boldsymbol{V} \end{bmatrix}$$

- Critical demand occurs when

$$\det \boldsymbol{J} = \det \begin{bmatrix} \boldsymbol{H} & \boldsymbol{M} \\ \boldsymbol{N} & \boldsymbol{K} \end{bmatrix} = 0,$$

- → monitor the determinant of the Jacobian matrix when running numerous load flows with increased loading.

Static Analysis – Voltage stability indices

- A number of voltage stability indices can be used:
- One is based on the classical dQ/dV criterion (hard to determine)

$$k_V = \frac{V_s - V_x}{V_s}, \text{ where } \left.\frac{d(Q_S - Q_L)}{dV}\right|_{V=V_x} = 0.$$

- Another proximity index is (moderately convenient to use)

$$k_{\Delta Q} = \frac{d(Q_S - Q_L)}{dV}\frac{V_s}{Q_s}$$

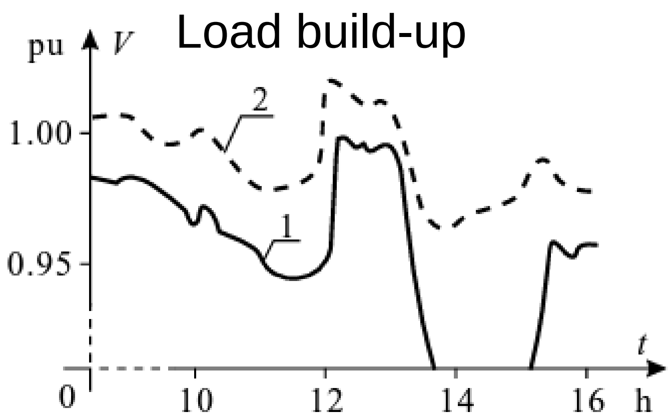
- Yet another proximity index is (most convenient to use)

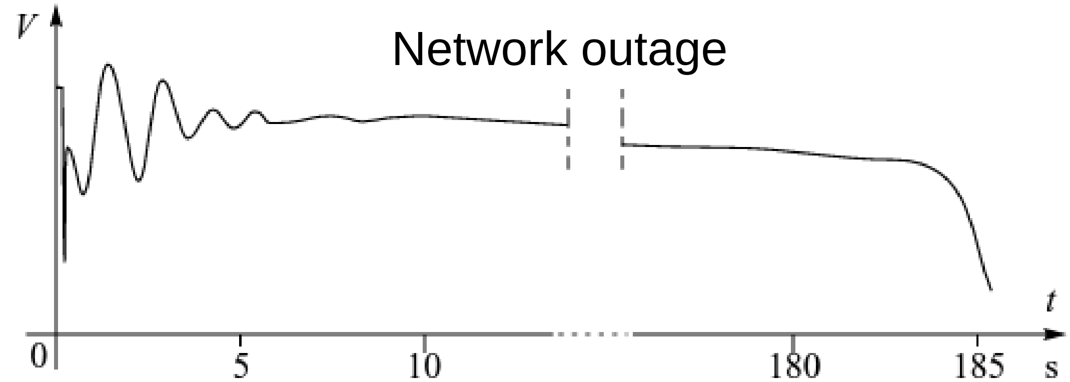
$$k_Q = \frac{dQ_G}{dQ_L}.$$

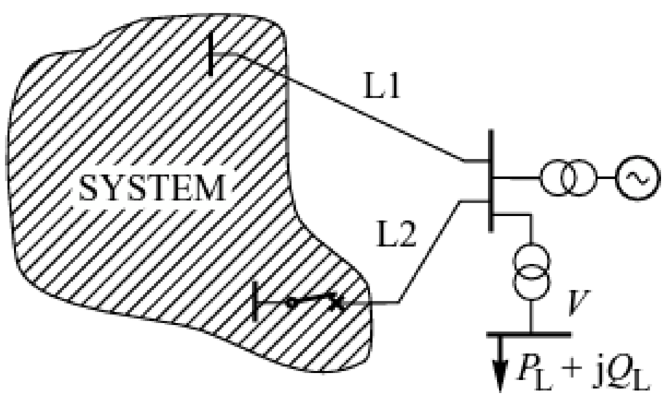
Under light load, $k_Q \approx 1$, near the critical load, $k_Q \rightarrow$ infinity.

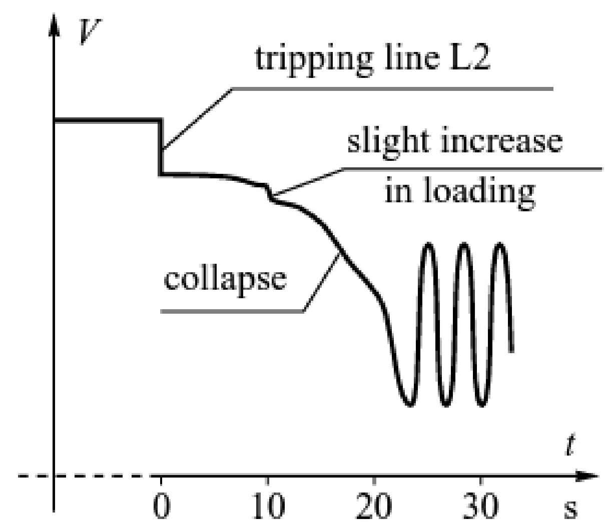
Dynamic Analysis

- The static load-voltage we have seen so far can only approximate the real system behavior under slow voltage variations.
- In practice, the above is a process that is influenced by load dynamics, control and protection equipment – more complicated.



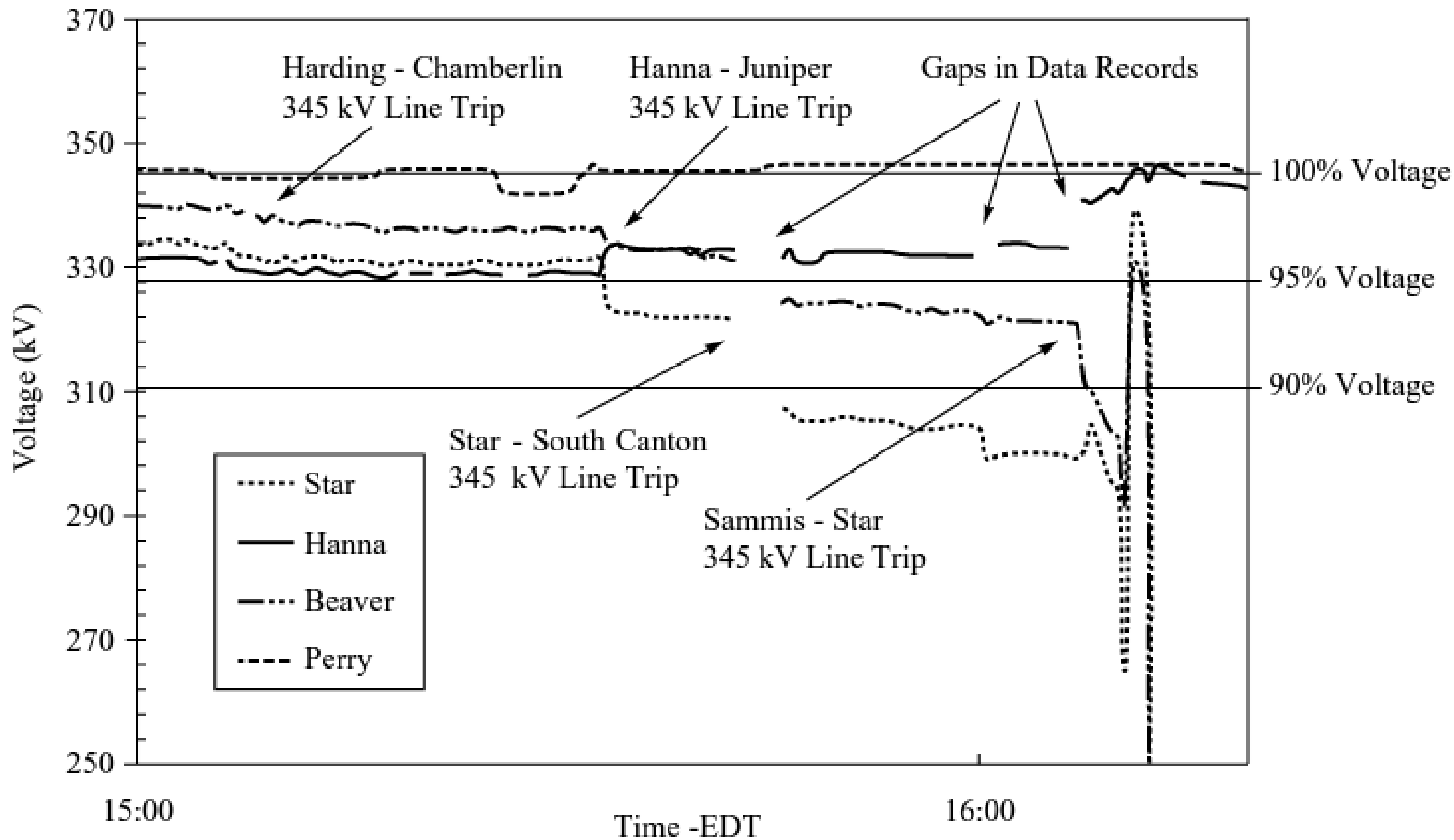






Examples of power system blackout
USA/Canada, 2003

- Cascaded tripping of transmission lines left 50 million customers without power.
- Each trip caused overload and lower voltages on other lines.



Prevention of voltage collapse

- Action is required at the network planning, operational and monitoring and control stages.
- During planning, reliability criteria must be satisfied for all possible contingencies on at least N-1 type (with maximum voltage drop not exceeded and sufficiently large stability margins)
- During operation and real-time monitoring and control, the desired voltage profile should be continuously maintained, In addition, adequate amounts of real and reactive power reserves at the generators should be maintained.
- Additional defenses include:
  - Emergency back-up reactive power reserve
  - Emergency increase in reactive power generation at the expense of reduction in real power
  - Reduction in real power demand by using OLTC transformers.
  - Under-voltage load shedding .