S8.1 Switch-mode inverter (one phase-leg, half bridge)

A general analysis of the switch-mode inverter (shown in the figure below) is to be done. The switching frequency \( f_s \) which is also the frequency of the triangular signal is 1450 Hz. The DC voltage, \( V_d \), is 600 V. Output voltage is sinusoidal voltage with a frequency equal to 50 Hz. The load is connected between the inverter leg A and the dc voltage midpoint O.

(a) Find the frequency modulation ratio, \( m_f \). Why is it chosen as an odd number?

(b) Calculate the output voltage (rms value of 1 harmonic), when the amplitude modulation ratio, \( m_a \), is equal to 0.8?

(c) Prove that \( V_A \) is \( m_a V_d / 2 \).

(d) When \( m_a \) varies from 0 to 1, the mo the linear domain. Why?

(e) Compute the rms value of the 5 most dominant harmonics of \( v_{A0} \) (at \( m_a = 0.8 \)), by using Table 8-1, page 207. Also indicate the frequencies at which these harmonics appear.

(f) Which frequencies are desirable for the switching frequency? List the advantages and disadvantages of low/high switching frequency.

S8.1.

a) Frequency modulation ratio \( m_f \) = \( \frac{1450}{50} \) = 29.

b) \( m_a = 0.8 \); \( V_{0,peak} = (0.8)(600) / 2 = 240 \text{ V} \); \( V_{01} = \frac{240}{\sqrt{2}} = 170 \text{ V} \).

c) See Eqs. (8-6) and (8-7) in the text.

d) Modulation is linear when \( m_a \leq 1 \) because \( v_{A0}(t) = m_a \frac{V_d}{2} \sin(\omega t) \).

e) Use Table 8-1 in text with \( m_a = 0.8 \). Amplitudes will be rms values.

- Fundamental = 50 Hz; Amplitude = 170 V (see part b above)
- \( h = m_f \cdot 50 = 1450 \text{ Hz} \); Amplitude = \( \frac{1}{\sqrt{2}} (0.818)(300) = 173.5 \text{ V} \)
- \( h = 3m_f \cdot 2 = 3\times 1450 \text{ Hz} \); Amplitude = \( \frac{1}{\sqrt{2}} (0.176)(300) = 37.3 \text{ V} \)
- \( h = m_f \cdot 2 = 2\times 1450 \text{ Hz} \); Amplitude = \( \frac{1}{\sqrt{2}} (0.171)(300) = 36.3 \text{ V} \)
- \( h = m_f \cdot 2 = 3\times 1450 \text{ Hz} \); Amplitude = \( \frac{1}{\sqrt{2}} (0.139)(300) = 29.5 \text{ V} \)
- \( h = 2m_f \cdot 1 = 2\times 1450 \text{ Hz} \); Amplitude = \( \frac{1}{\sqrt{2}} (0.314)(300) = 66.6 \text{ V} \)

d) \( m_f \) should be integer and odd. This produces a symmetrical \( v_A(t) \) and minimizes the harmonics.
S8.2 Switch-mode Inverter (single phase, full bridge)

The inverter from 8.A is expanded with another leg. It is PWM controlled with bipolar voltage switching. It operates at the same \( m_a \) and \( m_b \), and \( V_d \) still equals 600 V. Output voltage is sinusoidal and has a frequency of 50 Hz.

(a) Why is it advantageous to use full bridge instead of half bridge in case of higher power?

(b) Show that the peak value of the first harmonic component of the output voltage, \( P_{dc} \), equals \( m_b \) times \( V_d \) for this inverter.

(c) Why is this type of switching called Bipolar?

(d) Compute the rms value of the 5 most dominant harmonics of \( v_{oa} \) (at \( m_b=0.8 \)), by using Table 8.1, page 207. Also indicate the frequencies at which these harmonics appear.

S8.2.

a) With the same type of transistors, the full bridge converter will double the output power compared with the half bridge. For increasing power, it is also possible to parallel connect transistors in the half-bridge converter. Paralleling is demanding and normally a derating must be done to give room from unbalanced current sharing during both the switchings and the on-state intervals.

b) From Fig. 8.6 in the text, it can be seen that the duty cycle \( D \) approaches \( m_a \) at \( v_{control peak} \). As \( V_o = D V_d \), then \( V_{o1,peak} = m_a V_d \).

c) PWM with bipolar voltage switching is described in Ch. 7.7-1 of the text. \( T_{A+} \) and \( T_{B+} \) are controlled in the same way. \( T_{A-} \) and \( T_{B-} \) are controlled by an inverter replica of the control signal for \( T_{A+} \) and \( T_{B+} \). For a sinusoidal control signal, this results in the output voltage waveform shown in Fig. 8.12 of the text. For each switching cycle, the output voltage is both positive and negative, depending on the duty cycle. The output voltage waveform is bipolar and hence the name for the modulation algorithm.

d) \( V_{A0,1} = \frac{1}{\sqrt{2}} (0.8)(600) = 340V, \ 50 \text{ Hz} \)

\( V_{A0,26} = \frac{1}{\sqrt{2}} (0.818)(600) = 347V, \ 1450 \text{ Hz} \)

\( V_{A0,57} = \frac{1}{\sqrt{2}} (0.314)(600) = 133V, \ 2850 \text{ Hz} \)

\( V_{A0,59} = \frac{1}{\sqrt{2}} (0.314)(600) = 133V, \ 2950 \text{ Hz} \)

\( V_{A0,85} = \frac{1}{\sqrt{2}} (0.176)(600) = 75V, \ 4250 \text{ Hz} \)
S8.3 Bipolar single phase half bridge inverter

L = 15 mH
E_s = 230 V
f = 50 Hz
V_d = 800 V

(a) The frequency of the triangular signal is 750 Hz. Calculate the frequency modulation ratio, \( m_f \).

(b) Find \( m_i \) when the output voltage is 230 V?

(c) Find the angle, \( \alpha \), between the phasors, \( V_d \) and \( E_s \) for \( P = 1 \) kW and \( Q = 500 \) Var.

(d) For the same \( Q \), find \( \alpha \) and \( V_d \) when \( P = -1 \) kW (rectifier mode).

(e) Find \( \alpha \) and \( V_d \) for \( Q = 1 \) kVar and \( -1 \) kVar when \( P \) is kept equal to zero.

(f) Sketch the line on which the arrow head of voltage phasor \( V_d \) moves along for \( P = 0 \) kW and for \( P = 1 \) kW, and varying \( Q \).

S8.3,

a) \[ m_f = \frac{750}{50} = 15 \]

b) \[ \sqrt{2} \cdot 230 = m_i \cdot \frac{800}{50} \Rightarrow m_i = 0.813 \]

c) The circuit shown below is the equivalent circuit for the inverter in the frequency domain.

\[ V_{A,ph} = V_{L,ph} + E_{A,ph} \] where the subscript \( ph \) indicates that these are phasors.

Choose \( E_{A,ph} = 230 \) Vol. Then:

\[ I_{P,ph} = \frac{50}{E_{A,ph}} = 4.35 \text{ A in phase with } E_{A,ph} \]

\[ I_{Q,ph} = \frac{500}{E_{A,ph}} = 2.17 \text{ A lagging } E_{A,ph} \text{ by } 90^\circ \]

Phasor diagram shown below.

\[ V_{A,ph} = 230 + 435 \text{ A} \]

\[ V_{A,ph} = 230 + (5)(2.17 + j 4.35) = 241 + j 242 = 247 \text{ } e^{j53^\circ} \]

(d) When \( P = -1 \) kW, the phasor diagram shown above must be changed so that \( I_{P,ph} \) is reversed (i.e. negative). The new diagram is shown on the next page.
The voltage $V_{A_{ph}} = 241 - 221 = 242 \, \text{e}^{j45^\circ}$

e) $P = 0$ and $Q = \pm 1 \, \text{kVAR}$ : $I_{Q_{ph}} = \pm 4.25 \, \text{A}$

$$Q = \pm 1 \, \text{kVAR} ; \quad V_A = 250 \, \text{V} \quad \text{and angle} = 0 \; ; \quad Q = \pm 1 \, \text{kVAR} ; \quad V_A = 210 \, \text{V} \quad \text{and angle} = 0$$

f) Movement of $V_{A_{ph}}$ for $P = 0$ shown in part e. Movement for $P = 1 \, \text{kW}$ shown below.

S8.4 Single-phase full bridge converter

$$e(t) = \sqrt{2} E_0 \sin \omega t$$

The problem with ripple in the output current from a single-phase full bridge converter is to be studied. The first harmonic of the output voltage is given by $V_{o1}$ at $f = 47$ Hz. The load is given in the figure as $L = 100 \, \text{mH}$ in series with an ideal voltage source $e(t)$. It is assumed that the converter works in square wave mode.

(a) Calculate the value of $V_d$ which gives $V_{o1} = 220 \, \text{V}$.

(b) Calculate the peak value of the ripple current.

The conditions are the same as in (a), but the converter operates in sinusoidal PWM mode, bipolar modulation, $m_r = 21$ and $m_s = 0.8$.

(c) Which value of $V_d$ gives $V_{o1} = 220 \, \text{V}$?

(d) Explain why the ripple current has its peak value at the zero crossing of the first harmonic voltage, and find this value.

The conditions are the same as in (a), but the output voltage is given by voltage cancellation (see figure 8-17) and $V_d = 369 \, \text{V}$.

(e) Calculate the "overlap angle" $\alpha$ and peak value of the ripple current.

(f) Compare the values found in the three previous problems.
a) \( V_{o1, \text{ph}} = \frac{2}{\pi} V_d \). Eq. (8-36) in text. \( V_{o1, \text{ph}} = \sqrt{2} V_{o1} \). \( V_{o1} = 220 \text{V} \)

\[
V_d = \frac{2}{\pi} \sqrt{2} V_{o1} = \frac{2}{\pi} \sqrt{2} (220) = 244 \text{V}
\]

b) Ripple current is as shown in Fig. 8-19a.

\[
2 \cdot I_{\text{ripple, peak}} = \frac{1}{\omega L} \int_0^\pi (V_d - \sqrt{2} V_{o1} \sin(\omega t)) \, d(\omega t) = \frac{1}{\omega L} (\pi V_d - 2\sqrt{2} V_{o1})
\]

\[
I_{\text{ripple, peak}} = \frac{1}{2 \omega L} V_d \left( \pi - \frac{8}{\pi} \right)
\]

Put in numbers: \( \frac{1}{2 \pi (20 \pi (47) (0.01)} \left( \pi - \frac{8}{\pi} \right) (244) = 2.46 \text{A} \)

c) \( m_a = 0.8 \); \( V_{o1, \text{peak}} = (0.8) V_d = (0.8)(220) = \sqrt{2} \times 220 \); \( V_d = 389 \text{V} \)

d) 

The switching frequency, \( f_s \), is \( f_s = \frac{1}{T_s} = 987 \text{Hz} \). Period \( T_s \approx 1 \text{msec} \). As can be seen in the figure above, the output voltage across the inductor is positive for approximately one half of a period about 0.5 ms. In this interval the inductor voltage is approximately constant at \( V_d \) since \( V_o(t) = 0 \).

The peak of the ripple in the output current will be

\[
I_{\text{ripple, peak}} = \frac{V_d T_s}{2 L} = 1 \text{A}
\]
e) \[ V_{01} = \frac{4}{\pi} V_d \cos \beta ; \quad \sqrt{2} \cdot 220 = \frac{4}{\pi} (389) \cos \beta ; \quad \beta = 51^\circ \]
\[ \alpha = 180 - 2 \beta = 180 - 2(51) = 78^\circ \]

From the figure above: \[ \Delta I_1 = -\Delta I_3 ; \quad \Delta I_1 = \frac{1}{\alpha} \int \sqrt{2} \cdot 220 \sin(\omega t) dt \]
\[ \Delta I_1 = \frac{1}{\alpha} \left[ \frac{1}{\sqrt{2}} \right] \int_{0}^{\alpha/2} 220 \sin(\omega t) dt = \frac{1}{\alpha} \sqrt{2} \cdot 220 \left[ 1 - \cos(\alpha/2) \right] = 2.3 \text{ A} \]

\[ \Delta I_2 \]
\[ \Delta I_2 = \frac{1}{2} \int_{0}^{\alpha/2} \left[ \sqrt{2} \cdot 220 \sin(\omega t) \right] dt = \frac{1}{2} \left[ (389)(0.89) - (311)(0.78) \right] = 5.45 \text{ A} \]
\[ \Delta I_2 > \Delta I_1 ; \quad \text{Max} \; I_{\text{ripple, peak}} = 2.72 \text{ A} \]

f) FWM gives the lowest ripple. The two types of square are not much different from each other. Higher switching frequency gives even lower ripple.

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Problem 9.11

\[ V_{qs} \]

\[ I_{(a)} \quad \text{and} \quad I_{(b)} \]
\[ t_0 = 0, \; t_1 = 0.106 \mu s, \; t_{p1} = 0.356 \mu s, \; t_{p2} = 0.606 \mu s, \]
\[ t_2 = 0.722 \mu s, \; t_3 = 1.138 \mu s, \; t_4 = 1.627 \mu s \]
\[ f_s = \frac{1}{T_s (= t_4)} = 0.6146 \text{ MHz} \]
Note that \( t_4 = T_s \) is obtained by averaging the \( v_c(t) \) waveform over one time-period and equating it to \( V_o = 10V \).

\[
I_{L,\text{peak}} = 2.5A = 2.5 I_o,
\]
\[
V_{c,\text{peak}} = 30.0V = 2.0 V_d.
\]

Problem 9-12

\[
f_s = \frac{1}{t_5 = T_s} = 0.667 \text{ MHz}
\]

Note that \( t_5 = T_s \) is obtained by averaging the \( v_c(t) \) waveform over one time-period and equating it to \( V_o = 10V \).

\[
I_{L,\text{peak}} = 2.5A = 2.5 I_o,
\]
\[
V_{c,\text{peak}} = 30.0V = 2.0 V_d.
\]
Problem 7-14

For ZVS, \( I_o \geq \frac{V_d}{Z_o} \)

Equate \( I_{o,\min} = \frac{V_d}{Z_o} \) \( \therefore \) \( Z_o = \frac{V_d}{I_{o,\min}} \)

\( \therefore V_{\text{switch}}^\text{max} = V_d + Z_o I_{o,\max} \)

\[ = V_d \left(1 + \frac{I_{o,\max}}{I_{o,\min}}\right) \]

\[ = 40 \left(1 + \frac{20}{4}\right) = 240V \]