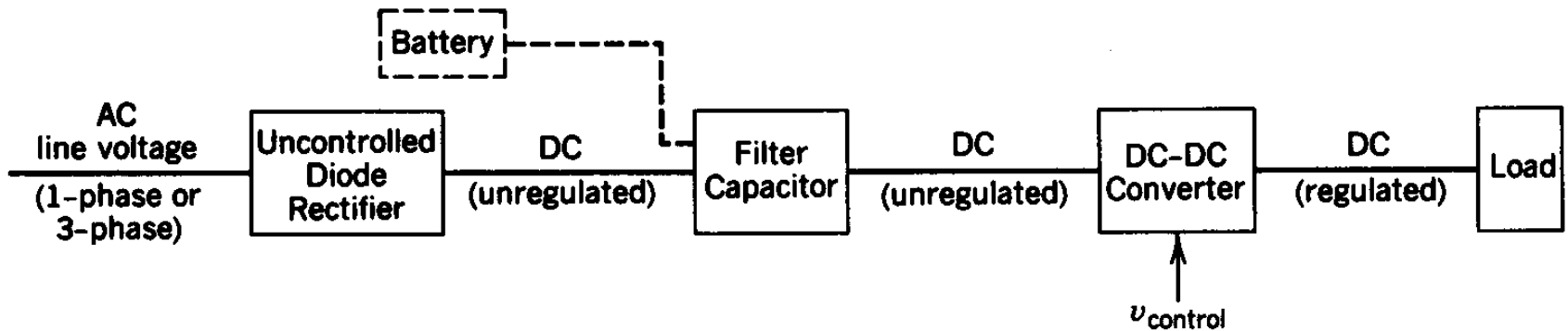


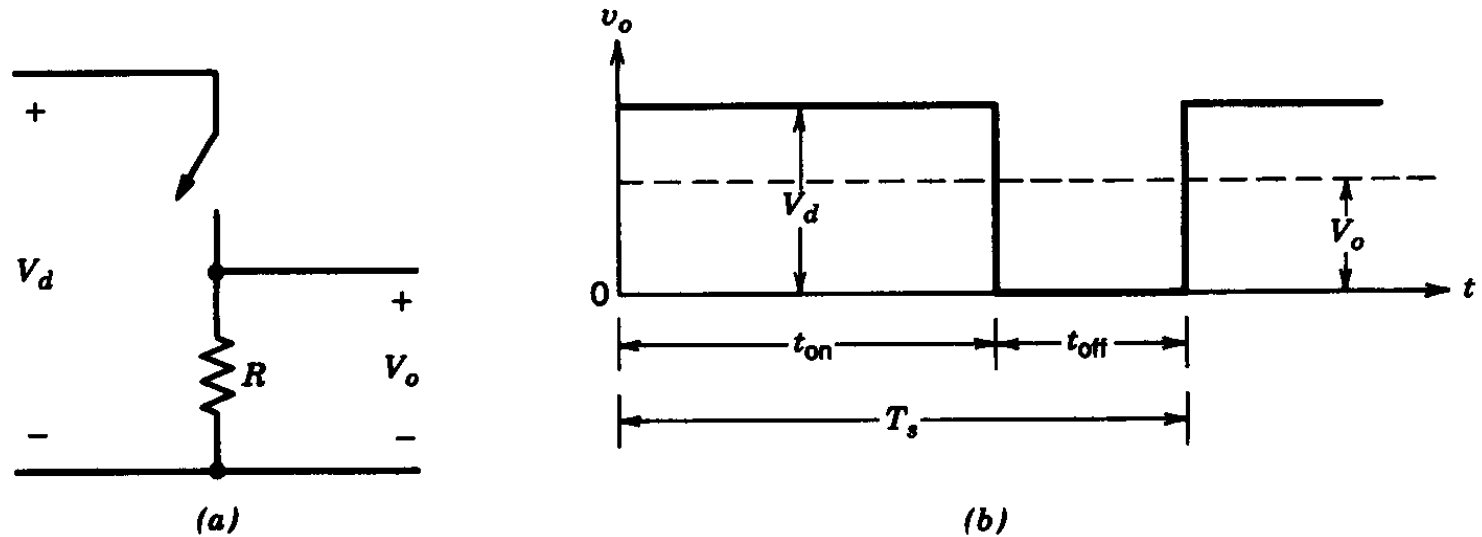
# DC-DC Switch-Mode Converters

EE 442/642

# Block Diagram of DC-DC Converters



# Stepping Down a DC Voltage – Basic Concept

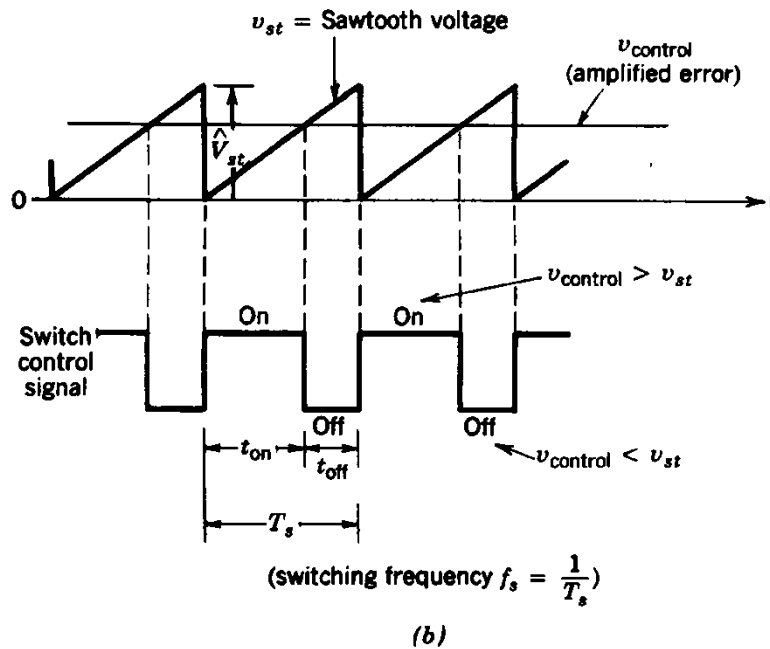
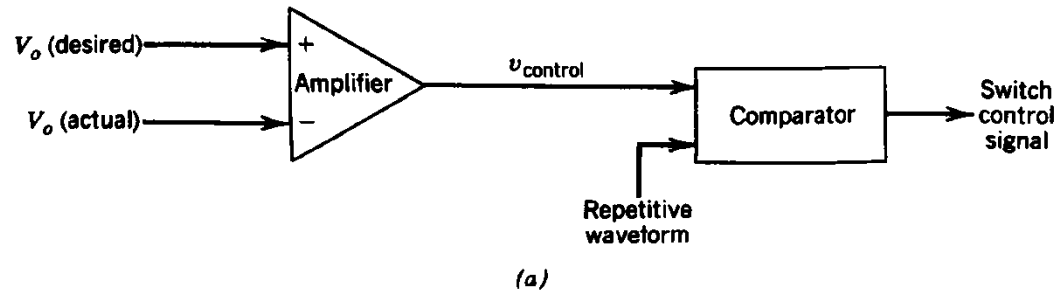


Switch Duty Ratio:  $D = \frac{t_{on}}{T_s}$

Average Output Voltage:  $V_o = \frac{t_{on}}{T_s} V_d = DV_d$

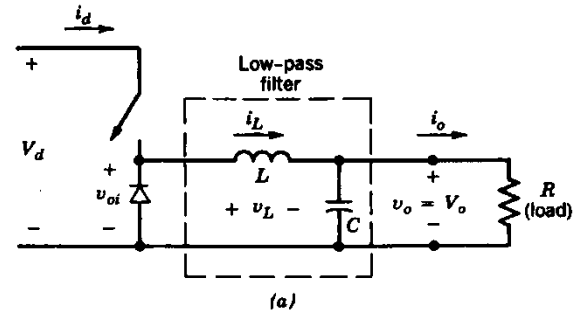
Any parasitic inductance in the above circuit will cause damage to the switch  
In addition, the output voltage contains significant ripple..

# Pulse-Width Modulation in DC-DC Converters

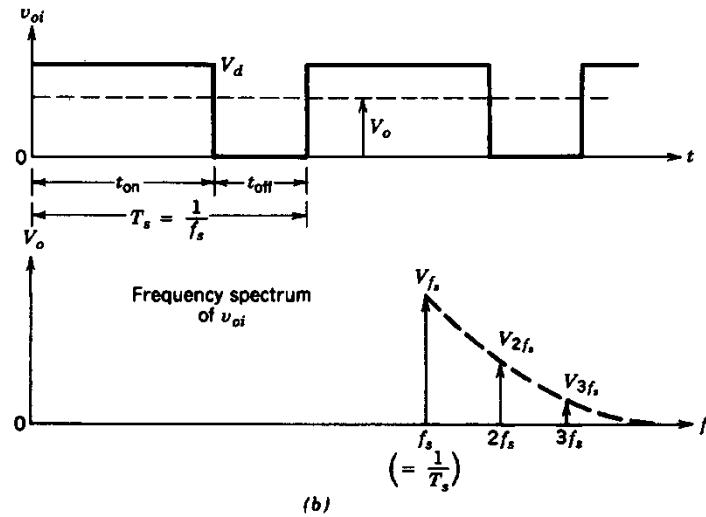


Duty Ratio:  $D = \frac{t_{on}}{T_s}$

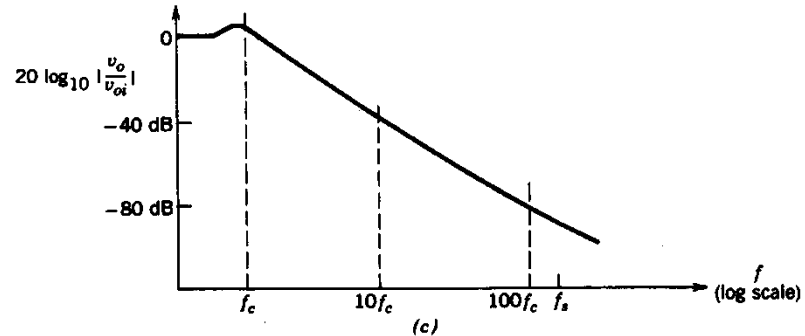
# Step-Down (Buck) DC-DC Converter



Output Voltage: 
$$V_o = \frac{t_{on}}{T_s} V_d = DV_d$$



Filter corner frequency ( $f_c$ ) should be much lower than the switching frequency ( $f_s$ )

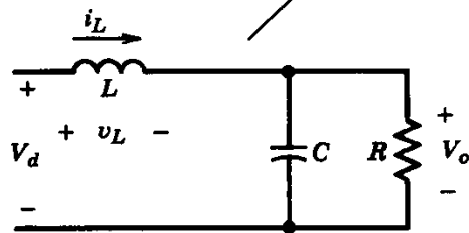
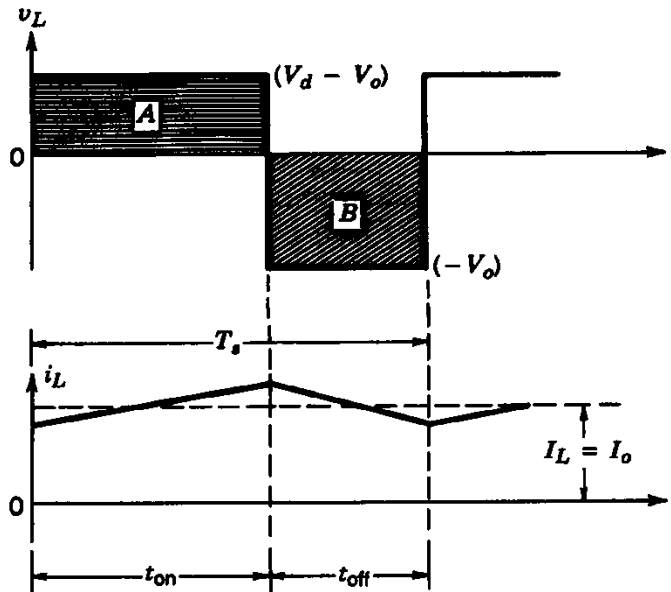


# Step-Down DC-DC Converter: Continuous Conduction Mode

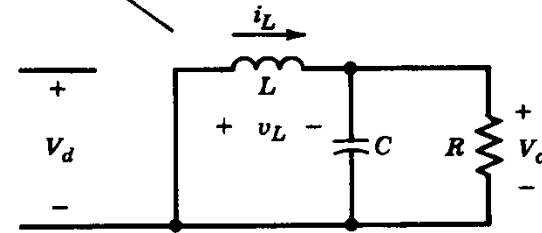
$$V_o = DV_d$$

$$V_d I_d = V_o I_o$$

$$I_o = \frac{I_d}{D}$$

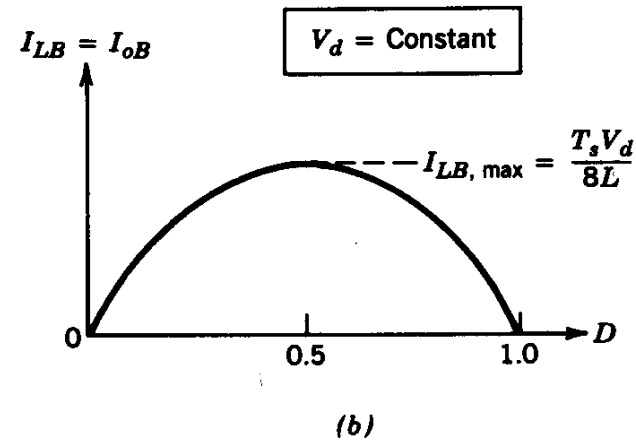
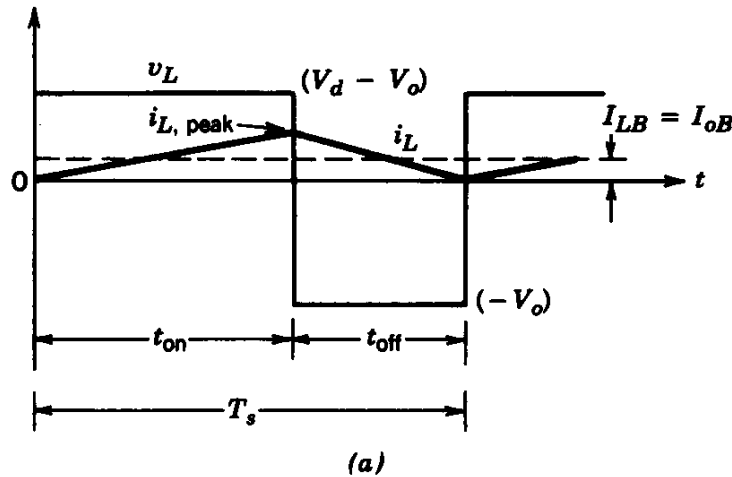


(a)



(b)

# Step-Down DC-DC Converter: boundary of Continuous/Discontinuous Conduction Mode



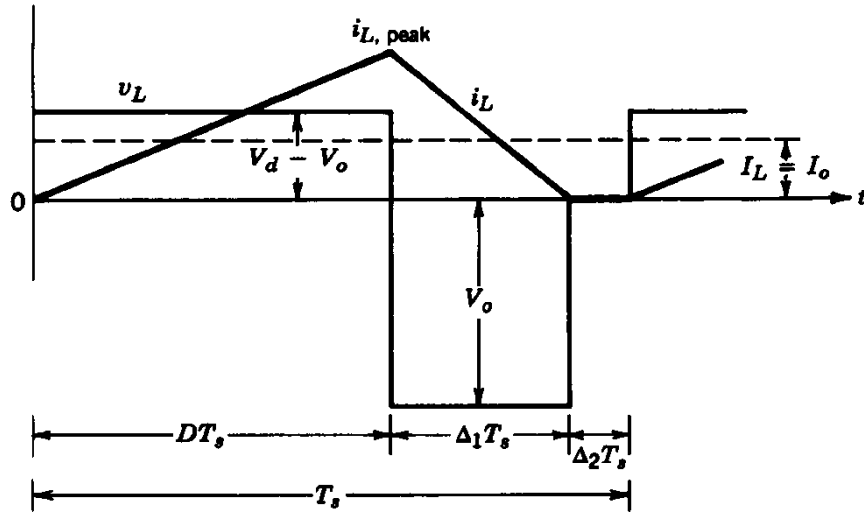
Output Voltage:

$$V_o = DV_d$$

Critical current below which inductor current becomes discontinuous:

$$\begin{aligned}
 I_{LB} = I_{oB} &= \frac{1}{2} i_{L, peak} = \frac{t_{on}}{2L} (V_d - V_o) = \frac{DT_s}{2L} (V_d - V_o) \\
 &= \frac{DT_s}{2L} (1 - D)V_d
 \end{aligned}$$

# Step-Down DC-DC Converter: Discontinuous Conduction Mode (Constant $V_d$ )



$$(V_d - V_o)DT_s + (-V_o)\Delta_1 T_s = 0$$

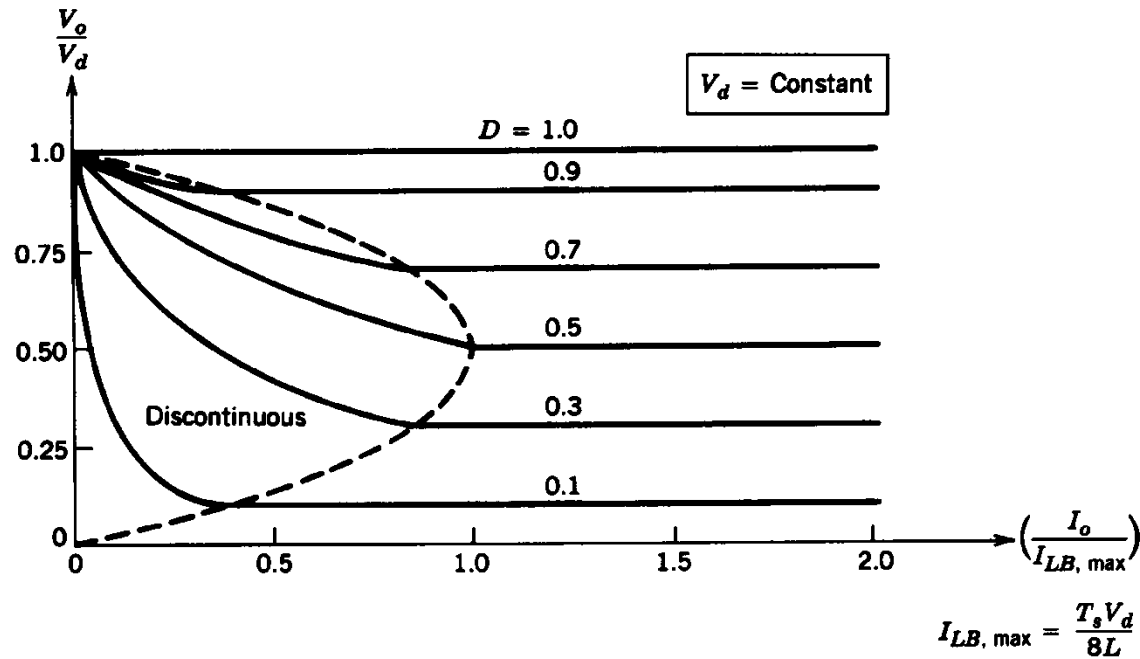
$$\Delta_1 = \frac{I_o}{4DI_{LB, \max}}$$

Output Voltage: 
$$\frac{V_o}{V_d} = \frac{D^2}{D^2 + I_o / (4I_{LB, \max})}$$

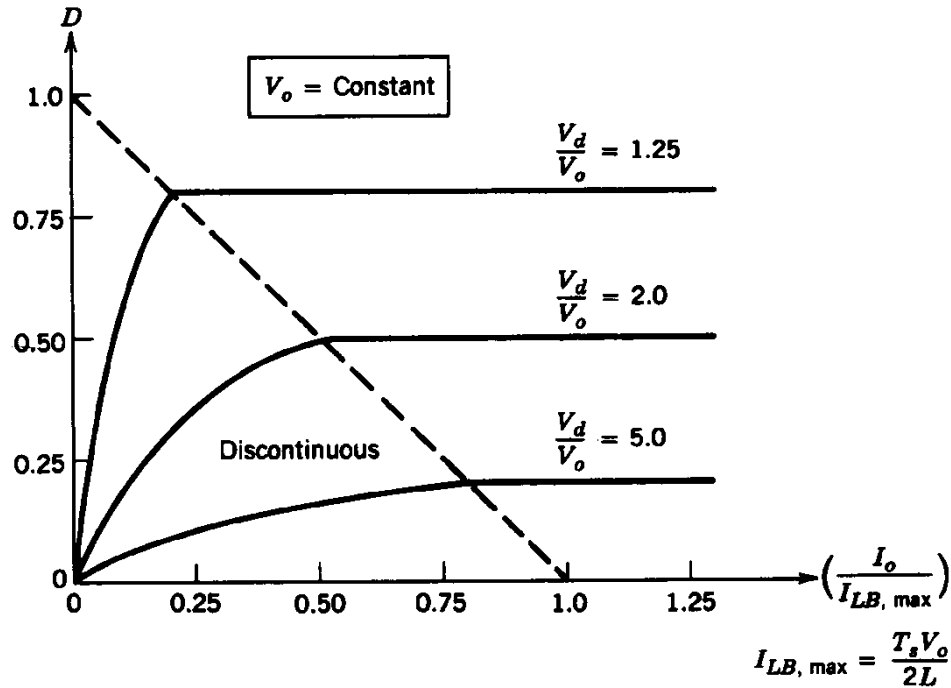
where 
$$I_{LB, \max} = \frac{T_s V_d}{8L}$$



# Step-Down DC-DC Converter: Limits of Cont./Discont. Conduction Mode (Constant $V_d$ )



# Step-Down DC-DC Converter: Limits of Cont./Discont. Conduction mode (Constant $V_o$ )



If output voltage is kept constant, 
$$I_{LB} = \frac{T_s V_o}{2L} (1 - D) \Rightarrow I_{LB, \max} = \frac{T_s V_o}{2L_s}$$

Duty ratio for a given current: 
$$D = \frac{V_o}{V_d} \left( \frac{I_o / I_{LB, \max}}{1 - (V_o / V_D)} \right)^{1/2}$$

# Step-Down Converter: Output Voltage Ripple

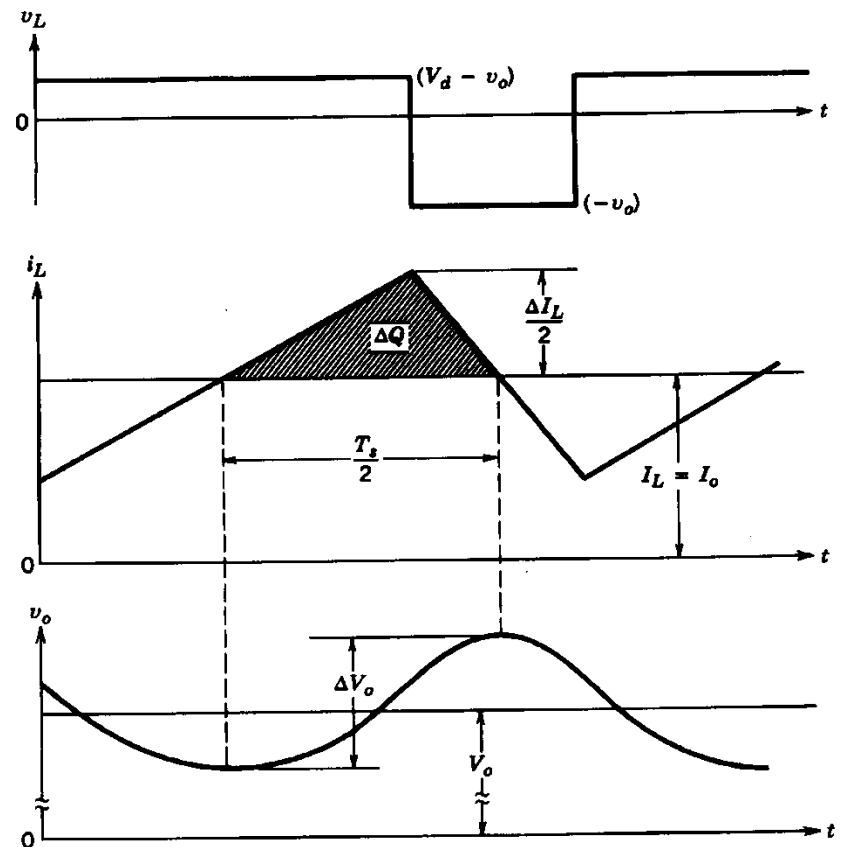
Consider continuous conduction mode.

Assume all the inductor ripple current flows through the capacitor (with the average current flows through the resistive load). Then

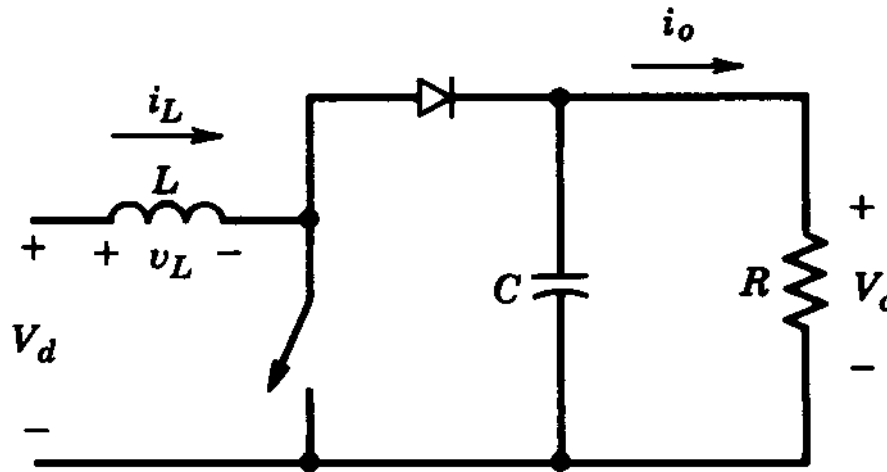
$$\Delta V_o = \frac{\Delta Q_c}{C} = \frac{1}{C} \left( \frac{1}{2} \frac{\Delta I_L T_s}{2} \right),$$

$$\Delta I_L = \frac{T_s V_o}{L} (1 - D),$$

$$\Rightarrow \frac{\Delta V_o}{V_o} = \frac{T_s^2}{8LC} (1 - D) = \frac{\pi^2}{2} (1 - D) \left( \frac{f_c}{f_s} \right)^2$$



# Step-Up (Boost) DC-DC Converter



- Output voltage must be greater than the input

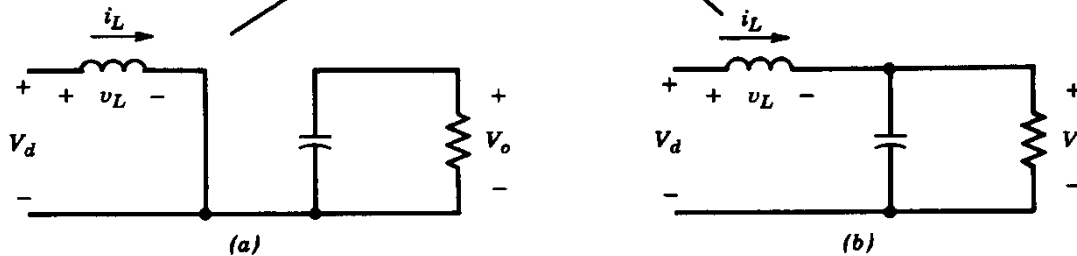
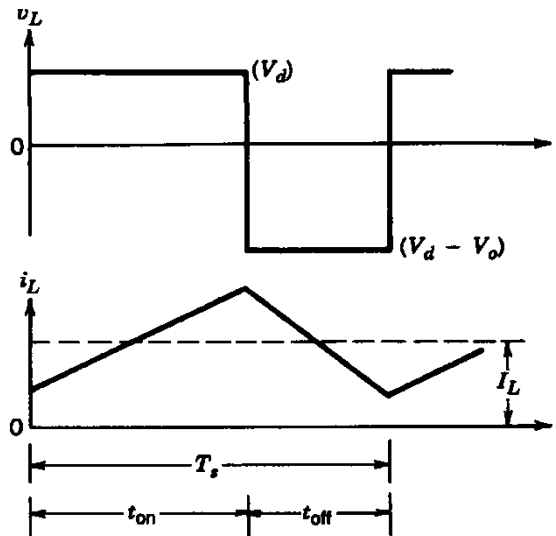
# Step-Up DC-DC Converter: Continuous Conduction

$$V_d t_{on} = (V_d - V_o) t_{off}$$

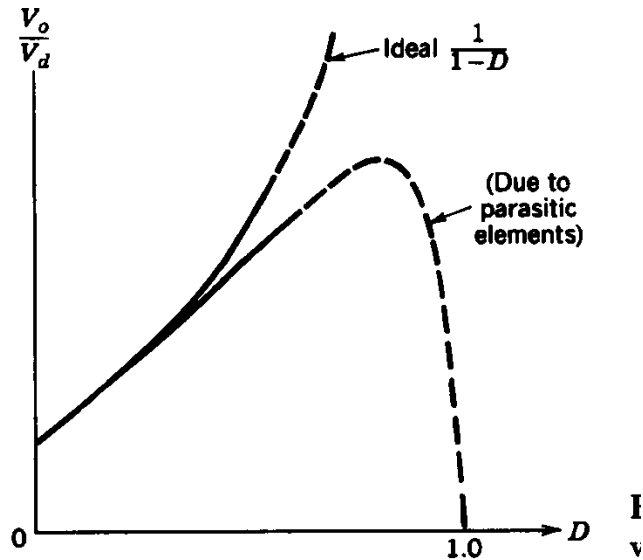
$$\Rightarrow V_o = V_d / (1 - D)$$

$$V_d I_d = V_o I_o$$

$$I_o = I_d (1 - D)$$



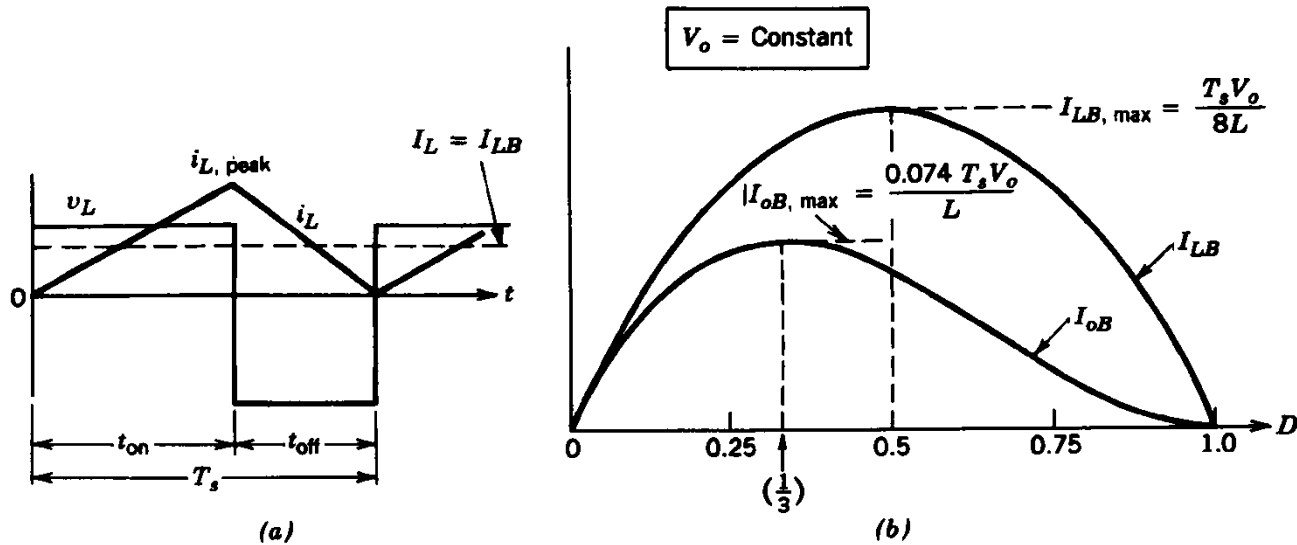
# Step-Up DC-DC Converter: Effect of Parasitics



Parasitic elements are due to losses in the inductor, capacitor, switch, and diode.

The duty-ratio is generally limited before the parasitic effects become significant.

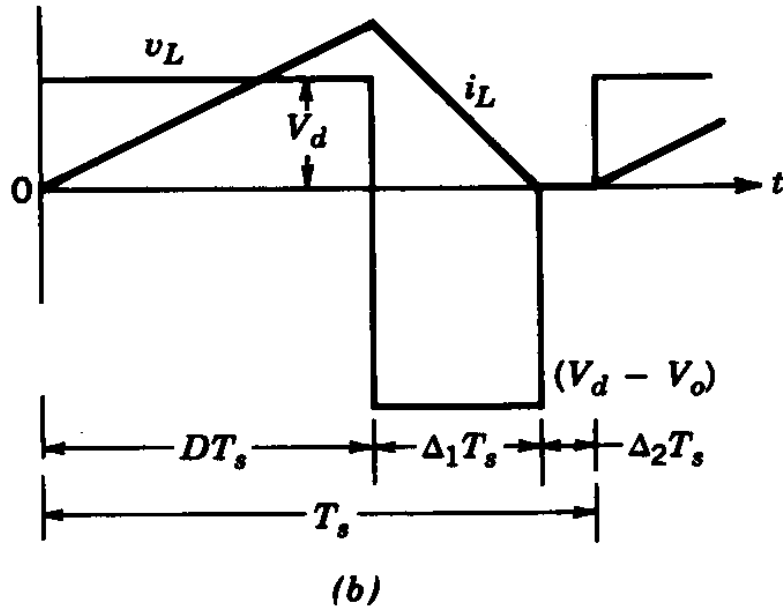
# Step-Up DC-DC Converter: Limits of Cont./Discont. Conduction



$$I_{LB} = I_{dB} = \frac{1}{2} i_{L, peak} = \frac{t_{on}}{2L} V_d = \frac{T_s V_o}{2L} D(1-D), \quad \Rightarrow I_{LB, max} = \frac{T_s V_o}{8L}$$

$$I_{oB} = (1-D)I_{dB} = \frac{T_s V_o}{2L} D(1-D)^2, \quad \Rightarrow I_{oB, max} = \frac{2}{27} \frac{T_s V_o}{L}$$

# Step-Up DC-DC Converter: Discontinuous Conduction (constant $V_o$ )



$$V_d DT_s + (V_d - V_o) \Delta_1 T_s = 0$$

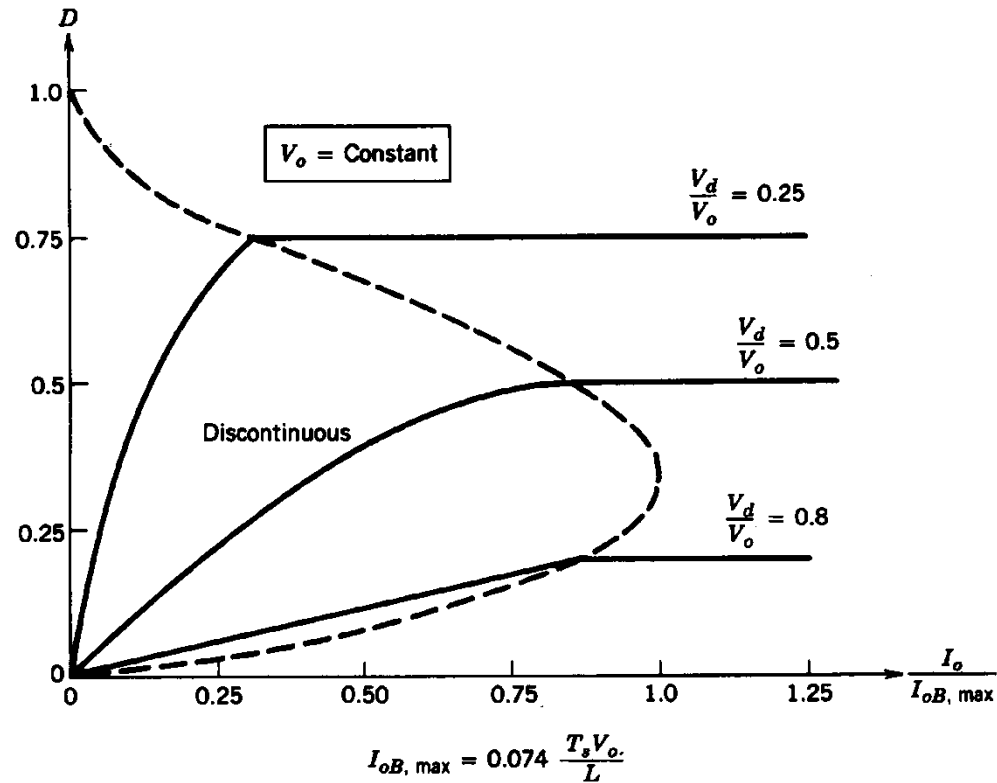
$$\Delta_1 = \frac{I_o}{D(T_s V_d / 2L)}$$

For constant  $V_o$  and variable  $V_d$ ,

$$D = \left( \frac{4}{27} \frac{V_o}{V_d} \left( \frac{V_o}{V_d} - 1 \right) \frac{I_o}{I_{oB, \max}} \right)^{1/2}$$



# Step-Up DC-DC Converter: Limits of Cont./Discont. Conduction (constant $V_o$ )



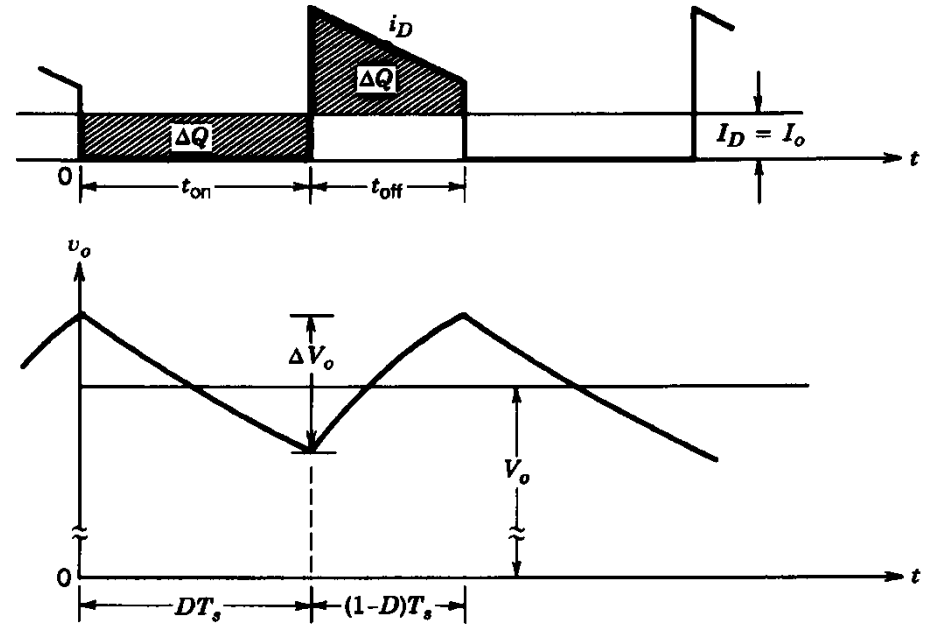
# Step-Up DC-DC Converter Output Ripple

Consider continuous conduction mode.

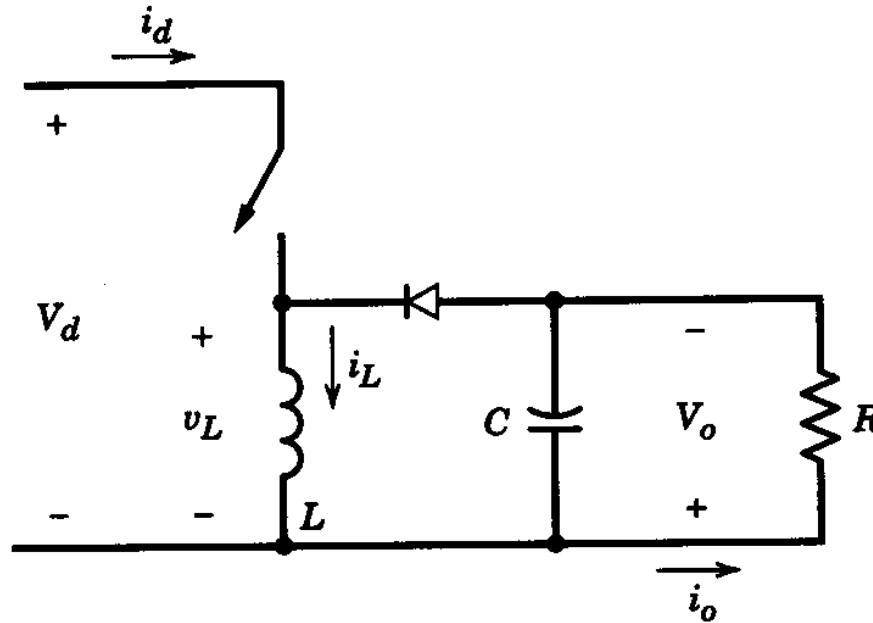
Assume all the inductor ripple current flows through the capacitor (with the average current flows through the resistive load). Then

$$\Delta V_o = \frac{\Delta Q_c}{C} = \frac{I_o DT_s}{C} = \frac{V_o}{R} \frac{DT_s}{C},$$

$$\Rightarrow \frac{\Delta V_o}{V_o} = \frac{DT_s}{RC} = D \frac{T_s}{\tau}$$

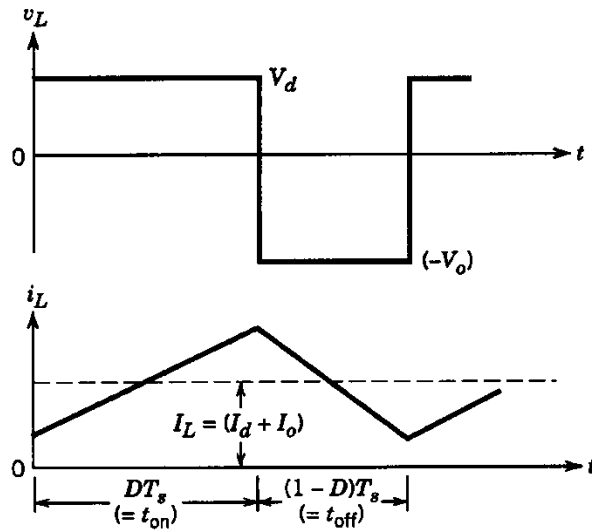


# Step-Down/Up (Buck/Boost) DC-DC Converter



- The output voltage can be higher or lower than the input voltage.
- Note the reverse polarity of the output voltage.

# Buck-Boost Converter: Cont. Conduction Mode

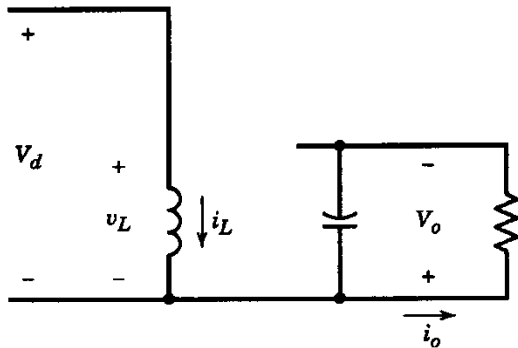


$$V_d t_{on} + (-V_o) t_{off} = 0$$

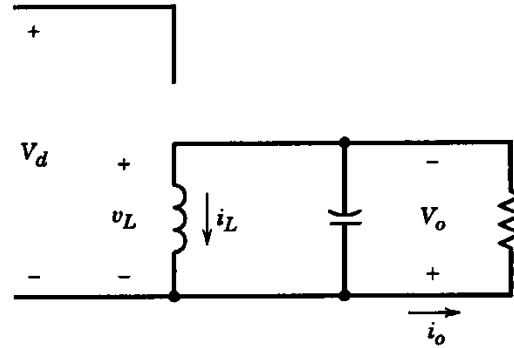
$$\Rightarrow V_o = V_d D / (1 - D)$$

$$V_d I_d = V_o I_o$$

$$I_o = I_d (1 - D) / D$$



(a)

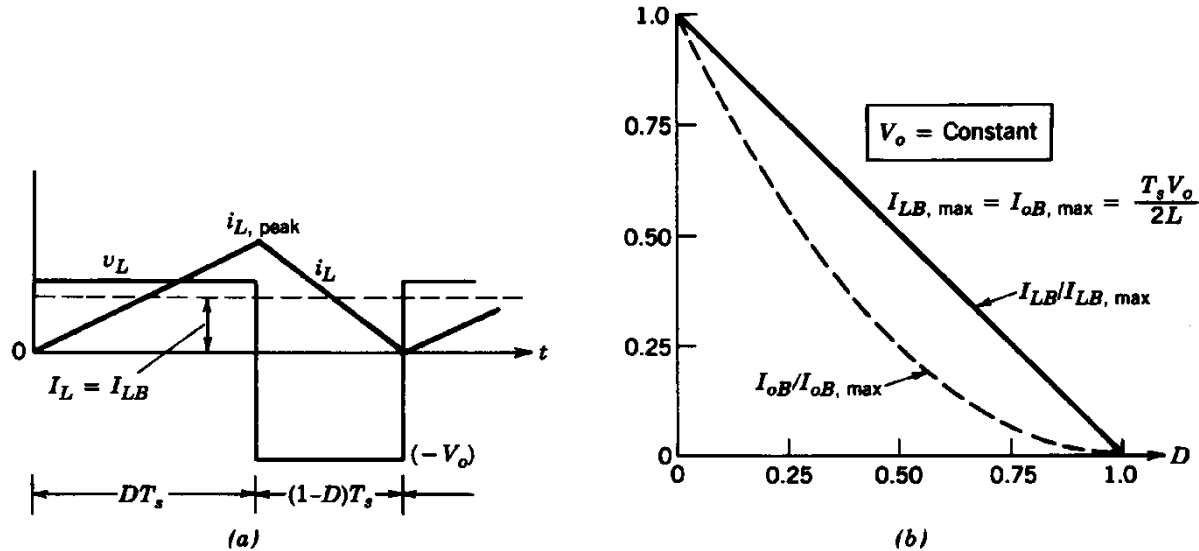


(b)

Note:

$$I_L = I_d + I_o$$

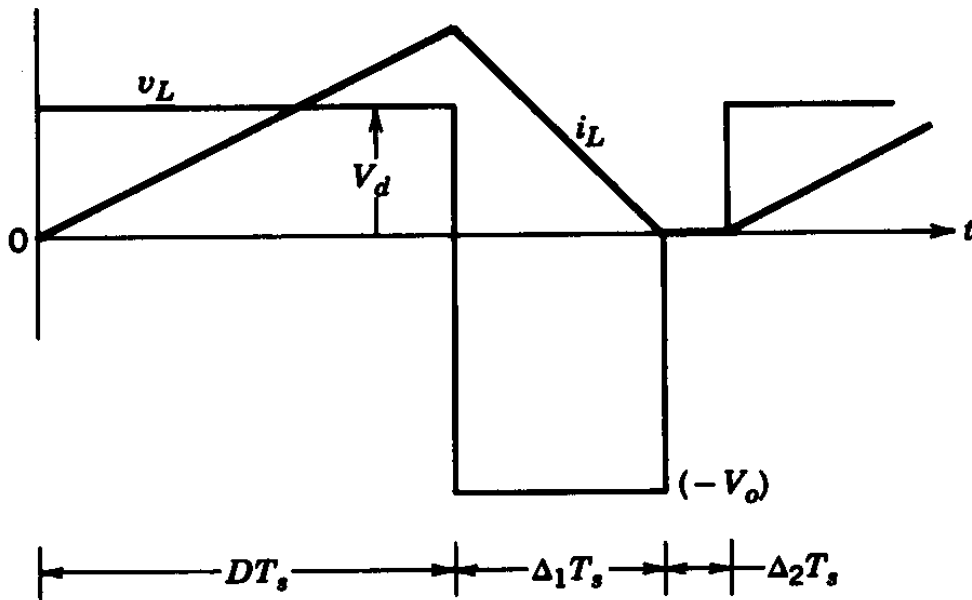
# Buck-Boost Converter: Limits of Cont./Discont. Conduction



$$I_{LB} = \frac{T_s V_o}{2L} (1-D), \quad \Rightarrow \quad I_{LB, \text{max}} = \frac{T_s V_o}{2L}$$

$$I_{oB} = \frac{T_s V_o}{2L} (1-D)^2, \quad \Rightarrow \quad I_{oB, \text{max}} = \frac{T_s V_o}{2L}$$

# Buck-Boost Converter: Disc. Conduction Mode



$$V_d DT_s + (-V_o) \Delta_1 T_s = 0$$

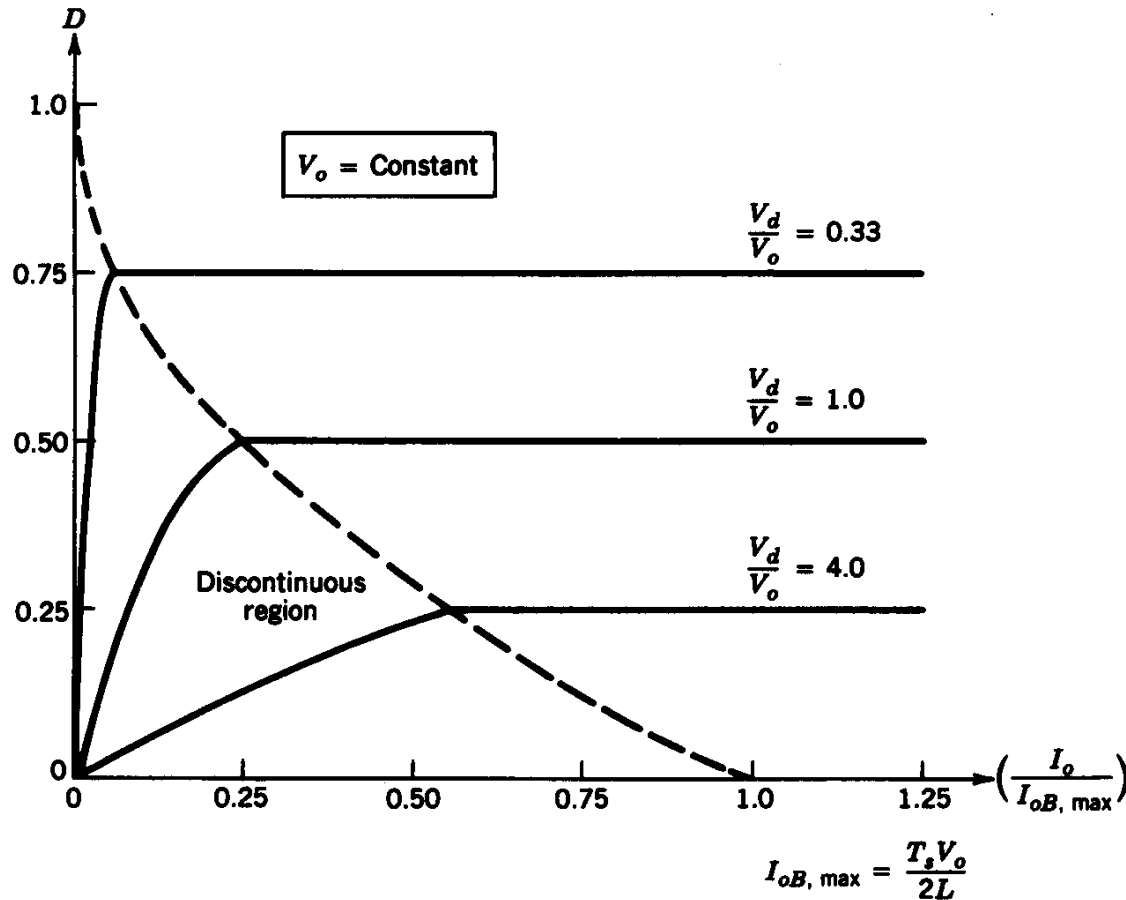
$$\Delta_1 = \frac{DI_o}{I_d}$$

$$I_L = \frac{V_d}{2L} DT_s (D + \Delta_1)$$

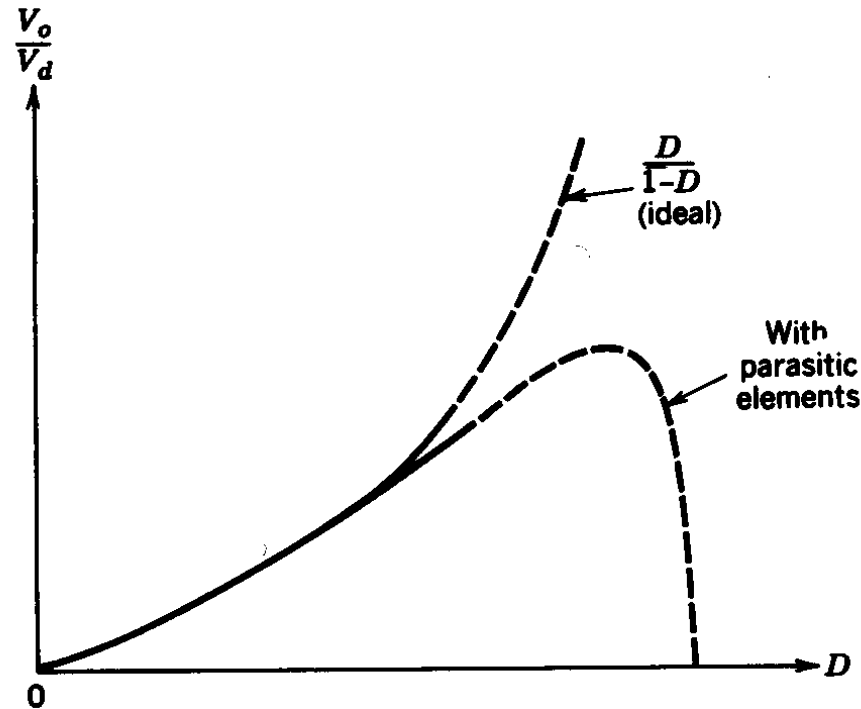
For constant  $V_o$  and variable  $V_d$ ,

$$D = \frac{V_o}{V_d} \left( \frac{I_o}{I_{oB,\max}} \right)^{1/2}$$

# Buck-Boost Converter: Limits of Cont./Discont. Conduction



# Buck-Boost Converter: Effect of Parasitics

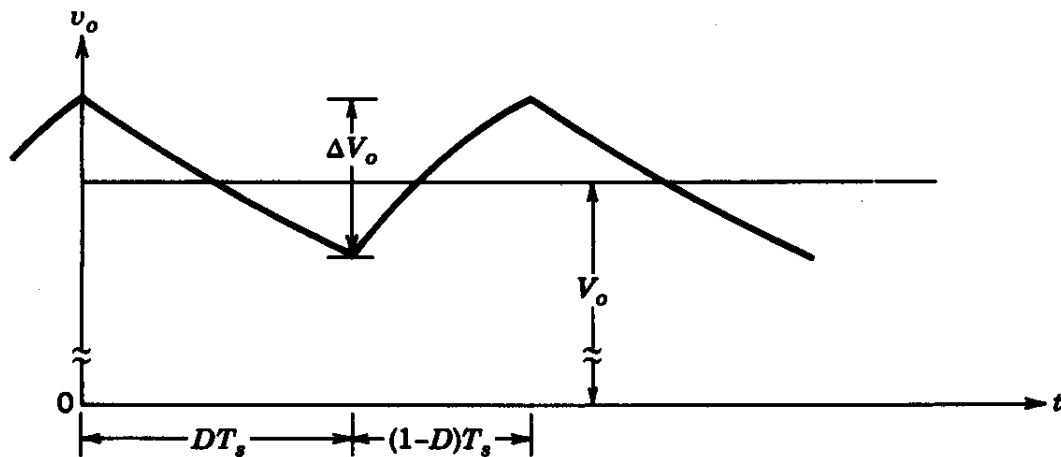
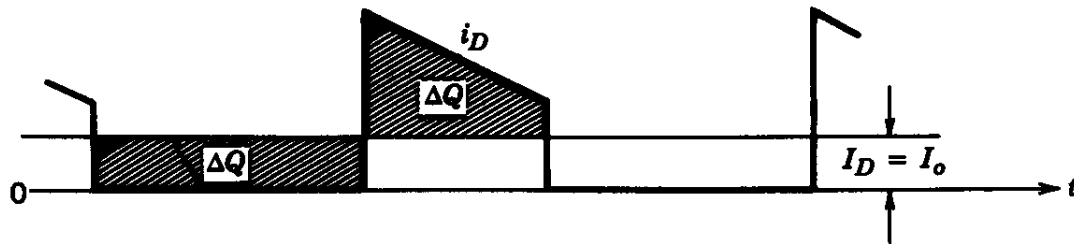


In practice, the duty-ratio is limited to avoid these parasitic effects from becoming significant.



# Buck-Boost Converter: Output Voltage Ripple

Assume all the ripple current component of the diode current flows through the capacitor and its average value flows through the load resistor.



$$\Delta V_o = \frac{\Delta Q_c}{C} = \frac{I_o DT_s}{C} = \frac{V_o}{R} \frac{DT_s}{C},$$

$$\Rightarrow \frac{\Delta V_o}{V_o} = \frac{DT_s}{RC} = D \frac{T_s}{\tau}$$

# Cuk DC-DC Converter (Buck-Boost Converter)

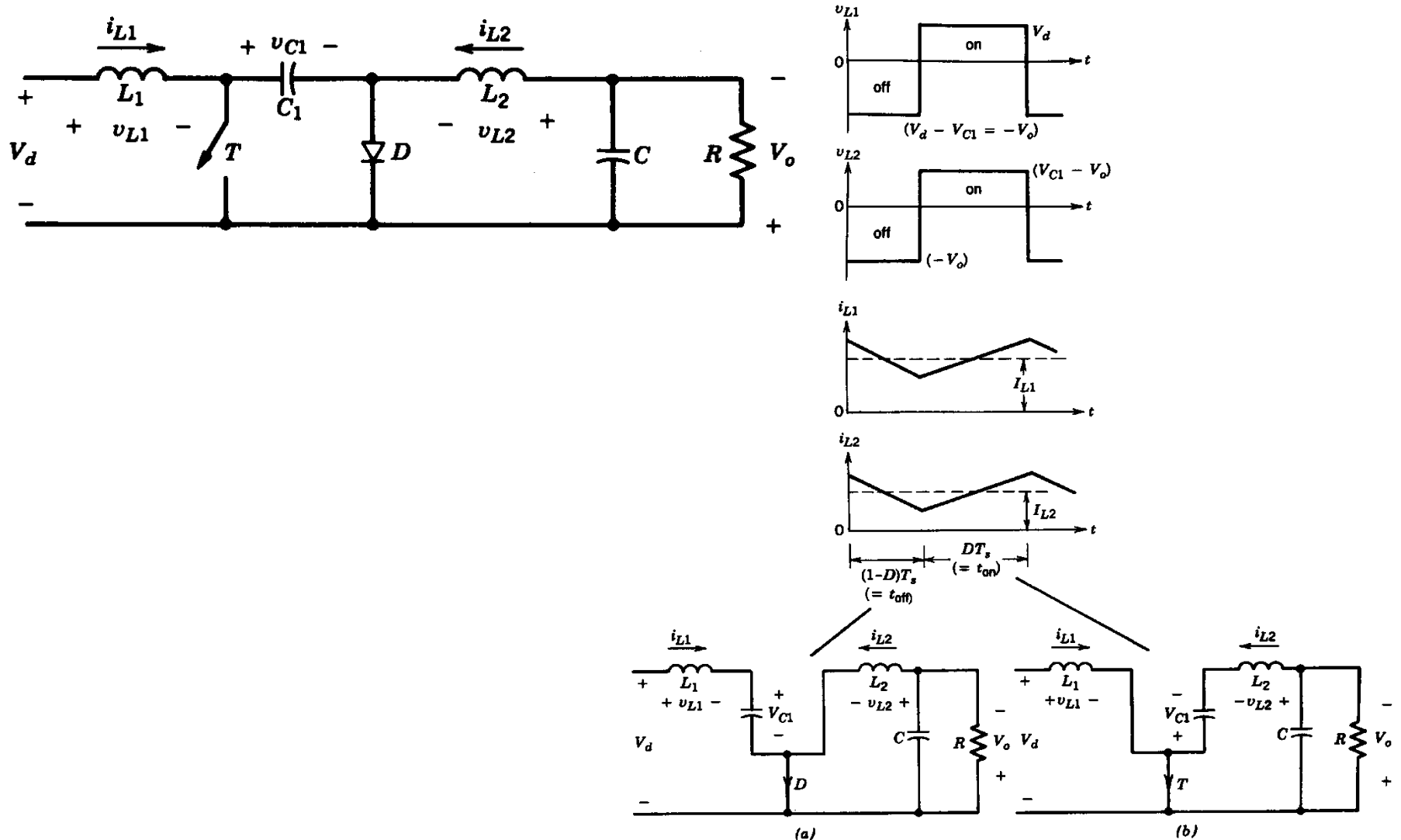
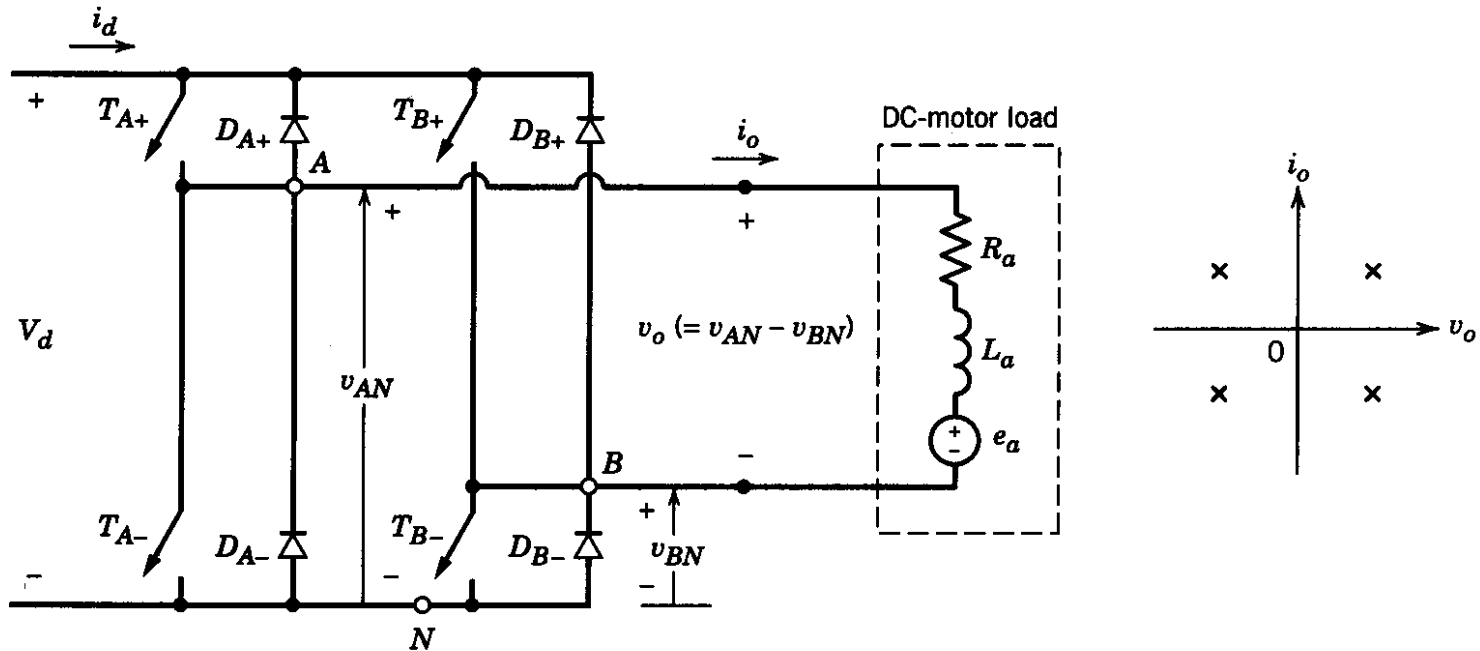


Figure 7-26 Cúk converter waveforms: (a) switch off; (b) switch on.

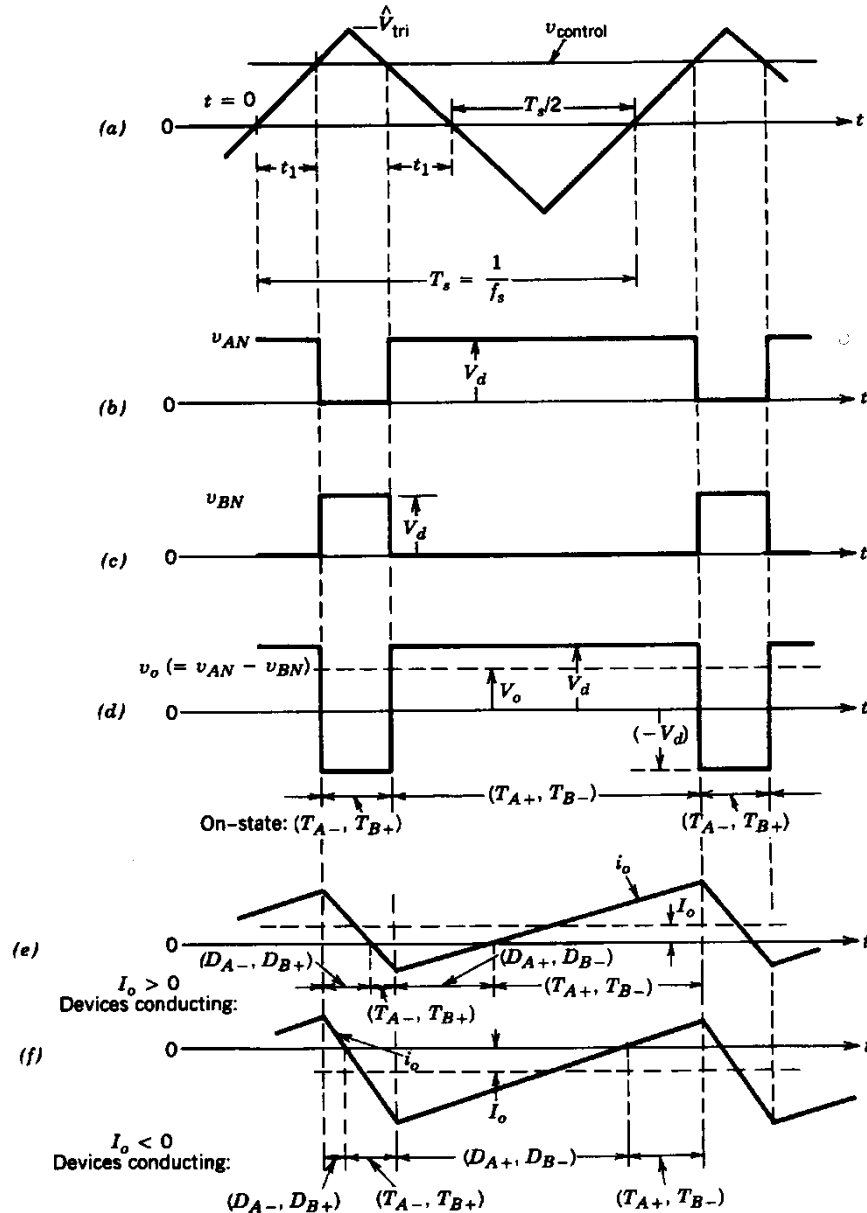
# Full-Bridge DC-DC Converter: Possible Operation in all Four Quadrants



Applications:

- DC Motor Drives
- DC-AC Conversion at power frequency (UPS)
- DC-AC Conversion at high frequency (switch-mode power supplies)

# Converter Waveforms: PWM with Bi-polar Voltage Switching



← triangular (rather than saw-tooth) waveform is compared to  $v_{control}$ .

When  $v_{control} > v_{tri}$ ,  $TA+$ ,  $TB-$  are switched ON, and  $(TB+, TA-)$  are switched OFF.

Duty cycle of  $(TA+, TB-)$ :  $D_1 = t_{on}/T_s = 0.5(1 + v_{control}/V_{tri})$

Duty cycle of  $(TB+, TA-)$ :  $D_2 = (1 - D_1)$

$V_o = V_{AN} - V_{BN} = D_1 V_d - D_2 V_d = (2 D_1 - 1) V_d = k \cdot v_{control}$

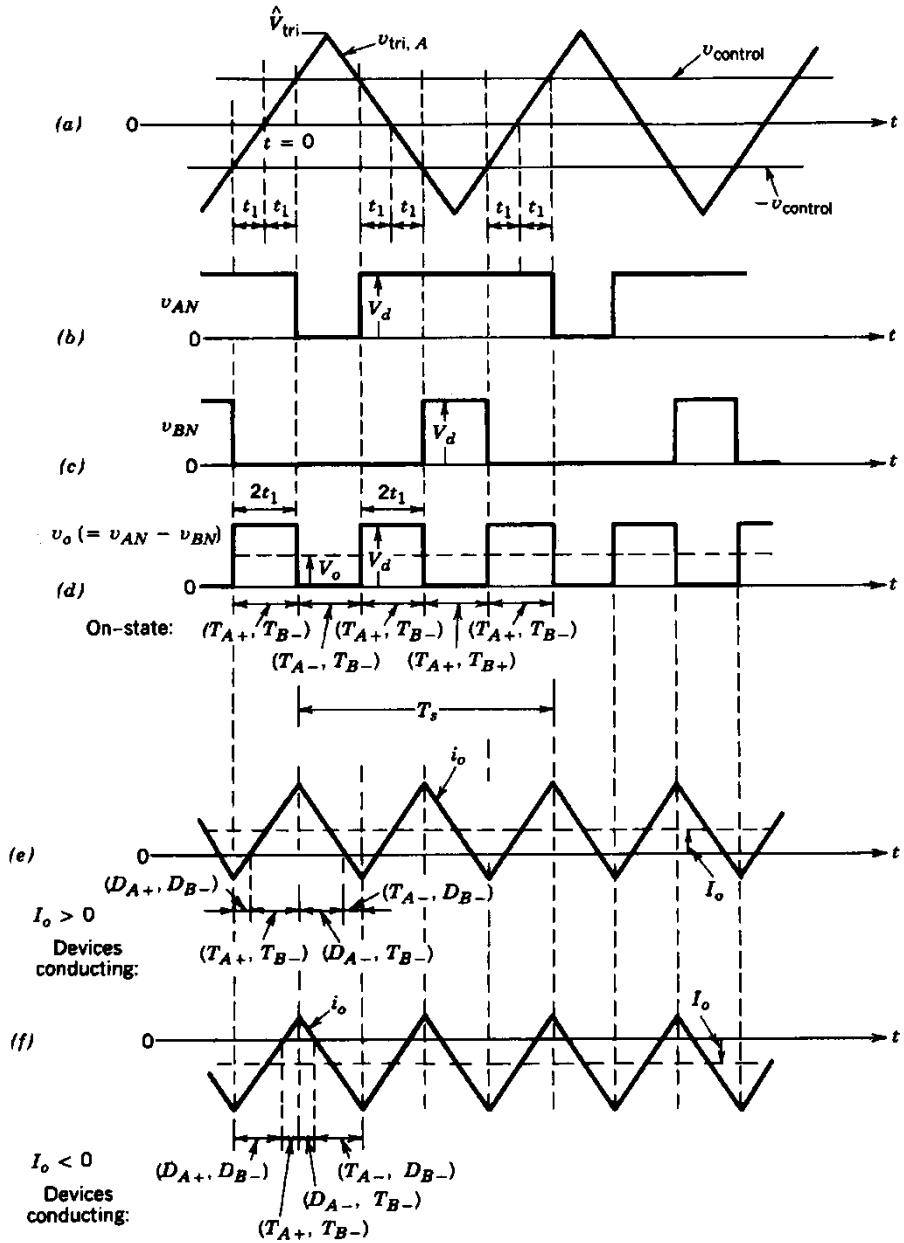
where  $k = V_d/V_{tri}$

As  $D_1: 0 \rightarrow 1$ ,  $V_o: -V_d \rightarrow V_d$ .

← Case where the average power flows from  $V_d$  to  $V_o$ .

← Case where the average power flows from  $V_o$  to  $V_d$ .

# Converter Waveforms: Uni-polar Voltage Switching



← triangular waveform is compared to  $v_{control}$  and  $-v_{control}$ .

When  $v_{control} > v_{tri}$ , TA+, TB- are switched ON. When  $-v_{control} > v_{tri}$ , TB+, TA- are switched ON.

Duty cycle of (TA+, TB-):  $D_1 = t_{on}/T_s$   
 Duty cycle of (TB+, TA-):  $D_2 = (1 - D_1)$

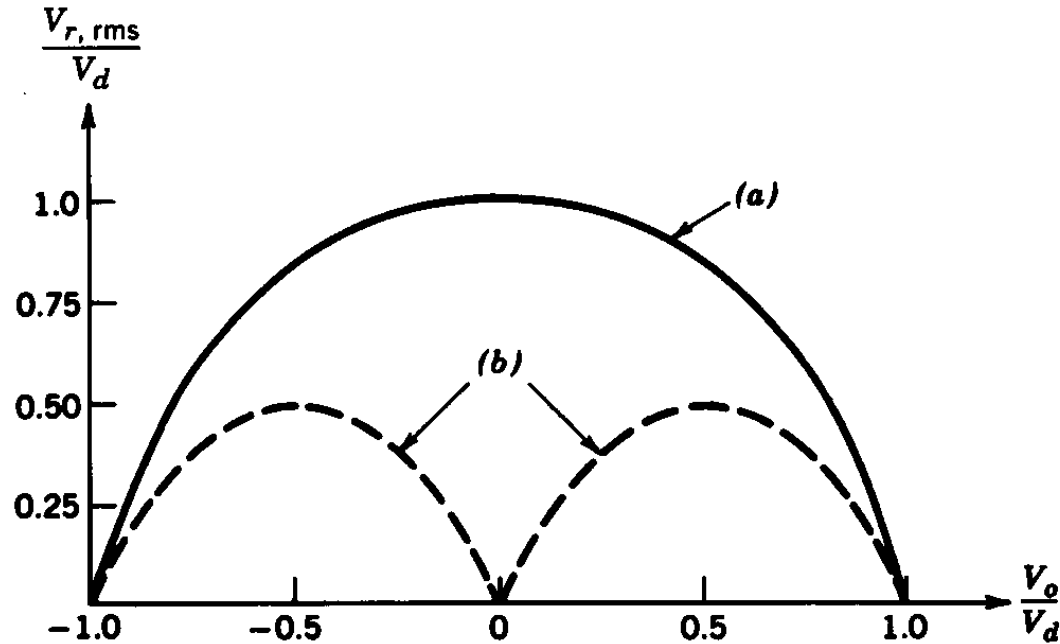
$$V_o = V_{AN} - V_{BN} = D_1 V_d - D_2 V_d = (2 D_1 - 1) V_d = k \cdot v_{control}$$

where  $k = V_d/V_{tri}$

← Case where the average power flows from  $V_d$  to  $V_o$ .

← Case where the average power flows from  $V_o$  to  $V_d$ .

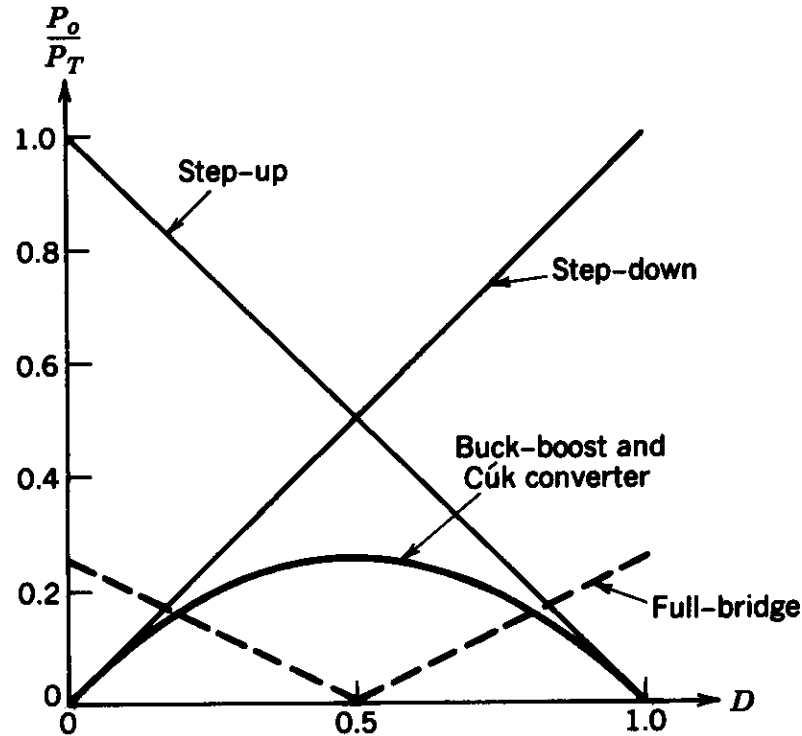
# Output Ripple in Full Bridge Converter



(a) Bipolar switching: 
$$V_{r,rms} = \sqrt{V_{o,rms}^2 - V_o^2} = 2V_d \sqrt{D_1 - D_1^2}$$

(b) Unipolar switching: 
$$V_{r,rms} = \sqrt{V_{o,rms}^2 - V_o^2} = \sqrt{6D_1 - 4D_1^2 - 2V_d}$$

# Switch Utilization in DC-DC Converters



Switch Utilization Ratio:  $P_o / P_T = (V_o I_o) / (V_T I_T)$

where  $V_T$  and  $I_T$  are the peak voltage and current ratings of the switch.