Review of Basic Electrical and Magnetic Circuit Concepts

EE 442-642

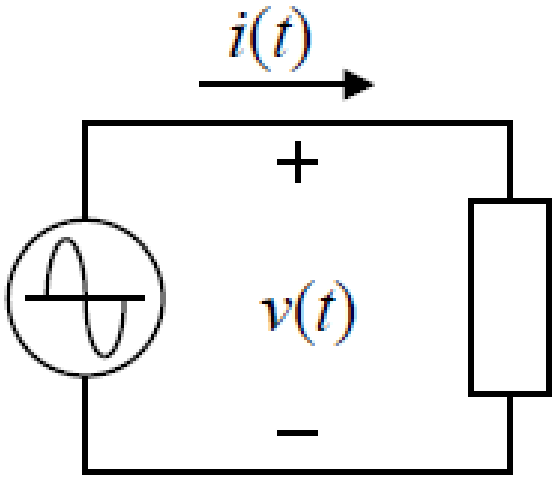
Sinusoidal Linear Circuits:

Instantaneous voltage, current and power, rms values

$$v(t) = V_m \cos(\omega t + \theta_v)$$

$$i(t) = I_m \cos(\omega t + \theta_i)$$

$$p(t) = v(t)\, i(t)$$



$$\theta = \theta_v - \theta_i \quad V_m = \sqrt{2}\,|V| \quad I_m = \sqrt{2}\,|I|$$

$$p(t) = \underbrace{|V|\,|I|\cos\theta\,\{1 + \cos 2(\omega t + \theta_v)\}}_{\text{energy flow into the circuit}} + \underbrace{|V|\,|I|\sin\theta\sin 2(\omega t + \theta_v)}_{\text{energy borrowed and returned by the circuit}}$$

Average (real) power, reactive power, apparent power, power factor

$$p(t) = |V||I|\{1+\cos 2(\omega t+\theta_v)\}\cos\theta + |V||I|\sin 2(\omega t+\theta_v)\sin\theta$$

$$p_R(t) = |V||I|\{1+\cos 2(\omega t+\theta_v)\}\cos\theta = \overline{P}\{1+\cos 2(\omega t+\theta_v)\}$$

$$p_X(t) = |V||I|\sin 2(\omega t+\theta_v)\sin\theta = S\sin\theta\sin 2(\omega t+\theta_v)$$

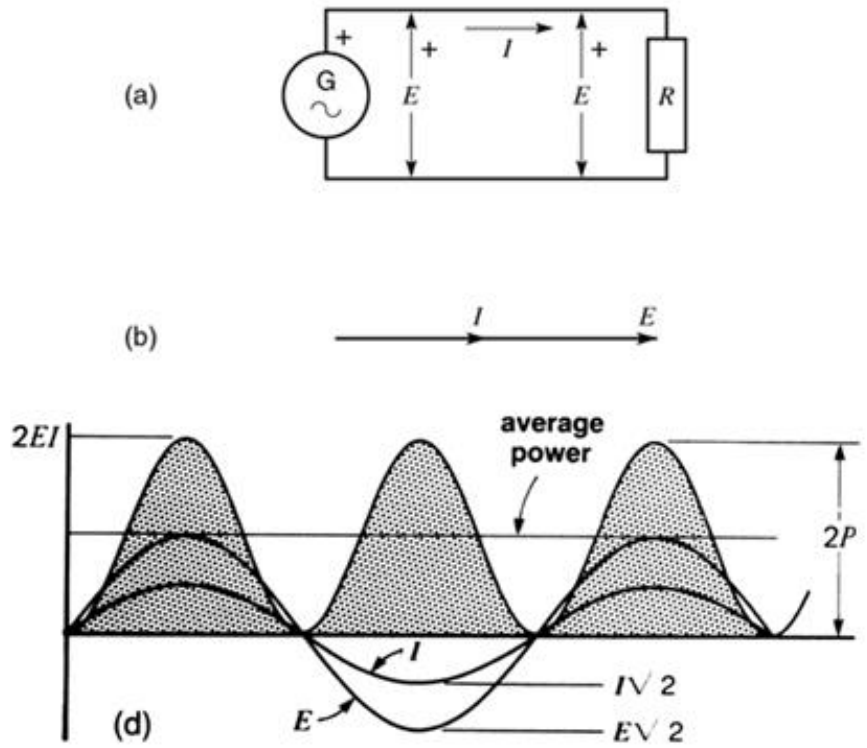
$$\overline{P} = |V||I|\cos\theta$$

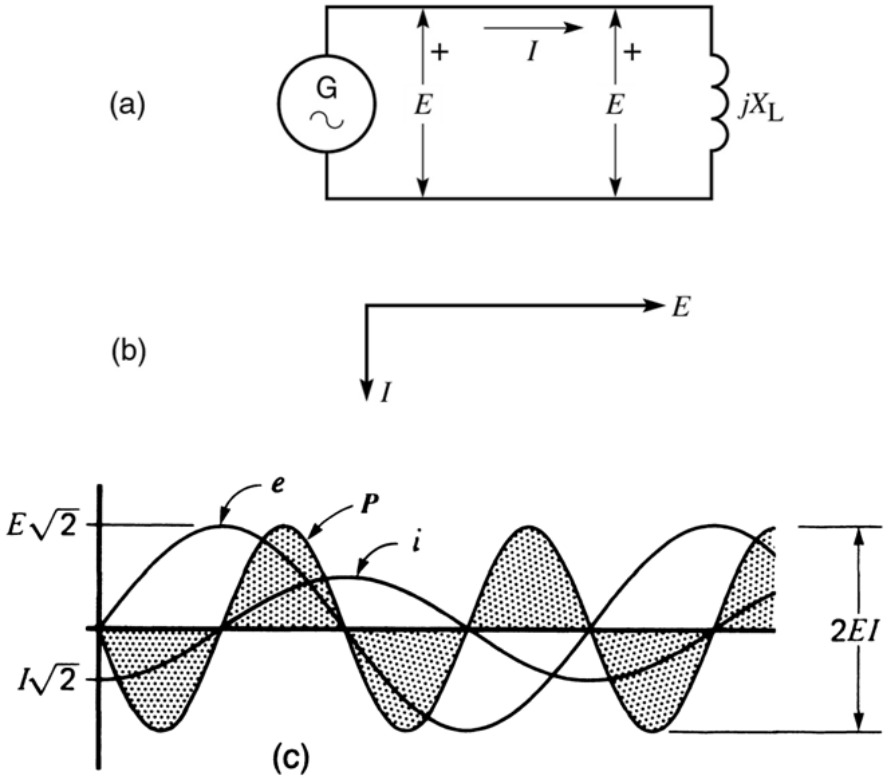
$$S = |V||I|$$

$$Q \equiv S\sin\theta = |V||I|\sin\theta$$

$$pf = \cos\theta = \frac{\overline{P}}{|V||I|}$$

Instantaneous power in pure resistive and inductive circuits





Phasor notation, impedance and admittance

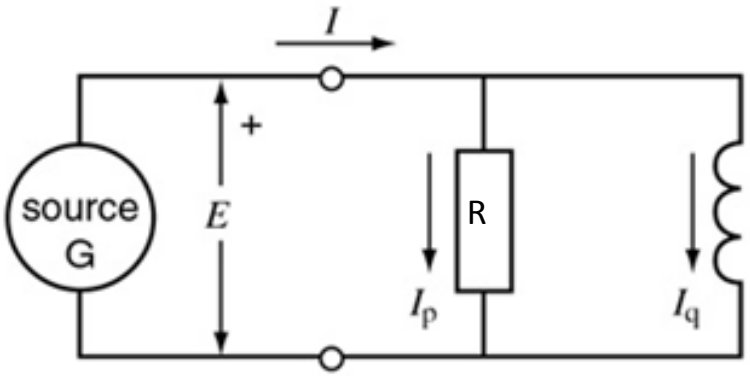
Transformation of a sinusoidal signal to and from the time domain to the phasor domain:

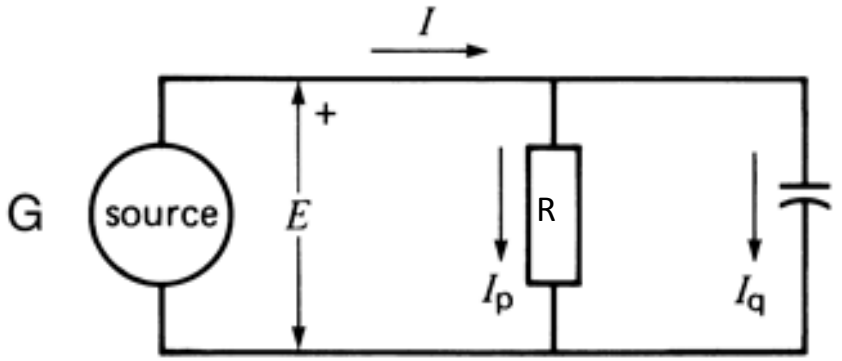
$$v(t) = \sqrt{2}|V|\cos(\omega t + \theta_v) \longleftrightarrow V = |V|\angle\theta_v$$

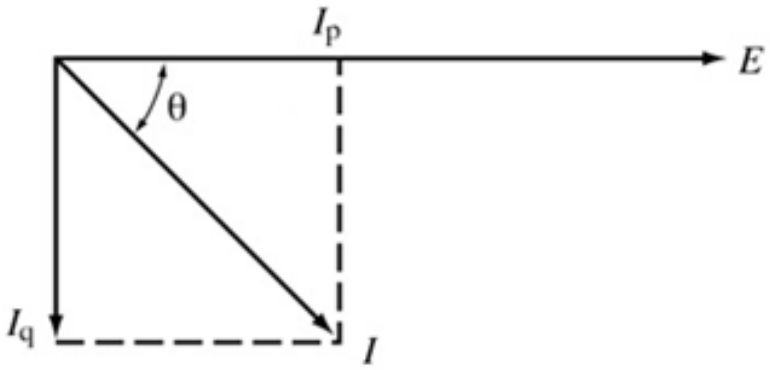
(time domain) (phasor domain)

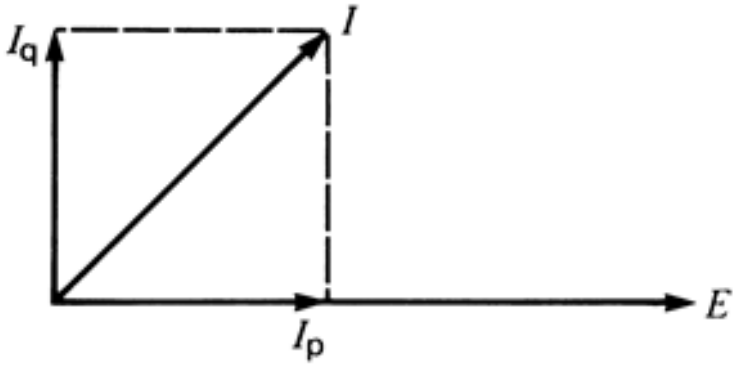
Element	Impedance	Admittance
R	$Z = R$	$Y = \frac{1}{R}$
L	$Z = j\omega L$	$Y = \frac{1}{j\omega L}$
C	$Z = -j\frac{1}{\omega C}$	$Y = j\omega C$

Resistive-Inductive, resistive-capacitive Load

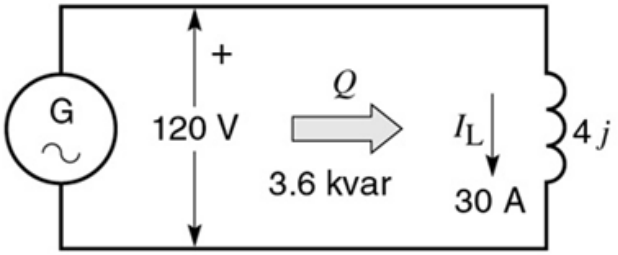


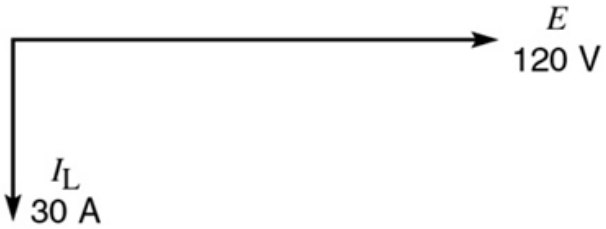


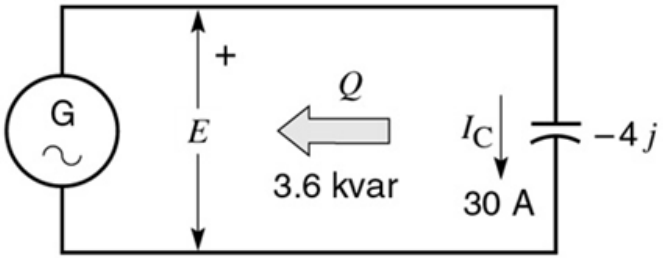


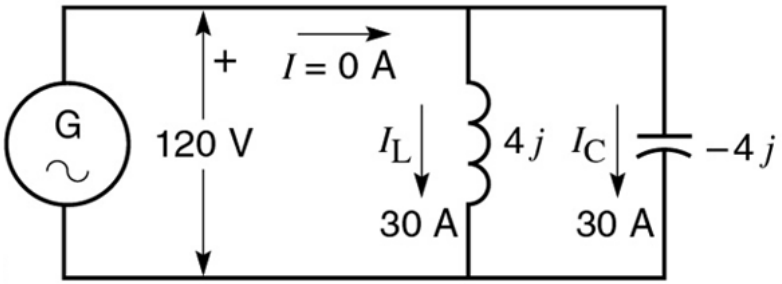


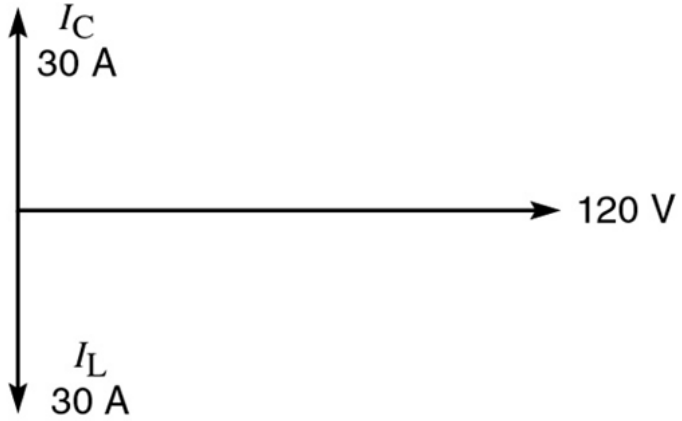
Power in inductive and capacitive circuits









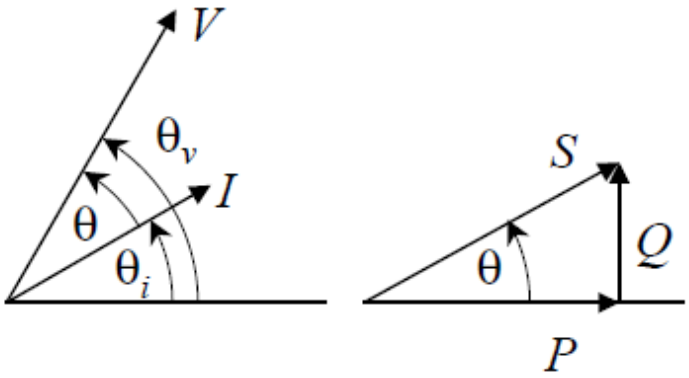


Complex Power, power triangle

$$V\,I^{*} = |V|\,|I|\angle(\theta_v - \theta_i) = |V|\,|I|\angle\theta = S$$

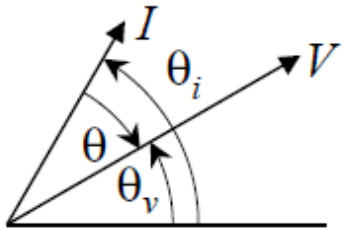
$$S = |V|\,|I|\cos\theta + j\,|V|\,|I|\sin\theta = P + jQ$$

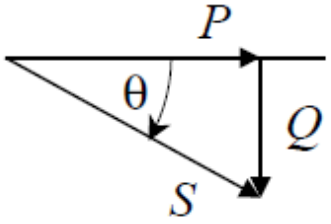
$$|S| = \sqrt{P^2 + Q^2}$$



Lagging Power Factor

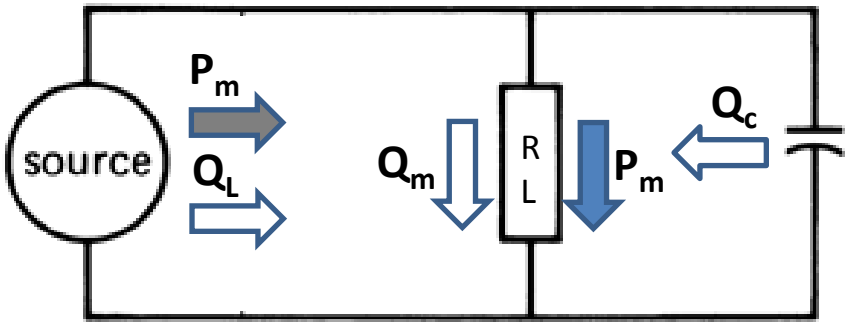
Leading Power Factor

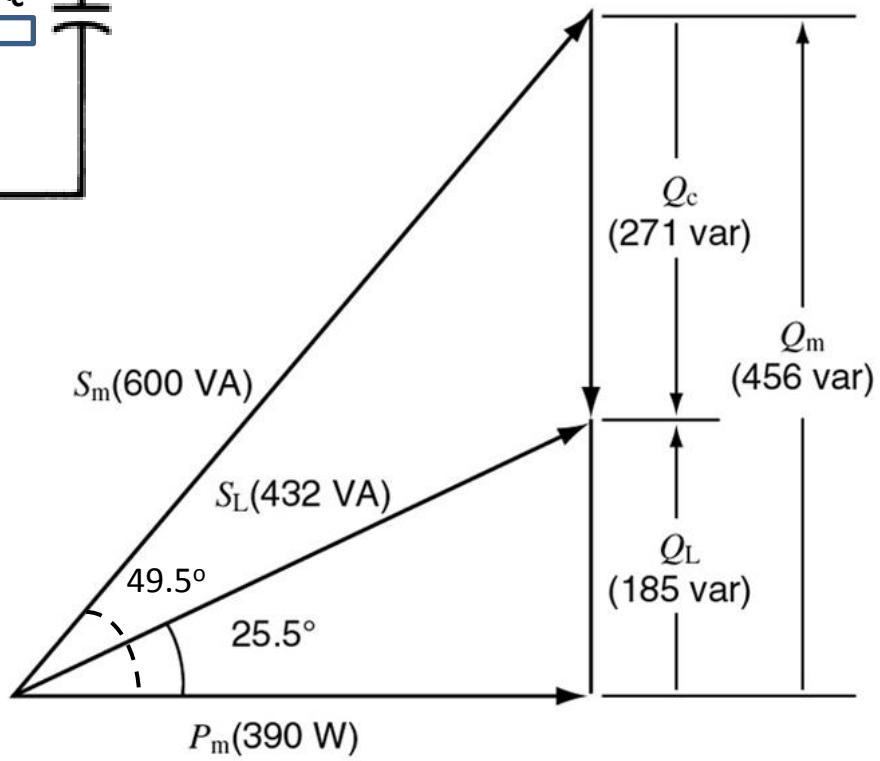




Example: Power Factor Correction

The power triangle below shows that the power factor is corrected by a shunt capacitor from 65% to 90% (lag).





Conservation of power

- At every node (bus) in the system,
  - the sum of real powers entering the node must be equal to the sum of real powers leaving that node.
  - The same applies for reactive power,
  - The same applies for complex power
  - The same **does not apply** for apparent power

- The above is a direct consequence of Kirchhoff's current law, which states that the sum of the currents flowing into a node must equal the sum of the currents flowing out of that node.

Balanced 3 Phase Circuits

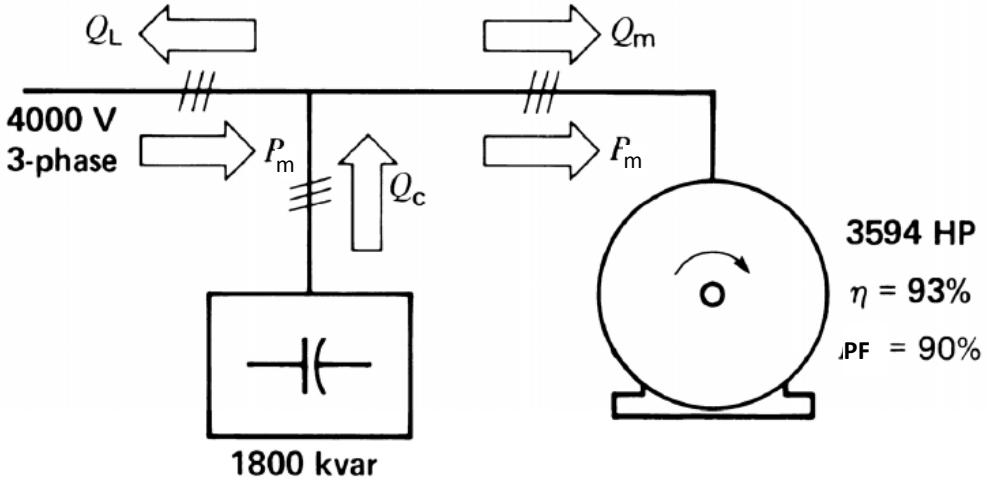
- Bulk power systems are almost exclusively 3-phase. Single phase is used primarily only in low voltage, low power settings, such as residential and some commercial customers.
- Some advantages of three-phase system:
  - Can transmit more power for the same amount of wire (twice as much as single phase)
  - Torque produced by 3ϕ machines is constant, easy start.
  - Three phase machines use less material for same power rating
- Real, reactive and complex power in balanced 3-phase circuits

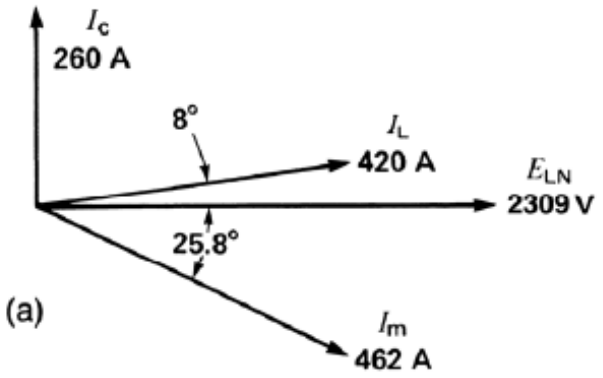
$$P_{3\phi} = 3\left|V_p\right|\left|I_p\right|\cos\theta = \sqrt{3}\left|V_{LL}\right|\left|I_L\right|\cos\theta$$

$$Q_{3\phi} = 3\left|V_p\right|\left|I_p\right|\sin\theta = \sqrt{3}\left|V_{LL}\right|\left|I_L\right|\sin\theta$$

$$S_{3\phi} = 3\,V_p\,I_p^* = \sqrt{3}\,V_{LL}\,I_L^*$$

Example: power factor correction In three-phase circuit



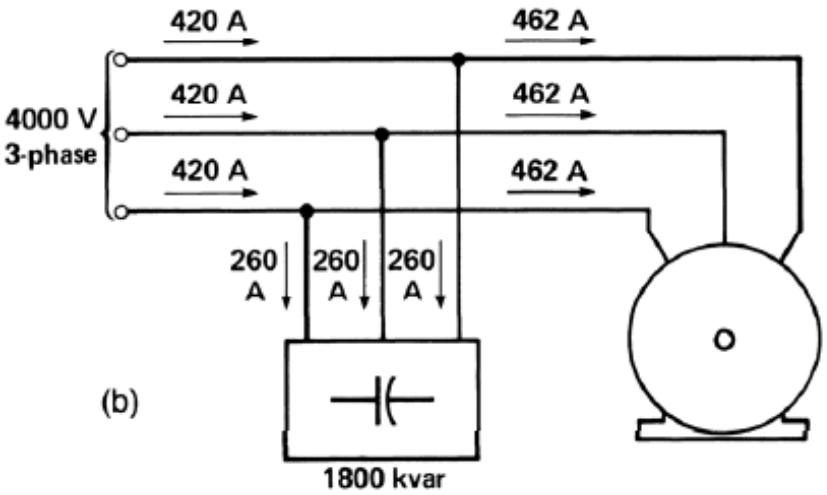


P_m = √3x4x0.462xcos(25.8°)= 2.88 MW

Q_m = √3x4x0.462xsin(25.8°)= 1.39 MVAR

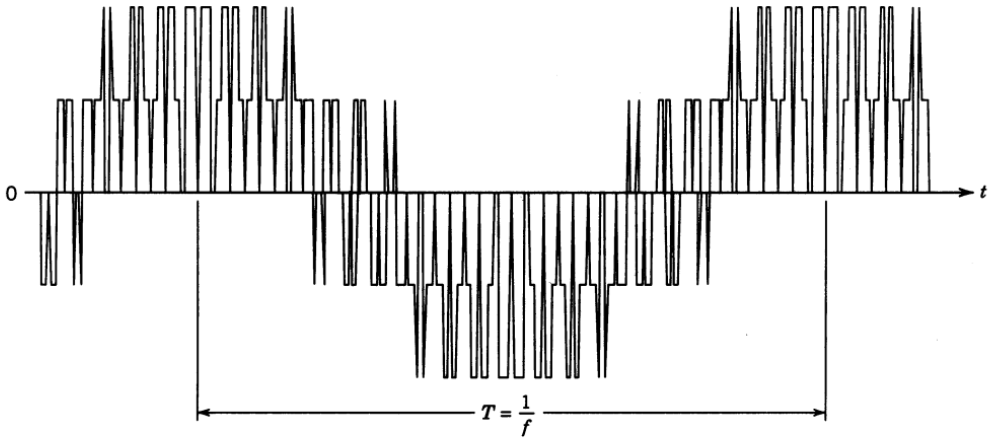
Q_c = 1.8 MVAR

$Q_L = Q_m - Q_c$ = - 0.41 MVAR

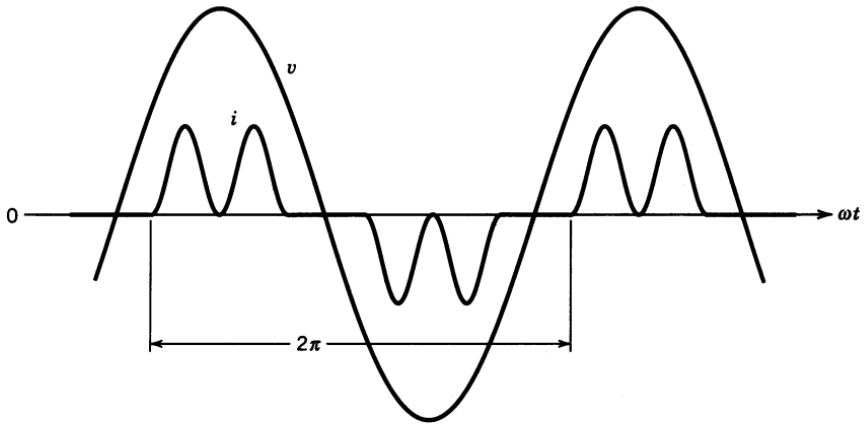


Power electronic circuits are non-linear

- Periodic waveforms but often not sinusoidal → analytical expressions in terms of Fourier components



(a)



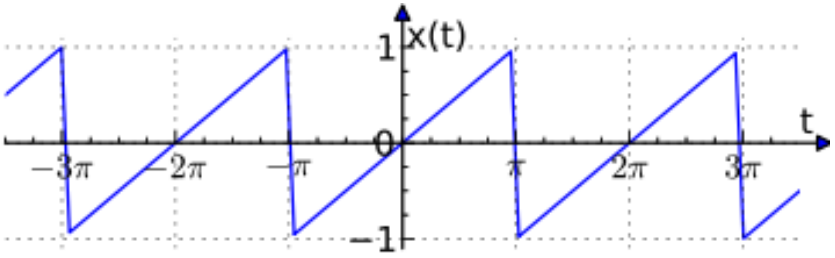
(b)

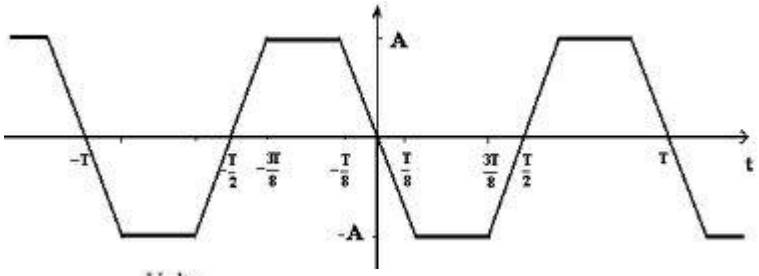
Fourier Analysis

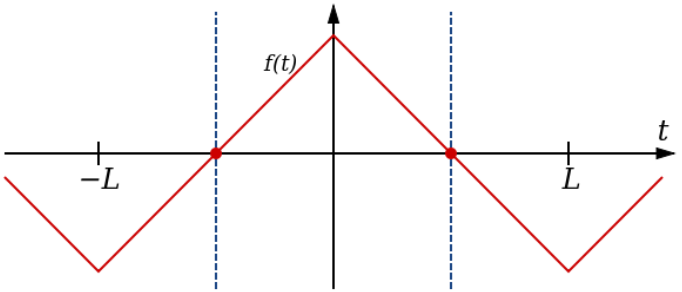
Table 3-1 Use of Symmetry in Fourier Analysis

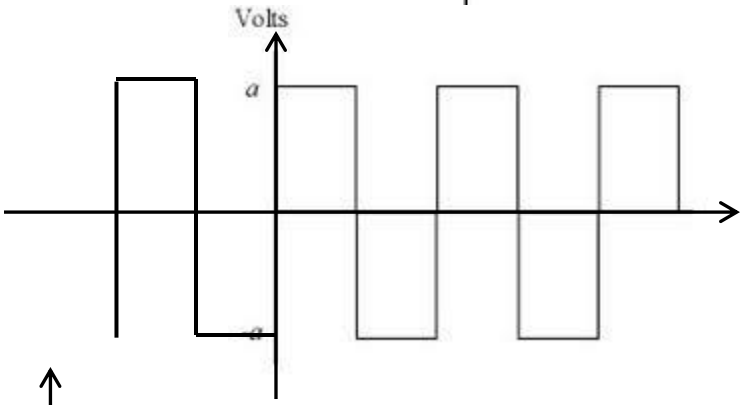
Symmetry	*Condition Required*	a_h and b_h
Even	$f(-t) = f(t)$	$b_h = 0 \quad a_h = \frac{2}{\pi}\int_0^{\pi} f(t)\cos(h\omega t)\, d(\omega t)$
Odd	$f(-t) = -f(t)$	$a_h = 0 \quad b_h = \frac{2}{\pi}\int_0^{\pi} f(t)\sin(h\omega t)\, d(\omega t)$
Half-wave	$f(t) = -f(t + \frac{1}{2}T)$	$a_h = b_h = 0$ for even h $a_h = \frac{2}{\pi}\int_0^{\pi} f(t)\cos(h\omega t)\, d(\omega t)$ for odd h $b_h = \frac{2}{\pi}\int_0^{\pi} f(t)\sin(h\omega t)\, d(\omega t)$ for odd h
Even quarter-wave	Even and half-wave	$b_h = 0$ for all h $a_h = \begin{cases} \frac{4}{\pi}\int_0^{\pi/2} f(t)\cos(h\omega t)\, d(\omega t) & \text{for odd } h \\ 0 & \text{for even } h \end{cases}$
Odd quarter-wave	Odd and half-wave	$a_h = 0$ for all h $b_h = \begin{cases} \frac{4}{\pi}\int_0^{\pi/2} f(t)\sin(h\omega t)\, d(\omega t) & \text{for odd } h \\ 0 & \text{for even } h \end{cases}$

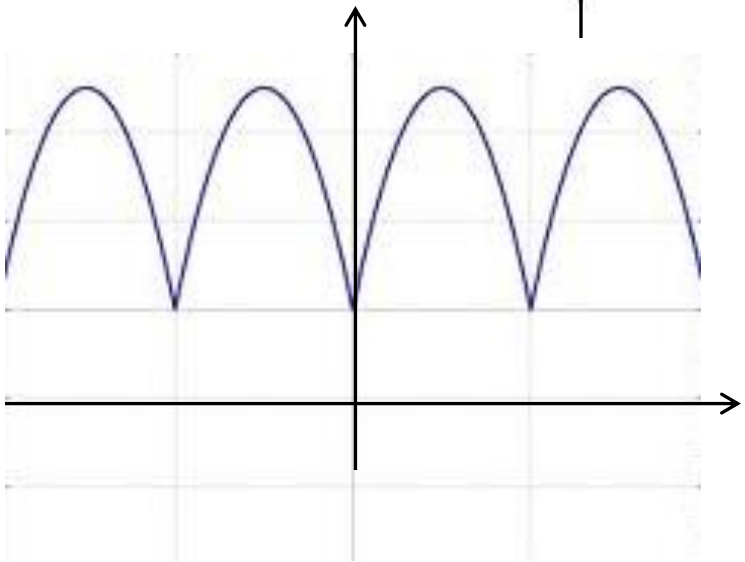
Example of simple non-sinusoidal periodic signals











Current decomposition

- Current decomposition of into fundamental (i_{s1}) and distortion current (i_{dis}):

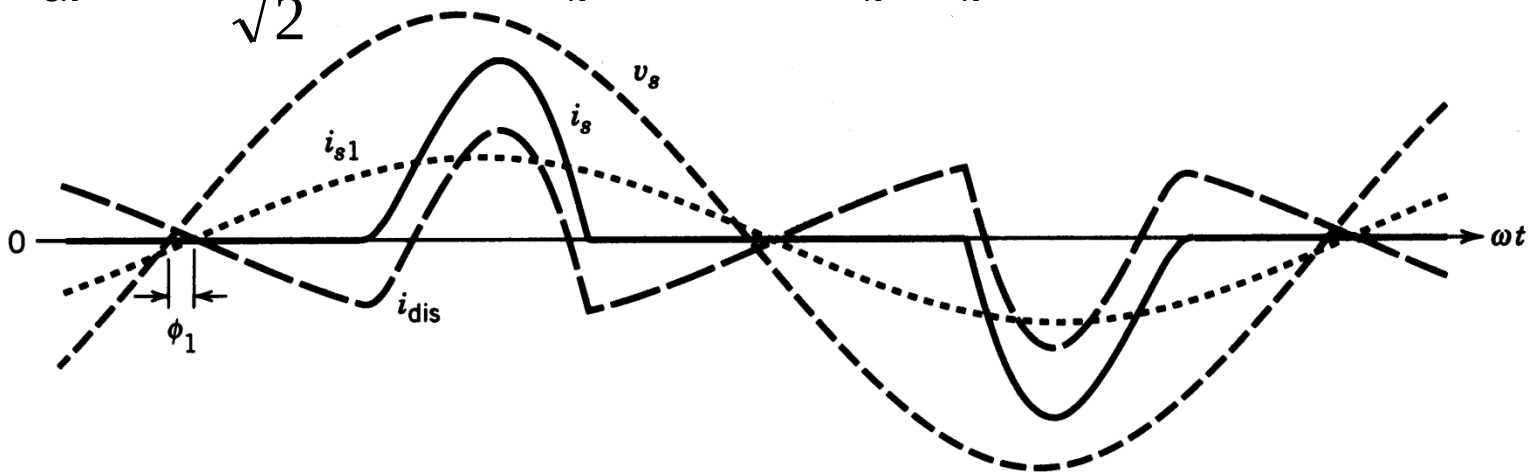
$$i_s(t) = i_{s1}(t) + \sum_{h \neq 1} i_{sh}(t) = i_{s1}(t) + i_{dis}(t)$$

where

$$i_{sh}(t) = a_h \cos(h\omega_1 t) + b_h \sin(h\omega_1 t) = \sqrt{2} I_{sh} \cos(h\omega_1 t - \theta_h)$$

Herein,

$$I_{sh} = \frac{\sqrt{a_h^2 + b_h^2}}{\sqrt{2}}, \qquad \theta_h = \arctan(b_h / a_h)$$



RMS Value and Total Harmonic Distortion

- The rms value of a distorted waveform is equal to the square-root of the sum of the square of the rms value of each harmonic component (including the fundamental).

$$I_s = \sqrt{I_{s1}^2 + \sum_{h \neq 1} I_{sh}^2} = \sqrt{I_{s1}^2 + I_{dis}^2}$$

- Total Harmonic Distortion

$$THD\,(\%) = 100\,\frac{I_{dis}}{I_{s1}} = 100\,\frac{\sqrt{I_s^2 - I_{s1}^2}}{I_{s1}}$$

Power and Power Factor

- Average (real) power and reactive powers: $P = \sum_{h=0,1,....} V_h I_{sh} \cos(\phi_h)$ $Q = \sum_{h=0,1,....} V_h I_{sh} \sin(\phi_h)$
- Apparent Power: $S = V_s I_s = \sqrt{P^2 + Q^2 + D^2}$
- Power factor: $PF = \frac{P}{S}$
- Case of sinusoidal voltage and non-sinusoidal current:

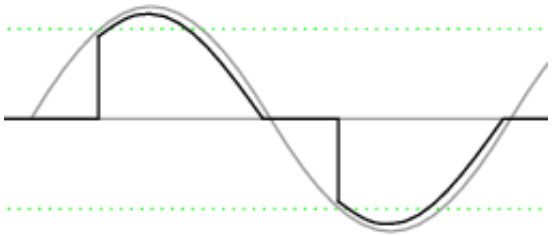
$$P = V_{s1} I_{s1} \cos(\phi_1), \quad Q = V_{s1} I_{s1} \sin(\phi_1)$$

$$PF = \frac{V_{s1} I_{s1} \cos(\phi_1)}{V_{s1} I_s} = \frac{I_{s1}}{I_s} \cos(\phi_1) = \frac{I_{s1}}{I_s} DPF = \frac{1}{\sqrt{1 + (THD)^2}} DPF$$

- Displacement Power Factor: $DPF = \cos(\phi_1)$

Example

A 460 V, 60 Hz AC source supplies power to a 14.1 Ω resistive load. The load current is delayed by 81 degrees by a back-to-back thyristor circuit as shown to the right. Compute the following:



a) rms values of the current

b) magnitude and phase angle of the 60 Hz current component.

c) Magnitude of the 3^{rd} and 5^{th} harmonic components,

d) Active power, fundamental reactive power.

e) Displacement power factor and overall power factor.

Solution:

a) 25.26 A,

b) 22 A, - 27 deg,

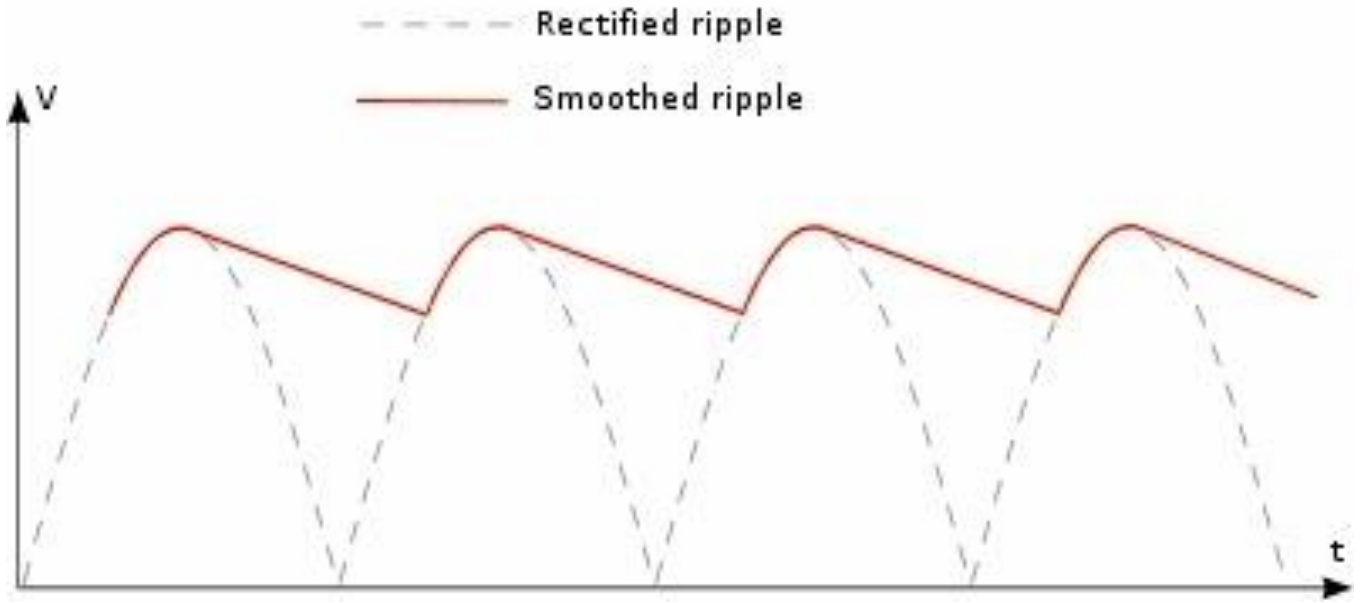
c) 8.9 A, 3.7 A,

d) 9 kW, 4.6 kVAR,

e) 89% and 77.4%

Ripple of DC Signal

- **Ripple factor** may be **defined** as the ratio of the root mean square (rms) value of the **ripple** signal to the absolute value of the DC component of the signal, usually expressed as a percentage.
- R**ipple**is also commonly expressed as the peak-to-peak value relative to the DC value.

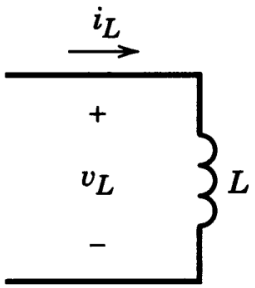


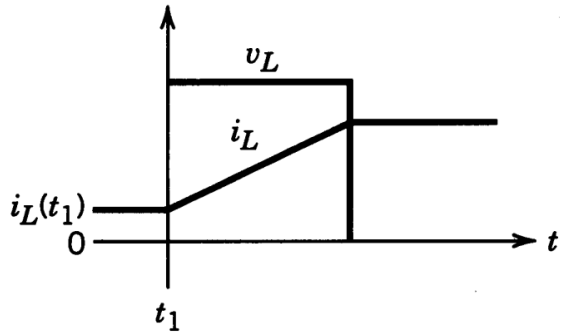
Current-voltage in an inductor and capacitor

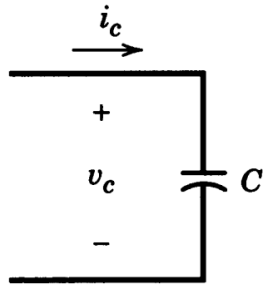
- In an inductor, the voltage is proportional to the rate of change of current.
- In a capacitor, the current is proportional to the rate of change of voltage.

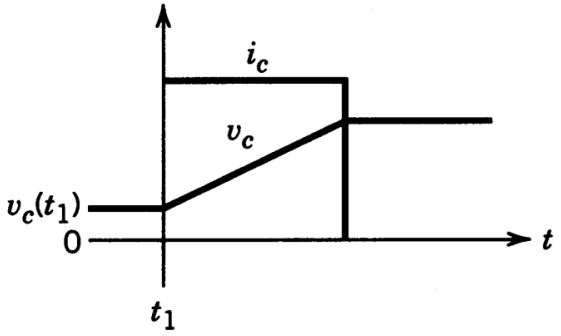
$$v = L\frac{d\,i}{d\,t} \qquad i = C\frac{d\,v}{d\,t}$$

$$i = \frac{1}{L}\int_{t_0}^{t} v(t)\,d\,t + i(t_0) \qquad v = \frac{1}{C}\int_{t_0}^{t} i\,d\,t + v(t_0)$$





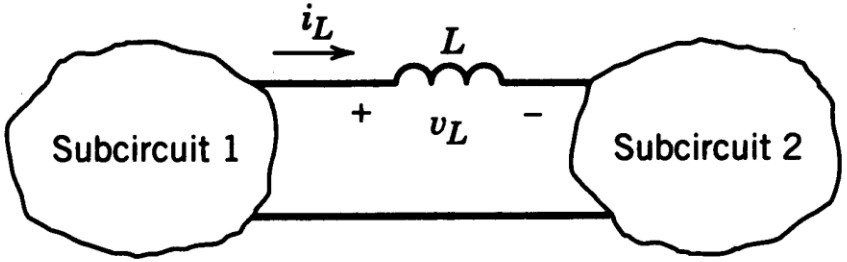




Inductor response in steady-state

- At steady-state, $i(t+T) = i(t) \rightarrow \int_{t_0}^{t_o+T} v\, d\, t = 0$

- Volt-seconds over T = 0 → Area "A" = Area "B"



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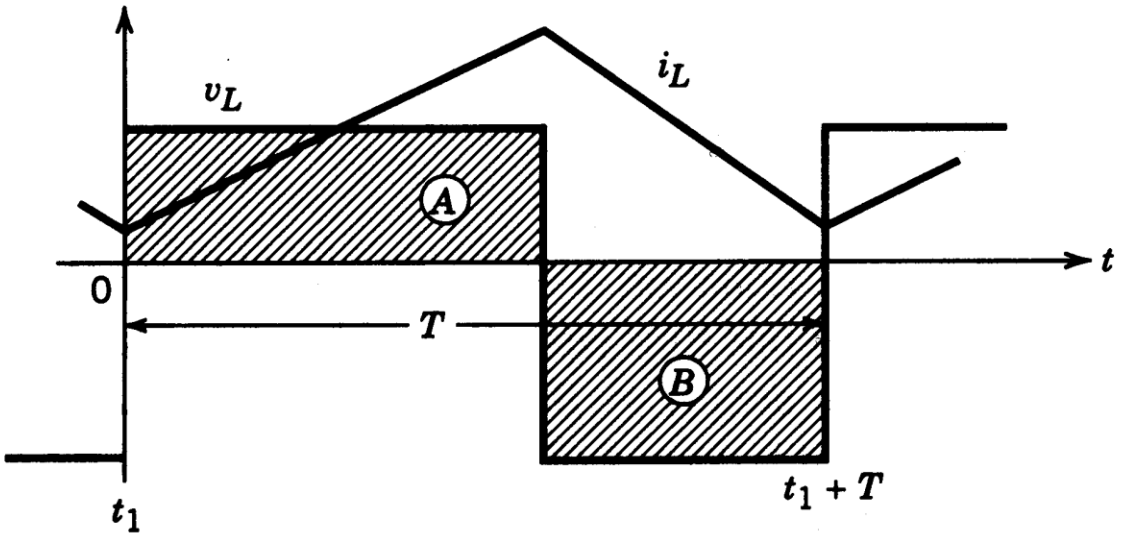

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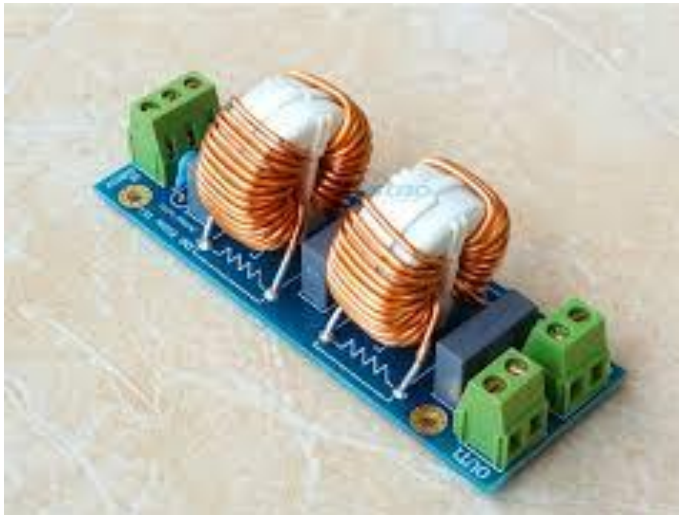
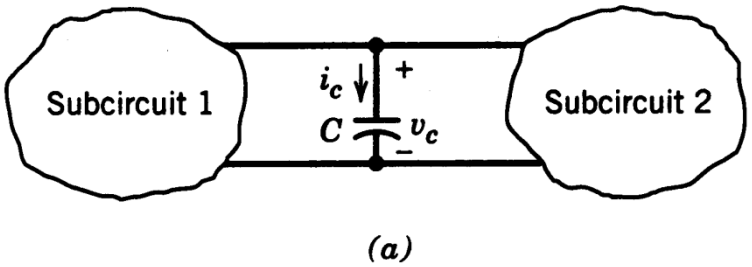


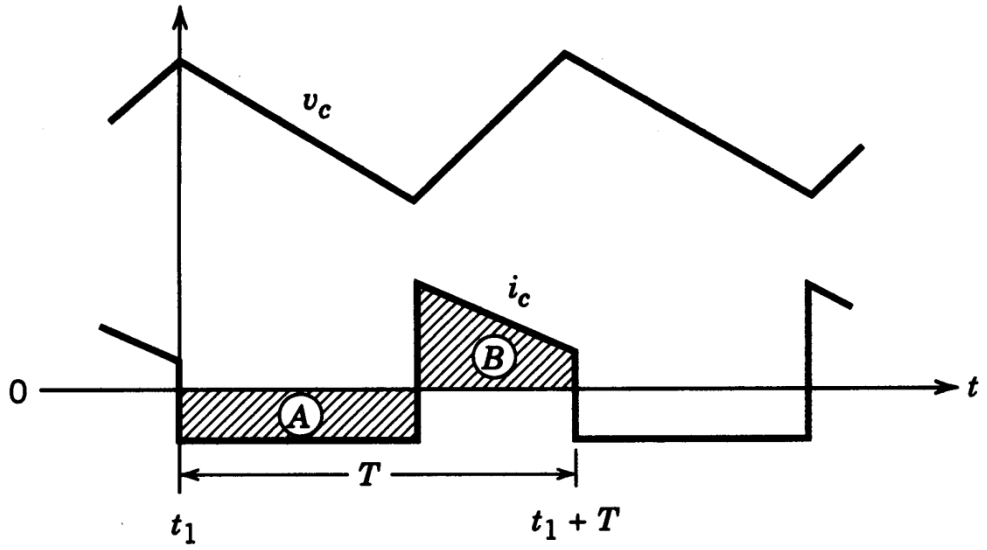
Capacitor response in steady-state

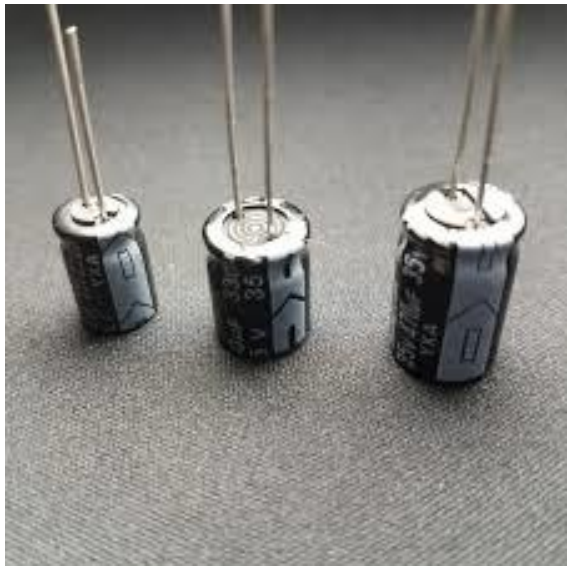
- At steady-state, $v(t+T) = v(t) \quad \rightarrow \quad \int_{t_0}^{t_o+T} i\, d\, t = 0$

- Amp-seconds over T = 0 → Area “A” = Area ”B”



(a)





Ampere's Law

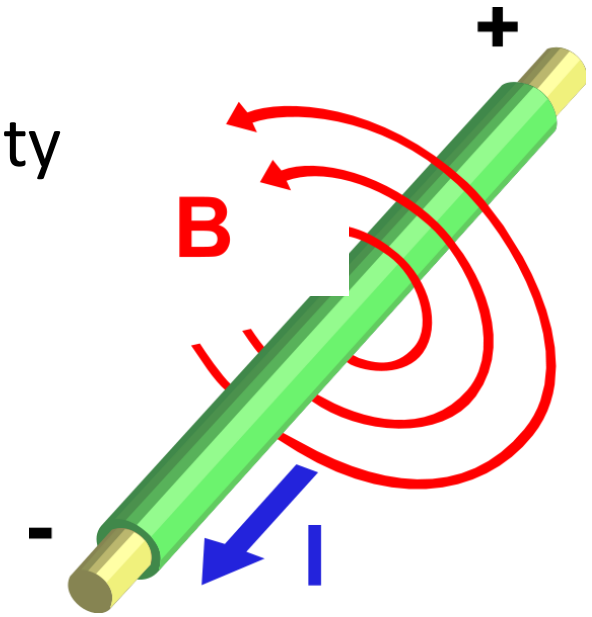
- Ampère's circuital law, discovered by André-Marie Ampère in 1826, relates the integrated magnetic field around a closed loop to the electric current passing through the loop.

$$\oint H.dl = I_{net}$$

where H is the magnetic field intensity

- At a distance *r* from the wire,

$$\oint H.dl = H.(2\pi r) = I$$



Magnetic Flux Density

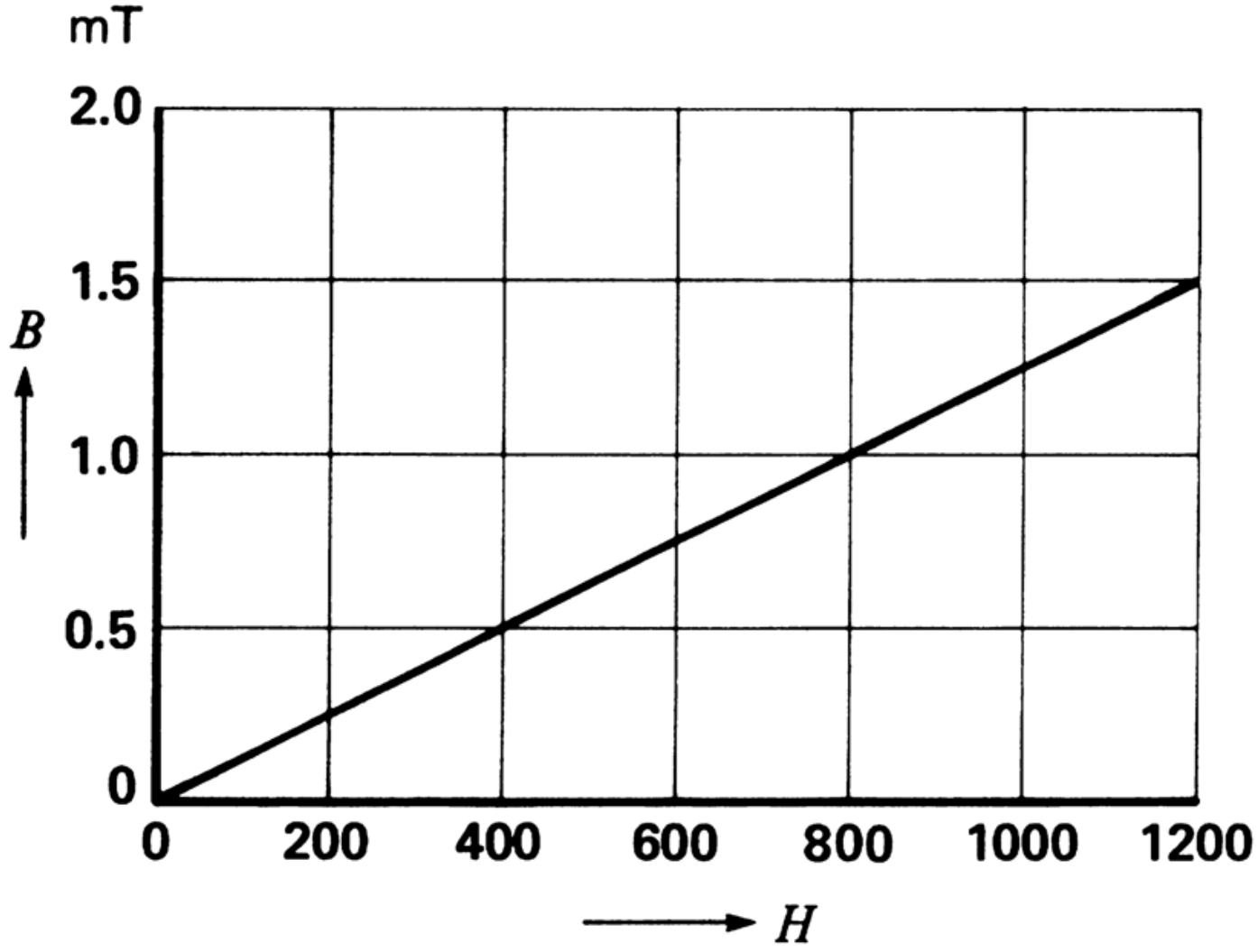
- Relation between magnetic field intensity H and magnetic field density B:

$$B = \mu H = (\mu_r \mu_0) H$$

where

  - μ_r is the relative permeability of the medium (unit-less),
  - μ_o is the permeability of free space (= $4\pi \times 10^{-7}$ H/m)

B-H Curve in air and non-ferromagnetic material



Magnetic Flux

- Magnetic flux is the total flux within a given area. It is obtained by integrating the flux density over this area:

$$\phi = \int B dA$$

- If the flux density is constant throughout the area, then,

$$\phi = BA$$

Ampere's Law applied to a magnetic circuit (Solid Core)

- Ampere's law:

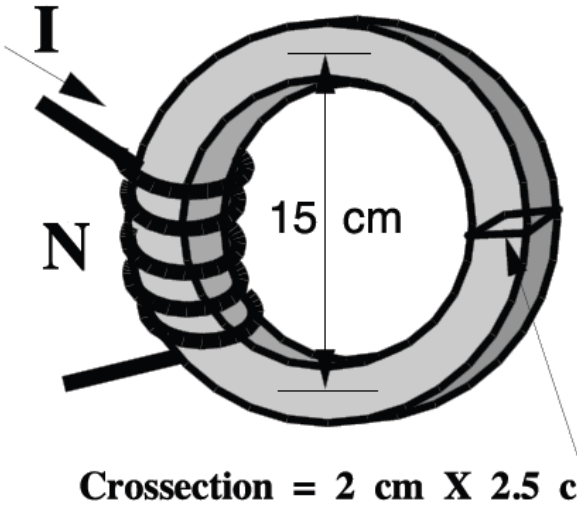
$$\oint H.dl = Hl = \frac{B}{\mu} l = NI$$

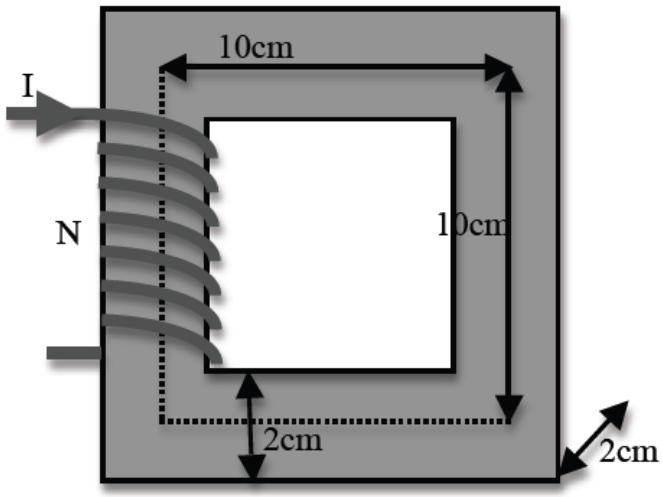
- Where *l* is the average length of the flux path. The *Magnetic* flux is:

$$\phi = \int BdA = BA$$

- Where A is the cross sectional area of the core. Hence,

$$NI = \phi(\frac{l}{\mu A}) = \phi\Re$$





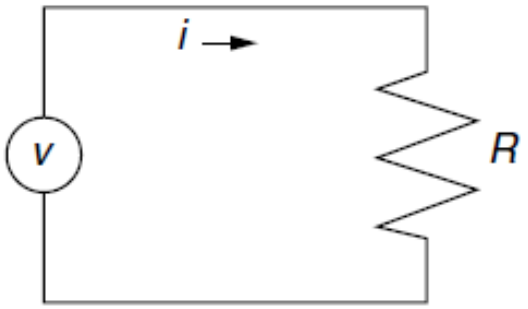
Analogous electrical and magnetic circuit quantities

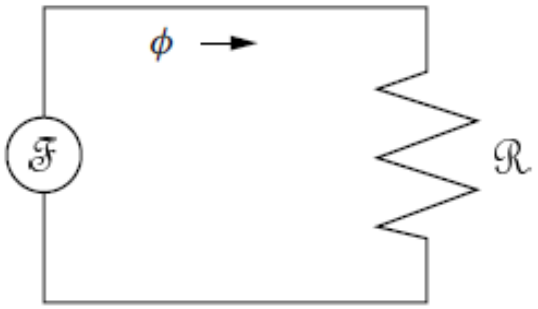
Electrical	Magnetic	Magnetic Units
Voltage v	Magnetomotive force $\mathcal{F} = Ni$	Amp-turns
Current i	Magnetic flux ϕ	Webers Wb
Resistance R	Reluctance $\mathcal{R}$	Amp-turns/Wb
Conductivity $1/\rho$	Permeability μ	Wb/A-t-m
Current density J	Magnetic flux density B	$\text{Wb/m}^2 = \text{teslas T}$
Electric field E	Magnetic field intensity H	Amp-turn/m

Electrical

Magnetic

EQUIVALENT CIRCUITS





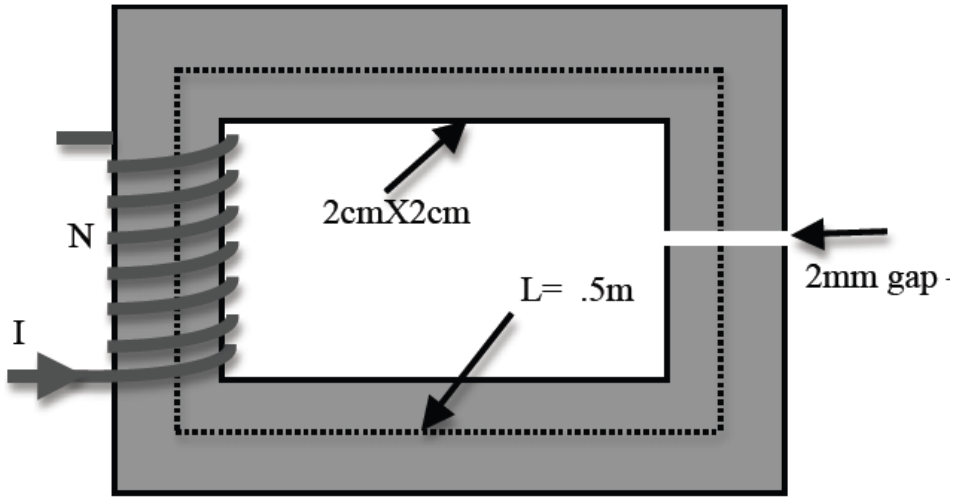
Ampere's Law applied to a magnetic circuit
(core with air gap - ignore leakage flux and fringing effect)

$$\oint H.dl = H_c l_c + H_a l_a = \frac{B}{\mu_r \mu_o} l_c + \frac{B}{\mu_o} l_a = NI$$

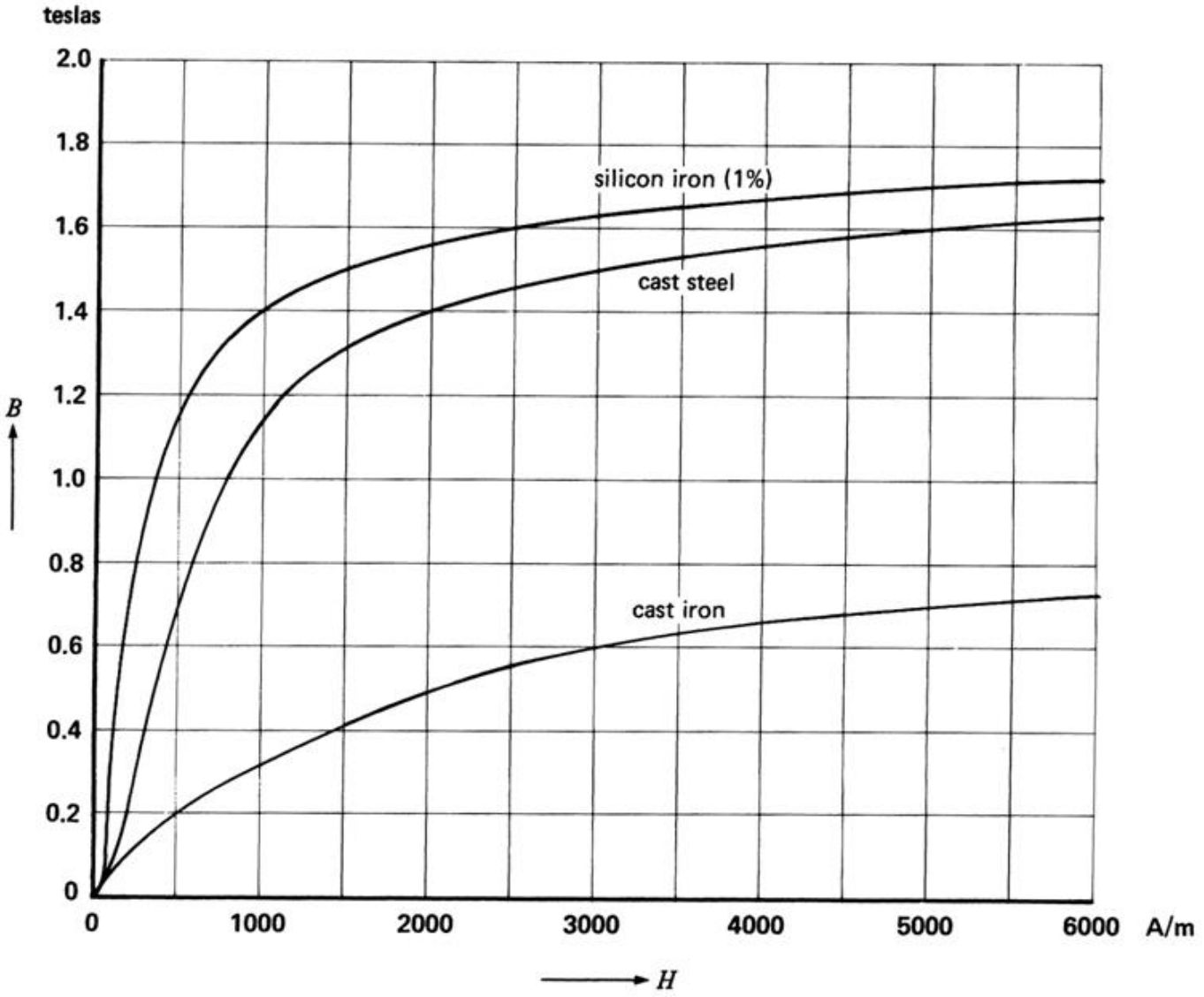
$$NI = \phi \Re$$

where

$$\Re = \left(\frac{l_c}{\mu_r \mu_o A} + \frac{l_a}{\mu_o A} \right)$$

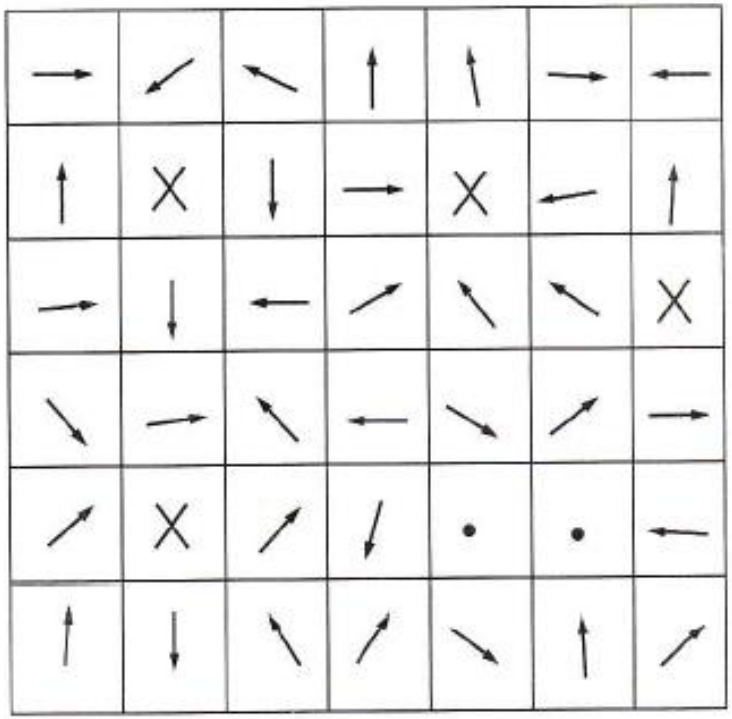


B-H Curve of Ferromagnetic materials

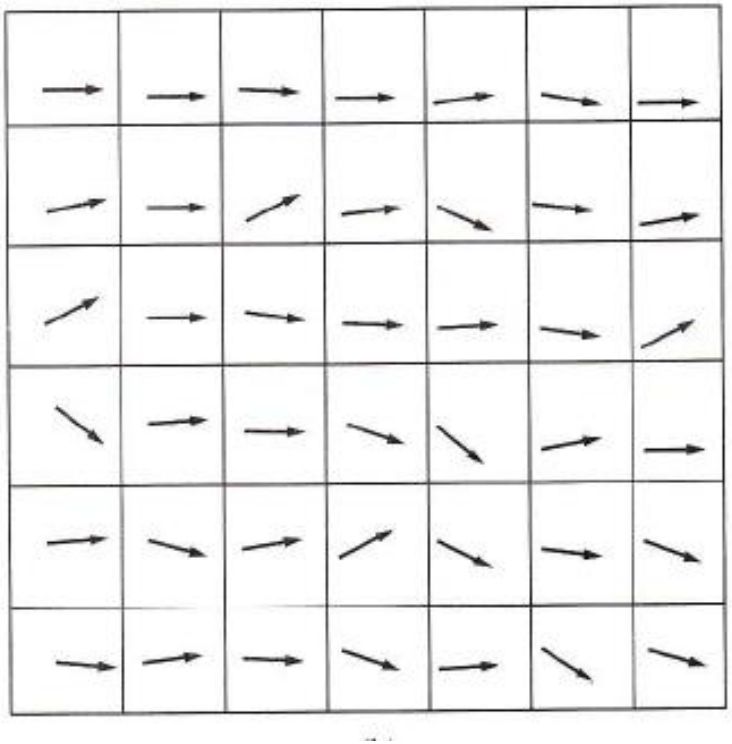


Orientation of magnetic domains without and with the presence of an external magnetic field

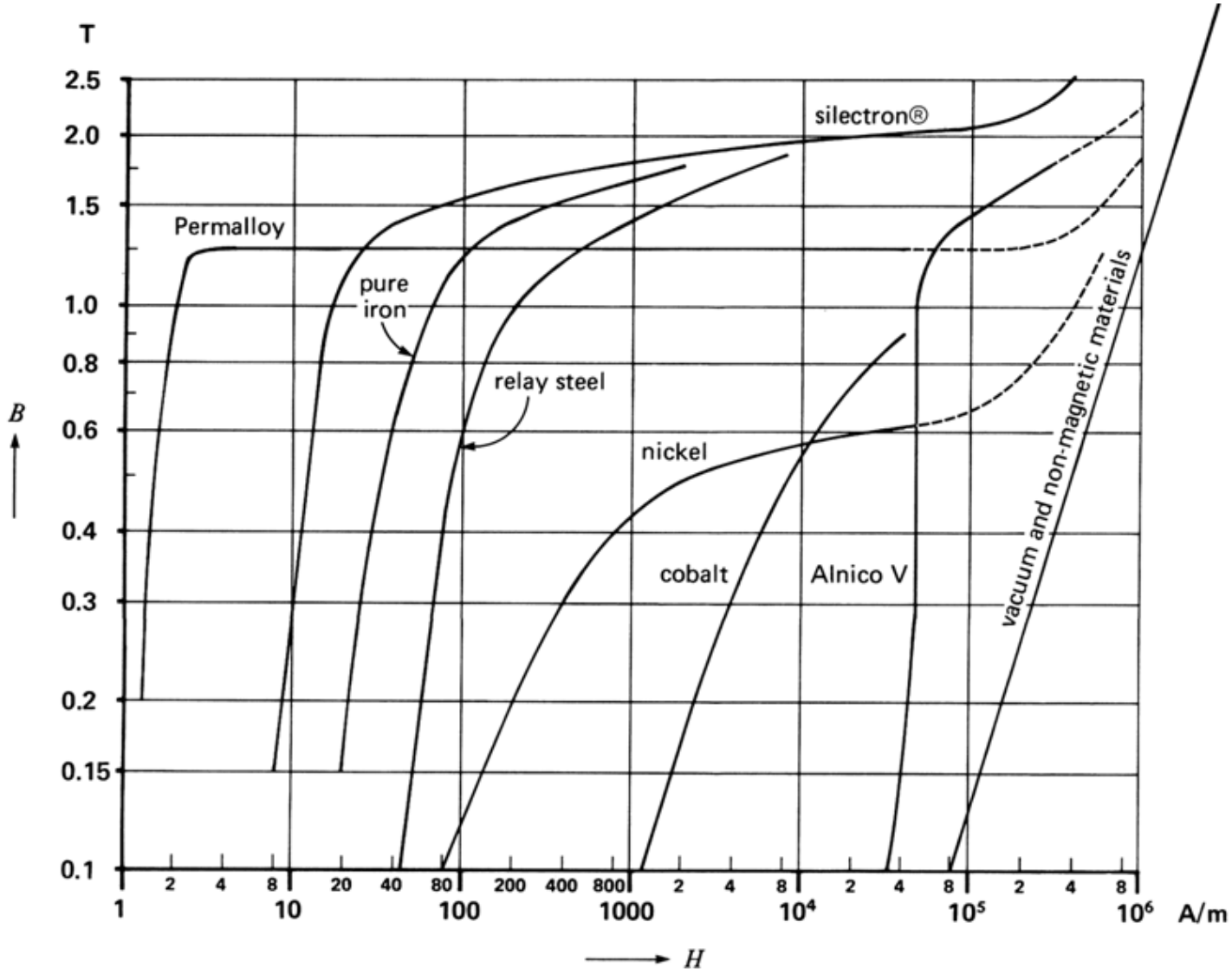
Without external magnetic field



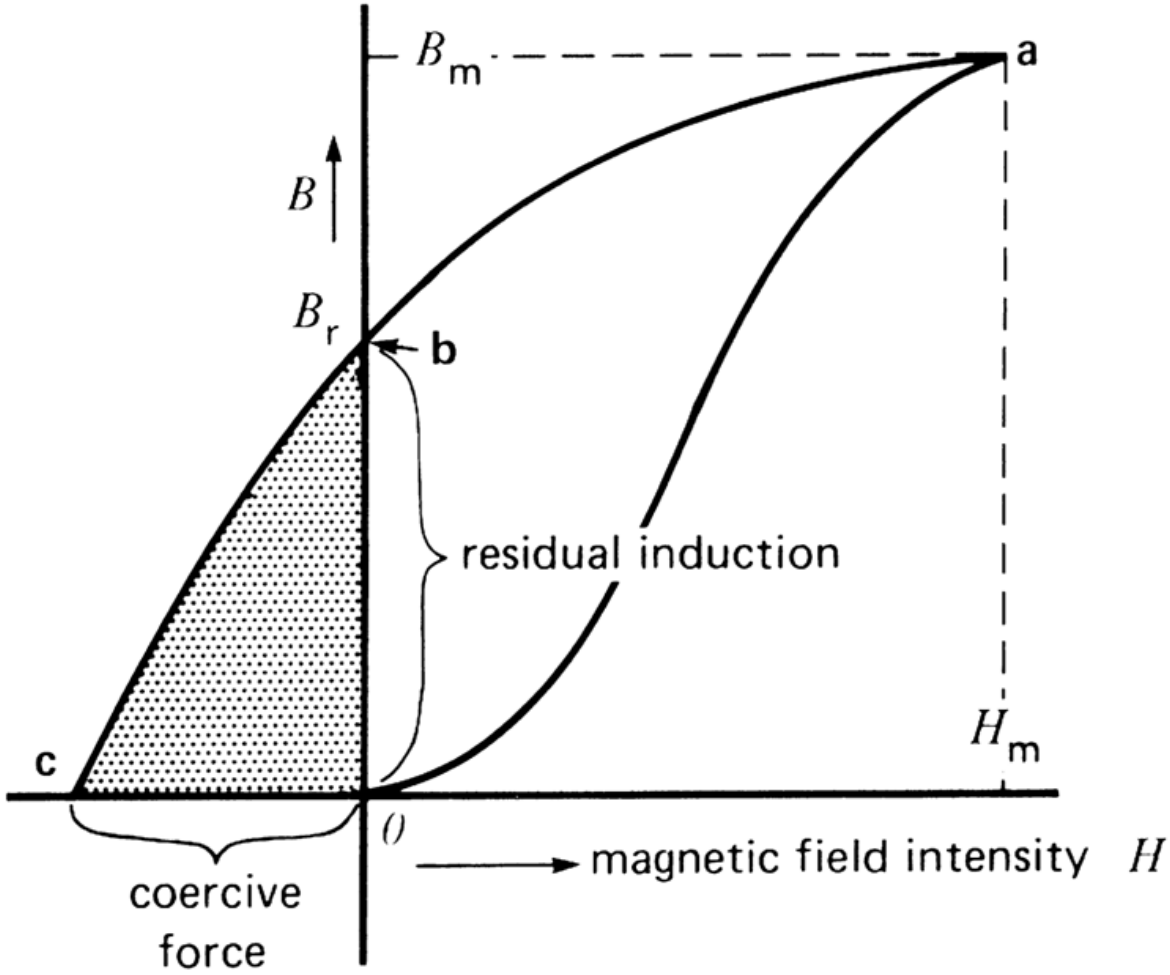
With external magnetic fiedl



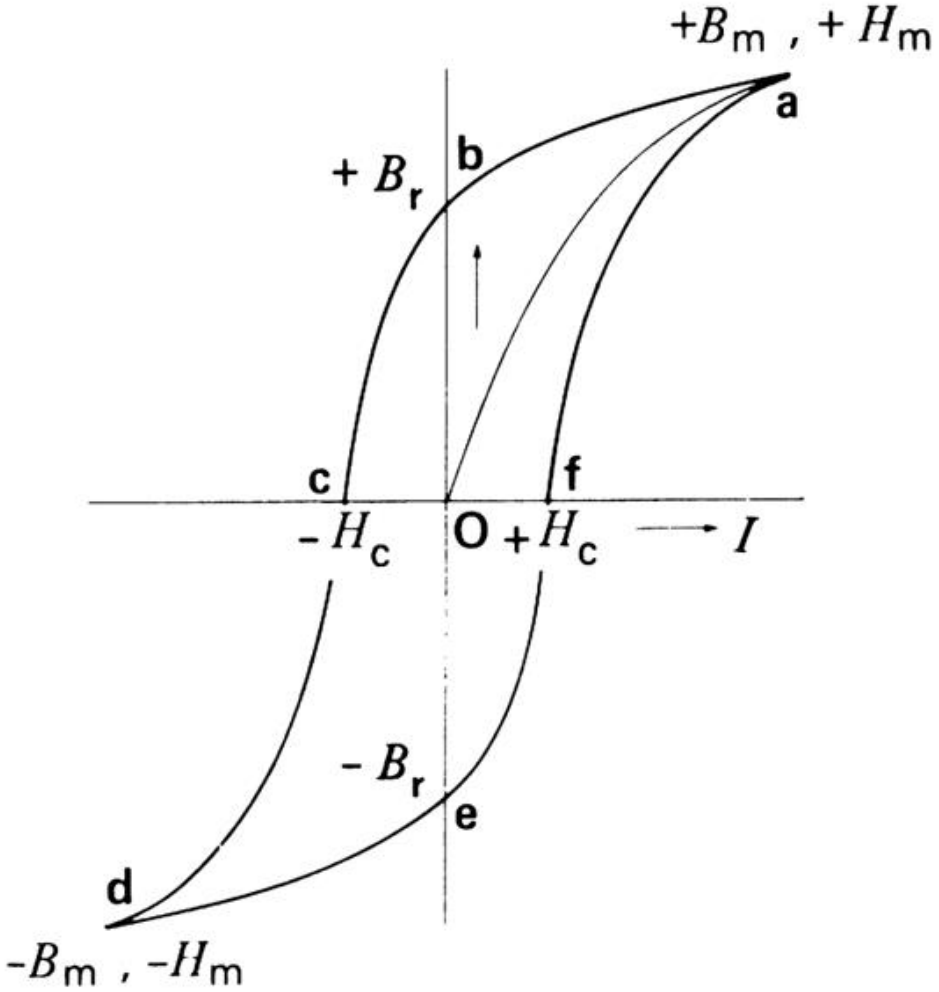
Saturation curves of magnetic and nonmagnetic materials



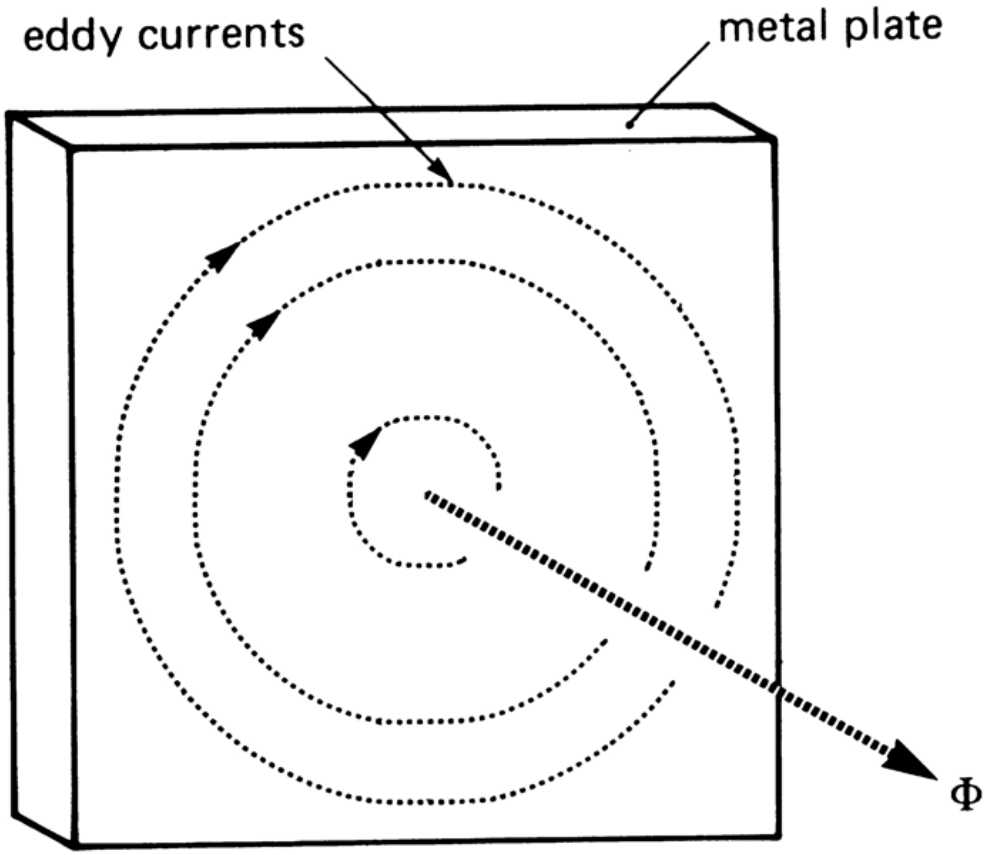
Residual induction and Coercive Force



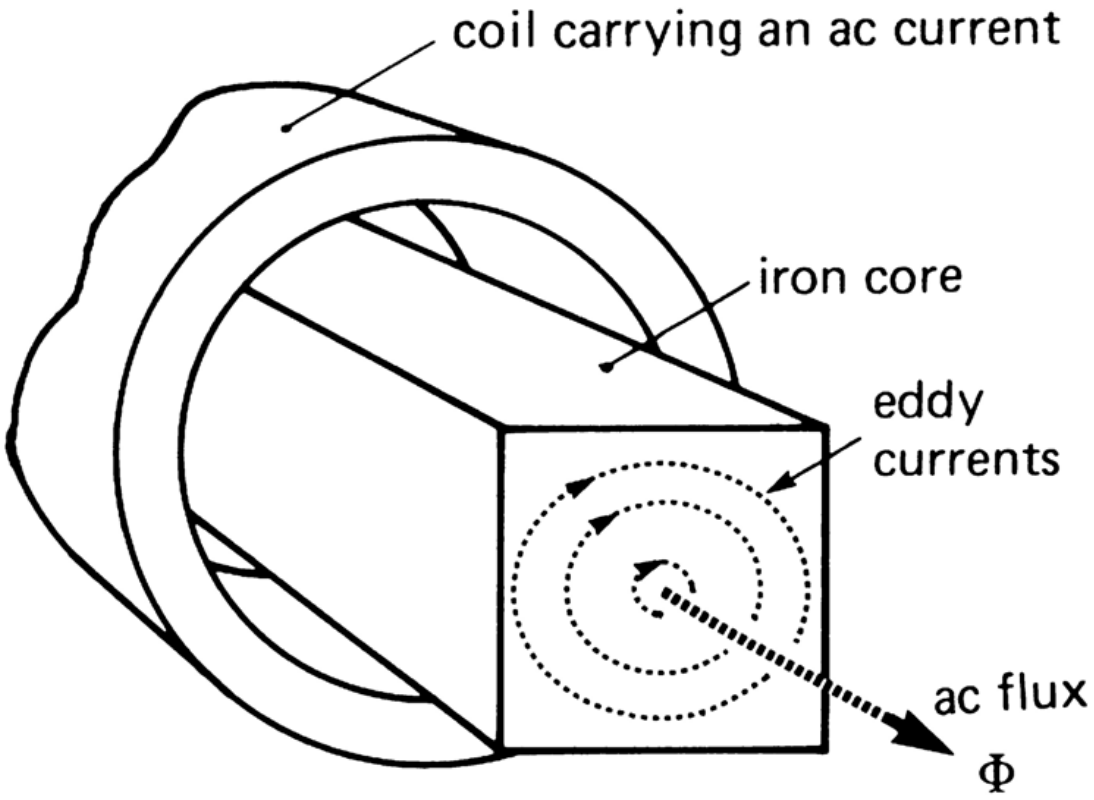
Hysteresis Loop traced by the flux in a core under AC current



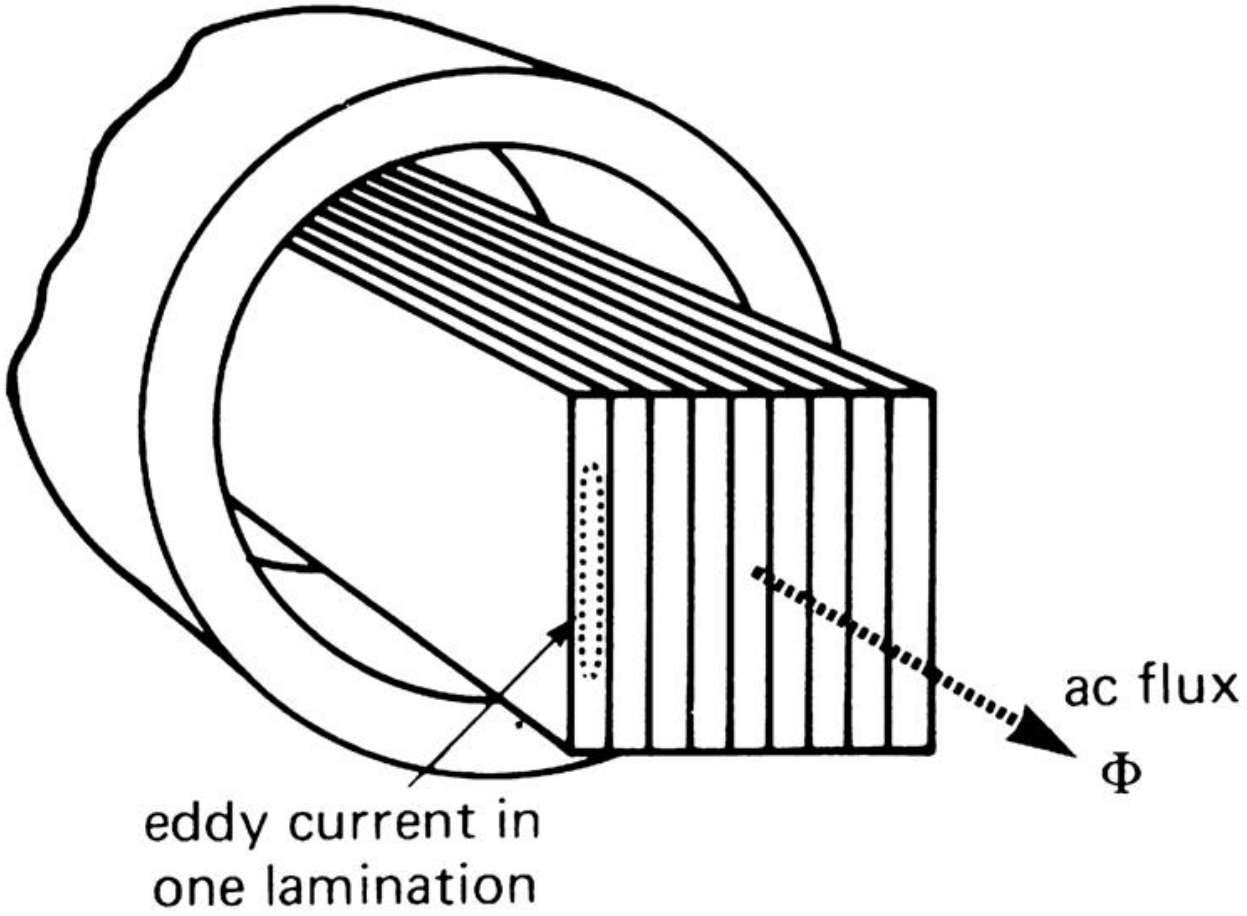
Eddy currents are induced in a solid metal plate under the presence of a varying magnetic field



Solid iron core carrying an AC flux (significant eddy current flow and power loss)



Core built up of insulated laminations minimizes eddy currents (and eddy current losses)



Faraday's Law

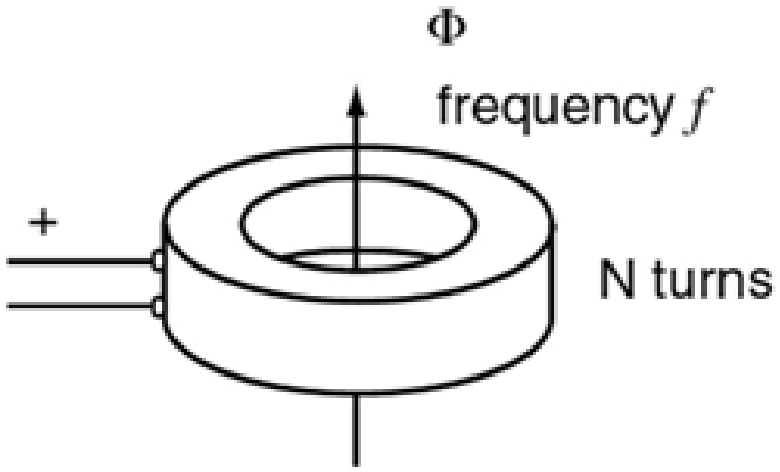
- Faraday's law of induction is a basic law of electromagnetism relating to the operating principles of transformers, electrical motors and generators. The law states that:

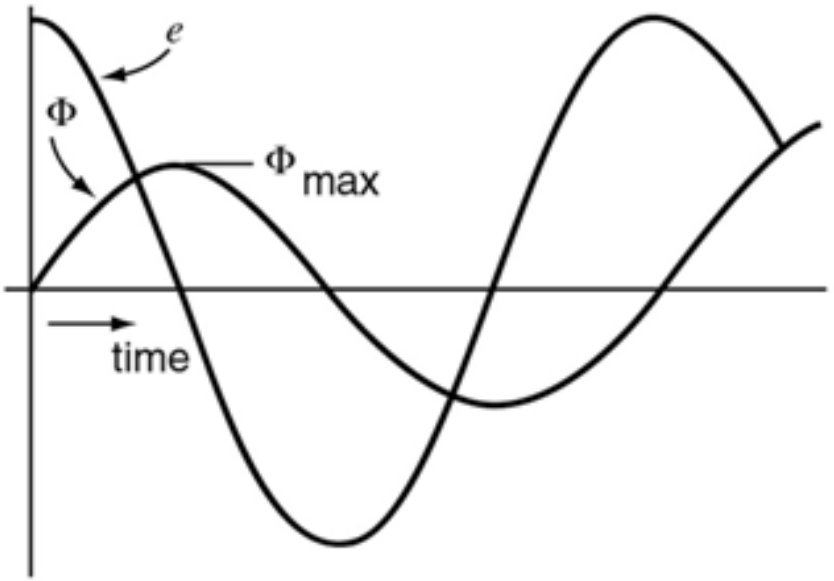
"The induced electromotive force (EMF) in any closed circuit is equal to the time rate of change of the magnetic flux through the circuit"

Or alternatively, "the EMF generated is proportional to the rate of change of the magnetic flux".

$$e = -N \frac{d\phi}{dt}$$

Voltage induced in a coil when it links a variable flux in the form of a sinusoid



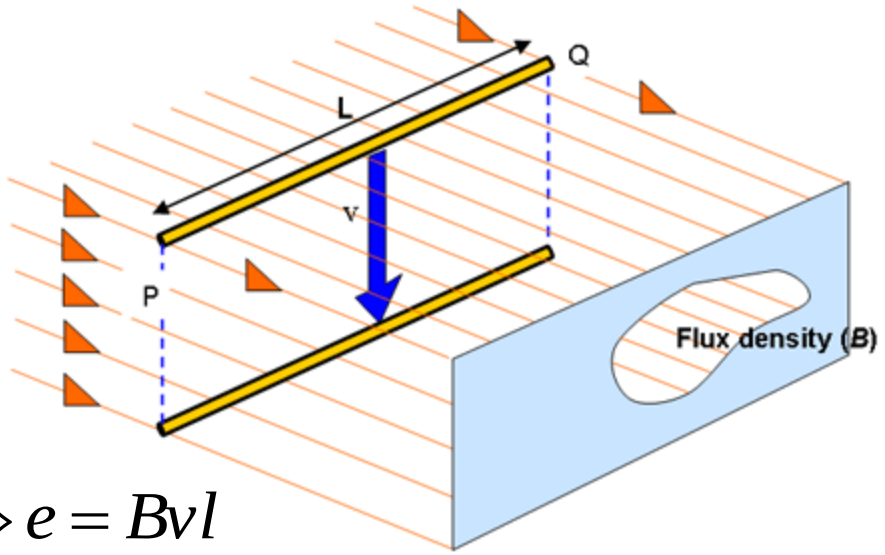


Induced voltage in a conductor moving in a magnetic field

- The voltage induced in a conductor of length *l* that is moving in a magnetic field with flux density *B*, at a speed *v* is given by

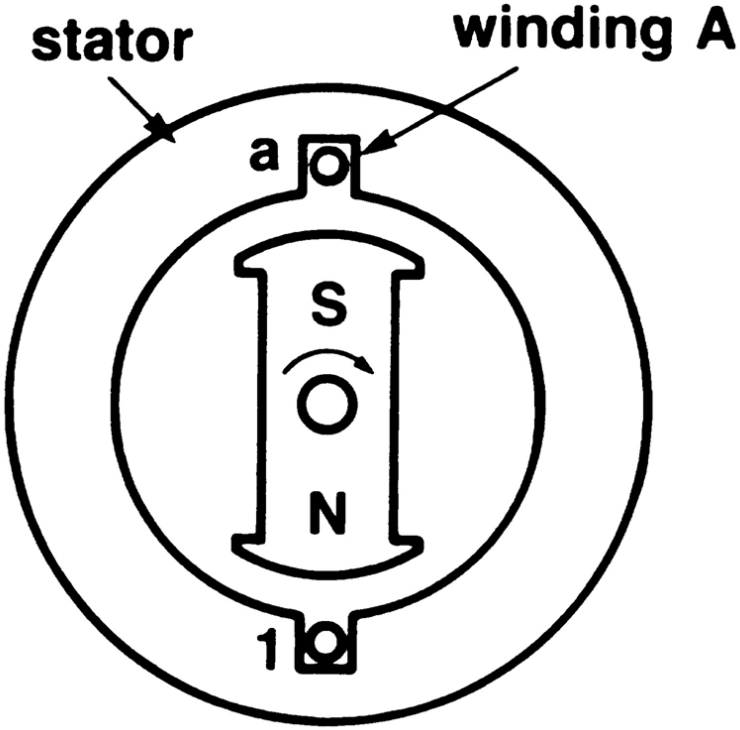
$$e = (vB\sin\theta)l\cos\phi$$

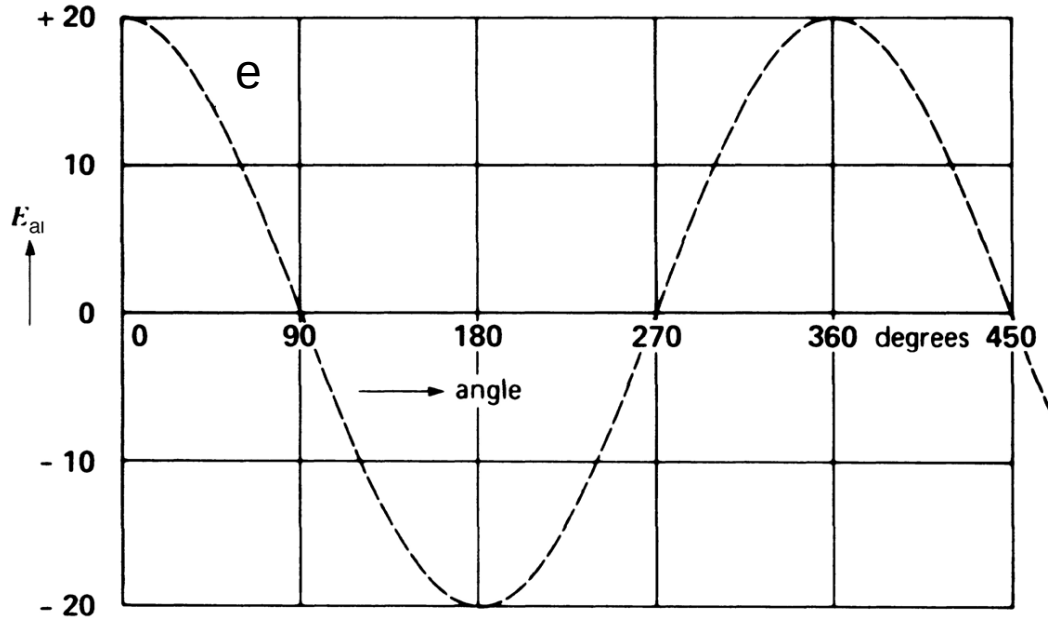
where θ is the angle between vxB and the velocity vector, and ϕ is the angle between vxB and the wire. The polarity of the induced voltage is determined by Lenz's Law.



$$\theta = 90\,\text{deg}. \quad and \quad \phi = 0\,\text{deg} \Rightarrow e = Bvl$$

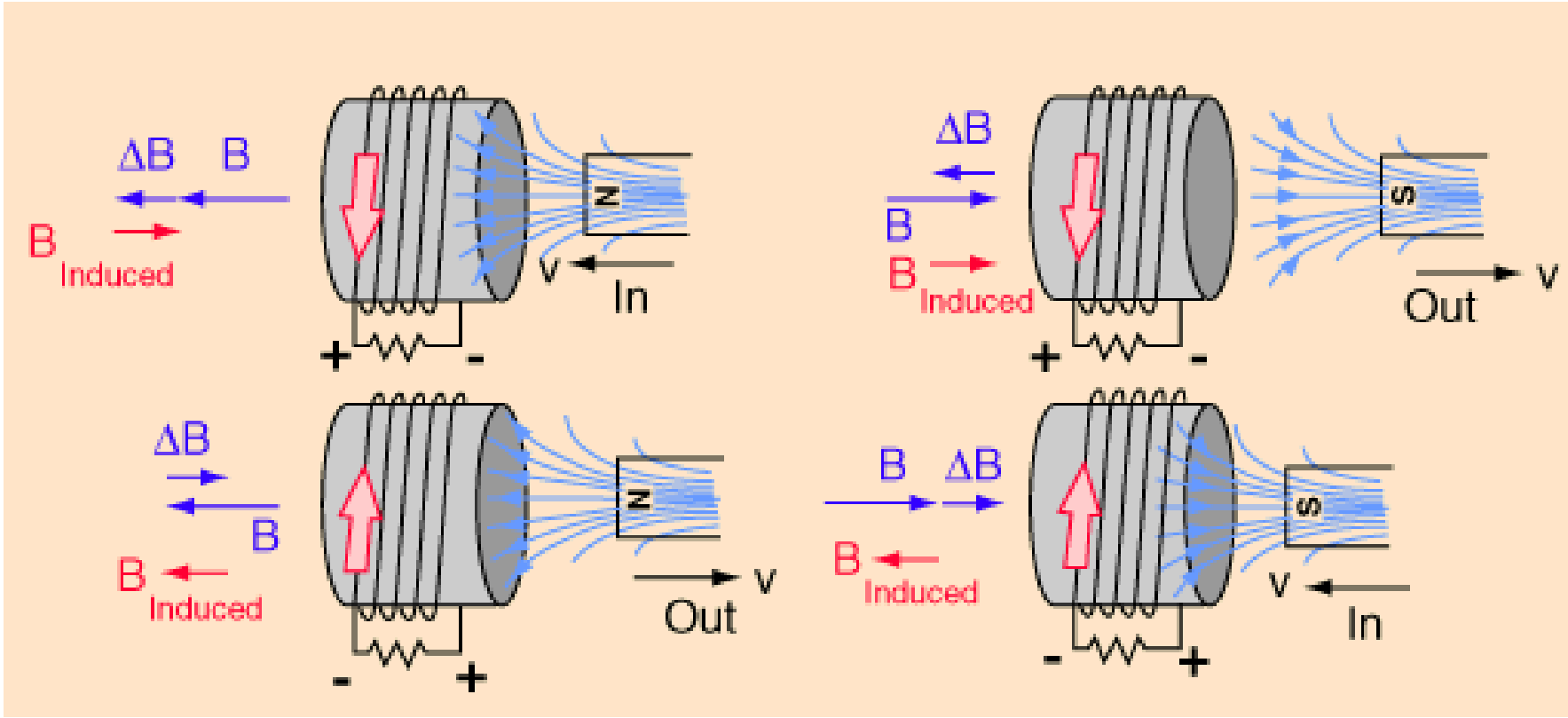
Induced voltage in a coil by a rotating magnet



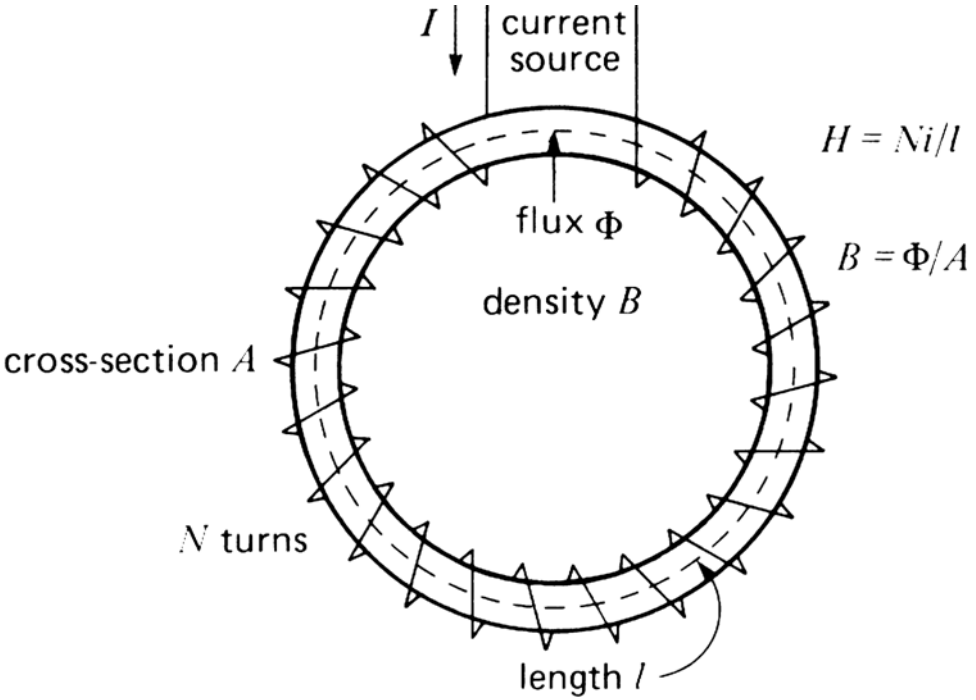


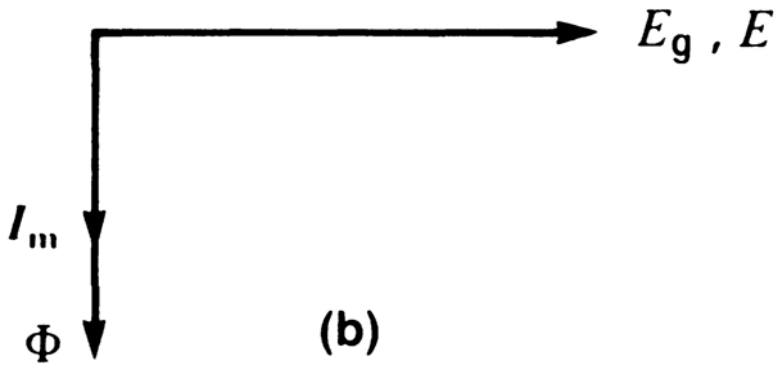
Lenz's Law

- The polarity of the induced voltage is such that it produced a current whose magnetic field opposes the change which produces it.



Inductance of a coil





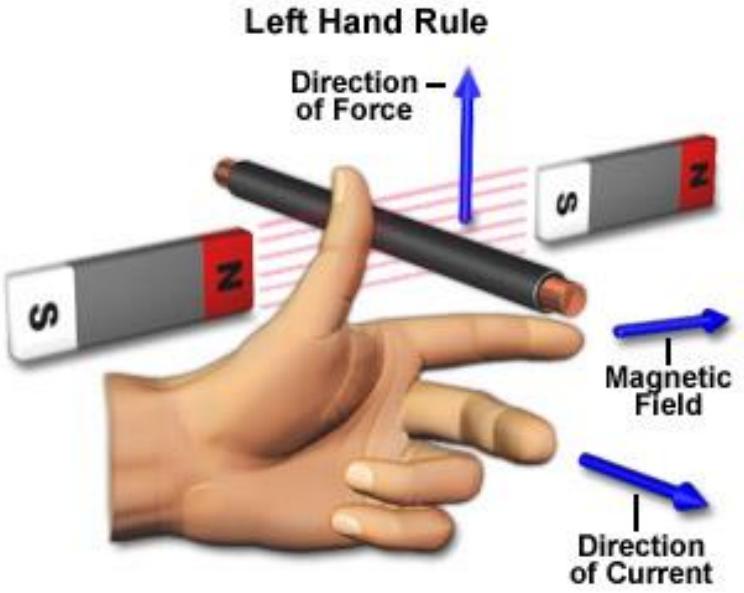
$$e = L\frac{di}{dt} = N\frac{d\phi}{dt} = N\frac{d(Ni\mu A/l)}{dt} = (N^2\mu A/l)\frac{di}{dt} \longrightarrow L = \frac{N^2\mu A}{l}$$

Induced force on a current-carrying conductor

- The force on a wire of length *l* and carrying a current *i* under the presence of a magnetic flux B is given by

$$F = Bil\sin\theta$$

where θ is the angle between the wire and flux density vector. The direction of the force is determined by the right hand rule



Transformers

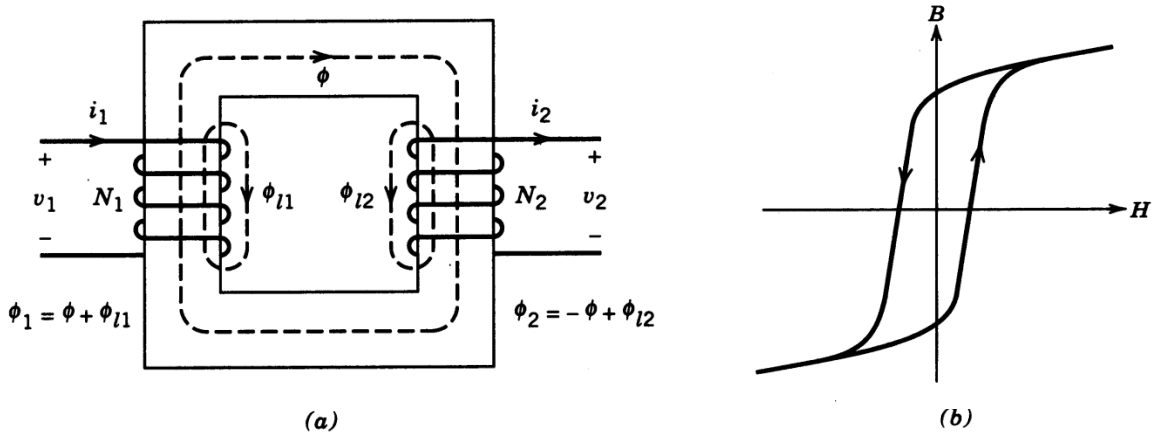


Figure 3-18 (*a*) Cross section of a transformer. (*b*) The *B–H* characteristics of the core.

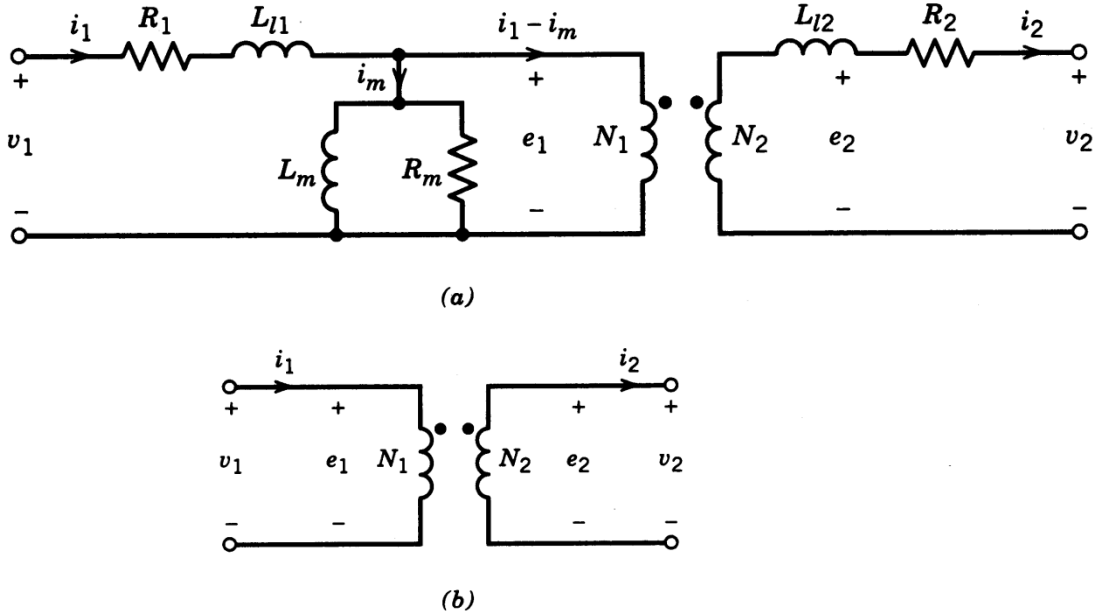


Figure 3-19 Equivalent circuit for (*a*) a physically realizable transformer wound on a lossless core and (*b*) an ideal transformer.

High-frequency vs. Low-frequency transformers

- For a given supply voltage, the flux density B in a transformer core is:
  - Inversely proportional to supply frequency
  - Inversely proportional to the cross-sectional area of the core.
- As the operating frequency increases, we can use less turns and a smaller core cross-sectional area. So a high-frequency transformer is smaller than a low frequency transformer of the same power rating.
- However, hysteresis losses in the core will increase with frequency if the flux density is kept constant. So for high frequency transformers, we "ditch" the laminated iron core and use a ferrite material. This needs to be operated at a lower flux density than iron but exhibits low hysteresis losses.

Example:

- 60 Hz, 120/24 V, 36 VA, B = 1.5 T, core loss = 1 W, N1/N2 = 600/120, weight = 500 g.
- 6 kHz, 120/24, 480 VA, B = 0.2 T, core loss = 1 W, N1/N2 = 45/9, weight = 100 g.