Solutions to Supplemental Problems

to accompany the 3rd Edition of

Power Electronics: Converters, Applications and Design

by

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Chapter 1 - Power Electronic Systems

S1.1.

In linear electronics, semiconductor devices are used in the middle of their linear amplification regions where both the voltage across the component and the current thru it are relatively large. This results in high power dissipation.

In power electronics, the semiconductor devices are used as switches. When the device is on (approximating a closed switch) the voltage across the device is very low (usually 1-3 volts maximum) and the current through it is large. The power dissipation, while substantial, is much less than operating in the linear amplification region at the same current level. When the device is off (approximating an open switch) the voltage across the component is large but the current is very small and the power dissipation in the off state can usually be considered as zero.

S1.2.

1. Advances in microelectronics enabling the fabrication of high performance controllers in both digital and analog forms.
2. Revolutionary improvements in the capabilities (voltage, current, power dissipation, switching speeds) of semiconductor devices which operate as the switches in power electronic converters.
3. Large expansion of the market for power electronic converters.

S1.3.

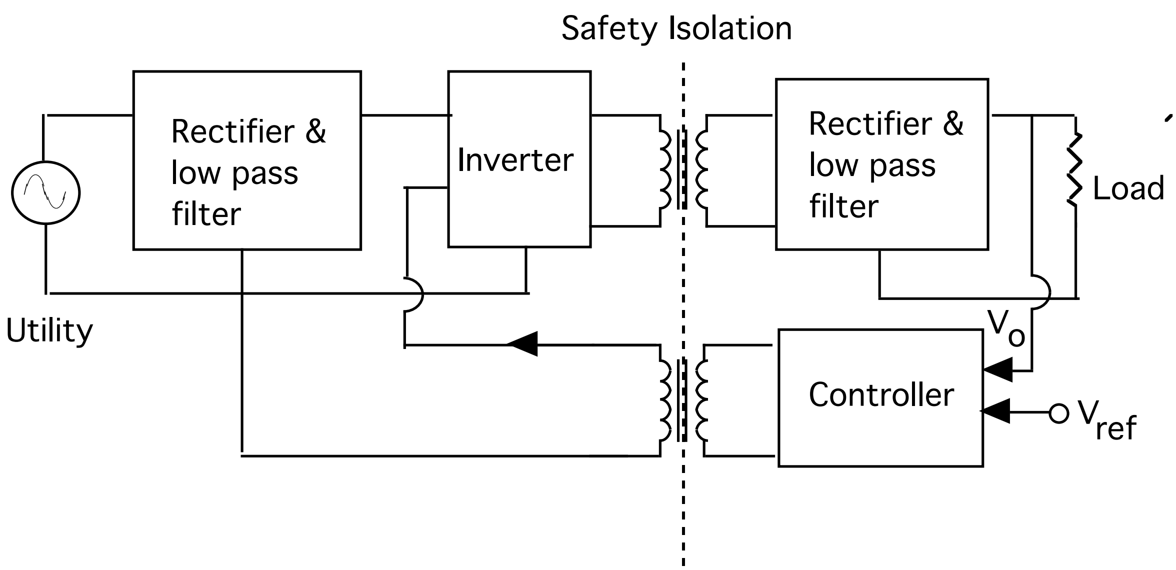
The table shown below characterizes the application areas in terms of the relative importance or priority the power electronics designer must place on each of the listed specifications. The assesments in the table are highly qualitative.

Application	Pwr Rating	Dynamics	Efficiency	Cost	Size and Weight
Residential	<10kW	slow	low priority	high priority	moderate priority
Commercial	<100 kW	fast	moderate priority	high priority	low priority
Industrial	all ranges	fast	moderate priority	moderate priority	low priority
Transportation	<1MW	moderate	high priority	moderate priority	moderate priority
Utility Systems	all ranges	moderate	high priority	moderate priority	low priority
Aerospace	<100kW	moderate	high priority	low priority	high priority
Telecommunications	<1kW	fast	modrate priority	moderate priority	moderate priority

S1.4.

a) Figure 1-3a is already diagrammed showing that the converter has two rectifiers, one inverter, a transformer and two energy storage capacitors.

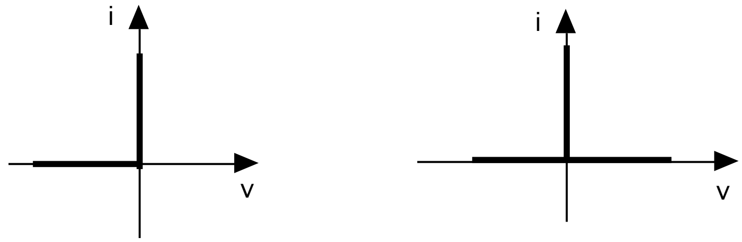
b) Block diagram shown below.



Chapter 2 - Overview of Semiconductor Power Switches

S2.1.

a) Ideal i-v curves for a diode and thyristor are shown below. A more complete figure for the diode is shown in Fig. 2-1 of the text and for the thyristor, Fig. 2-3 of the text.



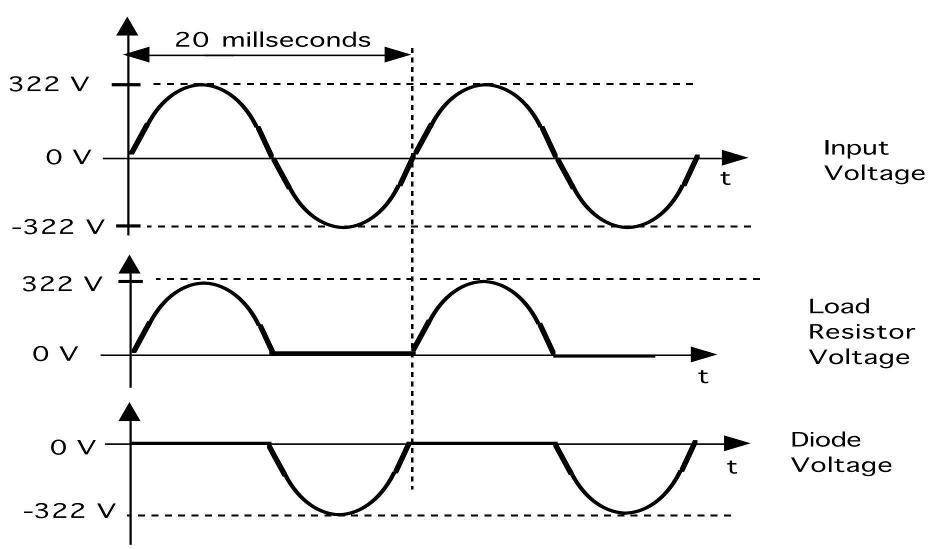
Ideal diode i-v curve Ideal thyristor i-v curve

b) Ideal characteristics are used when the basic operation of a converter circuit is being analyzed or designed from a top-down (system) view point. In this situation, idealized characteristics greatly simplify the effort with minimal loss of accuracy. The nonideal characteristics which are eliminated in the idealizations are second order effects that only have minor effects on the overall converter characteristics.

Real characteristics are used when the effects of the actual characteristics are to estimated. These effects are usually most important on the device itself and may cause the device maximum capabilities to be exceeded. For example the on-state resistance of a diode, a nonideal characteristic, causes power dissipation in the diode when it is in the on-state. An accurate knowledge of the power losses is necessary to correctly dimension the heat sink. If the power dissipation exceeds the maximum power rating of the diode, the internal temperature of the diode may exceed the maximum allowable value and at the very least, the reliability of the diode will be significantly reduced.

S2.2.

a) Diode voltage and load resistor voltage shown below.



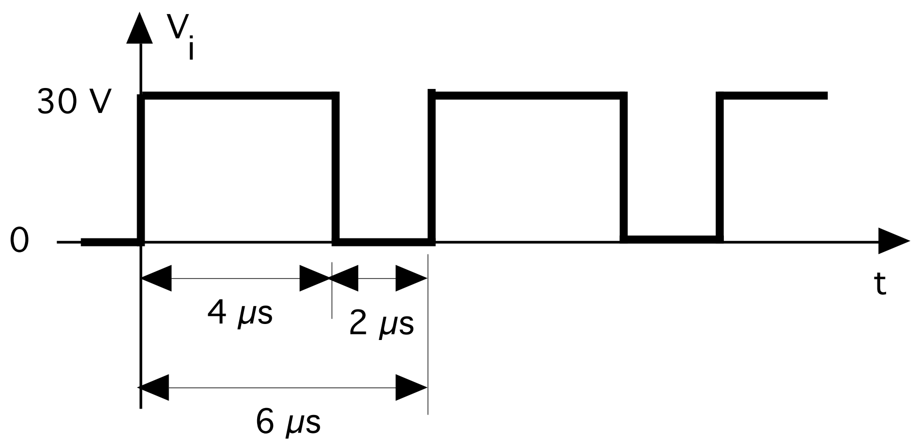
$$V_d = \frac{1}{0.02}\int_0^{0.01} 322\sin(2\pi x50t)dt = \frac{322}{\pi} = 102 \text{ volts}$$

b) Devices in parallel have the same voltage impressed across them. The device with the lower forward voltage across it for a given current, will conduct a larger current than the other paralleled devices. This means that the device carrying the most current will dissipate more power than the other paralled devices. Consequently the temperature will be larger in this device than in the other devices. This higher temperature will lead to an even larger share of the current going through this device. The end result may be that the current in this device may exceed the rating of the device.

Chapter 3 - Review of Basic Concepts

S3.1.

a) Input voltage shown below.



In the steady state, there is no average voltage across the inductor so the average of V_i equals the average of V_o. $< V_i > = \frac{(30)(4)}{6} = 20$ V $= V_o$.

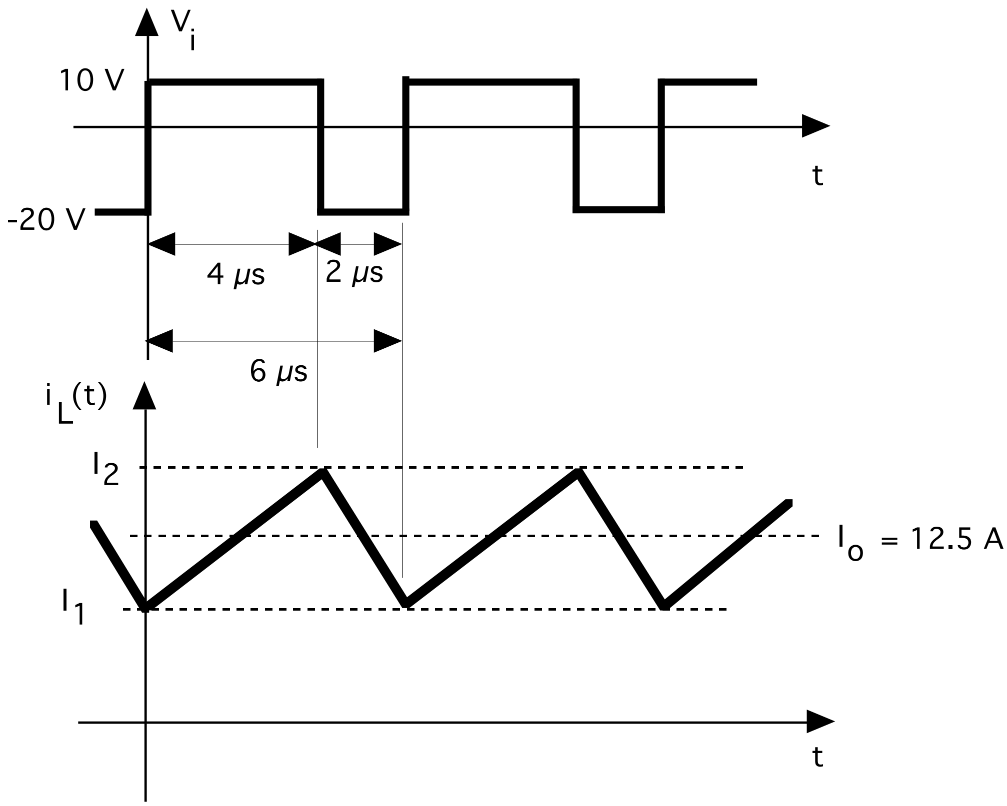
b) 250 W = $\frac{(20)^2}{R}$, $R = \frac{400}{250} = 1.6\ \Omega$; $(I_o)^2(1.6) = 250$; $I_o = \sqrt{\frac{250}{1.6}} = 12.5$A

c) Inductor voltage = $v_i(t) - V_o$ where it is assumed that the time-varying portion of the output voltage is very small (C very large). Hence inductor voltage and current waveforms are as shown below. For the inductor $v_L(t) = L\frac{di_L(t)}{dt}$. During the time intervals when the inductor voltage is constant, the inductor will change linearly with time.

During the 4 μs interval the inductor current = $I_1 + \frac{10\ v}{5\ \mu H} t = I_1 + 2x10^6\ t$
At the end of the 4 μsec interval $I_1 + 8 = I_2$ or $I_2 - I_1 = 8$ A

Average current = $I_o = 12.5\ A = \frac{I_1(6\mu s) + (8A)(4\mu s)(0.5) + (8A)(2\mu s)(0.5)}{6\ \mu s} = I_1 + 4A$

Hence $I_1 = 8.5$A and $I_2 = 16.5$ A



$<[i_L(t)]^2> = <[I_o + i_{ripple}(t)]^2 = [I_o]^2 + 2\, I_o < i_{ripple}(t)> + < [i_{ripple}(t)]^2$

$i_{ripple}(t)$ is shown below. As is clear from the diagram it has no average value.

Hence $< i_{ripple}(t)> = 0$. Thus

$<[i_L(t)]^2> = [I_o]^2 + < [i_{ripple}(t)]^2$;

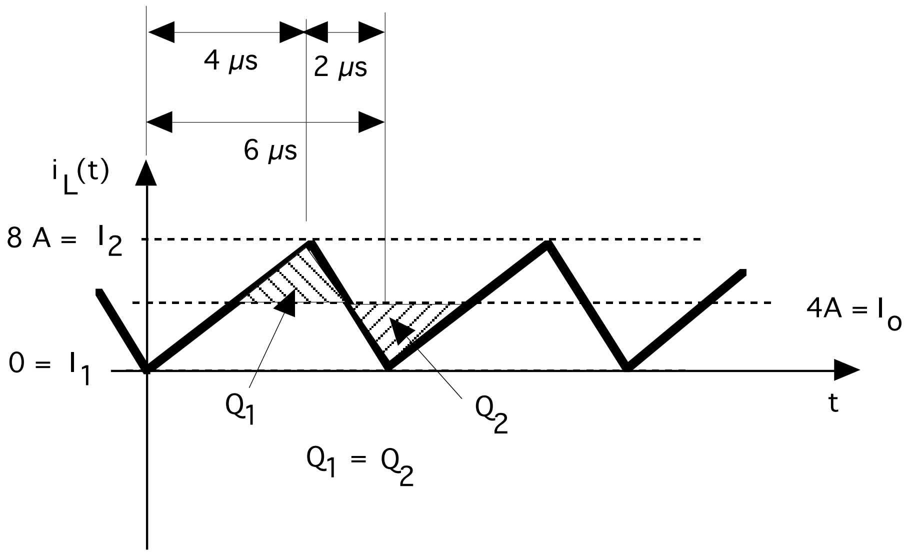
Mean square value of a triangular wave = $\{\text{base-to-peak}\}^2/3$; See solutions to problem 3-3e in existing solutions manual.

$$< [i_{ripple}(t)]^2 > : = \frac{[I_2 - I_1]^2/4}{3} = \frac{64/4}{3} = 5.33\ A^2$$

$$\text{To find } I_{L,rms} = \sqrt{[I_o]^2 + < [i_{ripple}(t)]^2} = \sqrt{(12.5)^2 + 5.33} = 12.71\ A$$

d) $80\ W = \frac{(20)^2}{R}$, $R = \frac{400}{80} = 5\Omega$; $(I_o)^2(5) = 80W$; $I_o = \sqrt{\frac{80}{5}} = 4A$

The inductor voltage waveform is unchanged from part b). The current waveform is as shown below.



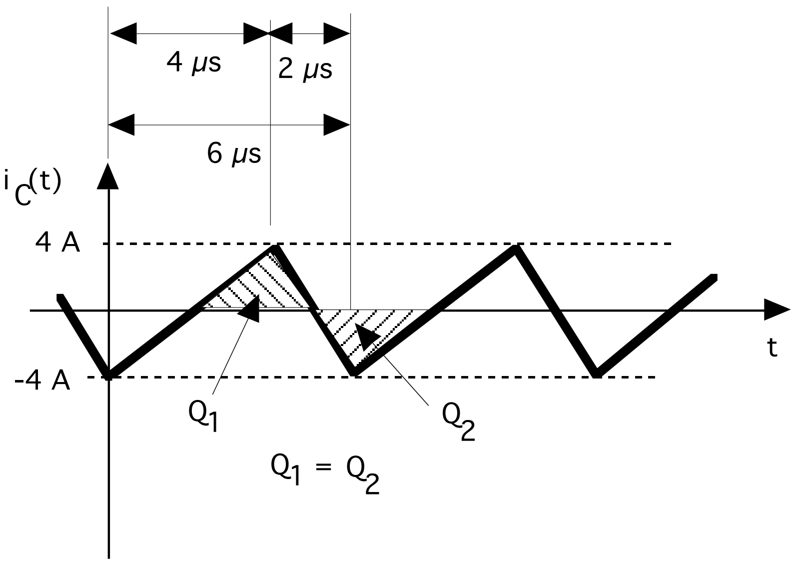
The ripple current $i_{ripple}(t)$]is unchanged from part b) since it is governed by the inductor voltage. Thus $< [i_{ripple}(t)]^2 > = 5.33\ A^2$.

$I_{L,rms} = \sqrt{(4)^2 + 5.33} = 4.62$ A

e) For part b) $\frac{I_{L,rms}}{I_{L,avg}} = \frac{12.71}{12.5} = 1.017$: For part c) $\frac{I_{L,rms}}{I_{L,avg}} = \frac{4.62}{4} = 1.16$

The ripple current in the inductor is independent of the load. Thus as the load resistance increases, the average current is reduced and the ratio of the rms current to the average current gets larger.

f) The current in the capacitor is equal to the inductor current minus the current in the load. The inductor current = $I_{L,avg} + i_{ripple}(t)$. The capacitor cannot conduct any dc current such as $I_{L,avg}$. Hence the capacitor current equals the ripple current through the inductor. The capacitor currents for parts b) and c) are the same and are shown below.
The rms ripple current in the inductor was found to be $\sqrt{5.33} = 2.31$ A in parts b) and c). Thus the rms capacitor current in both situations is equal to 2.31 A.



S3.2.

a) The waveform in Fig. 3-3a is a square wave. The rms value of the fundamental is given by (see solutions to prob. 3-3 in the solutions manual of the second or third edition, both are the same) $F_1 = \frac{4A}{(1.414)(\pi)} = 100$ amps where A is the base-to-peak amplitude of the square wave.
Solving for A = 110.06 amps.
The rms value of a square wave is equal to its ampliude. Thus $I_{rms} = 110.06$ amps.

b) For the waveform of Fig. 3-3b, $F_1 = \frac{4A\cos(\pi/6)}{(1.414)(\pi)}$ (see solutions to prob. 3-3. in the solutions of the second or third edition). $\frac{4A\cos(\pi/6)}{(1.414)(\pi)} = 100$ amps. Solving for the amplitude A = 126.9 amps.
The waveform in Fig. 3-3b is rectangular waveform which the signal is equal to zero for 1/3 of the time. Thus the rms current $I_{rms} = \frac{(126.9)^2\,(2)}{3} = 103.9$ amps.

c) The larger the fraction of a period that a periodic waveform has a value of zero, the smaller the rms value of the waveform will be compared to its base-to-peak amplitude.

S3.3.

a) Inductance $L = \frac{N^2}{R}$ where R is the reluctance of the magnetic path thru the core and gap combined. See Eq. 3-69 in text.

If the reluctance of the core can be neglected, then $R = \frac{l_{gap}}{\mu_o A_c}$; l_{gap} = 1 mm is the airgap length, $\mu_o = 4\pi x10^{-7}$ is the magnetic permeability of free space, and $A_c = 1\ cm^2$ is the area of the core and gap (ignoring flux fringing in the gap).

Thus $L = \frac{\mu_o A_c N^2}{l_{gap}} = \frac{(4\pi x10^{-7})(10^{-4})(100)^2}{10^{-3}}$ = 1.26 millihenries

b) $LI = N\phi$: Eq. 3-67 of text. $I_{max} = \frac{N A_c B_{max}}{L} = \frac{(100)(10^{-4})(0.3)}{1.26x10^{-3}}$ = 2.38 A

c) Maximum energy stored in inductor = $\frac{L (I_{max})^2}{2} = \frac{(1.26mH)(2.4)^2}{2}$ = 0.0036 joules

d) L = (0.00126)(4) ≈ 5 millihenries ; $I_{max} = \frac{(200)(10^{-4})(0.3)}{5x10^{-3}}$ = 1.2 amps

Maximum energy stored in inductor = $\frac{(5x10^{-3})(1.2)^2}{2}$ = 0.0036 joules

e) $W = \frac{(0.3)^2(10^{-4})(10^{-3})}{(2)(4\pi x10^{-7})}$ = 0.0036 joules. This method of estimating the energy stored in the airgap agrees with the value obtained using the inductance and maximum currents.

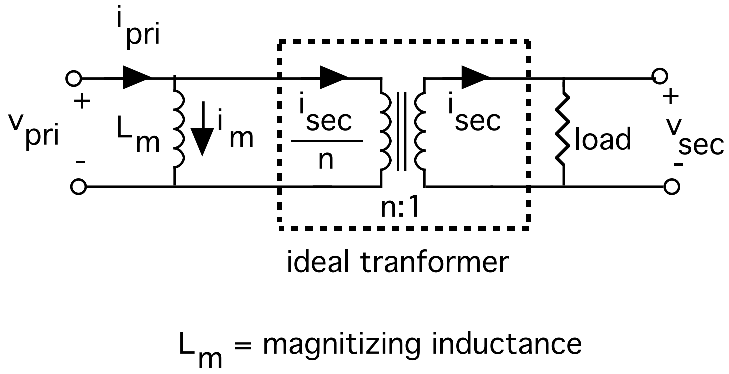
S3.4.

The figure reference in the problem statement should be Fig. P3-3b.

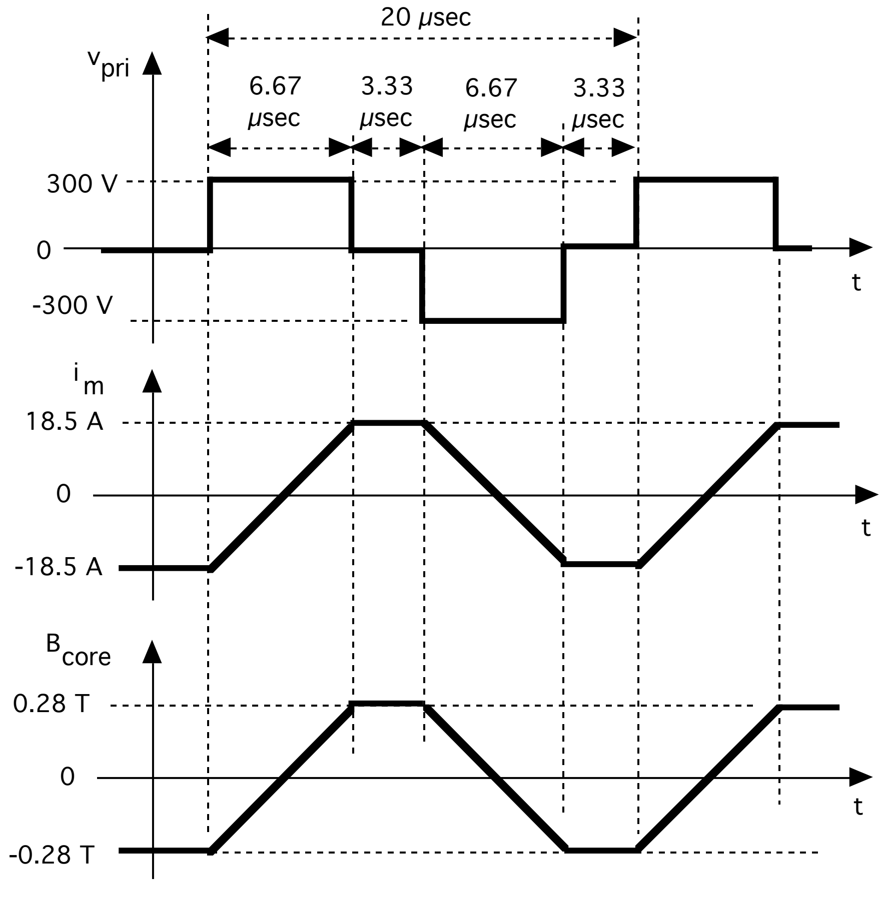
a) Transformer equivalent circuit (no leakage inductance) shown below. When there is no load, $i_{sec} = 0$, and the primary current is composed of only magnetizing current i_m.

$L_m \frac{di_m}{dt} = v_{pri}$; Ignoring reluctance of core, magnetizing inductance given by

$$L_m = \frac{\mu_o A_c N^2}{l_{gap}} = \frac{(4\pi x10^{-7})(3x10^{-4})(12)^2}{10^{-3}} = 54.4\ \mu\text{henries}$$



Voltage, flux density, and magnetizing current waveforms shown below. Amplitudes of current and flux density calculated in part b).



b) During the 6.67 μsec time interval when the primary voltage is 300 V, the magnetizing current is found from the equation $L_m \frac{di_m}{dt} = v_{pri}$. Integrating this equation , assuming t = 0 is at the start of the interval when the primary voltage steps up to 300 v, yields

$i_m(t) = i_m(0) + \frac{v_{pri}\, t}{L_m}$; Over the 6.67 μsec time interval, the total change in current

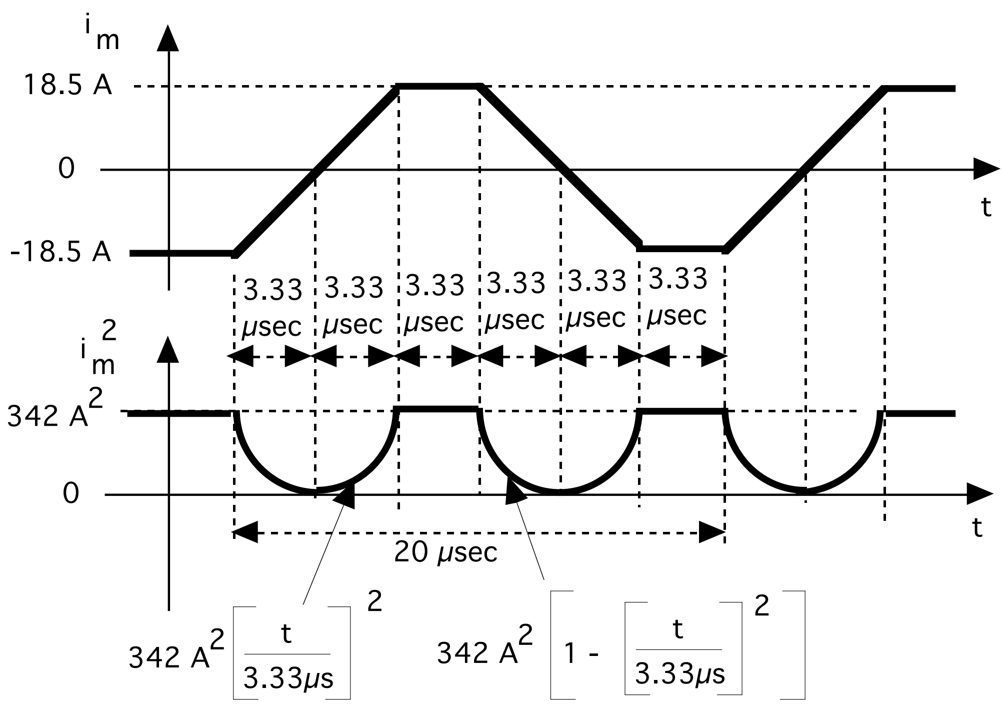
$\Delta i_m = \frac{(300v)(6.67\mu sec)}{54\mu henries}$ = 37 A. Hence the amplitude of the magnetizing current is 18.5 A.

Estimate the flux density using $L_m I_m = N_{pri} A_{core} B_{core}$ where I_m = the base-to-peak value of the magnetizing current. Using this yields for the base-to-peak flux density

$B_{core} = \frac{(54\mu H)(18.5A)}{(12)(3x10^{-4}\ m^2)} = 0.28\ T$

Note: This flux density exceeds the 0.2 T specified in the problem statement. The value of saturation flux density should be increased to 0.3 T.

c) The waveforms for the magnetizing current and the square of the magnetizing current versus time are shown below. The square of the current versus time are easily derived from the current versus time waveform. Also shown on the waveform are the time functions that apply to the square of the current during the time intervals that the current is changing linearly in time.



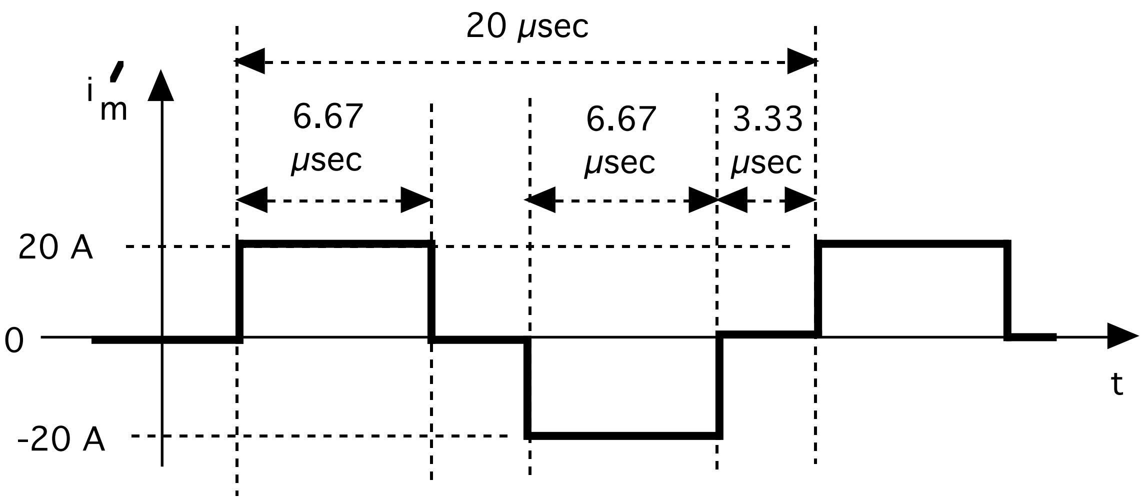
The area under one of the flat segments is $(342\ A^2)(3{,}33\mu s) = 0.00114\ A^2$-sec.

The area under one of the parabolic segments is $(342\ A^2)(3{,}33\mu s)/3 = 0.00038\ A^2$-sec.

$$<(i_m)^2> = \frac{(2)(0.00114) + (4)(0.00038)}{20\ \mu\text{sec}} = 190\ A^2\ ;$$

RMS value of the magnetizing current = $\sqrt{190}$ = 13.8 A

d) New component of primary current shown below.



RMS value of the new primary current component is given by

$$\sqrt{\frac{(20)^2(13.33\ \mu\text{sec})}{20\ \mu\text{sec}}} = 16.33\ A$$

e) The square of the total primary current can be written as

$(i_{m1}(t) + i_{m2}(t))^2 = (i_{m1}(t))^2 + 2i_{m1}(t)\ i_{m2}(t) + (i_{m2}(t))^2$;

$i_{m1}(t)$ is the magnetizing current shown in part a). $i_{m2}(t)$ is the new primary current component shown in part d). When the time average $<(i_{m1}(t) + i_{m2}(t))^2>$ is computed, the cross-product term $2i_{m1}(t)\ i_{m2}(t)$ averages to zero. Hence the rms value of the total primary current is;

$<(i_{m1}(t) + i_{m2}(t))^2> = \sqrt{190 + 267} = 21.4$ A

f) $L_m = \frac{\mu_o A_c N^2}{l_{gap}} = \frac{(4\pi x 10^{-7})(3x10^{-4})(12)^2}{10^{-4}} = 544\ \mu$henries

The magnetizing current waveform remains the same shape, but the amplitude is now 1.85 A.
The flux density waveform remains unchanged in both shape and amplitude.
The rms value of the magnetizing current is now 1.38 A.
The 20 A component of the primary current is unaffected by the change in gap length.
Thus the rms value of the total primary current is now =

$<(i_{m1}(t) + i_{m2}(t))^2> = \sqrt{1.9 + 267} = 16.4$ A

g) For the voltage waveform shown in part a) $v_{pri} = 300\text{ V} = N_{pri} A_{core} 2B_{sat} /(\Delta T)$ where

$\Delta T = 6.67\ \mu\text{sec} = \frac{1}{3f}$ where f is the frequency of the voltage waveform (50 kHz in part a). Letting f vary, but keeping the interval that the voltage is positive and equal to 300 V at 1/3 of the period yields at the edge of core saturation $N_{pri} A_{core} 2B_{sat} = \frac{300}{3f}$

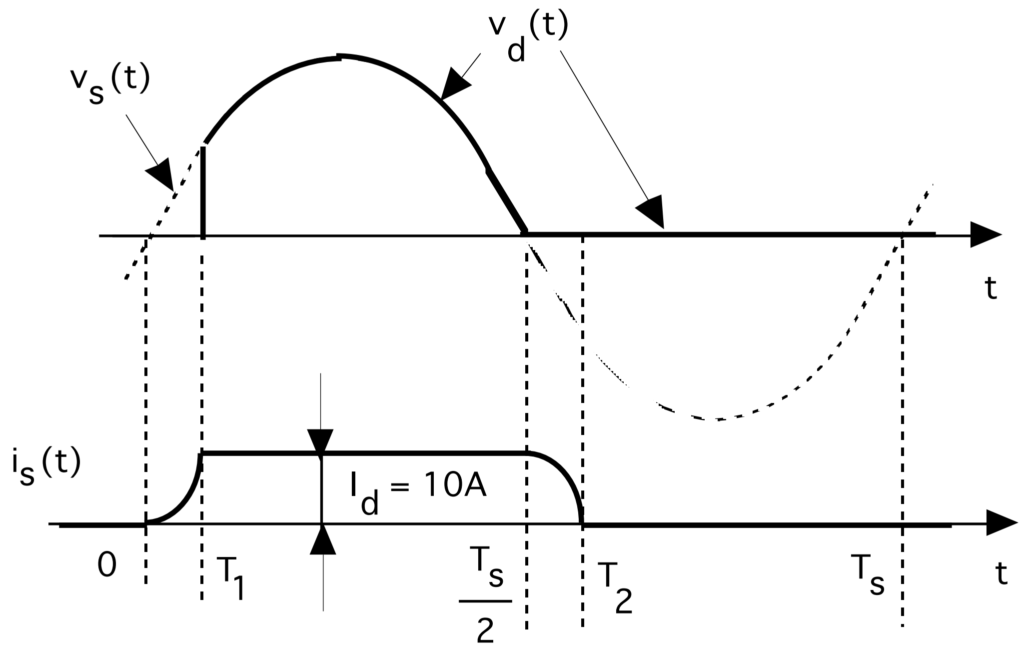
Solving for $f = \dfrac{300}{(12)(3\text{x}10^{-4})(2)(0.3)(3)} = 46.3$ kHz.

h) This lowest frequency does not depend on the length of the airgap. The derivation in part g) does not depend on the value of inductance and hence the length of the airgap.

Chapter 5 - Line-Frequency Diode Rectifiers

S5.1.

a) Voltage and current plots shown below.



b) For $v_s(t) < 0$, D_1 is off and D_2 conducts. Inductor current equals zero. As $v_s(t)$ crosses zero going positive, it takes a finite time for the current $i_s(t)$ to build up to 10 A. During this buildup time T_1, D_2 remains in conduction which keeps $v_d(t) = 0$, ie. creates a voltage notch.

As $v_s(t)$ crosses zero going negative, $v_d(t)$ will equal zero as diode D_2 turns on, so there will be no voltage notch. The average value of the the voltage waveform shown above with the voltage notch caused by L_s is smaller than the $L_s = 0$ waveform.

c) During time interval T_1 ; $v_L = L_s \frac{di_L}{dt}$; $\int_0^{I_d} di_L = I_d = \frac{1}{\omega L_s} \int_0^{\omega T_1} \sqrt{2}V_s \sin(\omega t)d(\omega t)$

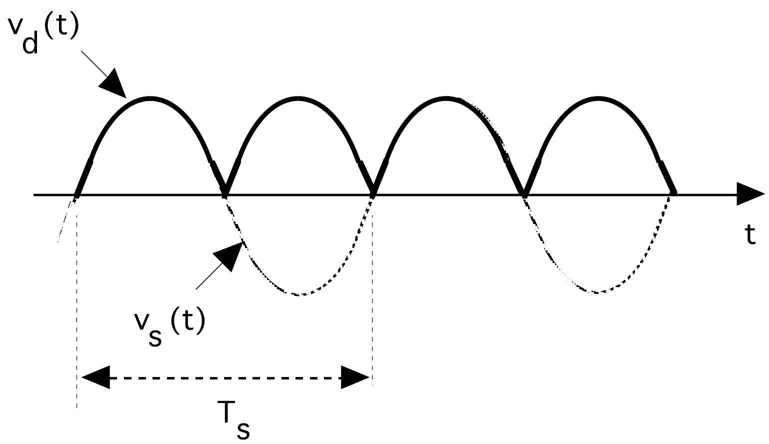
$\omega L_s I_d = \sqrt{2}\, V_s\{1 - \cos(\omega T_1)\}$; Put in numbers and solve: $\omega T_1 = 17.9$ degrees

$V_d = \frac{1}{2\pi} \int_{\omega T_1}^{\pi} \sqrt{2}V_s \sin(\omega t)d(\omega t)$ which evaluates to $V_d = 101$ V.

$P_d = (10A)(101\ V) = 1010$ watts

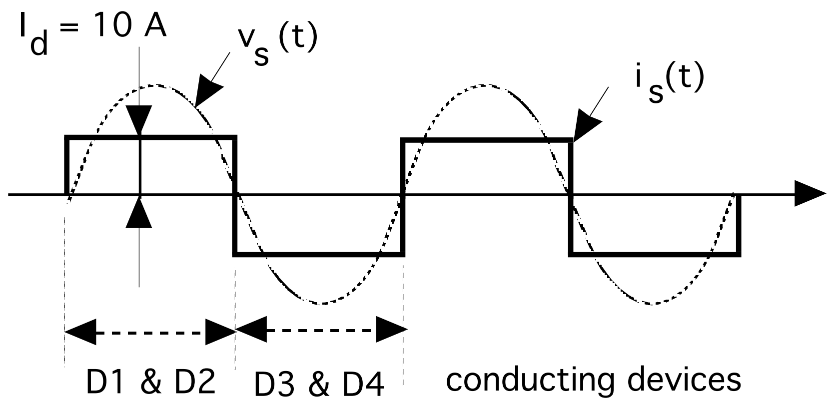
S5.2.

a) Waveform shown below.



b) $V_{do} = \frac{2}{T_s} \int_0^{T_s/2} \sqrt{2} V_s \sin(\omega t) dt = \frac{2\sqrt{2} V_s}{\omega T_s} \{\cos((0) - \cos(\omega T_s/2)\} = \frac{2\sqrt{2} V_s}{\pi}$

c) $i_s(t)$ = current supplied by the grid. Current plotted below



Diodes numbered per Fig. 5-10 in text.

d) $I = \sqrt{\frac{2}{T_s}\int_0^{T_s/2} I_d^2\, dt} = I_d$

e) $i_s(t)$ is an odd function of time. Hence even numbered harmonics are zero.

$I_{s1,peak} = \frac{2}{\pi}\int_0^{\pi} I_d \sin(\omega t) d(\omega t) = \frac{4\, I_d}{\pi}$; $I_{s1,rms} = I_{s1} = \frac{I_{s1,peak}}{\sqrt{2}} = \frac{4\, I_d}{\sqrt{2}\pi}$

f) Power Factor $PF = \frac{P}{I_s V_s}$; $P = V_s I_{s1}$ average power at fundamental frequency delivered by grid. $V_s I_s$ = rms power delivered by the grid.

$PF = \frac{I_{s1}\, V_s}{I_s\, V_s} = \frac{4 I_d}{\sqrt{2}\pi I_d} = 0.9$

g) $P_d = I_d V_d = (10A)\frac{2\sqrt{2}(230v)}{\pi} = 2.07$ kW ; $P_s = I_{s1} V_s = \frac{(4)(10A)(230V)}{\sqrt{2}\pi} = 2.07$ kW

h) The harmonics in the line current cause higher line currents and thus losses than if just the fundamental component was flowing. The harmonics add to the apparent power, just as a phase angle, making the apparent power larger than the active power.

S5.3.

a) $I_{sc} = \frac{V_s}{\omega L_s} = \frac{230}{(2\pi)(50)(1.22\text{x}10^{-3})} = 600$ A

b) $\frac{I_d}{I_{sc}} = \frac{10}{600} = 0.016$; From Fig. 5-17 $\frac{V_d}{(0.9)V_s} = 1.42$; $V_d = (1.42)(0.9)(230) = 294$ V

$P_s = P_d = I_d V_d = (10A)(294\text{ V}) = 2.94$ kW

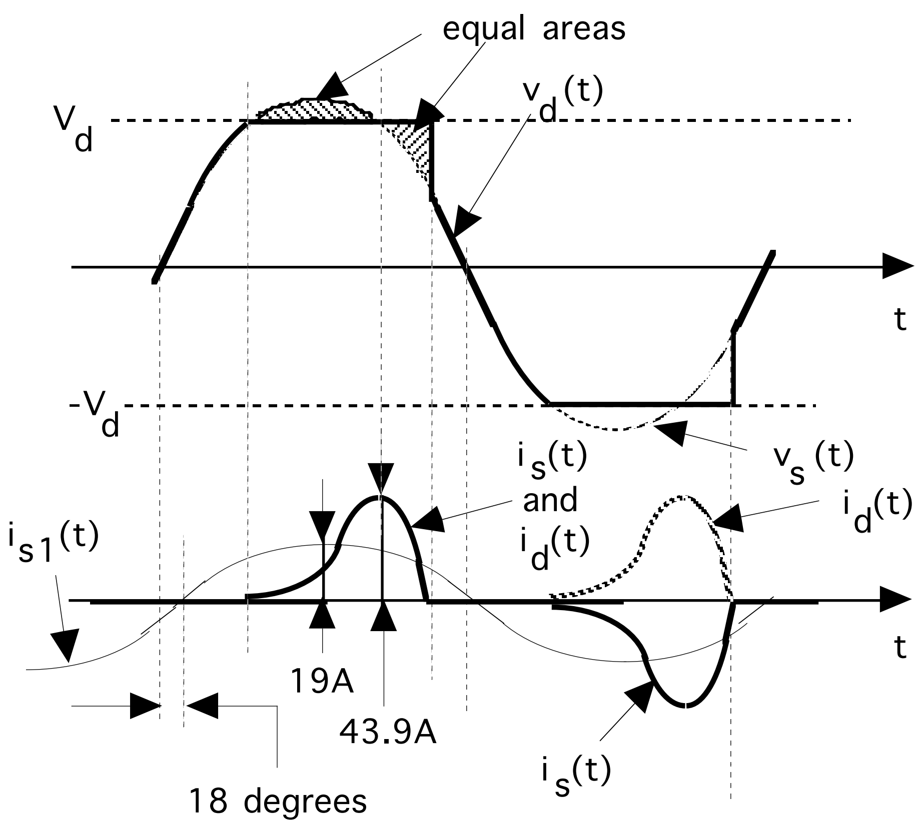
c) $S = \frac{P_d}{PF}$; From Fig. 5-18, Pf = 0.7 ; $S = \frac{2.94\text{kVA}}{0.7} = 4.2$ kVA

d) The line current is highlyh non-sinusoidal. Only the fundamental component of the line current can supply real power, the rest is reactive power. Even if the fundamental component of the line current is almost in phase with the line voltage, the need for apparent power will be high due to the highly non-sinusoidal line current, or the high content of the harmonic components of the line current.

e) $S = V_s\, I_s$; $I_s = \frac{4200\text{W}}{230\text{V}} = 18.3$ A ; From Fig. 5-19, the Crest Factor = 2.4 $\equiv \frac{I_{s,peak}}{I_s}$

$I_{s,peak} = (2.4)(18.3) = 43.9$ A

f) No ripple in $v_d(t)$ so it is constant at the value V_d. From Fig. 5-18, Depletion Power Factor DPF = 0.95. $\cos(\phi) = 0.95$; $\phi = 18$ degrees ;



$P = V\, I_{s1} \cos(\phi) = 2940$ W ; $I_{s1} = \frac{2940}{(230)(0.95)} = 13.46$ A ; $I_{s1,peak} = \sqrt{2}\; I_{s1} = 19$ A

S5.4.

g) $I_s = 18.3$ A ; Fuses are rated at 16 A. They may blow in this application.
Put a resistor of about 0.5 ohm in series with the line. may have to consider losses and cooling of the resistor.

Alternative is to use a series inductor with $\omega L = 0.5$ ohms or $l = 1.2$ mH. Larger and more expensive than a resistor but no losses.

h) P = 2.94 kW in an ohmic load ; $I_s = \frac{2940W}{230V} = 12.8$ A

16 A fuse will not blow. If a 13 a fuse were available, it also would not blow.

S5.5.

a) v_d and the line currents are shown in Fig. 5-32 in the text.

b) $$V_{do} = \frac{6}{\pi}\int_0^{\pi/6} \sqrt{2}V_{LL}\sin(\omega t)d(\omega t) = \frac{3\sqrt{2}V_{LL}}{\pi}$$

c) $$I_r = \sqrt{\frac{1}{T}\int_0^T i^2\, dt} = \sqrt{\frac{2}{T}\int_0^{(T/2)(2/3)} I_a^2\, dt} = \sqrt{\frac{2}{3}}I_a = 81.6 \text{ A}$$

d) $$I_{r1} = \frac{1}{\sqrt{2}}\frac{2}{\pi}\int_{\pi/6}^{5\pi/6} I_a\sin(\omega t)\, d(\omega t) = 0.78\, I_a$$

e) Eq. 3-33 of text. $I_s^2 = I_{s1}^2 + I_{s2}^2 + I_{s3}^2 + \ldots$; Now $I_{s2}^2 + I_{s3}^2 + \ldots$ can not be negative.

Thus $I_s > I_{s1}$ if $I_{sh} > 0$

f) The main advantage is a smoother dc voltage. Also the line current has less harmonic content (triple harmonics are missing).

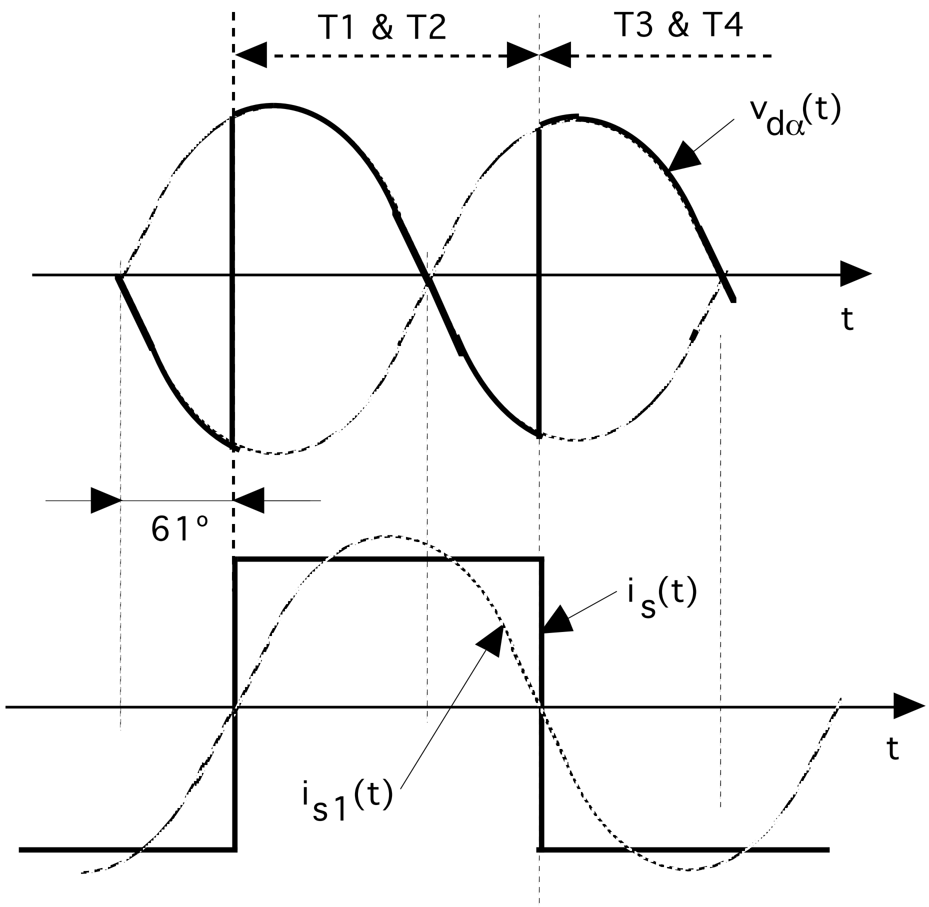
Chapter 6 - Line Frequency Phase-Controlled Rectifiers and Inverters

S6.1.

a) The thyristor will conduct when the voltage across it is in the forward direction and a gate current is provided, usually as a pulse.
The thyristor will stop conducting when the current through it crosses zero and tries to go negative.

b) This result is proven in the text, Eq. (6-6) with the aid of Fig. 6-6.

c) $(0.9)(320)\cos(\alpha) = 100$; $\cos(\alpha) = 0.48$; $\cos^{-1}(0.48) = 61.2$ degrees



d) Waveforms plotted below. Thyristors numbered as in Fig. 6-5.

e) $I_s = \sqrt{\frac{2}{T}\int_0^{T/2} I_d^2\, dt} = I_d = 10$ A rms; $i_s(t)$ is an odd function if $\omega t = \alpha$ is set as the time origin. Then from Table 3-1 in the text:

$$b_1 = \frac{2}{\pi}\int_0^{\pi} I_d \sin(\omega t)\, d(\omega t) = \frac{4}{\pi} I_d : i_{s1}(t) = \sqrt{2}\,\frac{2\sqrt{2}}{\pi} I_d \sin(\omega t - \alpha)$$

f) The phase angle between $i_{s1}(t)$ and v_s is 61° or 1.07 radians lagging.

g) $S = V_s I_s = 2.3$ kVA ; $P_s = V_s I_{s1} \cos(\alpha) = 1004$ watts ; $P_d = (100V)(10A) = 1000$ watts
P_s is not quite equal to P_d because of numerical rounding in the calculations.

h) Commutation means a change in the path for the source current. When a source inductance is considered, the commutation takes a finite amount of time.
$v = L\frac{di}{dt}$; $\int_{t_1}^{t_2} v\, dt = L_c \Delta I$; t_1 is the commutation start time and t_2 is the end of the commutation interval. $t_2 - t_1 = t_c$ = commutation time.
As can be seen, the voltage-time integral needed is proportional to $L_c\Delta I$, the product of the commutation inductance and the change in the current.

i) Eq. (6-6) in the text. $V_{d\alpha} = 0.9\, V_s \cos(\alpha)$; Current change $\Delta I = 2I_d$.
A voltage-time integral equal to $2\, L_c\, I_d$ is "lost" every half-cycle.
The average over one half-cycle is $\Delta V_{average} = \frac{2}{T} 2\, L_c\, I_d$; $T = 1/f$ where f is the frequency.

$\Delta V_{average} = 4\, f\, L_c\, I_d$ which is the same as Eq. (6-25) in the text. $\omega = 2\pi f$.

$$\Delta V_{average} = \frac{2\,\omega\, L_c\, I_d}{\pi} = V_{d\alpha} - V_d ;$$

$$V_d = 0.9\, V_s \cos(\alpha) - \frac{2\,\omega\, L_c\, I_d}{\pi} = \sqrt{2}\,\frac{2\sqrt{2}}{\pi} V_s \cos(\alpha) - \frac{2\, X_c\, I_d}{\pi}$$

j) See Eqs. (6-20) to (6-23) on p. 131 of text.

k) $\frac{2}{\pi} X_s I_d = \frac{2}{\pi}\,\omega\, (5\mathrm{x}10^{-3})(10) = \frac{2}{\pi}(2\pi)(60)(0.005)(10) = 1.2$ V = "Lost" voltage ΔV_{dn}

$$\frac{2\sqrt{2}}{\pi} V_s \cos(\alpha) = 100 + 1.2 \;\; ; \cos(\alpha) = 0.488 \;\; ; \alpha = 60.7^o$$

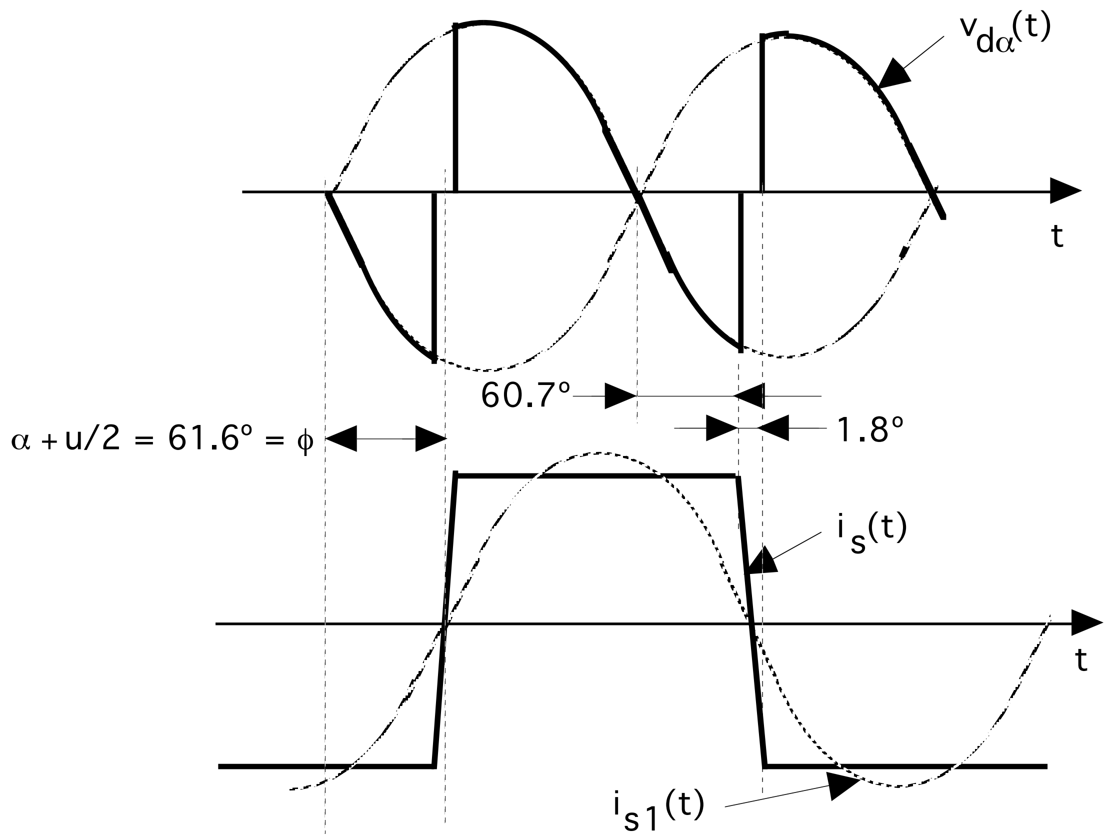
Trigger angle α must be smaller to provide 100V when the commutation inductor L_c is included compared to when L_c is assumed to be zero.

$$\cos(\alpha + u) = \cos(\alpha) - \frac{2\,\omega\, L_s\, I_d}{\sqrt{2}\, V_s} \quad ; \text{Eq. (6-24)}$$

$$\cos(\alpha + u) = 0.488 - \frac{(2)(2\pi)(60)(0.005)(10)}{\sqrt{2}(100)} = 0.488 - 0.0267 = 0.461$$

$$\alpha + u = 62.5^o \;\; ; u = 62.5 - 60.7 = 1.8^o$$

l)



S6.2.

a) With the gate currents constantly present, the thyristors behave as diodes. Hence the voltages V_d, V_{Pn}, and V_{Nn} are the same as shown in Fig. 5-32 of the text.

b) The currents in the thyristors will be the same as the currents in the diodes of Fig. 5-32.
c) A gate current is needed and there must be a positive voltage across the thyristor.

d) See Fig. 6-20 in the text.

e) $V_{do} = \frac{3\sqrt{2}}{\pi} V_{LL}$ (ideal diode rectification). Due to the firing delay of α radians, there is a voltage loss of A_α six times per line cycle or every π/3 radians..

$$A_\alpha = \int_0^\alpha \sqrt{2} V_{LL} \sin(\omega t)\, d(\omega t) = \sqrt{2}\, V_{LL}\{1 - \cos\alpha\}$$

Average voltage loss is $\frac{3\, A_\alpha}{\pi} = \frac{3}{\pi} \sqrt{2}\, V_{LL}\{1 - \cos\alpha\}$

$V_{d\alpha} = V_{do}$ - Average voltage loss $= \frac{3}{\pi} \sqrt{2}\, V_{LL}\cos\alpha$

f) $P_d = V_{d\alpha}\, I_d = \frac{3}{\pi} \sqrt{2}\, (230)\cos(60^\circ)\{10\} = 1.55$ kW ; $P_s = P_d$ since there are no losses.

g) The voltages will be the same as those shown in Fig. 6-20 of the text except that voltage transitions will not be as abrupt but instead will have a commutation plateau as shown in Fig. 6-25 or as shown in the solutions to Prob. S6.1. part l).

h) Due to the inductance, there is an additional voltage drop compared to the expression in Eq. 6-40 or the solution in Prob. S6.2. part e). There are six voltage drops per line cycle or every π/3 radians. The voltage-time integral is similar to that described in the solution to Prob. S.6.1. part h). Adapting those results to this situation, $\Delta I = I_d$ and $L_c = L_s$.

$$\Delta V_{average} = \frac{6}{T_s} L_s\, I_d = \frac{(3)(2\pi)f}{\pi} L_s\, I_d = \frac{3\omega}{\pi} L_s\, I_d$$

$$V_d = V_{d\alpha} - \Delta V_{average} = \frac{3}{\pi} \sqrt{2}\, V_{LL}\cos\alpha - \frac{3\omega}{\pi} L_s\, I_d$$

This is the same as Eq. (6-55) but derived in a different way to provide an alternative explanation.

i) $P_d = 1.55$ kW and $I_d = 10$ A ; $V_d = 155.25$ V

$$\frac{3\omega}{\pi} L_s\, I_d = \frac{(3)(2\pi)(50)}{\pi} (0.005\text{ H})(10\text{ A}) = 15\text{ V}$$

$1.35\, V_{LL}\cos\alpha = 155.25 + 15$; $\alpha = 56.75^\circ$

j) Eq. (6-62) ; $\cos(\alpha + u) = \cos\alpha - \frac{2\omega}{\sqrt{2} V_{LL}} L_s\, I_d$;

$$\frac{2\omega}{\sqrt{2}V_{LL}} \; L_s\, I_d = \frac{(2)(2\pi)(50)}{\sqrt{2}(230)} \; (.005)(10) = 0.097 \quad ;\alpha + u = 63.14^o \quad ; \quad u = 6.4^o$$

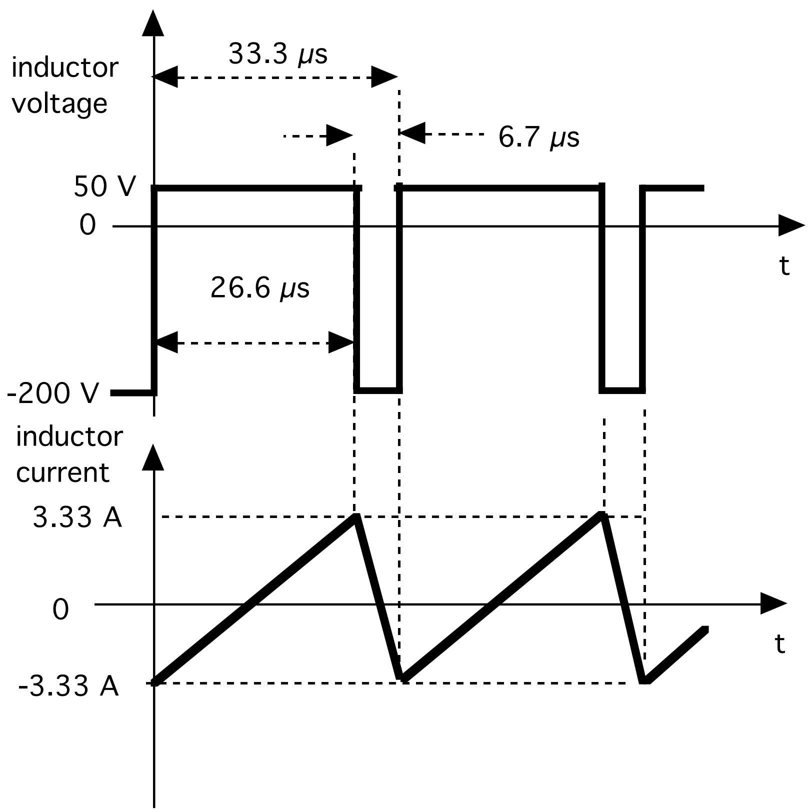
Chapter 7 - DC to DC Switch Mode Converters

S7.1.

a) $V_o = 200\text{ V} = (0.8)(V_d)$; Hence $V_d = \frac{200}{0.8} = 250\text{V}$

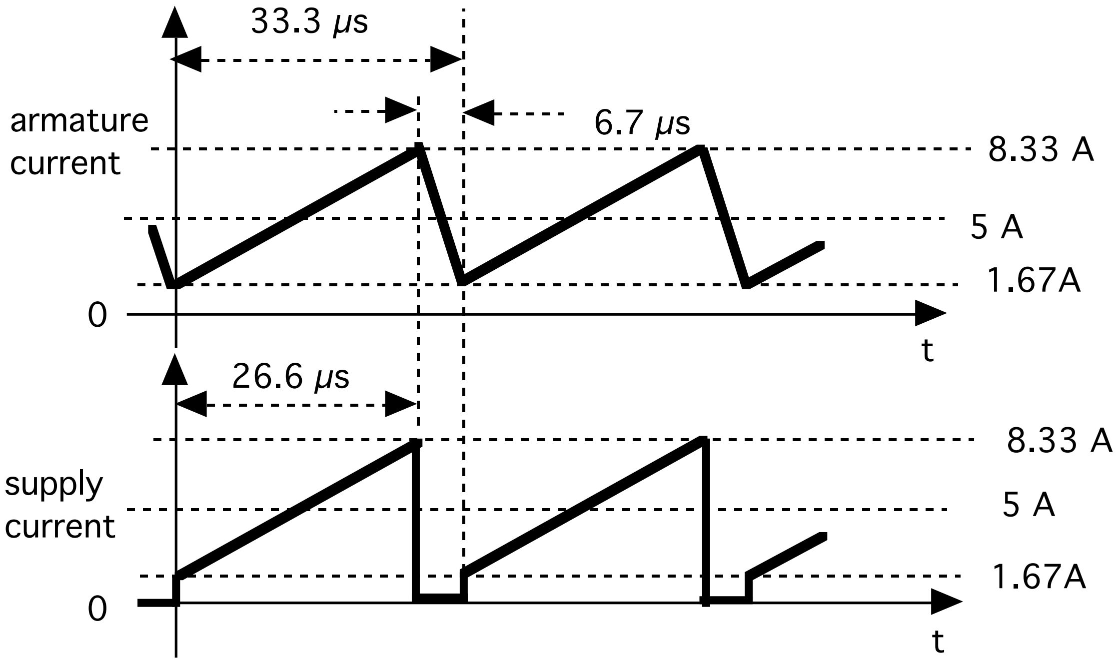
b) Voltage across the inductor is shown below.

Use $L\frac{di}{dt} = v$ to find the ripple current where v is the voltage across the inductor which is given above. Resulting ripple current shown below.



c) The armature current is composed of a dc current of 5 A and the 3.33 A base-to-peak ripple current shown above in part b). Thus the minimum current is 5 - 3.33 = 1.67 A. The peak current is 5 + 3.33 = 8.33 A.

d) The total armature current (dc plus ripple) is shown below.
The current i_d is equal to the armature current when the switch is closed and equal to zero when the switch is open. The resulting i_d is also shown below.



e) The base-to-peak ripple current is 3.33 A. Assuming that the armature voltage remains at 200 V, then the ripple current remains independent of the dc armature current. Hence the boundary between continuous and discontinuous conduction occurs at a dc current of 3.33 A.

f) Armature voltage $V_o = \dfrac{V_d D^2}{D^2 + [2I_o/(T V_d)]}$: see Eqs. (7-7) and (7-17) in text.

Input voltage V_d = 250 V, I_o = 2A. D = 0.8, T = 33.3 μsec (30 khz)

Evaluating $V_o = \dfrac{(250)(0.8)^2}{(0.8)^2 + (2)(2)/[(33.3\mu s)(250)]} = 239$ V

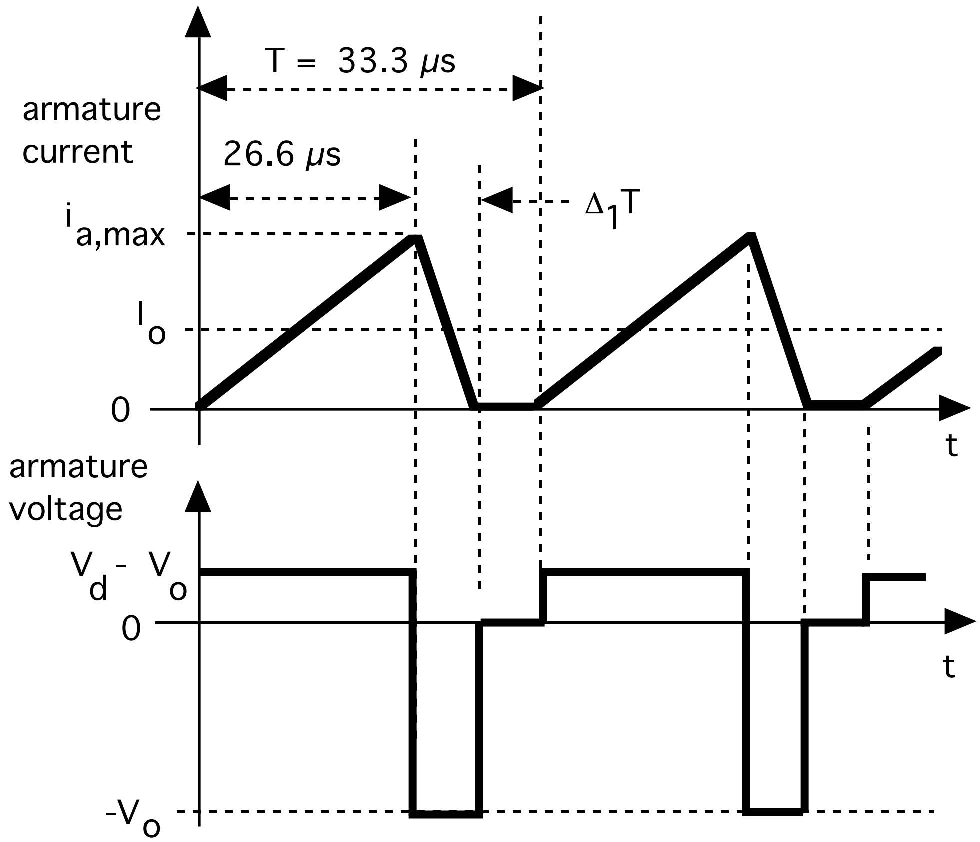
g), h) and i) Waveforms shown on next page.

Peak armature current $i_{a,max} = \dfrac{2 V_o I_o}{D V_d}$: see Eqs. (7-7), (7-11) and (7-16) in text.

Evaluating $i_{a,max} = \dfrac{(2)(2)(239)}{(0.8)(250)} = 4.8$ A

$\Delta_1 T = \dfrac{2 I_o L}{D V_d}$; See Eqs.(7-7) and (7-16) in text.

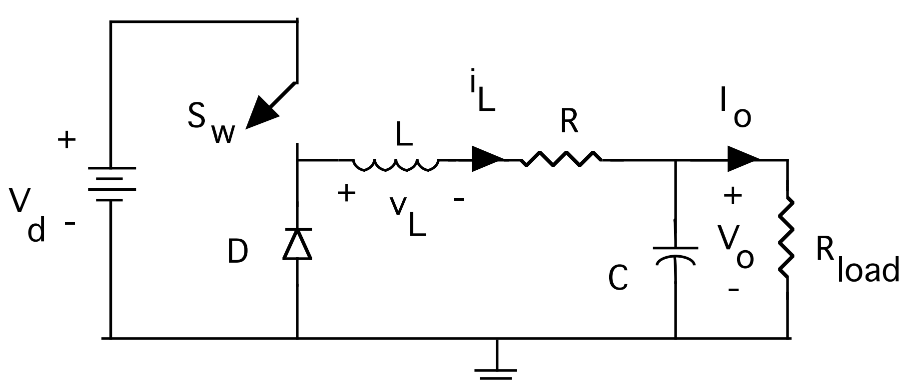
Evaluating $\Delta_1 T = \dfrac{(2)(2)(0.0002)}{(0.8)(250)} = 4\ \mu\text{sec}$



S7.2.

a) Rated output power = (20V)(25A) = 500 W

b) Equivalent load resistance $R_{load} = V_oI_o$ = 20V/25A = 0.8 Ω
Converter circuit shown below.



c) In an ideal step-down converter (R = 0),$V_o = DV_d$. With nonzero R, this is must be modified to $DV_d = V_o + I_oR$. Putting in numbers and solving for D gives

$20 \text{ V} + (25)(0.2) = 25 = D V_d = 48D$; $D = 25/48 = 0.521$

Note: No switching frequency specified in problem statement. Assume 50 kHz switching frequency for the rest of this problem.

d) At the boundary between continuous and discontinuous conduction, the average inductor current (dc load current I_o) is one-half the peak current in the inductor.

When the switch is closed:

$$L\frac{di_L}{dt} \approx L\frac{2I_o}{DT} = V_d - V_o - I_oR \text{ ; } T = \text{period of switching waveform.}$$

$$\text{Solving for } I_o = \frac{(V_d - V_o)DT}{1+RDT/(2L)}$$

When switch is open:

$$L\frac{di_L}{dt} \approx L\frac{2I_o}{(1 - D)T} = V_o + I_oR \text{ ; Solving for } I_o = \frac{V_o(1 - D)\,T}{2L[1 - RT(1 - D)/(2L)]}$$

Set the two expressions for I_o equal to each other and solve for V_o yields:

$$V_o = DV_d\left\{1 - \frac{R(1 - D)T}{2L}\right\} :$$

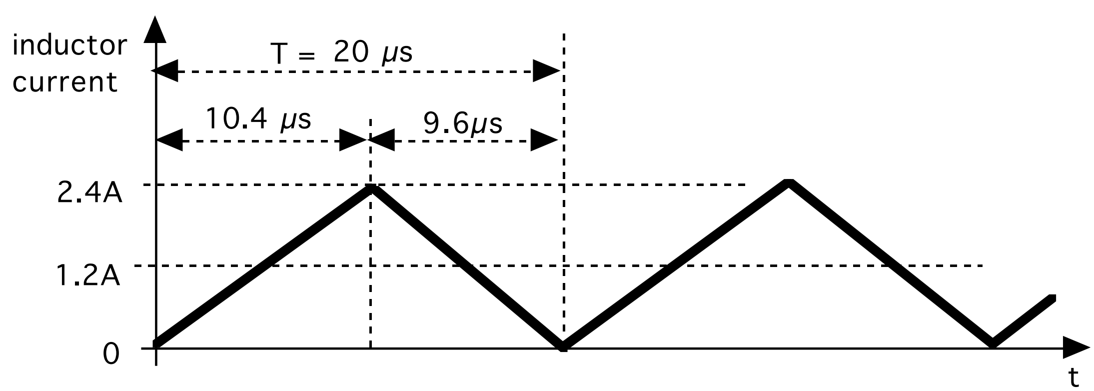
$$\text{Evaluating } V_o = (0.52)(48)\left\{1 - \frac{(0.2)(1 - 0.52)(2\text{x}10^{-5})}{(2)(10^{-4})}\right\} = 25 \text{ V.}$$

Put V_o = 25 V into expression for I_o (S_w open) and evaluate;

$$I_o = \frac{(48 - 25)(0.52)(2\text{x}10^{-5})}{1 + (0.2)(2\text{x}10^{-5})/[(2)(10^{-4})]} = 1.2 \text{ A}$$

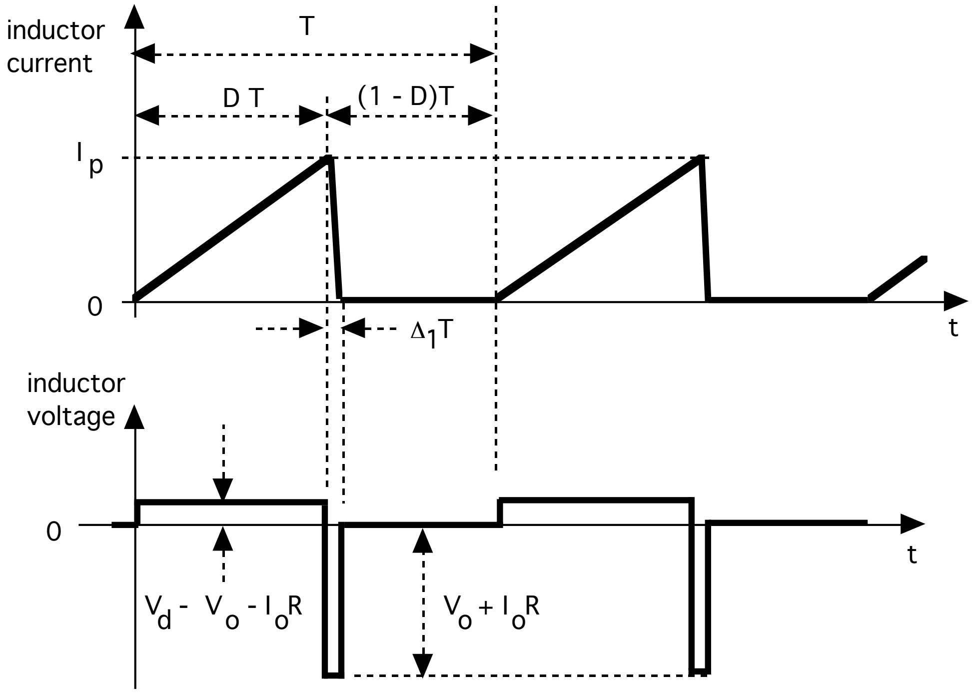
Power $P_o = (25)(1.2) = 30$ watts ; Equivalent load resistance = 25V/1.2A = 20.8 ohms

e) Inductor current at edge of continuous conduction (part d) shown below. Peak inductor current is equal to $2I_o = 2.4\text{A}$.



f) $V_o I_o = P_o = 1$ Watt

Converter will operate in the discontinuous mode with the inductor voltage and current waveforms taking on the following forms.



$$I_p = \frac{(V_d - V_o - I_o R)DT}{L} \quad ;$$

$(V_d - V_o - P_o R/V_o)DT = \Delta_1 T(V_o + P_o R/V_o)$; average inductor voltage = 0

$$I_o = \frac{I_p(DT/2 + \Delta_1 T/2)}{T} = \frac{I_p(D + \Delta_1)}{2} = \frac{P_o}{V_o}$$

$$\frac{P_o}{V_o} = \frac{(D + \Delta_1)}{2} \frac{(V_d - V_o - \frac{P_o}{V_o}R)DT}{L}$$

$$V_o^2 - V_d V_o + P_o R + \frac{2P_o L}{(D + \Delta_1)DT} = 0 \; ; \; V_o = \frac{V_d}{2} + \frac{1}{2}\sqrt{V_d^2 - 4P_o R\left[1 + \frac{2L}{R(D + \Delta_1)DT}\right]}$$

Evaluate V_o assuming $\Delta_1 << D$.

$$V_o = \frac{48}{2} + \frac{1}{2}\sqrt{(48)^2 - (4)(0.2)\left[1 + \frac{(2)(10^{-4})}{(0.2)(0.52)^2(2x10^{-5})}\right]} = 47.1 \text{ V.}$$

$I_o = \frac{1W}{47.1V} = 21$ mA : Equivalent load resistance $= \frac{47.1V}{0.021A} = 2{,}218$ ohms.

$$I_p = \frac{(0.021)(2)}{0.52} \approx 84 \text{ mA}$$

Check assumption that $\Delta_1 << D$.

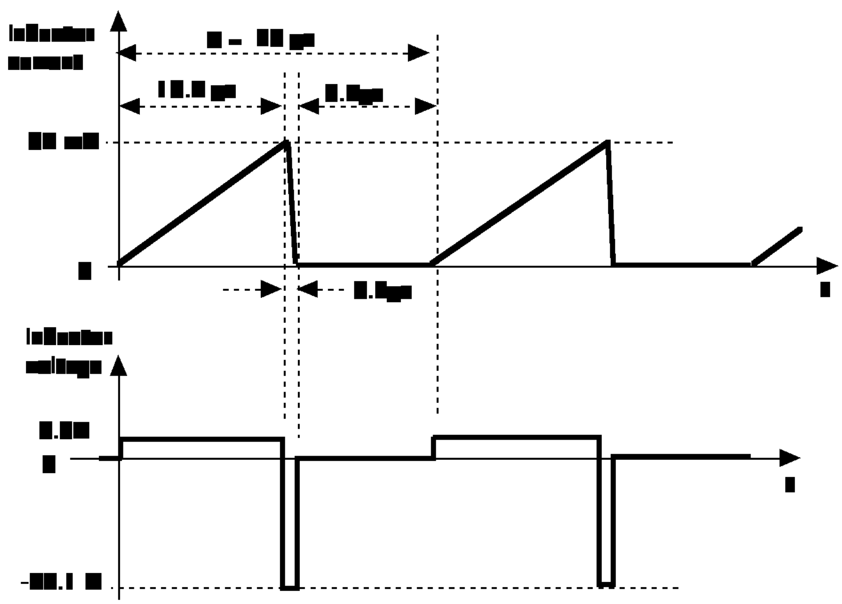
$$\Delta_1 = \frac{(V_d - V_o - P_o R/V_o)D}{(V_o + P_o R/V_o)} = \frac{[48 - 47.1 - (0.2)(0.021)](0.52)}{(47.1 + (0.2)(0.021))} = 0.01 << D = 0.52$$

$\Delta_1 T = (10^{-2})(2x10^{-5}) = 0.2\ \mu\text{sec}$

When switch is closed, inductor voltage = $(V_d - V_o - P_o R/V_o) = (48 - 47.1 - 0.2/47.1) = 0.9$ V

When switch is open, inductor voltage = $V_o + P_o R/V_o = 47.1 + 0.2/47.1 = 47.1$ V

g) Inductor voltage and current shown below for case where $P_o = 1$ watt



h) Switch on;

$$L\frac{di_L}{dt} = V_d - V_o - i_L R \ ; \ i_L(t) = A + B\,e^{-t/\tau} \ ; \ \tau = L/R = 10^{-4}/0.2 = 5x10^{-4} \text{ sec}$$

$$i_L(0) = 0 = A + B \ ; \ L\frac{di_L(0)}{dt} = -BL/\tau = -BR = (V_d - V_o) \ ; \ B = -(V_d - V_o)/R$$

$$i_L(t) = \frac{V_d - V_o}{R}\{1 - e^{-t/\tau}\} \ ;$$

$$i_L(10.4\mu s) = \frac{48 - 47.1}{0.2}\{1 - e^{-(10.4\mu s/500\mu s)}\} = 4.5(1 - 0.9792) = 93 \text{ mA}$$

Switch off:

$$L\frac{di_L}{dt} = -V_o - i_L R \ ; \ i_L(t) = A + B\,e^{-t/\tau} \ ; \ \tau = L/R = 10^{-4}/0.2 = 5x10^{-4} \text{ sec}$$

$$i_L(0) = 93 \text{ mA} = A + B \ ;$$

$$L\frac{di_L(0)}{dt} = -BL/\tau = -BR = -V_o - i_L(0)R = -47.1 - (0.093)(0.02) = -47.1 ;$$

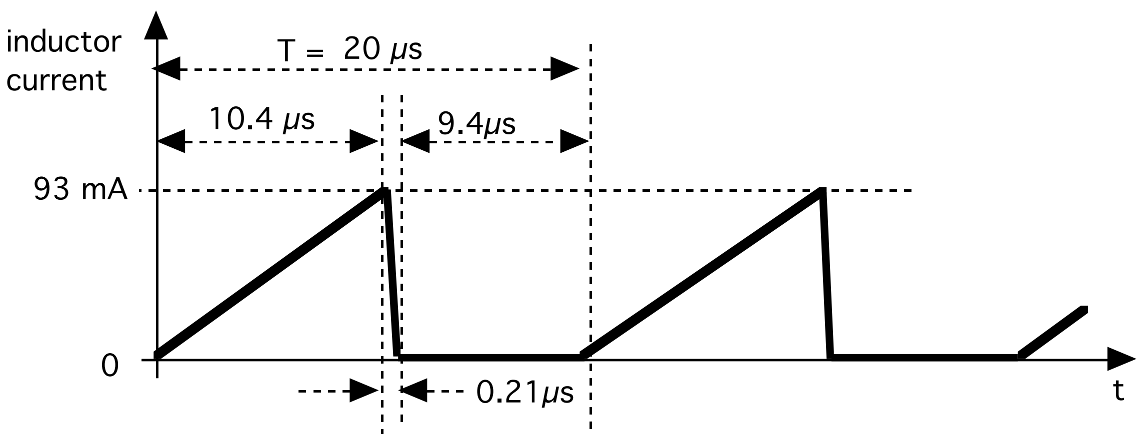
$B = 47.1/0.2 = 235.5$; $A = -235.407$

$i_L(t) = -235,4 + 235,5\ e^{-t/\tau}$; At $t = \Delta_1 T$, inductor current = 0.

$0 = -235.4 + 235.5\exp(-\Delta_1 T/\tau)$;

$\Delta_1 T = \tau \ln\{235.5/235.4\} = (5x10^{-4})(4.25x10^{-4}) = 0.21\ \mu sec$

i) Accurate inductor current plot shown below.



j) The inductor current versus time curves from part g) and part i) are nearly identical.

S7.3

a) Advantages
Bipolar PWM has a more simple circuit implementation. Only one control voltage and comparator needed in addition to the triangular wave generator.
Unipolar PWM has twice the ripple frequency as bipolar PWM and hence lower rms ripple component.

Disadvantages.
Bipolar PWM has half the ripple frequency compared to unipolar PWM and hence higher rms ripple component.
Unipolar PWM has more complicated circuitry because two control voltages are needed and thus two comparators, plus the triangle wave generator.

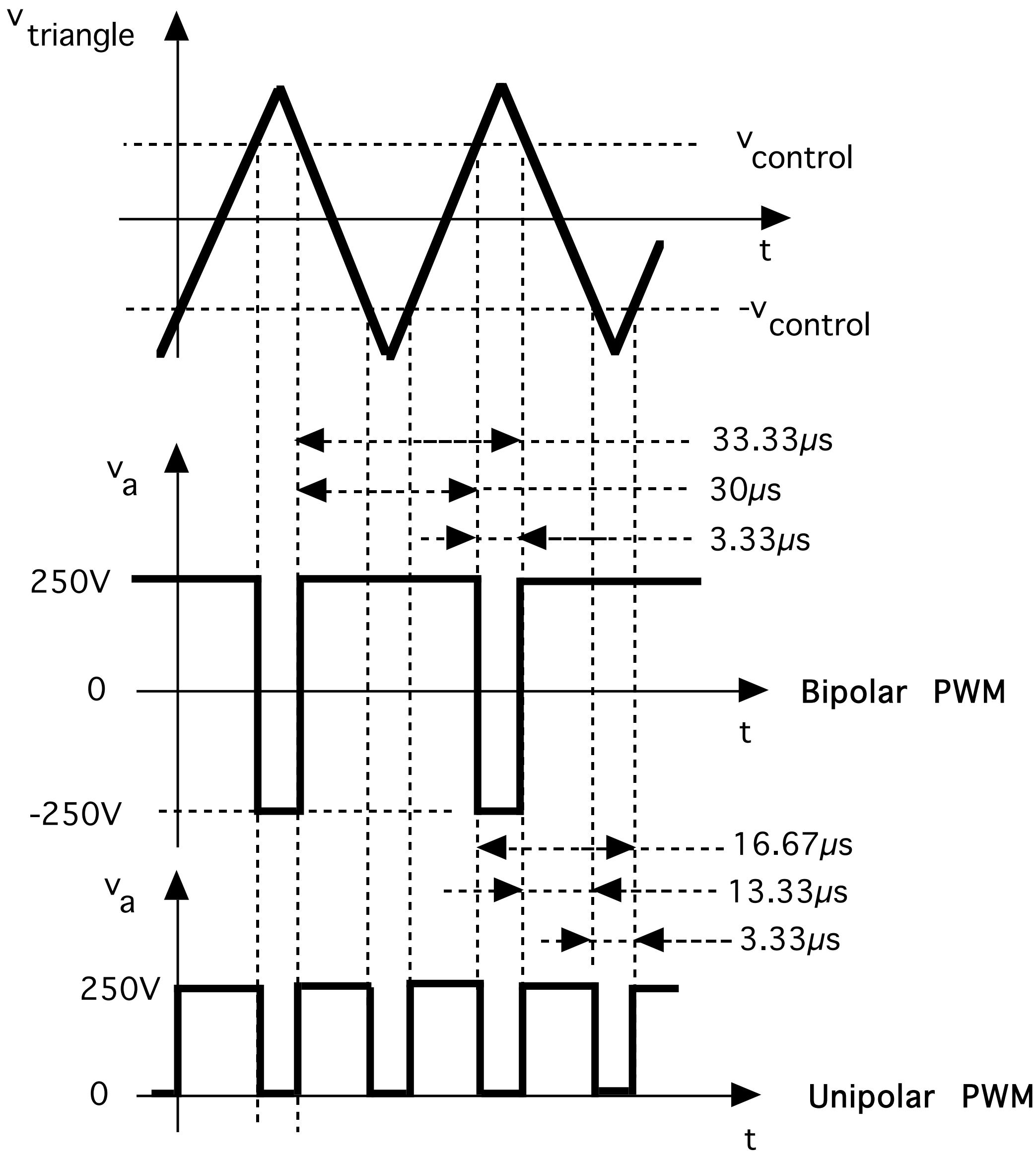
b) For both types of PWM, $V_o = (2D_1 -1))V_d$;

$$D_1 = \frac{V_o + V_d}{2V_d} = \frac{200 + 250}{(2)(250)} = 0.9 ; D_2 = 1 - D_1 = 1 - 0.9 = 0.1$$

Bipolar PWM ripple frequency = 30 kHz.

Unipolar PWM ripple frequency = 60 kHz

Waveforms for output voltage shown below.



c) Armature voltage = 50 V for 13.33μs.

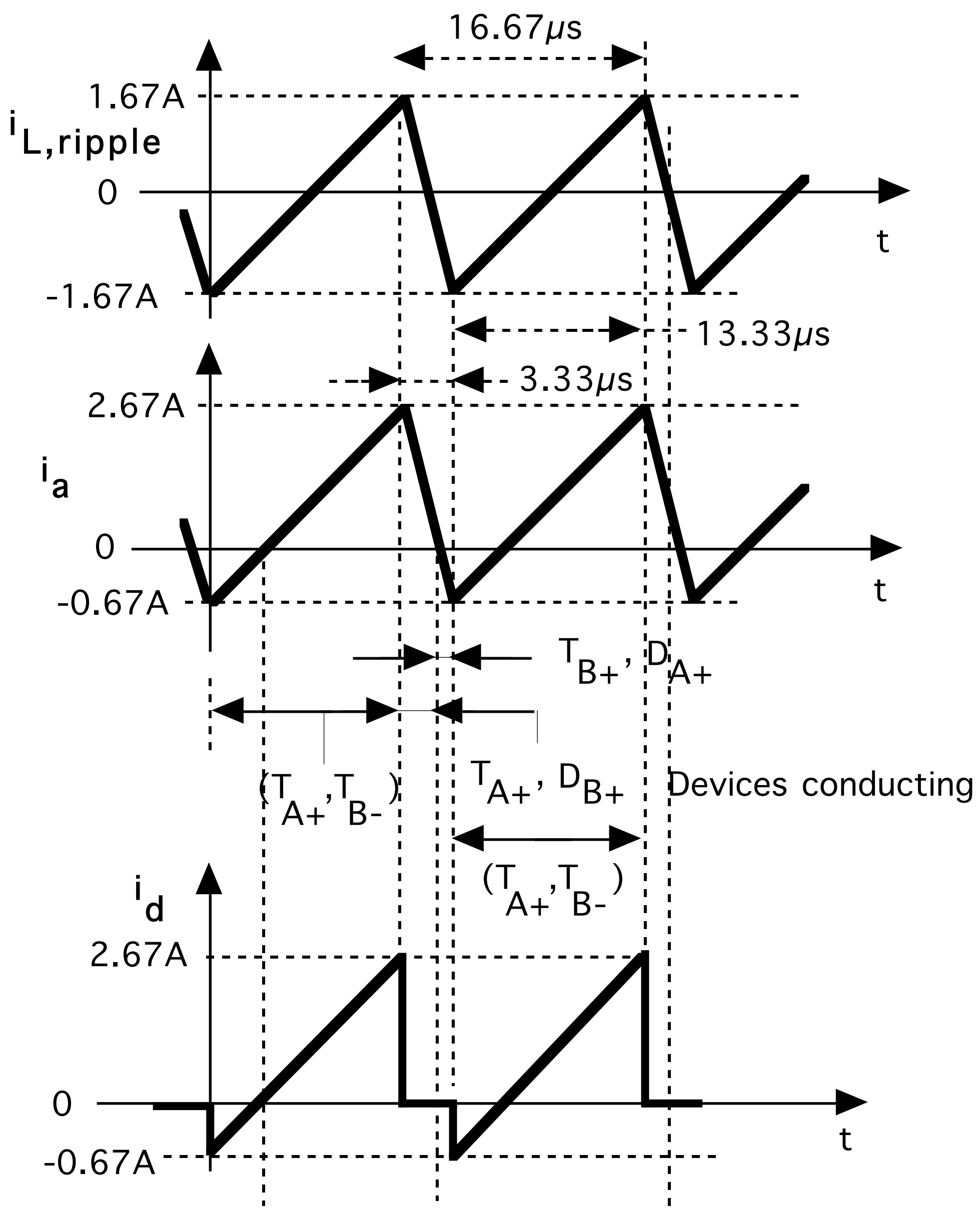
Total change in inductor (armature) current = $\frac{(250\text{v} - 200\text{V})(13.33\mu\text{s})}{(0.2\text{mH})}$ = 3.33 A

Ripple current has zero average as shown on next page and thus goes from -1.67A to 1.67A

Total armature current must have 1 A average. Thus total armature current is obtained by adding 1 A average (dc) current to the ripple current. Resulting instantaneous armature current shown on next page.

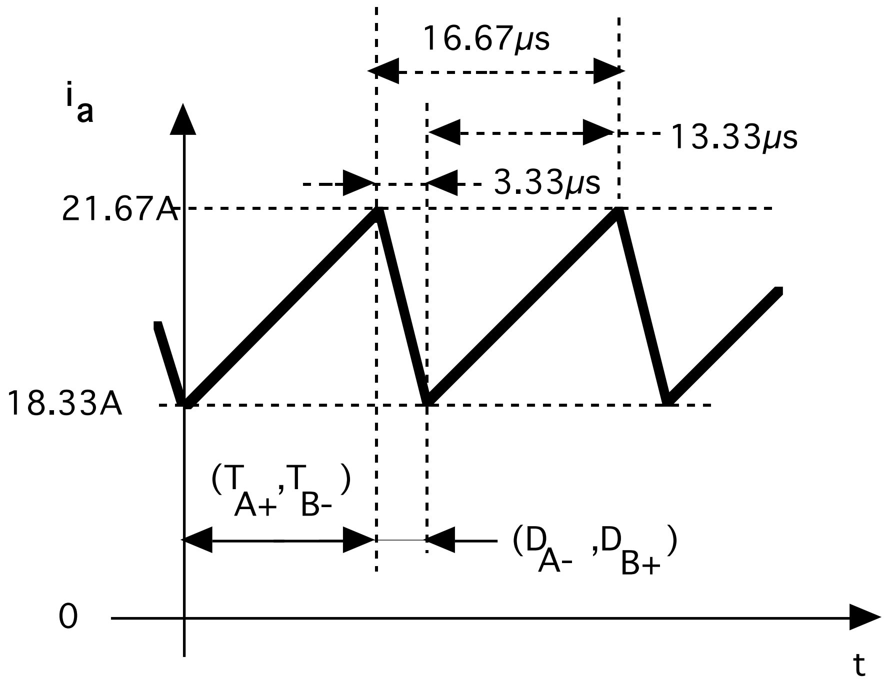
Maximum instantaneous armature current = 1.67 + 1 = 2.67 A.
Minimum instantaneus armature current = 1 -1.67 = -0.67 A

The source current $i_d(t)$ equals the armature current $i_a(t)$ when $v_a > 0$. When $v_a = 0$, $i_d = 0$ and armature current circulates through the upper or lower switches. The plots also indicate which power semiconductor devices are conducting at a given time. Refer to Fig. 7-29 of the text for further details.



d) If the average armature current I_a = 20 A, ripple current rides on top of a 20 A dc current. Hence plot of instantaneous armature current will be as shown on next page.
Maximum instantaneous armature current = 20+1.67 = 21.67 A
Minimum instantaneous armature current = 20 -1.67 = 18.33A

Instantaneous armature current never goes negative. D_{A+} and D_{B-} never conduct. Similarly T_{A-} and T_{B+} never conduct. Devices which do conduct and when indicated on the armature current plot on next page.



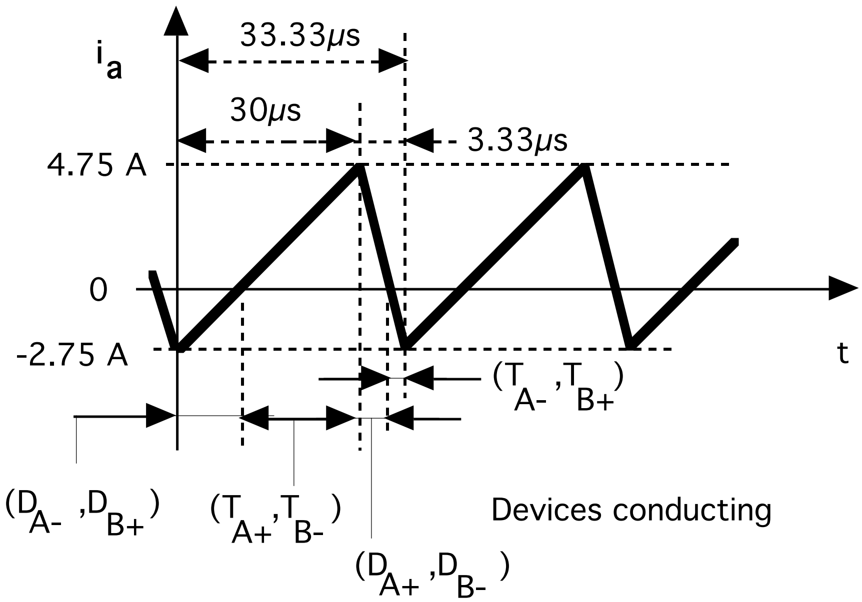
e) Armature voltage = 250 V for 30 μs. Resulting change in the armature current

$\Delta i_a = \dfrac{(250V - 200V)(30\mu s)}{0.2mH}$ = 7.5 A. Armature ripple current will have zero average and a base-to-peak amplitude of 7.5/2 = 3.75 A. Total instantaneous armature current will be composed of the ripple current riding on top of 1A of dc current.

Maximum instantaneous armature current = 1 + 3.75 = 4.75A

Minimum instantaneous armature current 1 - 3.75 = -2.75 A

Armature current, which is also equal to the source current $i_d(t)$, is shown below. The status of conduction in the semiconductor power switches is also indicated.



S7.4.

a) **Note: No switching frequency specified in problem statement. Assume 50 kHz.**

At the edge of CCCM-DCM:

$V_o = DV_d$. $D = \frac{10V}{40V} = 0.25$. Average output current $I_o = \frac{DT_s(V_d - V_o)}{2L}$

$$I_o = \frac{(0.25)(2x10^{-5})(40 - 10)}{(2)(5x10^{-5})} = 1.5 \text{ A}$$

b) For $I_o = 1.5A/10 = 0.15$ A, converter operating in DCM. Output voltage given by

$V_o = \frac{V_d D^2}{D^2 + \frac{2I_o L}{T_s V_d}}$: Eqs. (7-7) and (7-17) ; Solving for $D = \sqrt{\frac{2I_o L V_o}{T_s V_d (V_d - V_o)}}$

Evaluating: $D = \sqrt{\frac{(2)(0.15)(5x10^{-5})(10)}{(2x10^{-5})(40)(40 - 10)}} = 0.079$

c) $V_o(0.15 + 1\%) = \frac{(40)(0.079)^2}{(0.079)^2 + \frac{(2)(0.1515)(5x10^{-5})}{(2x10^{-5})(40)}} = 9.91$ V

$$V_o(0.15\ \text{-}1\%) = \frac{(40)(0.079)^2}{(0.079)^2 + \frac{(2)(0.1485)(5\text{x}10^{-5})}{(2\text{x}10^{-5})(40)}} = 10.06\ \text{V}$$

d) For a duty cycle of 25%, V_o is independent of I_o and thus appears as an ideal voltage source of 10 V. For a duty cycle of 7.9%, the voltage changes by 10.06 -9.91 = 0.015 V for a current change of 3 mA. (0.015V/(.003A) = 5 ohms. Converter appears as an ideal voltage source of 10V in series with a 5 ohm resistor.

S7.5.

a) 1kW = (48V)I_o ; I_o = 20.83 A ; $D = \frac{48}{100} = 0.48$; switch on-time $= \frac{0.48}{8\text{x}10^4} = 6\text{x}10^{-6}$ sec

$$\Delta i_L = \frac{(100\text{V} - 48\text{V})(6\text{x}10^{-6}\ \text{sec})}{4\text{x}10^{-5}\ \text{H}} = 7.8\ \text{A}$$

Inductor current is composed of a dc component of 20.83 A and a 3.9 A (base-to-peak, zero average) ripple current.

$I_{L,peak}$ = 20.83 + 3.9 = 24.73 A;

$$I_{L,rms} = \sqrt{I_o^2 + <(I_{L,ripple})^2>}\ ;\ <(I_{L,ripple})^2> = \frac{(3.9)^2}{3} = 5.07\ \text{A}^2$$

$$I_{L,rms} = \sqrt{(20.83)^2 + 5.07} = 20.95\ \text{A}$$

b) For L = 10 μH, the peak-to-peak ripple current is for times that of part a). Thus Δi_L = 31.2 A

$$I_{L,peak} = 20.83 + 15.6 = 36.43\ \text{A}\ ;\ <(I_{L,ripple})^2> = \frac{(15.6)^2}{3} = 81.1\ \text{A}^2$$

$$I_{L,rms} = \sqrt{(20.83)^2 + 81.2} = 22.7\ \text{A}$$

c) $$\frac{\text{cost } 10\mu\text{H}}{\text{cost } 40\mu\text{H}} = \frac{[10^{-5}][22.7][36.43]}{[4\text{x}10^{-5}][20.95][24.73]} = 0.399\ ;\ \text{Inductor ratio} = 0.25$$

d) Inductor = 2.5 μH ; Δi_L = 4 times larger than for 10 μH; Hence Δi_L = (31.2 A)(4) = 125 A

$$I_{L,peak} = 20.83 + 125/2 = 83.3\ \text{A}\ ;\ <(I_{L,ripple})^2> = \frac{(62.5)^2}{3} = 1302\ \text{A}^2$$

$$I_{L,rms} = \sqrt{(20.83)^2 + 1302} = 41.7\ \text{A}$$

$$\frac{\text{cost } 2.5\mu\text{H}}{\text{cost } 40\mu\text{H}} = \frac{[2.510^{-6}][41.7][83.3]}{[4\text{x}10^{-5}][20.95][24.73]} = 0.42\ ;\ \text{Inductor ratio} = 0.25$$

Chapter 8 - Switch-Mode DC-AC Inverters

S8.1.

a) Frequency modulation ratio $m_f = \frac{1450}{50} = 29$.

b) $m_a = 0.8$; $V_{o,peak} = (0.8)(600/2) = 240$ V ; $V_{o1} = \frac{240}{\sqrt{2}} = 170$ V.

c) See Eqs. (8-6) and (8-7) in the text.

d) Modulation is linear when $m_a \leq 1$ because $v_{Ao}(t) = m_a \frac{V_d}{2} \sin(\omega t)$.

e) Use Table 8-1 in text with $m_a = 0.8$. Amplitudes will be rms values

Fundamental = 50 Hz Amplitude = 170 V (see part b above)

$h = m_f$ (50x29 = 1450 Hz) Amplitude $= \frac{1}{\sqrt{2}}(0.818)(300) = 173.5$ V

$h = 3m_f \pm 2$; (3x29±2)x50 = 4,450 Hz and 4,250 Hz ; Amplitude $= \frac{1}{\sqrt{2}}(0.176)(300) = 37.3$V

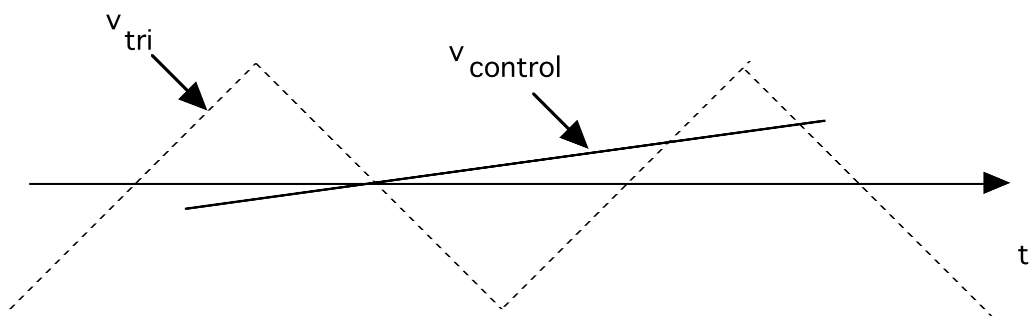
$h = 3m_f$; 3x29x50 = 4,350 Hz ; Amplitude $= \frac{1}{\sqrt{2}}(0.171)(300) = 36.3$V

$h = m_f \pm 2$; (29±2)x50 = 1550 hz and 1350 Hz ; Amplitude = $= \frac{1}{\sqrt{2}}(0.139)(300) = 29.5$V

$h = 2m_f \pm 1$; (2x29±1)x50 = 2950 Hz and 2850 Hz: Amplitude $= \frac{1}{\sqrt{2}}(0.314)(300) = 66.6$V

f) m_f should be integer and odd. This produces a symmetrical $v_o(t)$ and minimizes the harmonics.

Additionally when there is a zero-crossing in the control voltage, the triangular waveform should have a negative derivative when the sinusoidal control waveform has a positive derivative as shown in the figure below. Otherwise there may be underfined switchig instances if $v_{control}$ has the same derivative as v_{tri}. This may occur at overmodulation.



S8.2.

a) With the same type of transistors, the full bridge converter will double the output power compared with the half bridge. For increasing power, it is also possible to parallel connect transistors in the half-bridge converter. Paralleling is demanding and normally a derating must be done to give room from unbalanced current sharing during both the switchings and the on-state intervals.

b) From Fig. 8-6 in the text, it can be seen that the duty cycle D approaches m_a at $v_{control,peak}$. As $V_o = DV_d$, then $V_{o1,peak} = m_a V_d$.

c) PWM with bipolar voltage switching is described in Ch. 7-7-1 of the text. T_{A+} and T_{B-} are controlled in the same way. T_{A-} and T_{B+} are controlled by an inverter replica of the control signal for T_{A+} and T_{B-}. For a sinusoidal control signal, this results in the output voltage waveform shown in Fig. 8-12 of the text. For each switching cycle, the output voltage is both positive and negative, depending on the duty cycle. The output voltage waveform is bipolar and hence the name for the modulation algorithm.

d) $V_{Ao,1} = \frac{1}{\sqrt{2}}(0.8)(600) = 340\text{V}$; 50 Hz

$V_{Ao,29} = \frac{1}{\sqrt{2}}(0.818)(600) = 347\text{V}$; 1450 Hz

$V_{Ao,57} = \frac{1}{\sqrt{2}}(0.314)(600) = 133\text{V}$; 2850 Hz

$V_{Ao59} = \frac{1}{\sqrt{2}}(0.314)(600) = 133\text{V}$; 2950 Hz

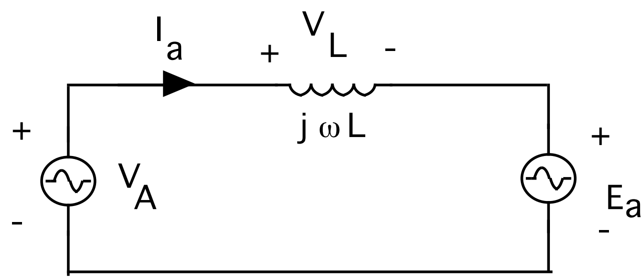
$V_{Ao,85} = \frac{1}{\sqrt{2}}(0.176)(600) = 75\text{V}$; 4250 Hz

S8.3.

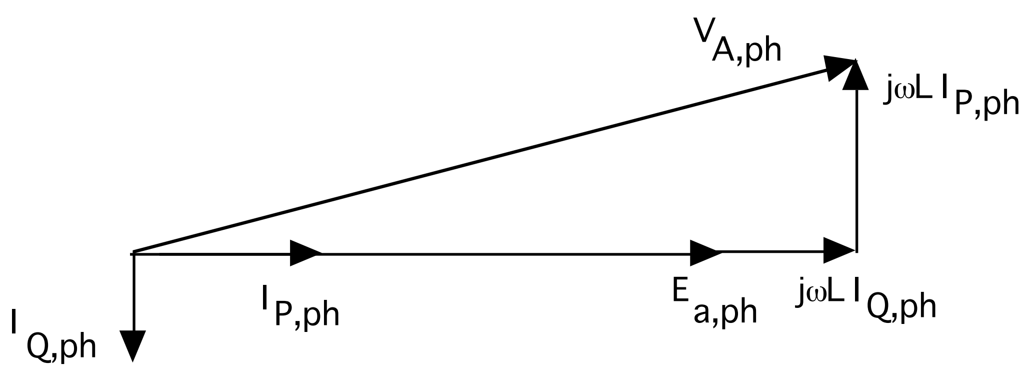
a) $m_f = \frac{750}{50} = 15$

b) $\sqrt{2}\,230 = m_a \frac{800}{2}$; $m_a = 0.813$

c) The circuit shown below is the equivalent circuit for the inverter in the frequency domain.

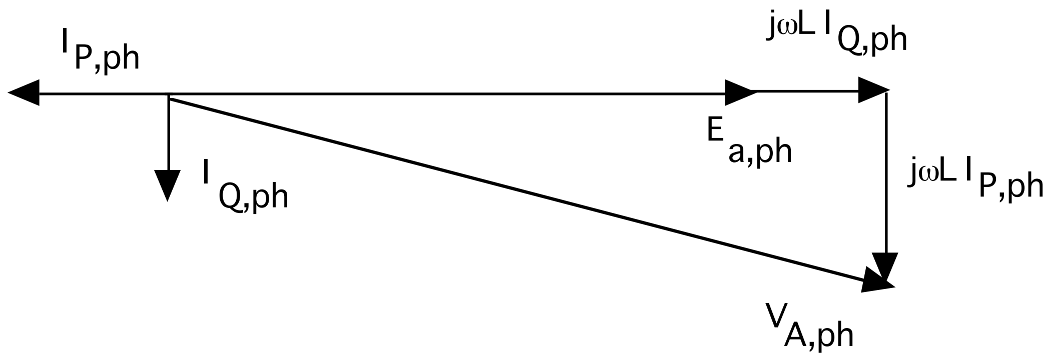


$V_{A,ph} = V_{L,ph} + E_{a,ph}$ where the subscript ph indicates that these are phasors.
Choose $E_{a,ph}$ = 230 V real; Then
$I_{P,ph} = P/E_{a,ph}$ = 100/230 = 4.35 A in phase with $E_{a,ph}$
$I_{Q,ph} = Q/E_{a,ph}$ = 500/230 = 2.17 A lagging $E_{a,ph}$ by 90º
Phasor diagram shown below.



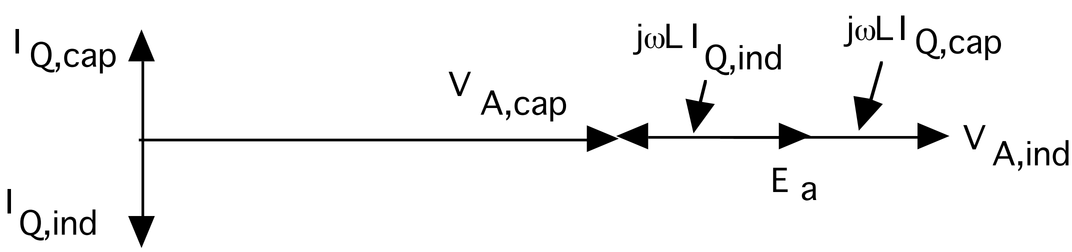
$V_{A,ph} = 230 + \omega L\,|\,I_{Q,ph}| + j\omega L\,|\,I_{P,ph}|$; $\omega L = (2\pi)(50)(0.015) = 5$ ohms
$V_{A,ph} = 230 + (5)\{2.17 + j4.35) = 241 + j21 = 242\, e^{j5^{\circ}}$

d) When P = -1 kW, the phasor diagram shown above must be changed so that $I_{P,ph}$ is reversed (i.e. negative). The new diagram is shown on the next page.



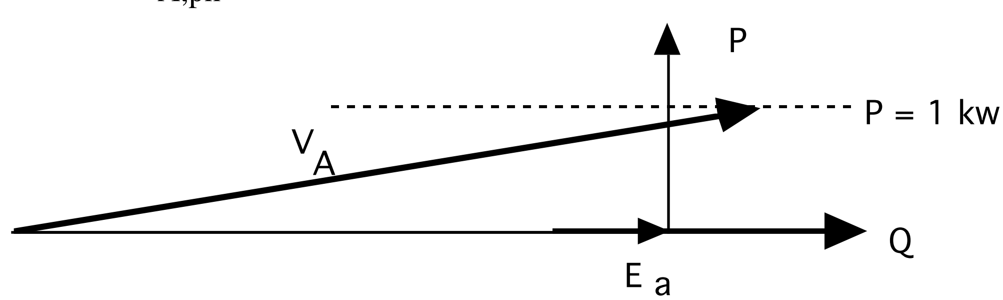
The voltage $V_{A,ph} = 241 - j21 = 242\ e^{-j5^o}$

e) $P = 0$ and $Q = \pm\ 1$ kvar ; $I_{Q,ph} = \pm\ j4.25$ A



$Q = +1$kvar ; $V_A = 250$ v and angle = 0 ; $Q = -1$kvar ; $V_A = 210$ v and angle = 0

f) Movement of $V_{A,ph}$ for $P = 0$ shown in part e). Movement for $P = 1$ kW shown below.



S8.4.

a) $V_{o1,\ ph} = \frac{4}{\pi}\ V_d$; Eq. (8-36) in text. $V_{o1,\ ph} = \sqrt{2}\ V_{o1}$; $V_{o1} = 220V$

$V_d = \frac{\pi}{4}\sqrt{2}\ V_{o1} = \frac{\pi}{4}\sqrt{2}\ (220) = 244V$

b) Ripple current is as shown in Fig. 8-19a.

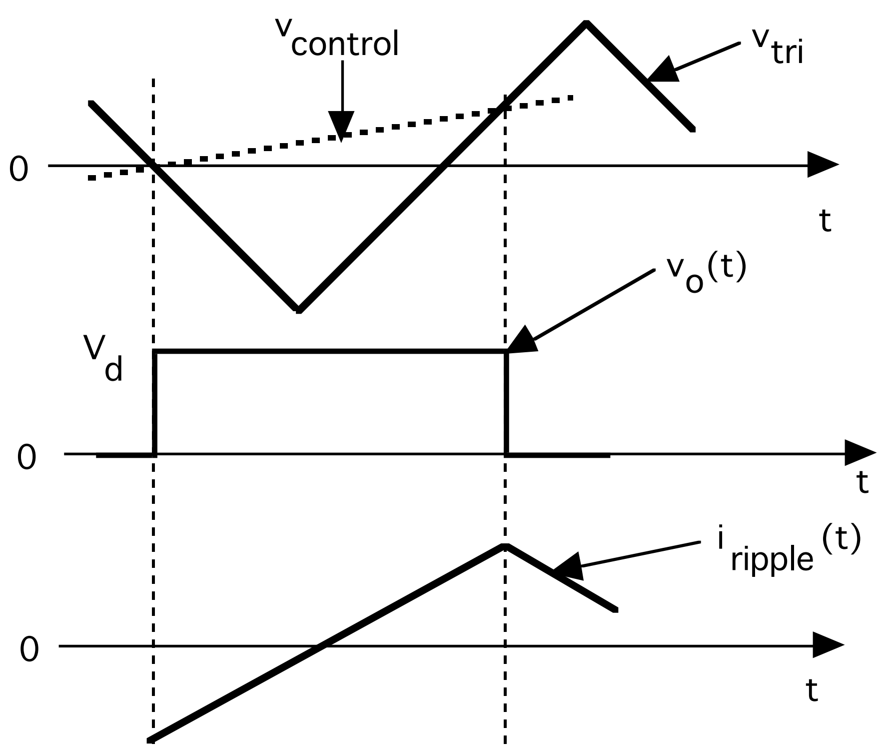
$$2\, I_{ripple,peak} = \frac{1}{\omega L}\int_0^{\pi}\{V_d - \sqrt{2}\, V_{o1}\sin(\omega t)\}\, d(\omega t) = \frac{1}{\omega L}\{\pi V_d - 2\sqrt{2}\, V_{o1})$$

$$I_{ripple,peak} = \frac{1}{2\omega L} V_d \{\pi - \frac{8}{\pi}\} ;$$

Put in numbers: $= \frac{1}{(2)(2\pi)(47)(0.1)}\ \{\pi - \frac{8}{\pi}\}\ (244) = 2.46$ A

c) $m_a = 0.8$; $V_{o1,peak} = (0.8)V_d = (0.8)(220) = \sqrt{2}\, 220$; $V_d = 389$ V

d)

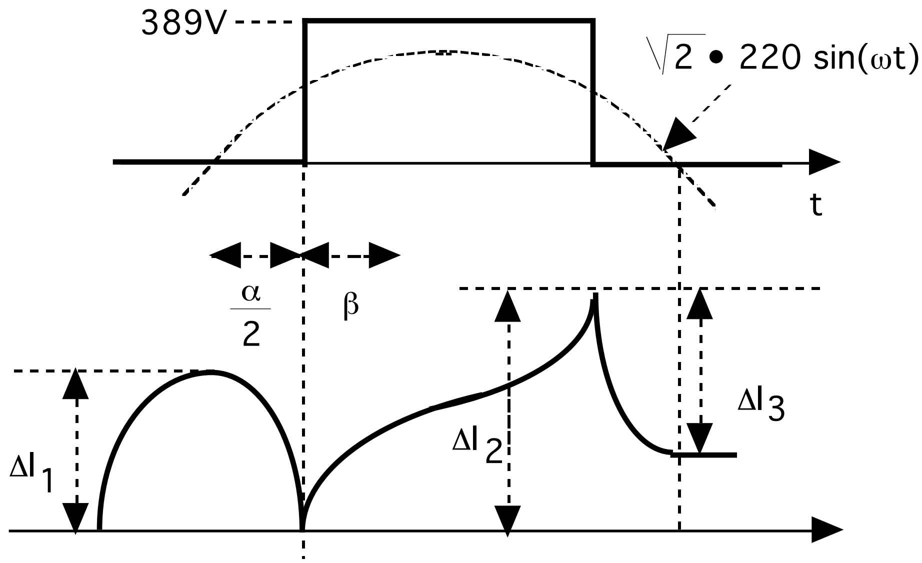


The switching frequency, f_s, is $f_s = 21x47 = 987$ Hz. Period $T_s \approx 1$ msec. As can be seen in the figure above, the ovoltage across the inductor is positive for approximately one half of a period about 0.5 ms. In this interval the inductor voltage is approximately constant at V_d since $v_o(t) \approx 0$.

The peak of the ripple in the output current will be $I_{ripple,peak} \approx \frac{V_d T}{4L} = 1$ A

e) $\hat{V}_{o1} = \frac{4}{\pi} V_d \cos\beta$; $\sqrt{2}(220) = \frac{4}{\pi}(389)\cos\beta$; $\beta = 51^o$

$\alpha = 180 - 2\beta = 180 - (2)(51) = 78^o$



From the figure above: $\Delta I_1 = -\Delta I_3$; $\Delta I_1 = \frac{1}{L}\int_0^{\alpha/2} \sqrt{2}(220)\sin(\omega t)dt$

$$\Delta I_1 = \frac{1}{\omega L}\int_0^{\alpha/2} \sqrt{2}(220)\sin(\omega t)d(\omega t) = \frac{1}{\omega L}\sqrt{2}(220)[1 - \cos(\alpha/2)] = 2.3A$$

$$\frac{\Delta I_2}{2} = \frac{1}{\omega L}\int_{\alpha/2}^{90} [V_d - \sqrt{2}(220)\sin(\omega t)]\, d(\omega t) = \frac{1}{\omega L}\{(389)(0.89) - (311)((0.78)\}$$

$$\Delta I_2 = 2\frac{1}{(2\pi)(47)(0.1)} \{(389)(0.89) - (311)((0.78)\} = 5.45 \text{ A}$$

$\Delta I_2 > \Delta I_1$; Max $I_{ripple,peak} = 2.72$ A

f) PWM gives the lowest ripple. The two types of square are not much different from each other. Higher switching frequency gives even lower ripple.

Chapter 10 - Switching DC Power Supplies

S10.1.

a) $V_o = V_d \dfrac{N_2\, D}{N_1\,(1 - D)}$: Solve $\dfrac{N_1}{N_2}$: $\dfrac{N_1}{N_2} = \dfrac{V_d\, D}{V_o\,(1 - D)}$

Evaluate: $\dfrac{N_1}{N_2} = \dfrac{(300)(0.4)}{(6)(1 - 0.4)} = 33.3$

b) $I_{m,max} - I_{m,min} = \dfrac{V_d\, D\, T_s}{L_m}$; Eq. (10-9) in text.

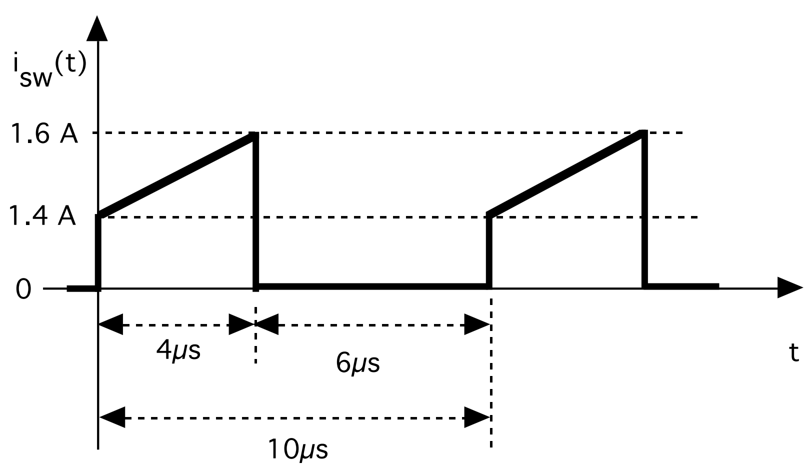
$L_m = \dfrac{(300V)(0.4)(10^{-5}\text{sec})}{0.2A} = 6 \text{ mH}$

c) $I_{sw,max} = \dfrac{I_o N_2}{(1 - D)N_1} + \dfrac{N_1(1 - D)\, T_s\, V_o}{2\, L_m\, N_2}$: Eq. (10-12) of text. Note turns ratio correction in the I_o term. Evaluating:

$$I_{sw,max} = \frac{N_2(30\text{ A})}{(33.3)N_2\,(1 - 0.4)} + \frac{(33.3)N_2(1 - 0.4)(10^{-5})(6)}{(2)(6\text{x}10^{-3})N_2} = 1.5 + 0.1 = 1.6\text{ A}$$

d) Switch current plotted below. During the 4 μs interval, $i_{sw}(t) = i_{sw}(0) + \dfrac{V_d t}{L_m}$

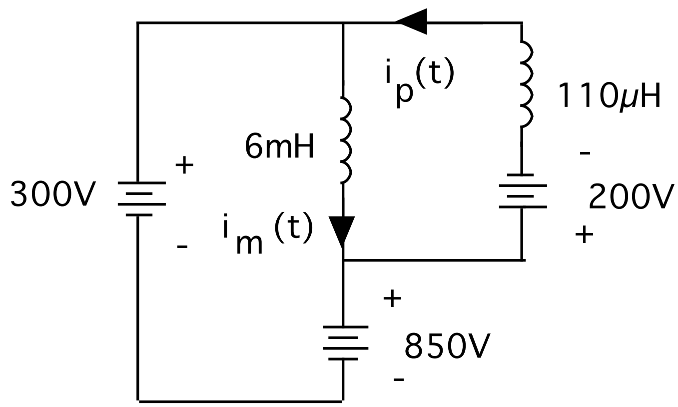
At t = 4 μs. $i_{sw}(4\mu s) = 1.6\text{ A} = i_{sw}(0) + \dfrac{(300)(4\text{x}10^{-6})}{6\text{x}10^{-3}}$; $i_{sw}(0) = 1.4$ A



e) $V_{sw} = \dfrac{V_d}{1 - D}$; Eq. (10-13) in text ; $V_{sw} = \dfrac{300}{1 - 0.4} = 500V.$

S10.2.

a) Equivalent circuit during the switch off-state interval shown below. The turns ratio N_1/N_2 = 33.3 as in problem S10.1. The 110 μH inductor is the 0.1 μH leakage inductance reflected to the primary side of the transformer. The 200 V supply is the 6 V output voltage reflected to the primary side of the transformer. Now $i_p(t=0) = 0$ so the magnetizing current $i_m(t=0)$ which equals 1.6 A must flow somewhere, and that will be through the zener diode which will be in breakdown at the start of the off-state interval. The 850V supply represents the zener diode in breakdown.



$(6\text{mH})\frac{di_m}{dt} = -550\text{V}$; $i_m(t) = 1.6 - 9.16\text{x}10^4\ t$; $(110\mu\text{H})\frac{di_p}{dt} = 350\text{V}$; $i_p(t) = 3.18\text{x}10^6\ t$

At $t = t_{rise}$ $i_m(t_{rise}) = i_p(t_{rise})$ and the zener will fall out of breakdown. The magnetizing current will then flow as usual through the transformer primary only.

Equating the two currents at $t = t_{rise}$ and solving yields $t_{rise} = 0.5\ \mu$s

Since $i_2 = i_p\frac{N_1}{N_2}$, this is also the risetime of i_2;

b) Power dissipated in zener = (0.5)(850v)(1.6A)(0.5 μsec)(10^5Hz) = 34 watts

S10.3.

a) $D_{max} = \frac{1}{1 + \frac{N_3}{N_1}}$; Evaluating $D_{max} = \frac{1}{1 + 1} = 0.5$

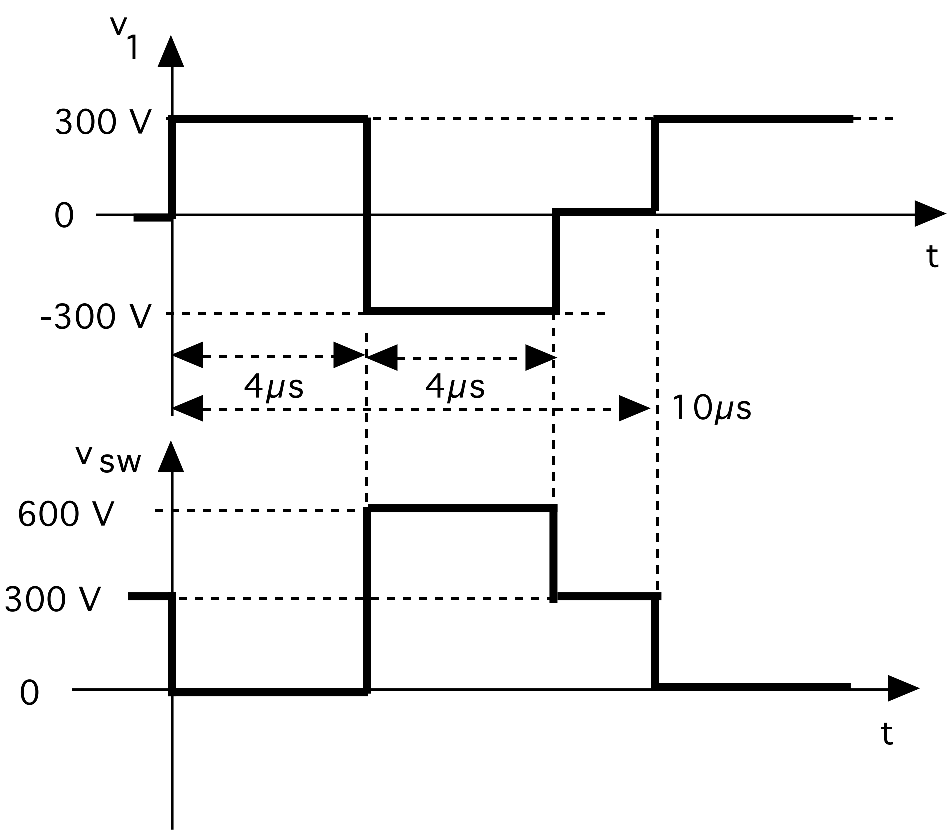
b) $\frac{V_o}{V_d} = \frac{N_2 D}{N_1}$; Eq. (10-16) in text. ; $\frac{N_1}{N_2} = \frac{(300v)(0.4)}{6V} = 20$

c) $V_{d,min} = \frac{N_1 V_o}{N_2 D_{max}} = \frac{(20)(6)}{0.5} = 240$ V

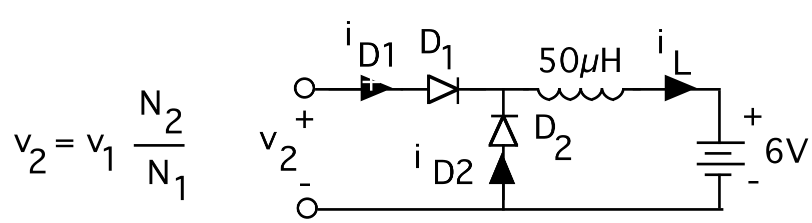
d) Voltage across the transistor during $T_{off} = V_d - v_1$: $v_1 = \frac{N_1 V_d}{N_3}$; See Fig. 10-11c in text.

$V_{sw} = 2V_d = 600$ V. $N_1 = N_3$

e) $v_1(t)$ plotted qualitatively in Fig. 10-11c in text. The time t_m is given by Eq. (10-20) in text. Using the numbers specified in the problem statement, $t_m = 4\ \mu s$. $v_1(t)$ plotted below. $v_{sw}(t) = V_d - v_1(t)$. Resulting switch voltage versus time also plotted below.



f) Equivalent circuit of secondary side of N_1:N_2 shown below.



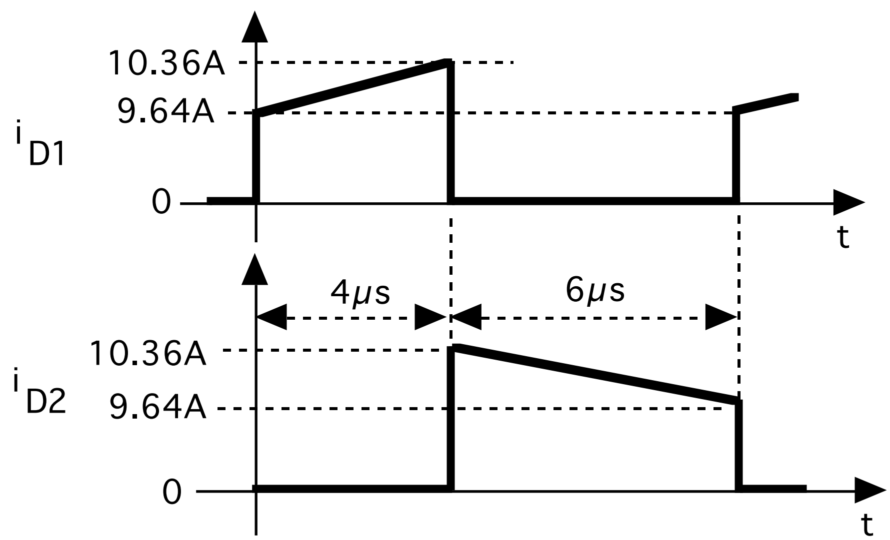
During t_{on} interval, v_2 = 15V and D_1 conducting. $(50mH)\frac{di_{D1}}{dt} = 15V - 6V$

$\Delta i_{D1} = \frac{(9V)(4\mu s)}{50\mu H} = 0.72$ A : $i_{D1} = i_L$ in this interval.

During t_{off} interval, v_2 = -15V and D_1 off, D_2 on. $(50mH)\frac{di_{D2}}{dt} = -6V$

$\Delta i_{D2} = \frac{(6V)(6\mu s)}{50\mu H} = 0.72$ A ; $i_{D2} = i_L$ in this interval.

Inductor current has an average value of 10 A and a ripple current (zero average) of 0.36A base-to-peak. Diode currents plotted below.



g) During t_{on} : $(15mH)\frac{di_m}{dt} = 300V$; $i_m(t) = i_m(0) + 2x10^4 t$; $i_m(4\mu s) - i_m(0) = 80mA$

During $t_{on} < t < t_m$: $(15mH)\frac{di_m}{dt} = -300V$; $i_m(t) = 80mA - 2x10^4 t$

At $t = t_{on} + t_m$, $i_m = 0$ and stays there until v_1 goes positive again. $t_m = 4\mu s$

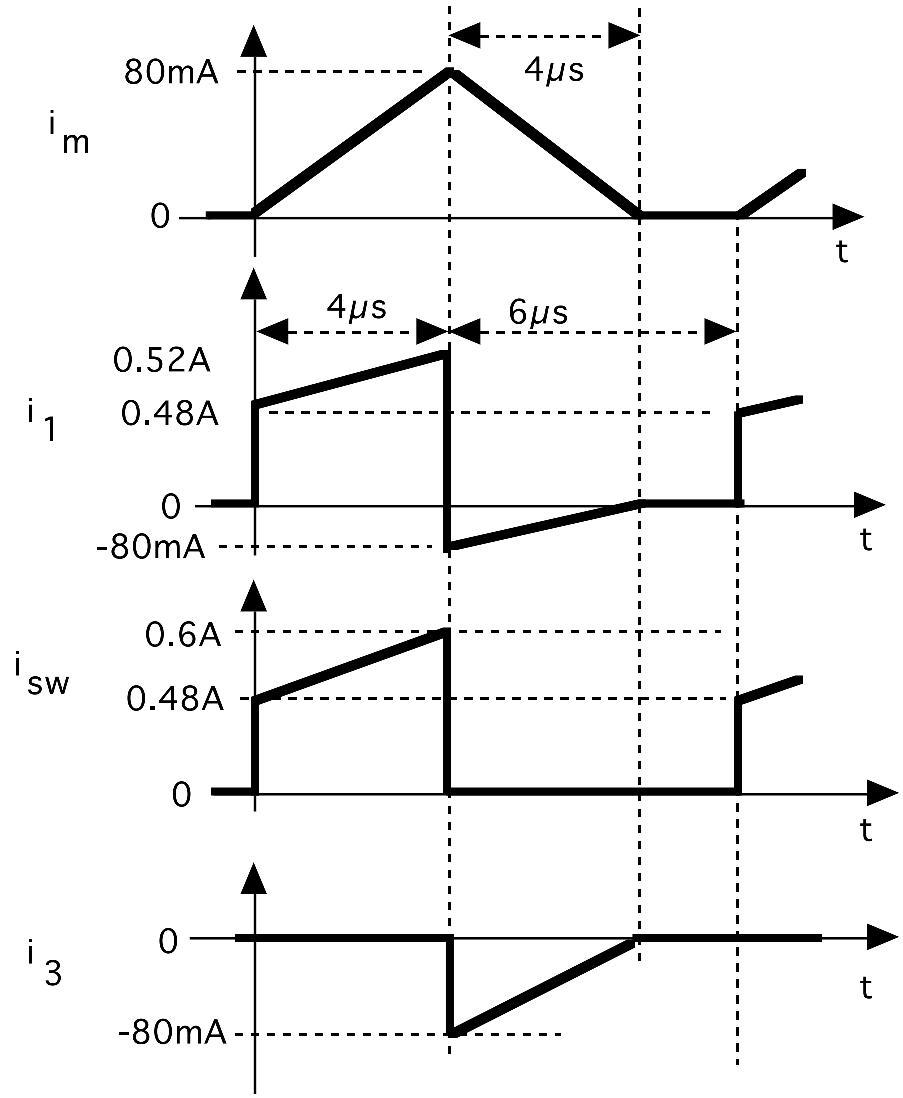
$i_{sw} = i_m + i_1$; Nonzero only during t_{on} interval.

During t_{on} interval;.$i_1 = \frac{N_2}{N_1} i_2 = 0.05\, i_2$; During t_m interval $i_1 = - i_m$.

During t_m interval, $i_2 = 0$ because D_1 reverse-biased. Thus i_1 couples to N_3 coil and

$i_3 = -\frac{N_1}{N_3} i_1 = i_m$ since $N_1 = N_3$.

Curves of i_m, i_1, i_{sw}, and i_3 shown on next page.

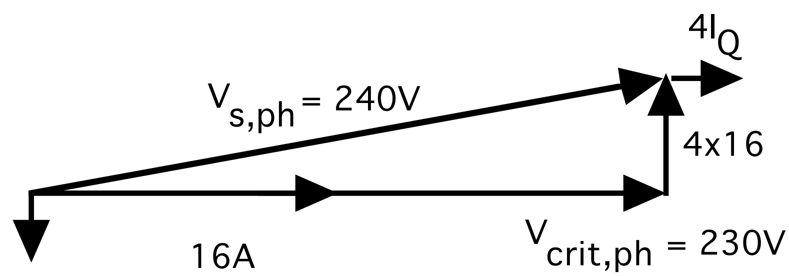

80mA
4μs
i m
0
t
4μs
6μs
0.52A
i 1
0.48A
0
-80mA
t
0.6A
i sw
0.48A
0
t
0
i 3
t
-80mA

Chapter 11 - Power Conditioners and UPS

S11.1.

a) $P = (370)(10) = 3.7$ kW ; $I_{P,UPS} = \frac{3700}{230} = 16$ A

$V_{s,ph} = jI_{s,ph} + V_{crit,ph}$; subscript ph implies these are phasor quanitities.



$230 + 4\ I_Q = v$; $v^2 + (4x16)^2 = (240)^2$; $v = 231.3$; $I_Q = \frac{1.3}{4} = 0.325$ A

$Q = (230)(0.325) = 75$ var inductive

b) $P = 4.5$ kW ; $I_{P,UPS} = \frac{4500}{230} = 19.6$ A

$v = \sqrt{(220)^2 - (4x19.6)^2} = 205.5$; $230 + 4I_Q = 205.5$

$I_Q = -6.1$ A (capacitive) ; $Q = 1406$ var (capacitive)

c) $P = (450)(0.5) = 225$ watts ; $I_{P,UPS} = \frac{225}{230} = 1$ A

$v = \sqrt{(220)^2 - (4x1)^2} = 220V$; $I_Q = 0$; $Q = 0$

d) $P = 225$ W ; $I_{P,UPS} = 1$ A

$v = 4\sqrt{(220)^2 - (4x1)^2} = 240V$; $I_Q = \frac{240 - 230}{4} = 2.5$ A ;

$Q = (230)(2.5) = 575$ var inductive

S11.2.

a) $V_d(0.8) = \sqrt{2}\ (220)$; $V_d = 212$ V

b) $I_P = \frac{1000}{120} = 8.3$ A

c) $I_c = (0.05)(8.3) = 0.4$ A ; $C = \frac{I_c}{\omega V} = \frac{0.4}{(2\pi)(6)(120)} = 9.2\ \mu F$

d) $(2\pi)(3000) = \frac{1}{\sqrt{L_f C_f}}$; $L_f = \frac{1}{(9x10^{-6})(2\pi)^2(3000)^2}$ = 0.3 mH

e) $V_{3h} = \frac{0.5}{100}\ V_1 = (.005)(120) = 0.6$ V   no reduction

$V_{5h} = \frac{0.3}{100}\ V_1 = (.003)(120) = 0.36$ V

$V_{20kHz} = \frac{1}{\sqrt{2}}\ (0.314)(212) = 47$ V ; 0.314 factor from Table 8-1 in text.

At 20 kHz $\omega L_f = 38\ \Omega$; $\frac{1}{\omega L_f} = 0.8\ \Omega$; $V_{f,20kHz} = \frac{0.8}{37.2}(47) = 1$ V

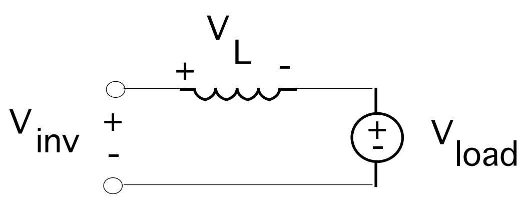
f_c = 3 kHz may give some amplification at no load, but not significant. At full load, the filter works well.

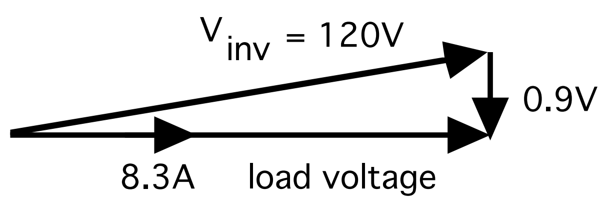
f) $I_f = 47/38.8 = 1.2$ A at 20 kHz.

g) $X_L = (2\pi)(60)L = 0.11\ \Omega$; $X_c = \frac{1}{2\pi C} = 288\ \Omega$

h) No load ; V_o = 120 V

1 kW load: $V_{load} = V_{inv} - j\omega LI$





$V_L = (8.3)(0.11) = 0.9$V ; $V_{load} = 120$V

S11.3.

a) $V_d = \frac{\sqrt{2}(120)}{0.8} = 212$ V

b) $I_P = \frac{1000}{120} = 8.3$ A

c) $\omega L_f = (0.04)(120)$; $L_f = \frac{(0.04)(120)}{(2\pi)(60)} = 13$ mH

d) $C_f = \frac{1}{L_f(2\pi)^2\ (3000)^2} = 0.2\ \mu$F

e) See solution to prob. S11.2 part e).

f) At 20 kHz, $\omega L_f = (2\pi)(2x10^4)(0.013) = 1640$ ohms

At 20 kHz, $\frac{1}{\omega C_f} = \frac{1}{(2\pi)(2x10^4)(2x10^{-7})} = 40$ ohms

Filter voltage at 20 kHz = 47 V; see solutions to Prob. S11.2 part e).

$I_f = \frac{47V}{1640 + 40} = 0.028$ A

g) No load; $V_{load} = 120$ V

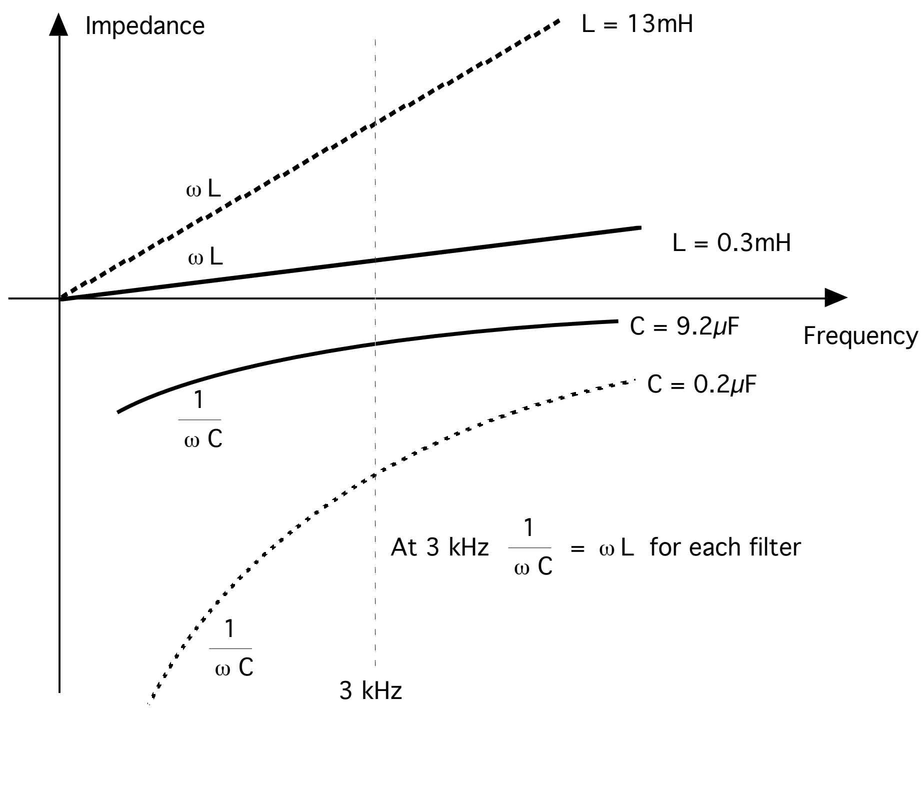
Full load: $V_{load} = V_{inv} - j\omega L_f I$, See equivalent circuit in solution to Prob. S11.2 part e).

Voltage drop across inductor specified as 4% of inverter output voltage.

$V_{load} = 120)\sqrt{1^2 + (0.04)^2} = 120.1$ V

S11.4.

a) The two filters have the same voltage damping, but the 60 Hz voltage drop is significantly different. See the impedance plots below.



b) At 60 Hz, the filter curent is mainly capacitive.
$I_{c,11.2}$ = 0.4 A = filter current Prob. S11.2
$I_{c,11.3} = (120)((2\pi)(60)(2x10^{-7}) = 9$ mA = filter current Prob. S11.3
Even 0.4 A 90° out of phase with 8.3 A is negligible.

c) Transient performance at rapid load changes is much better when the inductance is small.

d) and e) must be done using Pspice.

f) The transients in the output are much larger when the inductance is large. It is fairly easy to design a regulator when the inductance is small compared to when the inductance is large.

Chapter 12 - Introduction to Motor Drives

S12.1

Note: Gear coupling ratio a=1/6.25

M=3000Kg, J_m=0.3Kgm^2, J_{drum}=45Kgm^2, a=1/6.25, V=2m/s, θ=20^0

a) ω_{drum}= linear speed/radius=V/(D/2)=2/0.25=8 rad/s

   a= ω_{drum}/ω_m

   $\omega_m = \omega_{drum}$/a = 8*6.25=50rad/s

b) J_{Tot} = ?
   $J_{Tot} = J_m + a^2 J_{drum} = 0.3 + (1/6.25)^2 * 45 = 1.452$Kgm^2

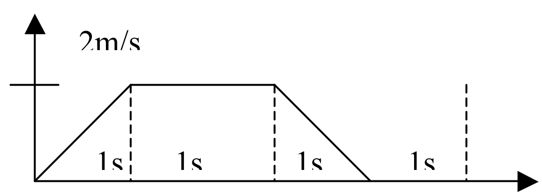
c) $T_{m,0}$ at standstill = ?

   F = M g cos (90-θ) = 3000*9.8* cos 70 = 10055.39 N
   T_l=F*r = 10055.39*0.5/2 = 2513.84N-m
   $T_{m,0}$ = a*T_l = 2513.84/6.25 = 402.215N-m

d) $T_{m,2}$ at 2 m/s uphill speed = ?

   $T_{m,2} = T_{m,0}$ = 402.215 N-m
   No increase in torque needed to maintain constant speed

e)



For acceleration
$T_{em} = a* d\omega_l/dt * [J_m+a^2 J_l] + a*T_{wl}$
ω_l at (V=2 m/s) = 2/(0.5/2)= 8 rad/sec
$d\omega_l/dt$ =8 rad/sec^2
T_{em1}=(1/6.25)*8*[0.3+1/(6.25)^2*45]+402.215 =404.073 N-m
$P_1 = T_{em1} * \omega_m$ = 404.073*50=20203.67W

For constant speed

$T_{em2}=T_{m,0}=402.215$ N-m

$P_2=402.215 *50=20110.75W$

For deceleration

$T_{em3}= -a* d\omega_l/dt * [J_m+a^2J_l] + a*T_{wl}$

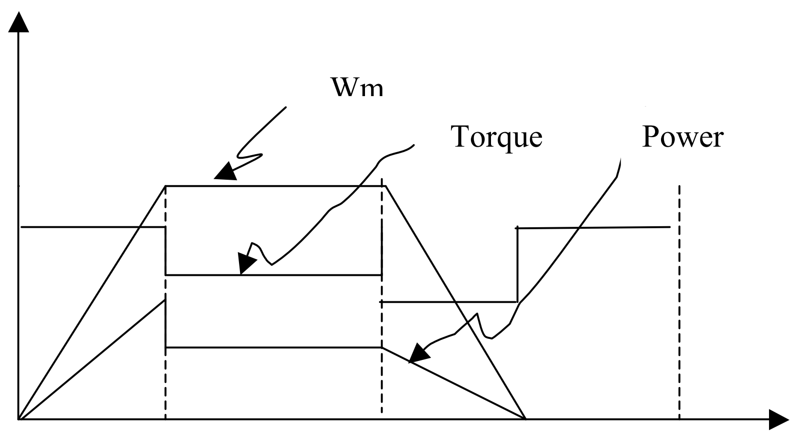
$= -(1/6.25)*8*[0.3+1/(6.25)^2*45]+ 402.215=400.356$N-m

P3=400.356*50=20017.82W

For standstill

$T_{em4}=T_{m,0}=402.215$ N-m

$P_4=402.215*0=0W$



S12.2

Full Drum ω_m=v/r=50/0.6=83.33 rad/sec

T_l=F*r=80*0.6=48 N-m

P=ω_m *T_l=48*83.33=4000W

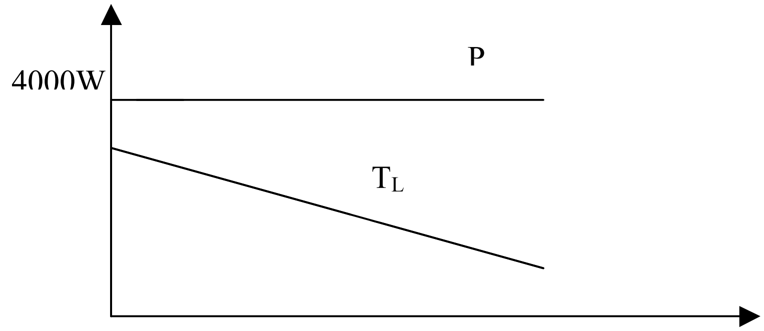
Empty Drum ω_m=N/r=50/0.1=500 rad/sec

T_l=F*r=80*0.1=8 N-m

P=ω_m *T_l=8*500=4000W

It can be noted that Power= Force * Velocity

As force 80 N-m and Velocity 50m/s both are constant,
Power=80*50=4000W is also constant



S12.3

M=1200Kg,V=50Km/hr, P_t=5000W

a) V_t=210V, I_t=?

   $I_t=P_t/V_t$=50000/210=23.809A

b) N_m=6000rpm, ω_m=? T_m=?

   $\omega_m=2*\pi*6000/60$=628.31rad/sec
   $T_m=P_t/\omega_m$=5000/628.31=7.957N-m

c) T_m at uphill=?
   F=M*g*cos(90-θ)=1200*9.8*cos 88=410.41N
   r=v/ω_m=50000/(3600*628.31)=0.022m
   T_m=F*r=410.41*0.022=9.029N-m

d) Maximum slope=?
   Maximum torque = 10*Torque in b)
   9*T_m=71.613N-m (Torque for uphill only)
   F=71.613/0.022=3255.136 N
   Cos (90-θ)=F/(M*g)= 3255.136/(1200*9.8)=0.276
   slope=$\theta=16.07^0$

Chapter 13 - DC Motor Drives

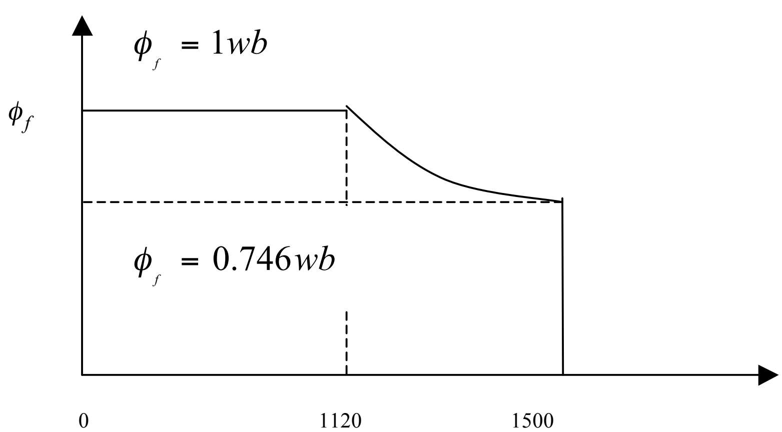
S13.1

R_a=0.5 ohm, V_{tr}=220 Volts, I_{ar}=30 A, P_r=6.15Kw
N_r=1120 rpm, ϕ_r =1 wb at I_{fr}=1A

a) The time constant of the field winding is large compared to the time constant of the armature current. So keeping the field at its maximum makes a short response time.

b) $E_a=V_{tr}-I_{ar}*R_a$=205 Volts
Also $E_a=K_e*\omega_m*\phi=2*\pi*K_e*N_r*\phi_{rf}$
$K_e = 205*60/(2*\pi*1120)=1.747$

c) From 1120 rpm to 1500 rpm V_t and E_a will be constant
E_a=205 V
$\phi_f = E_a/(K_e*\omega_m) = E_a/(K_e*2*\pi*N)$
ϕ_f = 1120/N   from 1120 to 1500 rpm
$\phi_f = \phi_{fr}$ from 0 to 1120 rpm
Large flux combined with maximum armature current gives the highest torque.



d) See Fig. 13.5 in text.

e) Iar=30A
   If we keep Iar constant 0.7Tload will be achieved at $0.7\phi_{fr}$

$$\omega_m = \frac{E_a}{K_e\phi} = \frac{205}{1.747 \times 0.7} = 167.34 \text{ rad/sec}$$

$$0.7 = \frac{\omega_r}{\omega_m} = \frac{2\pi \times 1120}{\omega_m}$$

$$\omega_m = 10053.096$$

$$N_{\max} = \frac{\omega_m \times 60}{2\pi} = 1598 rpm$$

S13.2

a) Traction as electric vehicles and locomotives
b) Where full torque is needed for all speeds, like a lift.
c) A permanent magnet motor provides rated flux at any operation. In a PM motor there is no power supply for the field current and no field losses.

S13.3

a) L_a=1mH, Buck converter V_d=300V

I_a at 50% torque, when ϕ is rated, is 50% of I_{ar}

$I_{a1}=0.5I_{ar}=0.5x30=15A$

N_1=0.4x1120=448 rpm

$E_{a1}=K_e\omega_{m1}=1.747x2\pi x448/60=81.91$ volts

$V_{t1}=E_{a1}+I_{a1}R_a=81.91+15x0.5=89.41$ volts

For fast increase in armature current, D=1.0 (Buck Converter)

$$V_d=E_{a1}+I_{a1}R_a+L_a\frac{di_a}{dt}$$

$L_a\frac{di_a}{dt}$=300-89.41=210.59 volts [Assuming E_a and I_aR_a constant during I_a increase, as mechanical time constant is large ω_m and Ea will change very slowly and I_aR_a drop is small compared to Ea]

$$dt = \frac{(30-15)\times 1\times 10^{-3}}{210.59} = 71.22\times 10^{-6} \text{ sec}$$

b) V_t=220 V (From problem 13.1)

$$La\frac{di_a}{dt}=300-220=80V$$

$$dt=\frac{(30-15)\times 1\times 10^{-3}}{80}=187.5\times 10^{-6}\sec$$

S13.4

a) Same as 13.3 a)

b)

$$\begin{aligned}E&=\frac{1}{2}LI^2-\frac{1}{2}L(0.5I)^2\\&=\frac{1}{2}(10^{-3})(30^2)-\frac{1}{2}(10^{-3})(0.5\times 30)^2\\&=0.3375\text{joules}\end{aligned}$$

c) Available acceleration torque =0.5 x T_{rated}

Speed change = 40% to 60%

Assuming B=0

$$Tem=J\frac{d\omega_m}{dt}$$

$$\begin{aligned}dt&=J\frac{d\omega_m}{T_{em}}=J\frac{d\omega_m}{K_t\phi_f I_{ar}}\\&=\frac{4\times 0.2\times 2\pi\times 1120}{0.5\times 1.747\times 60\times 1\times 30}\\&=3.58\sec\end{aligned}$$

Assumption of step change in I_a is valid. From prob. 13-2a), we know that current change required only 71 μ sec.

d)

$$N = 0.6 \times 1120 = 672 \text{ rpm}$$

$$Ea = K_e \omega_m = 1.747 \times 2\pi \times 672 / 60 = 122.86 \text{ volts}$$

Here $V = 0$ as current should decrease

$$Vt = Ea + IaRa$$

$$= 122.86 + 30 \times 0.5 = 137.86 \text{ volts}$$

$$L \frac{di_a}{dt} = 137.86$$

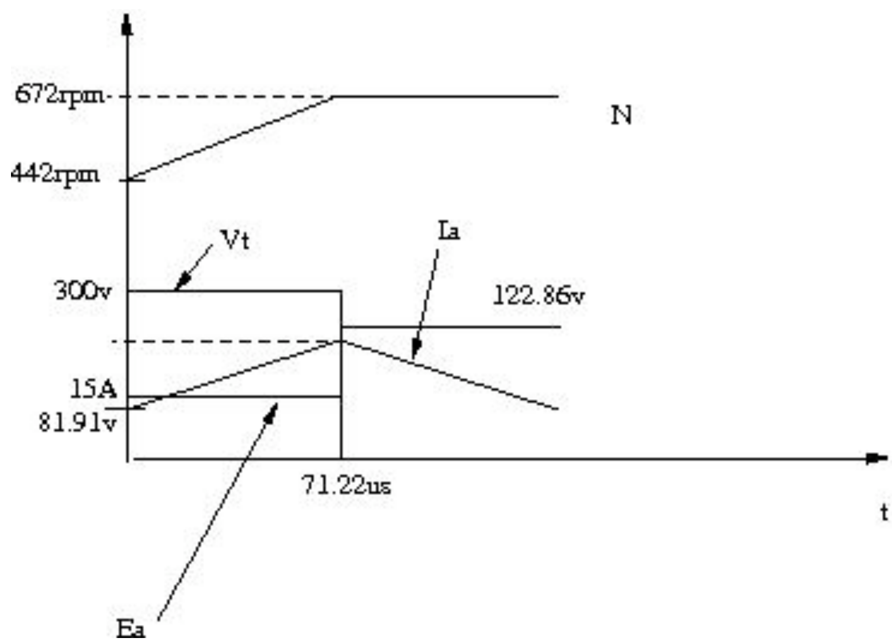
$$dt = \frac{(30 - 15) \times 10^{-3}}{137.86} = 108.806 \times 10^{-6}$$

e)

$$W = \frac{1}{2} J \omega_1^2 - \frac{1}{2} J \omega_2^2$$

$$= \frac{(2\pi)^2 \times 4}{2} \left(\frac{0.6^2 \times 1120^2}{60^2} - \frac{0.4^2 \times 1120^2}{60^2} \right) = 5502.41 \text{joules}$$

f)



g) Ignore I_aR_a drop

$$T_{em} = K_t \phi I_{ar} = 1.747 \times 1 \times 30 = 52.41 N-m$$

$$T_{em} = J \frac{d\omega_m}{dt}$$

$$0.7 T_{em} = J \frac{d\omega_m}{dt}$$

$$\frac{d\omega_m}{dt} = \frac{0.7 \times 52.41}{7} = 5.241 \text{ rad/sec}$$

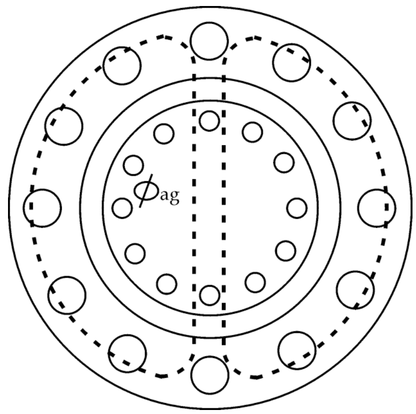
$$Ea = 1.747 \omega_m$$

$$\frac{dEa}{dt} = 1.747 \frac{d\omega_m}{dt} = 1.747 \times 5.241 = 9.156$$

Chapter 14 - Induction Motor Drives

S14.1

a)



The airgap flux, φ_{ag} is shown in the figure. It is the flux which is common for both the stator and the rotor windings.
For a two-pole (one pole-pair) induction motor, the airgap field will rotate with f revolutions per s or $\omega_s = 2\pi f[rad/s]$

b)
$$e_{ag}(t) = N_s \cdot \frac{d\varphi_{ag}(t)}{dt} \qquad (1)$$

From $$v = L\frac{di}{dt} = N\frac{d\varphi}{dt} \qquad (2)$$
$$L \cdot i = N \cdot \varphi \qquad (3)$$

$$N_s \cdot \varphi_{ag}(t) = L_m \cdot i_m(t) \qquad (4)$$

That is, the magnetizing current, i_m (t) is in phase with the airgap flux, φ_{ag} (t).
From eq. 1, the airgap flux is lagging the airgap voltage by 90°.

$$e_{ag}(t) = L_m \cdot \frac{di_m(t)}{dt} \Rightarrow \overline{E}_{ag} = j\omega L_m \overline{I}_m \qquad (5)$$

The equivalent circuit is like in Figure 14-2 of the textbook. The airgap flux is in phase with and proportional to the magnetizing current.

c)
The fields in the rotor have the same magnitude as the airgap field. However, due to the relative motion between the airgap and the rotor, this field is seen by the rotor as rotating at a speed:

$$\omega - \omega_r = \omega_{sl}$$

The angular speed ω_{sl} is the slip speed and the slip frequency f_{sl} is expressed as:

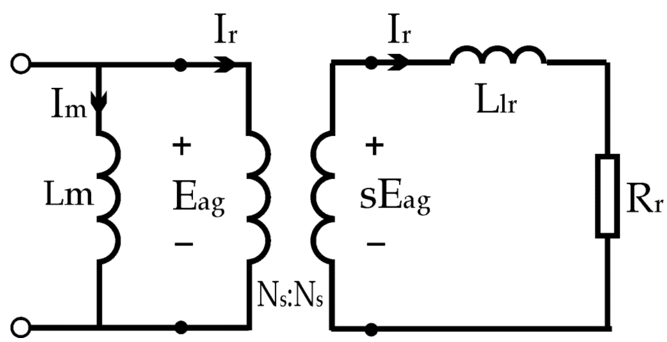
$$\omega_{sl} = 2\pi f_{sl} \tag{6}$$

d)
Voltage balance for the rotor is (referred to the stator winding):

$$e_r = N_s \frac{d\varphi_{rag}}{dt} = L_{lr} \frac{di_r}{dt} + R_r i_r \tag{7}$$

$$\overline{E}_r = j\omega_{sl} L_{lr} \overline{I}_r + R_r \overline{I}_r \tag{8}$$

$$\overline{E}_r = j\omega_{sl} N_s \overline{\Phi}_{ag}$$



e)
The frequency of the rotor flux and the rotor voltage is:

$$f_{sl} = \frac{f - f_r}{f} \cdot f = s \cdot f$$

From Eq 1: $\overline{E}_r = s \cdot \overline{E}_{ag}$. This is due to the relative motion between stator and rotor, which affects the rate of variation of the flux linkage as seen by the rotor.

f)
The equivalent number of turns of windings in the rotor of the fig. above is the same as for the stator. The rotor current will then keep the same value when referred to the stator.
Per phase power from the stator is

$$E_{ag} \cdot I_r \cdot \cos\varphi_r$$

Per phase power in the rotor circuit is

$$sE_{ag} I_r \cos\varphi_r$$

Since φ_r is the same in these two equations, the power on the rotor side is s times smaller than the power on the stator side. The missing power is the electrical motor power P_{em}.

$$P_{em} = (1-s)\cdot E_{ag} \cdot I_r \cdot \cos\varphi_r$$

Dividing both sides of eq (8) by the slip s = $\frac{\omega_{sl}}{\omega_s}$, we set the per. phase equivalent circuit of Fig. 14-2 of the text-book.

g)

Per-phase rotor losses are: $P_{loss} = R_r I_r^2$

Per-phase motor power is: $P_{em} = R_r \cdot \frac{f - f_{sl}}{f_{sl}} I_r^2$

h)

Stator is connected to a (stiff) voltage. Flux is according to Eq. 1. The magnetizing current is supplied from the grid as shown in Eq. 5.

When increasing the load torque, the T_{em} must also increase. T_{em} is produced by the resistive part of the rotor current, which then is (almost) proportional to the load torque.

S14.2

a)

Fig. 14-2 in the text book and solution of S 14.1

b)

This solution represents a straightforward way to calculate the voltages and currents of an induction motor.

1. Start: Assume E_{ag}^1 = per phase rated voltage
2. Calculate and find the V_s^1 resulting from the assumption at point 1
3. Multiply all voltages and currents by $\frac{V_s}{V_s^1}$, where V_s is the per phase rated voltage

$$\overline{I}_m^1 = \frac{\overline{E}_{ag}^1}{jX_m} = -j14.67A$$

$$f_{sl,rated} = \frac{1500-1464}{1500} \cdot 50 = 1.2Hz$$

$$\overline{I}_r^1 = \frac{\overline{E}_{ag}^1}{R_r \cdot \frac{1}{s} + jX_{lr}} = 17.59 - j0.28 = 17.6e^{-j0.92^\circ}$$

$$\overline{I}_s^1 = 17.59 - j14.95 \quad A$$

$$\overline{V}_s^1 = \overline{E}_{ag}^1 + \overline{I}_s^1(R_s + jX_{ls}) = 233e^{j1.1} \quad V$$

$$\overline{E}_{ag} = 208 \quad V$$

c)

$$\frac{V_s}{V_s^1} = 0.945 \quad \overline{I}_m = 13.86 \quad A$$

d)
The rotor current is lagging 0.92° and the magnetizing current is lagging 90°. The angle between these two currents is $\sigma = 89.08°$

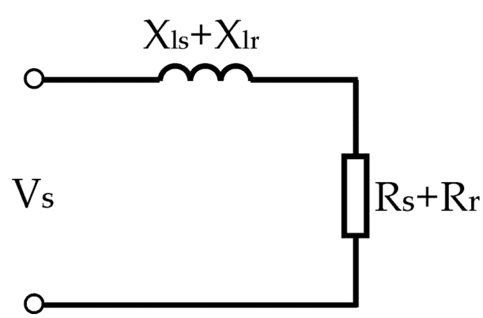
$$\sin\sigma = 0.99987$$

e)
Power from line:
$P_s = 3 \cdot \mathrm{Re}(\overline{V}_s \cdot \overline{I}_s^*) = 10797 \quad W$
losses in stator
$P_{sloss} = 3 \cdot R_s \cdot I_s^2 = 428.5 \quad W$
losses in rotor
$P_{r.loss} = 3R_r \cdot I_r^2 = 248.9 \quad W$

$$P_{em} = 3R_r \cdot (\frac{f - f_{sl}}{f_{sl}}) \cdot I_r^2 = 10123 \quad W$$

The sum of these 3 components is very close to P_s.

f)
At low speed, s ≈ 1.
Then the rotor impedance is much less than X_m. Thus X_m can be omitted.
An approximate equivalent circuit for low speed is



$$\overline{I}_s = \frac{\overline{V}_s}{R_s + R_s + j(X_{ls} + X_{lr})} = 238.6e^{-j48.3°} A = 10.9\,pu$$

g)
At start, the frequency of the rotor currents is approximately equal to the line frequency.

h)
A deep bar rotor will have a larger resistance at start. Thus the rotor current will be less than calculated above, and also more resistive. The torque per Ampere will increase.

i)
By frequency control, the stator frequency is ramped up, and the slip frequency in rotor will be equal to that which induces enough currents to generate $T_{em} = T_{load}$.
This means that the rotor currents are almost resistive. If T_{em} is kept below rated torque, the motor currents will also be below their rated values when the motor is started by a variable frequency converter. To achieve this, the magnetizing current should be kept close to its rated value. Therefore, the airgap voltage

$$E_{ag} = j\omega L_m$$

must increase when the converter frequency increases.

j)
When s is small, $\frac{R_r}{s}$ ωL_{lr} as calculated above.
Then approximately

$$E_{ag} = I_r \quad \frac{f}{f_{sl}} \cdot R_r$$

$$I_r = \frac{E_{ag}}{f} \cdot \frac{1}{R_r} \cdot f_{sl}$$

$$\frac{E_{ag}}{f} = \frac{208}{50} = 4.16 \quad \text{is kept constant}$$

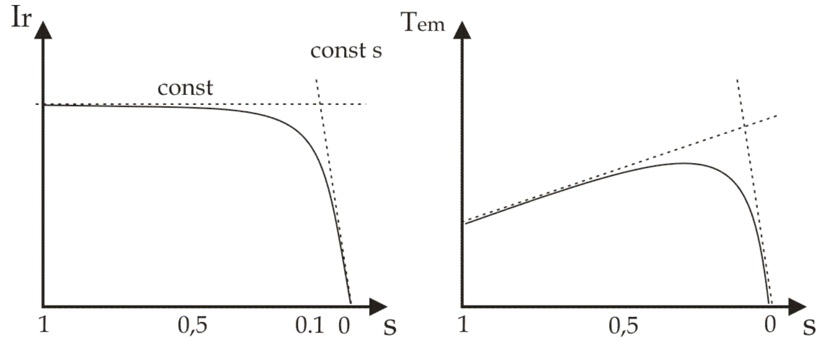
$$I_r = \frac{4.16}{0.3} \cdot f_{sl} = 13.87 \cdot f_{sl}$$

$$T_{em} = \frac{P_{ag}}{\omega} = \frac{3R_r \cdot \frac{f}{f_{sl}} \cdot I_r^2}{2\pi f}$$

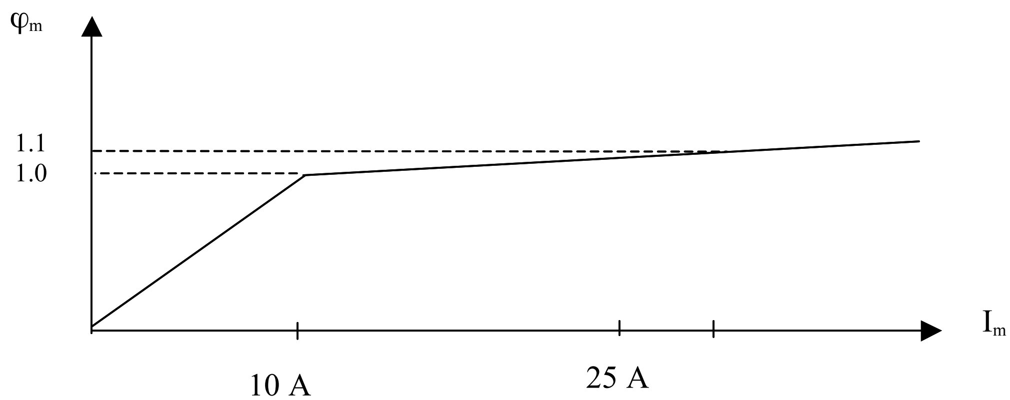
$$T_{em} = \frac{3 \cdot 0.3 \cdot (13.87)^2}{2\pi} f_{sl} = 27.6 \cdot f_{sl}$$

k)

l)



S14.3



Airgap flux as function of magnetizing current.

a)
Torque is proportional to $\varphi_m \cdot I_r$.

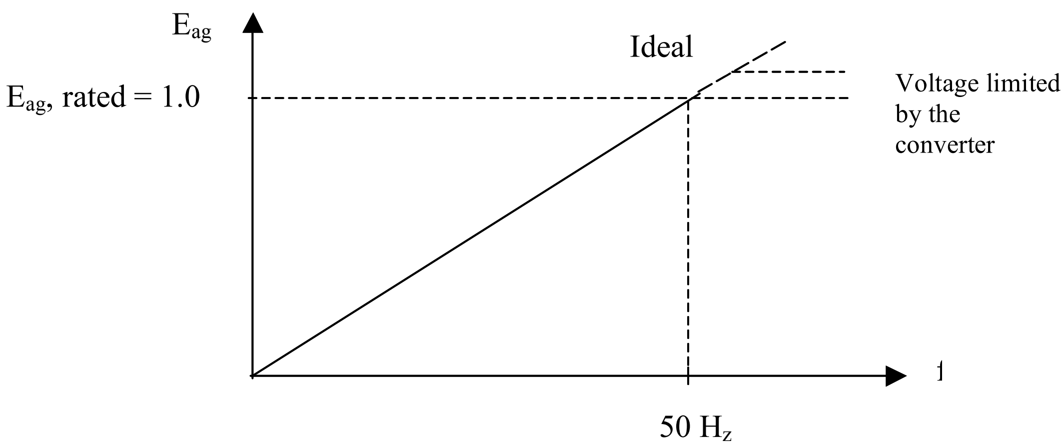
$$T_{em} \quad k_4 \varphi_{ag} I_r \qquad \text{(Eq. 14-25)}$$

The magnetizing current to keep the flux at its rated value is relative low. There is a long (10s of ms) timeconstant to change φ_{ag} · I_r can be changed in less than a ms. To keep the induction motor ready for a fast response, φ_{ag} should be controlled to its rated value and I_r to the instantaneous torque command.

b)
From Eq. 14-6, Eq. 14-8:

$$E_{ag} = k_3 \varphi_{ag} \cdot f = k \cdot f$$

φ_{ag} is kept constant to its rated value if E_{ag} is proportional to the stator frequency f.



c)
Eq. 14-17 and 14-18c combined:

$$T_{em} = k^1 \frac{I_r^2}{\varphi_{sl}}$$

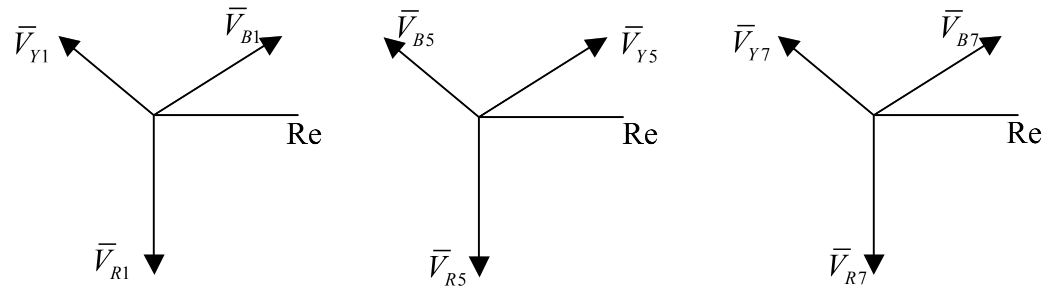
The torque is given by I_r and f_{sl}. This is independent of f and V_s. the ratio between I_r and f_{rl} depends on the airgap flux, φ_{ag}.

S14.4

a)

$$V_R = V_{s1} \sin \omega_1 t + 0.2 V_{s1} \sin \omega_5 t + 0.14 V_{s1} \sin \omega_7 t$$
$$V_Y = V_{s1} \sin(\omega_1 t - 120) + 0.2 V_{s1} \sin[\omega_5 t - (5 \times 240)] + 0.14 V_{s1} \sin[\omega_7 t - (7 \times 120)]$$
$$V_B = V_{s1} \cos(\omega_1 t - 240) + 0.2 V_{s1} \sin[\omega_5 t - (5 \times 120)] + 0.14 V_{s1} \cos[\omega_7 t - (7 \times 240)]$$

b)



All phasors rotate counterclockwise.

c)

ϕ_{ag5} rotates at a speed of $5\omega_{syn1}$ in clockwise direction and ϕ_{ag7} rotates at at speed of $7\omega_{syn1}$ in the counterclockwise direction.

$$\phi_{ag1} = \frac{V_{s1}}{k\omega_{syn1}}$$

$$\therefore \quad \phi_{ag5} = \frac{V_{s5}}{k\omega_{syn5}} = \frac{0.2V_{s1}}{5k\omega_{syn1}} = 0.04\phi_{ag1}$$

$$\therefore \quad \phi_{ag7} = \frac{V_{s7}}{k\omega_{syn7}} = \frac{0.14V_{s1}}{7k\omega_{syn1}} = 0.02\phi_{ag1}$$

d) 5H and fundamental ➔this will produce torque components that pulsate at sixth harmonic frequency.
7H and fundamental ➔ same as above
Total interaction ➔ produce a torque that pulsates at sixth harmonic frequency.

e) See Eq.14-49 in text.

S14.5

a)

$$\omega_r = (1-s)\omega_s \qquad (9)$$

$$P_{em} = P_r \frac{1-s}{s} \qquad \text{(from Eq. 14 - 18a)} \qquad (10)$$

Fan torque at ω_s is T_s

Torque at other speeds:

$$T_{fan} = T_s \cdot \left(\frac{\omega_r}{\omega_s}\right)^2 = T_s\left(1-s\right)^2 \tag{11}$$

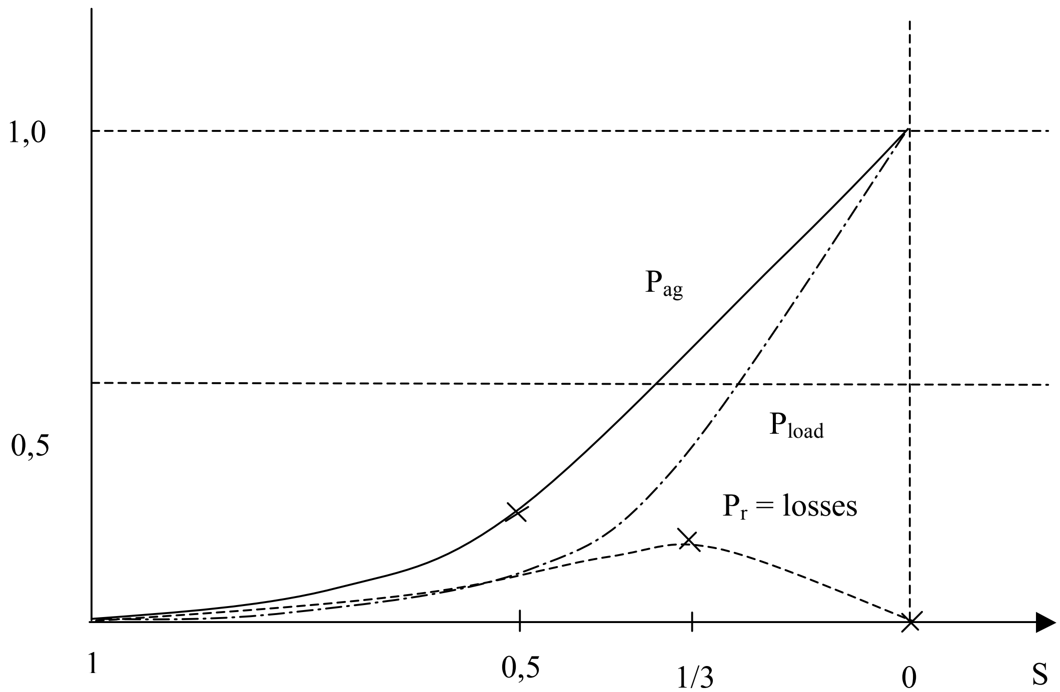
$$P_{load} = T_{fan} \cdot \omega_r = T_s \cdot \omega_s \left(1-s\right)^3 = P_{em} \tag{12}$$

Rotor losses P_r :

$$P_r = P_{em} \cdot \frac{s}{1-s} = T_s \omega_s s(1-s)^2 \tag{13}$$

$$P_{ag} = T_{fan} \cdot \omega_s = P_{load} + P_r \qquad \text{(from Eq. 14 – 18)} \tag{14}$$

This can also be seen from combining (3), (4) and (5).



b)
Max value of P_r = rotor losses is found by derivation of (5). By this derivation, max loss is for s = 1/3.

T_r is the torque at rated speed (which is lower than ω_s).
(Depending upon rated slip).

From (5):

$$P_{r,ats=1/3} = T_s\omega_s \frac{1}{3}(\frac{2}{3})^2 = T_s\omega_s \frac{4}{27}$$

c)
Assume 1kW = $T_s\omega_s$
Max power loss is 148 W (for s = 1/3)

$$P_{r,s=3\%} = 1kW \cdot \frac{0,03}{1-0,03} = 31W$$

The rated or max losses in this low slip motor is only 31W. If a motor of rated slip 3% should be used, it must be 4.8 times larger than the power at rated speed 4.8kW.

d)
$$P_{r,s=10\%} = 1kW \cdot \frac{0,1}{0,9} = 111W$$
Using this 10% slip motor, it must only be 1,33 times larger 1.33 kW
So far this application, a high efficiency motor is not the best.

S14.6

a)
$$e_{ag}(t) = N_s \cdot \frac{d\varphi_{ag}(t)}{dt}$$
Voltage and flux are sinusoidal, with frequency f, so that related phasors are expressed as:
$$E_{ag} = N_s\omega \cdot \Phi_{ag} = N_s \cdot 2\pi f \cdot \Phi_{ag} = k \cdot f \cdot \Phi_{ag}$$
$$V_s = E_{ag} = k \cdot f \cdot \Phi_{ag}$$

b)
For a given current, electromagnetic torque is proportional to the flux. It is therefore desirable to operate at the highest possible flux level, in order to get the maximum torque out of the machine, given the current limitation. However, the flux cannot be increased above a certain limit, due to saturation of the magnetic iron, and the associated losses. Moreover, the flux cannot be increased above the limit defined by the available stator voltage, as stated by the equations above. Insulation limit of the stator windings is also of concern, when trying to operate the machine above its rated voltage. Machines are usually designed to reach rated voltage at rated stator frequency, when the flux is controlled to its rated value.

c)
From equations in a), when the voltage is fixed at its rated value, the flux is inversely proportional to the frequency:

$\Phi_{ag} = V_{s,rated}/(k*f)$

d)
Expression for the electromagnetic torque as a function of airgap flux and slip is derived in the textbook (from eq. 14-21 to eq. 14-27):

$$T_{em} = k_6 \{V_{s,rated}\}^2 f_{sl}$$

Using the result in c):

$$T_{em} \approx k_6 \{V_{s,rated}\}^2 f_{sl} = k_7\{V_{s,rated}\}^2 f_{sl}/f^2$$

e)
The assumption is valid when the frequency of the rotor quantities is small, that is when the slip is small. A rule of thumb is to consider the slip small when it is less than twice its rated value.

f)

g)
According to the previous equation, the electromagnetic torque is decreasing with the square of the frequency:

$$T_{em} \propto 1/f^2 \rightarrow P \propto 1/f$$

Rotor current is proportional to the airgap flux (see eq. 14-26 in textbook):

$$I_r \propto \Phi_{ag} \rightarrow I_m \propto 1/f$$

Rotor copper losses will therefore be proportional to $1/f^2$.

h)
Normally, the machine is dimensioned based on losses considerations. Therefore, optimum utilization is when losses are close to the rated value. This means that the rotor current should be kept constant, allowing for more torque availability.

i)
From eq. 14-25 in the textbook: $T_{em} \approx k_4 \Phi_{ag} I_r$

If the rotor current is kept constant, and the airgap flux is proportional to 1/f in order to keep the terminal voltage constant, then:

$$T_{em} \propto 1/f \rightarrow P = \text{constant}$$

From eq. 14-26 in the textbook:

$$I_r \cong k_5 \cdot \Phi_{ag} \cdot f_{sl} \Rightarrow f_{sl} \quad f$$

The slip must increase proportionally to the frequency, in order to keep the rotor current constant.

(j)

The control strategy described above can be applied until the pull-out torque is reached. The pull-out torque value decreased with the square of the operating frequency. Therefore, above a threshold frequency (typically about two times the rated speed) is no longer possible to keep the rotor current constant by linearly increasing the slip. The slip must be kept constant, resulting in an available torque proportional to $\frac{1}{f^2}$, as in g).

The only way to get more torque out of the machine, is to increase the flux level, resulting in a terminal voltage above the rated value. This is possible if the machine is controlled by a converter with enough available voltage (insulation is normally not a serious issue, since the windings must anyway be able to withstand the high voltage peaks resulting from the converter switching), provided that the cooling system is able to remove the additional heat caused by the increased iron losses.

Chapter 15 - Synchronous Motor Drives

S15.1

Input parameters shown below.

$K_E = 0.012V_{(L-L)} / rpm$, $R_S = 0.83\Omega / phase, L_{LS} = 1.0mH / phase, L_S = 9.5mH / phase$

a)

$$E_{fa} = \frac{K_E N}{\sqrt{3}} = \frac{60 K_E \omega_s}{2\pi\sqrt{3}}$$

$$\therefore \omega_s = \frac{E_{fa} \times 2\pi\sqrt{3}}{60 K_E}$$

$$T_{em} = \frac{P_{em}}{\omega_s} = K_T I_s = \sqrt{2} K_T I_a$$

$$\omega_s = \frac{P_{em}}{\sqrt{2} K_T I_a}$$

$$\therefore \frac{E_{fa} \times 2\pi\sqrt{3}}{60 K_E} = \frac{P_{em}}{\sqrt{2} K_T I_a}$$

$$K_T = \frac{60 K_E P_{em}}{\sqrt{2} I_a \times 2\pi\sqrt{3} E_{fa}} = \frac{3 E_{fa} I_a K_E \times 60}{\sqrt{2} I_a \times 2\pi\sqrt{3} E_{fa}} = \frac{\sqrt{3} K_E \times 60}{\sqrt{2} \times 2\pi}$$

$$K_T = \frac{\sqrt{3}}{\sqrt{2} \times 2\pi} \times 0.012 \times 60 = 0.1403$$

b) Figure 15.2(b) in textbook. (E_{af} -Rotor Field and $E_{ag,a}$ -Air gap field) c)

$$P_{em} = 3E_{fa}I_a \qquad for\ \delta = 90^0$$

$$\therefore E_{fa}I_a = 300/3 = 100W$$

$$E_f = K_E n = 3000 \times 0.00692 = 20.76\ Volts$$

$$I_a = 4.817A$$

$$\omega = \frac{p}{2}\omega_s = \frac{2\pi np}{2\times 60} = \frac{2\pi \times 3000 \times 2}{2\times 60} = 314.16\ rad/\sec$$

$$E_{sa} = jI_a\omega L_a = I_a\omega(L_S - L_{LS}) = 4.817\times 314.16(8.5\times 10^{-3})$$

$$= j12.86\ Volts$$

$$E_{ag,a} = E_{fa} + E_{sa} = 20.76 + j12.86 = 24.42\angle 31.77^0$$

$$V_a = E_{ag,a} + (R_S + j\omega L_{LS})I_a$$

$$= 20.76 + j12.86 + (0.83 + j314.16\times 10^{-3} \times 1)4.817$$

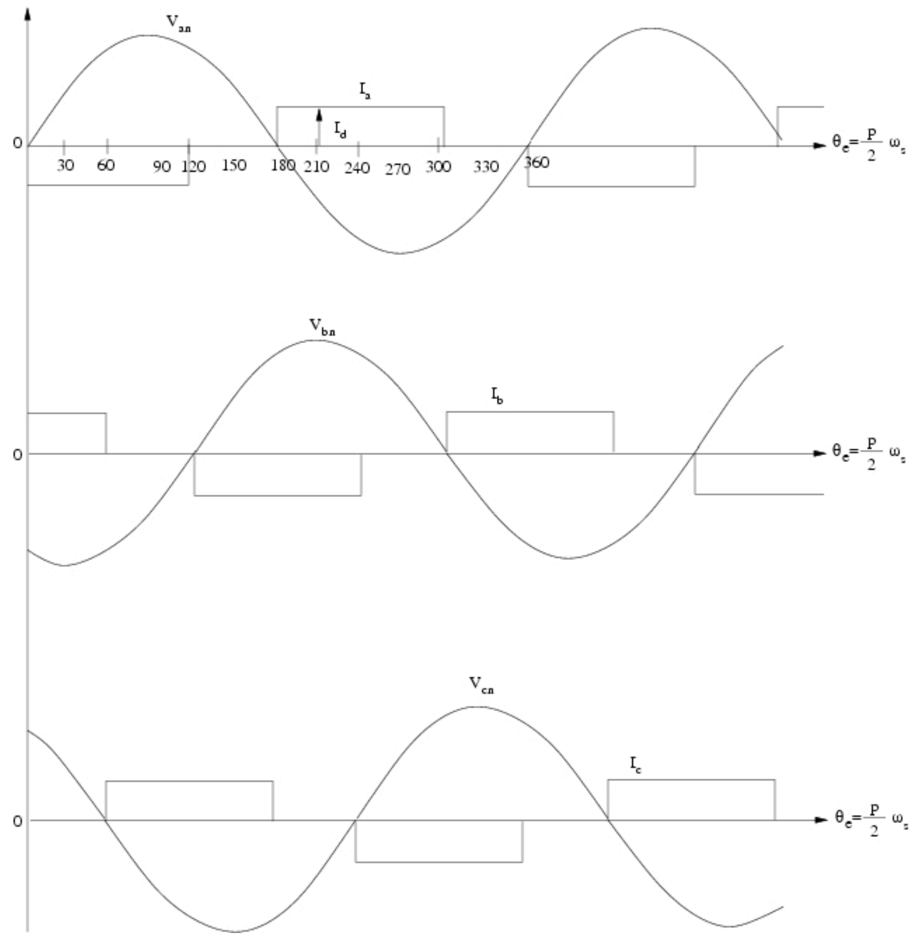
$$= 24.758 + j14.37\ Volts = 28.61\angle 30.14^0$$

Refer figure 15-3 in textbook with V_a leading $E_{ag,a}$ by 30.14^0

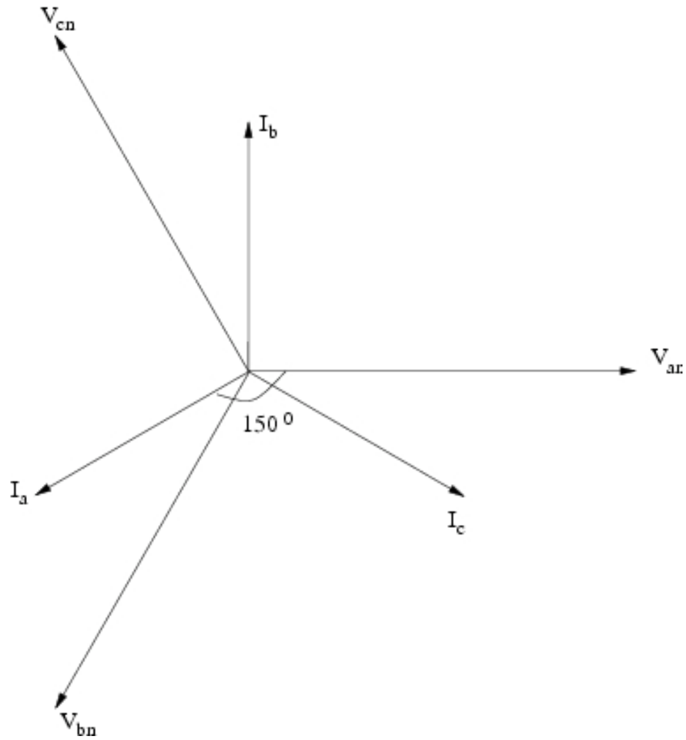
S15.2

a) Refer last part of section 15-5 in textbook.

b) Phase sequence a-b-c for clockwise direction. So for counterclockwise direction phase sequence a-c-b. Order for firing pulses (a+ b-)(b-c+)(c+ a-)(a- b+)(b+ c-)(c- a+)



d) If we assume current direction to be out of motor, phasor diagram will be as below, else the currents will be negative i.e. I_a will lead V_{an} by 30^0 and so forth.



e) As speed is half E_{fa}=0.5pu $\angle 0^0$ and synchronous reactance will be 0.9/2=0.45pu. And let's assume that V_{an} is leading E_{fa} by $\theta_1{}^0$

$V_{an} = E_{fa} - jI_a X_s$

Negative sign because our current direction is out of motor.

$$Van\angle\theta_1 = 0.5\angle 0 - 0.45\angle 150 + 90 + \theta_1 = 0.5\angle 0 - 0.45\angle(240 + \theta_1)$$

After equating real and imaginary parts we get $\theta_1{}^0 = 51^0$

flux ϕ_{fa} will lag E_{fa} by 90^0E, so $\delta = 171^0$ and power factor is 0.866 leading.

Chapter 16 - Residential and Industrial Applications

S16.1

a) Integral half-cycle control is typically less expensive than phase-angle control and generates fewer harmonics. So it is preferable for resistive heating or melting load.

b)

$$m = 1000, P_m = 800W, P_n = 250W, n = ?$$

$$P_n = \frac{n}{m} P_n$$

$$n = \frac{mP_n}{P_m} = \frac{1000 \times 250}{800}$$

$$n = 312.5 \quad 313$$

c) Switching large amounts of current can result in line voltage variations which affect ambient lighting or other equipment.

Chapter 17 - Electric Utility Applications

S17.1

a)

$$V_{LL(A)} = 156KV, I_d = 1.25KA, V_{reactance} = 0.04pu, L_s = ?$$

$$I_s = 2\times\sqrt{\frac{2}{3}}\times I_d = 2\times\sqrt{\frac{2}{3}}\times 1.25 = 2.04KA$$

$$I_s\omega L_s = \left(156/\sqrt{3}\right)\times 0.04 = 3.602KV$$

$$2.04\times 2\times\pi\times 60\times L_s = 3.602$$

$$L_s = 4.6835mH$$

$$\omega L_s = 120\times\pi\times 60\times L_s = 1.7655\Omega$$

b)

$$\gamma_{\min} = 180, V_{LL(B)} = 0.96V_{LL(A)}, I_d = 1.00KA, U_{dB} = ?U_{dA} = ?\alpha_A = ?$$

$$VLL(B) = 0.96\times 156 = 149.76KV$$

$$\omega L_s = 1.7655\Omega \text{ From } 17.1(a)$$

$$U_{dB} = 2\times\left[1.35V_{LL(B)}\cos\gamma_{\min} - \frac{3\omega L_s}{\pi}\times I_d\right]$$

$$= 2\left[1.35\times 149.76\cos 18^0 - \frac{3\times 1.7655}{\pi}\times 1.00\right]$$

$$= 381.19KV$$

Assume $R_{dc} = 0.02pu$ (Typical value in DC transmission line)

$$U_{dA} = U_{dB} + I_d R_{dc} = 381.19 + 0.02(156/\sqrt{3}) = 383KV$$

Also

$$U_{dA} = 2\times\left[\frac{3\sqrt{2}}{\pi}V_{LL(A)}\cos\alpha_A - \frac{3\omega Ls}{\pi}\times I_d\right]$$

$$383 = 2\times\left[\frac{3\sqrt{2}}{\pi}\times 156\cos\alpha_A - \frac{3\times 1.7655}{\pi}\times 1.00\right]$$

$$210.67\cos\alpha_A - 1.685 = 191.5$$

$$\cos\alpha_A = 0.917$$

$$\alpha_A = 23.508^0$$

c)

$$\mu_A = ?, \mu_B = ?, Id = 1KA, V_{LL(B)} = 149.76KV$$

$$\cos(\alpha_A + \mu_A) = \cos\alpha_A - \frac{2\omega L_s}{\sqrt{2}V_{LL(A)}} I_d$$

$$\cos(23.508 + \mu_A) = \cos 23.508 - \frac{2\times 1.7655}{\sqrt{2}\times 156}\times 1$$

$$\cos(23.508 + \mu_A) = 0.9$$

$$\mu_A = 2.333^0$$

Now

γ_B=180-α_B-μ_B

18=180-α_B-μ_B

α_B=162-μ_B

$$\cos(\alpha_B + \mu_B) = \cos\alpha_B - \frac{2\omega L_s}{\sqrt{2}V_{LL(B)}} I_d$$

$$\therefore \cos(162 - \mu_B + \mu_B) = \cos(162 - \mu_B) - \frac{2\times 1.7655}{\sqrt{2}\times 149.76}\times 1$$

$$-0.951 = \cos(162 - \mu_B) - 0.0166$$

$$\mu_B = 2.868^0$$

d)

$$\phi_A = \alpha_A + \frac{\mu_A}{2}$$

$$= 23.508 + \frac{2.333}{2}$$

$$= 24.674^0$$

$$\phi_B = \alpha_B + \frac{\mu_B}{2}$$

$$= (162 - \mu_B) + \frac{\mu_B}{2} = 162 - \frac{\mu_B}{2}$$

$$= 162 - \frac{2.868}{2}$$

$$= 160.566^0 \quad -----\text{ inverter operation}$$

e)

$$I_{p1} = ?, I_{q1} = ?, I_d = 1.25KA$$

RMS value of fundamental phase current

$$I_1 = 0.78 \times 2 \times I_d$$
$$= 0.78 \times 2 \times 1.25$$
$$= 1.95KA$$

Fundamental active current

$$I_{p1} = I_1 \cos\phi_A = 1.95 \times \cos 24.674^0 = 1.771KA$$

Fundamental reactive current

$$I_{q1} = I_1 \sin\phi_A = 1.95 \times \sin 24.674^0 = 0.814KA$$

S17.2

a) HVDC converters are line commutated, so the fundamental component of current lags voltage and it needs reactive power from the grid. Refer figure 6-26 for rectifier and figure 6-32 in textbook for inverter operation.

b) Both should be as small as possible

c) There should be sufficient turn off time for the thyristors. Also high values of firing and extinction angle will cause consumption of large reactive power.

d) Transformer with tap changer can control ac voltage V_{LL} supplied to converter, which in turn will control DC voltage V_{dc} to keep delay angle and extinction angle within certain limits

e) On the inverter side the goal is to keep extinction angle constant at $\gamma = \gamma_{\min}$

f) Reactive power supplied by capacitors and passive filters depend upon the grid voltage. So in case of grid voltage reduction the reactive power supplied by them reduces. Also during rapid reduction in the active power of the HVDC converter, the reactive power generated by passive filters will exceed the reactive power required by converters, which will result into system over voltage.

g) FACTS components can control reactive power irrespective of grid voltage and real power transfer, so FACTS components are more useful.

S17.3

Refer section 6-4-2 for rectifier and 6-3-4 for inverter (its for single phase, but similar things will happen for 3 phase) in textbook.

S17.4

a) 12 cells/24 V Battery, V_{min}/cell=1.6V, V_{max}=2.27V
Range of battery ➔1.6x12 to 2.27x12 = 19.2V-27.24 V

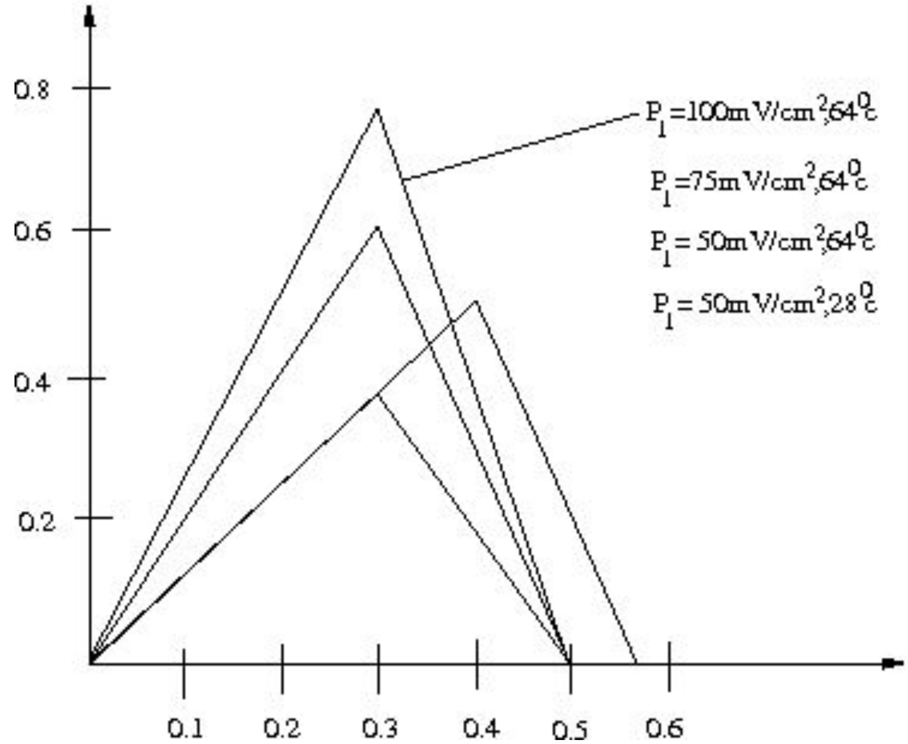
b) For V_{min} % change=(24-19.2)x100/24=20%
V_{max} % change=(27.24-24)x100/24=13.5%

c) At P_1=100mv/cm^2 at 64^{0}C, P=VI=0 to 0.3x2.5=0 to 0.75 and then decrease at V=0.5,P=0

At P_1=75mv/cm^2 at 64^{0}C, P=VI=0 to 1.9x0.3=0 to 0.57 and then decrease at V=0.5,P=0

At P_1=50mv/cm^2 at 64^{0}C, P=VI=0 to 1.25x0.3=0 to 0.375 and then decrease at V=0.48,P=0

At P_1=50mv/cm^2 at 28^{0}C, P=VI=0 to 1.25x0.4=0 to 0.5 and then decrease at V=0.56,P=0



d) At 0^0c provide current of 1.1 A cell voltage is approximately 0.57V and at 64^0c it is 0.4 V, so maximum number of cells required $=27.24/0.4=68.1\approx69$

e) When battery voltage is close to maximum, voltage per photovoltaic cell = 27.24/69 = 0.4 volts. At this voltage currents for each conditions are 2.22A, 1.6A, 1.26A, 0.962A respectively.

So energy available = Time $\times$ Voltage $\times$ Current
$E=(15/60)\times(69\times0.4)\times(2.22+1.6+1.26+0.962)$ =41.68W-H

f) When battery voltage is close to minimum, voltage per photovoltaic cell = 19.2/69 = 0.278 volts. At this voltage currents for each conditions are 2.52A, 1.85A, 1.26A and 1.22A respectively.

$E=(15/60)\times(69\times0.278)\times(2.52+1.85+1.26+1.22)$ =32.85W-H

g) Step down converter

h) For maximum power voltage in each case is 0.4volts/cell, and currents are 2.22A, 1.6A, 1.26A, 0.962A respectively.

$E_{max}=(15/60)\times(69\times0.4)\times(2.22+1.6+1.26+0.962)=41.68$W-H

i) 41.68W-H

j) With step down chopper we can operate all photovoltaic arrays at maximum power, number of parallel arrays can be reduced for the same charging per hour.

Chapter 19 - Semiconductor Physics

S19.1.

Junction will effectively disappear when the intrinsic carrier density n_i equals the lowest doping density in the pn junction structure (both the p-side and the n-side have the same doping level in this problem). Thus

$$n_i(T_i) = N_d = 10^{14} = 10^{10} \exp\left[- \frac{q E_g}{2k} \left\{ \frac{1}{T_i} - \frac{1}{300} \right\} \right]$$

Solving for T_i using E_g = 1.1 eV, k = 1.4x10^{-23} [1/°K] yields

T_i = 262 °C or 535 °K.

S19.2.

a) $V_{bar} = (0.01A)\, R_{bar} : \; R_{bar} = \frac{L}{A\sigma}$;

$\sigma = q\mu_n N_d = (1.6x10^{-19})(1500)(10^{15}) = 0.24$ mhos-cm

$R_{bar} = \frac{1}{(0.42)(0.24)} = 10\Omega$; $V_{bar} = (0.01A)(10\Omega) = 100mV$

b) As the temperature increases, the intrinsic carrier density increases which means higher conductivity or equivalently lower resistivity. This in turn means lower resistance in the bar and thus lower voltage drops at elevated temperatures, assuming the current remains constant.

c) At $T_{0.5}$ the resistance is 50% of the room temperature resistance. This means that the conductivity at $T_{0.5} = 2\sigma(25\ °C) = 0.48$ mhos-cm $= (q\,\mu_n n + q\,\mu_p p) = \sigma(T_{0.5})$.

$pn = n_i^2$; $p + N_d = n$; Solving for p and n yields:

$$p = \frac{1}{2}\sqrt{N_d^2 + 4n_i^2} - \frac{N_d}{2} : \; n = \frac{N_d}{2} + \frac{1}{2}\sqrt{N_d^2 + 4n_i^2}$$

$$\sigma(T_{0.5}) = q\,\mu_p\left[\frac{1}{2}\sqrt{N_d^2 + 4n_i^2} - \frac{N_d}{2}\right] + q\,\mu_n\left[\frac{N_d}{2} + \frac{1}{2}\sqrt{N_d^2 + 4n_i^2}\right] = 2\,q\mu_n N_d$$

Solving for n_i yields: $n_i = \frac{N_d}{2}\sqrt{\frac{(3\mu_n + \mu_p)^2}{(\mu_n + \mu_p)^2} - 1}$

$$n_i = \frac{10^{15}}{2}\sqrt{\frac{((3)(1500) + 500)^2}{(1500 + 500)^2} - 1} = 1.15x10^{15}\ cm^{-3}$$

$$n_i(T_{0.5}) = 1.15x10^{15} = 10^{10} \exp\left(\left\{ \frac{qE_g}{2k}\left[\frac{1}{298} - \frac{1}{T_{0.5}}\right]\right\}\right.$$

$$\frac{qEg}{2k} = \frac{(1.6x10^{-19})(1.1)}{(2)(1.4x10^{-23})} = 6250 \quad ; \quad 1.15x10^{15} = 10^{10} \exp\left(\left\{ 6250 \left[\frac{1}{298} - \frac{1}{T_{0.5}}\right]\right\}\right.$$

$$(6250) (1/298 - 1,/T_{0.5}) = \ln(1.15x10^{15}) = 11.65 \; ; \; \frac{1}{298} - \frac{11.65}{6250} = \frac{1}{T_{0.5}}$$

$$\frac{1}{T_{0.5}} = 0.00034 - 0.0019 = 0.0015 \; ; \; T_{0.5} = 667\ °K \text{ or } 394\ °C$$

Chapter 20 - Power Diodes

S20.1.

a. $I_{rr} = \{2I_F\tau\,(di_R/dt)\}^{1/2}$; $t_{rr} = \{2I_F\tau\,(di_R/dt)^{-1}\}^{1/2}$

$di_R/dt = V_d/L_\sigma = (100V)/(100nH) = 10^9$ A/sec

$I_{rr} = [(2)(100)(5x10^{-8})(10^9)]^{1/2} = 100$ A

$t_{rr} = [(2)(100)(5x10^{-8})/(10^9)]^{1/2} = 100$ nsec

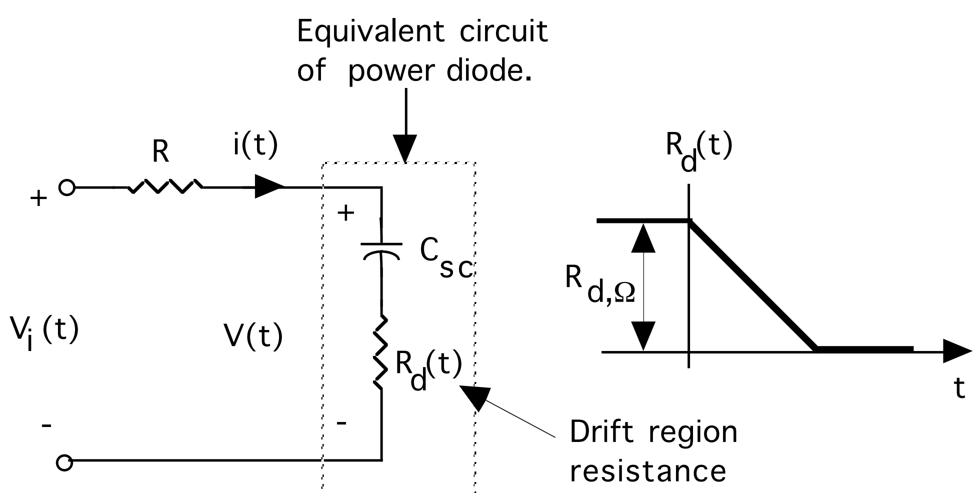
b. Snappiness factor S = 1 means that the rate of fall from I_{rr} to zero is the same as rate from I_F to I_{rr}. This rate is 10^9 A/sec.

Thus $V_{rr} = V_d + L_\sigma(di/dt) = 100\ V + (10^{-7})(10^9) = 200\ V < 400\ V$ rating.

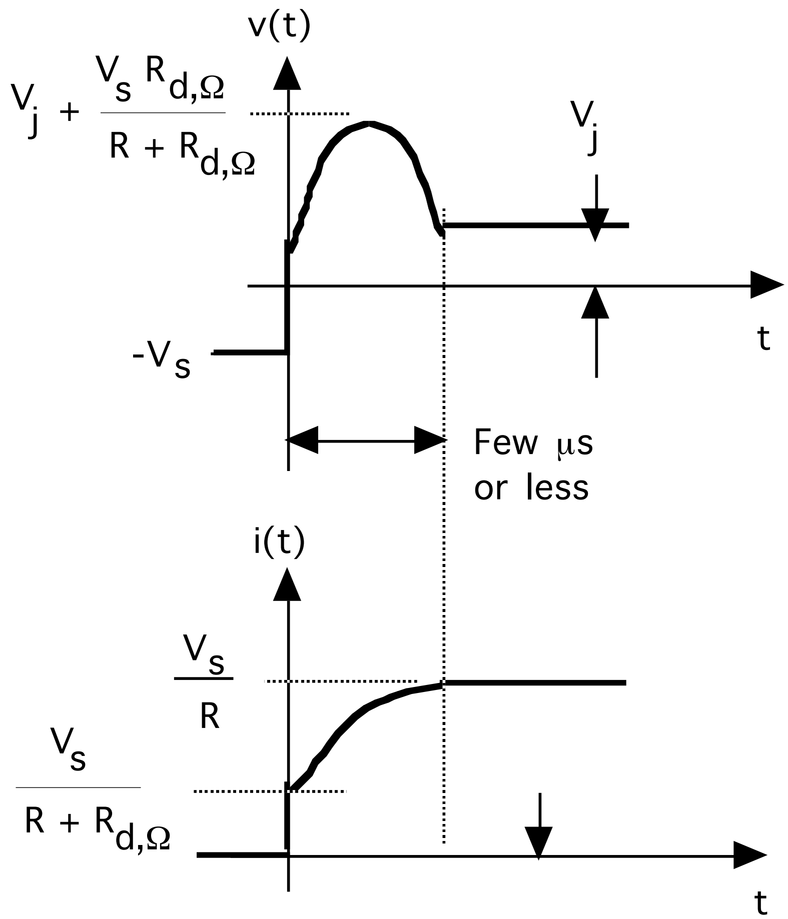
No snubber is needed.

S20.2.

Approximate model for signal level diode circuit shown below. This circuit produces thesimple RC circuit transient having values as shown in problem statement.
The power diode has approximate equivalent circuit shown below.



The equivalent circuit shown above for the power diode produces the following waveforms.



S20.3.

a. The punch-thru diode has a drift region which is about one-half that of the standard diode. This means that the diffusion length of the excess carriers, which is dependent on the carrier lifetime can be smaller for the punch-thru diode which then translates into a shorter carrier lifetime for carriers in the punch-thru diode.

b. The punch-thru diode will have a larger forward recovery voltage because of the larger ohmic resistance (due the much smaller doping level in the punch-thru drift region) of the drift region before conductivity modulation is fully developed.

c. The diffusion length scales as the square root of the carrier lifetime. Since the drift region length of the punch-thru diode is about one-half that of the standard diode, the lifetime in the punch-thru diode is one fourth that of the standard diode. The reverse recovery time t_{rr} is proportional to the square root of the carrier lifetime, so t_{rr} for the punch-thru diode will be about one half that of the standard diode.

S20.4.

a. Input voltage is a 250 V base-to-peak square wave. After full-wave rectification, a constant DC voltage of 250 volts is applied to the load. Each diode must block 250 V. With a 25% factor of safety, the blocking voltage rating should be (250)(1.25) = 312 V.

Current through load = (10^5 Watts)/(250 V) = 400 A = current through each diode. Each diode off for 50% of the time, so average forward curent = 200 A. Inclusion of 25% factor of safety yields (200)(1.25) = 250 A for maximum forward current rating.

b. Instantaneous power dissipated in each diode = [1+ (0.002)(400)][400] = 720 Watts
Average power dissipation in each diode = 360 W since each diode off 50% of the time.
$T_{j,max} = \langle P_{diode} \rangle [R_{\theta,j-a} + R_{\theta,c-a}] + T_a$; Assume ambient temperature T_a = 25 °C
$R_{\theta,c-a}$ = (150 - 25)/360 - 0.1 = 0.347 - 0.1 ≈ 0.25 °C/Watt

S20.5.

The forward current in a pn junction diode is inversely proportional to the carrier lifetime (see eq. 19-25, p. 520 of Power Electronics, 2nd edition, by Mohan, Undeland, and Robbins). Keeping everything constant except the carrier lifetime, means that the lifetime must be reduced by a factor of two in order to increase the current by a factor of two.

S20.6.

a. Initially assume the diode has a non-punch-thru structure.
$BV_{BD} = 1.3x10^{17}/2x10^{14}$ = 650 V. $W(BV_{BD}) = (10^{-5})(650)$ = 65 microns
Drift region length of 100 microns is somewhat longer than it needs to be. Diode is definitely a non-punch-thru structure.

b. $I_s = q\ A\ (n_i)^2\ \{[D_p\tau_p]^{.5}/(N_a\tau_p) + [D_n\tau_n]^{.5}/(N_d\tau_n)\}$

$I_s = q\ A\ (n_i)^2[D_n\tau_n]^{.5}/(N_d\tau_n)$ since $N_a >> N_d$

Good design practice requires $[D_n\tau_n]^{.5} = W_d$ = 100 um;

Hence $\tau_n = (10^{-2})^2/(40) = 2.5x10^{-6}$ sec

$I_s = (1.6x10^{-19})(1)(10^{20})(10^{-2})/[(2x10^{14})(2.5x10^{-6})] = 3x10^{-10}$ A

c. $V = V_j + R_{on}I_F$; $V_j = kT/q\ \ln(I_F/I_s) = (0.026)\ln(500/3x10^{-10})$ = 0.61 V

$R_{on}^{-1} = q(\mu_n + \mu_p)n_bA/W_d = (1.6x10^{-19})(900)(10^{17})(1)/(10^{-2}) = 1.4x10^3$

R_{on} = 0.0007 Ω ; $I_F\ R_{on} = (500)(7x10^{-4})$ = 0.35 V

Voltage drop = 0.61 + 0.35 = 0.96 V

S20.7.

$$I_{rr} \approx \sqrt{2\,\tau\, I_F \frac{di_R}{dt}} \;\; ; \;\; \tau \approx \frac{W_d^{\,2}}{(kT/q)(\mu_n + \mu_p)} = \frac{(5x10^{-3})^2}{(0.025)(1500 + 500)} = 50 \text{ nsec}$$

Assume switch turns on instantaneously so that entire V_d = 500 V is dropped across the 1 μH inductor. Hence $\frac{di_R}{dt} = V_d/L = 500/10^{-6} = 5x10^8$ A/sec.

$$I_{rr} \approx \sqrt{(2)(5x10^{-8})(200)(5x10^8)} = 100 \text{ A.}$$

$$t_{rr} \approx \sqrt{\frac{2\,\tau\, I_F}{\frac{di_R}{dt}}} = \sqrt{\frac{(2)(5x10^{-8})(200)}{5x10^8}} = 0.2\ \mu\text{sec}$$

S20.8.

a. $W_d = (2)(BV_{BD})/E_{BD} = (2)(1500)/3x10^5 = 100\ \mu m$; $N_d = BV_{BD}\left[\frac{2\,\varepsilon}{q\,W_d^2}\right]$

$$N_d = \frac{(2)(12)(9x10^{-14})(1500)}{(1.6x10^{-19})(10^{-2})^2} = 2.3x10^{14}\ cm^{-3}$$

b. Carrier lifetime $\tau = W_d^{\,2}/(D_n + D_p) = (10^{-2})^2/(39+13) \approx 2\ \mu$sec

$I_{rr} = \sqrt{2\,\tau\, I_f\ (di_R/dt)}$; $t_{rr} = \sqrt{\frac{2\,\tau\, I_f}{di_R/dt}}$; Circuit diagram of step-down converter showing the value of I_F inadvertently left out of problem statement. Cannot proceed any further without value for I_F.

Chapter 21 - BJTs

S21.1.

a. $BV_{CEO} = 100\text{ V} = \beta^{-1/4}\, BV_{CBO}$; $BV_{CBO} = (100)\,(20^{1/4}) = 217\text{ V}$

Treat base-collector junction as a p^+-n^- step junction where $BV = 1.3\text{x}10^{17}/N_d$

$N_d = 1.3\text{x}10^{17}/217 = 6\text{x}10^{14}\text{ cm}^{-3}$

b. Drift region length = $L_d = 10^{-5}\, BV_{BD} = (10^{-5})(217) = 22$ microns

c. $L_d^2 = D\,\tau$; $D = D_n + D_p = (13 + 39) = 52\text{ cm}^2/\text{V-sec}$

$\tau = (2.2\text{x}10^{-3})^2(39 + 13) = 15$ microseconds

S21.2.

a. Treat base-collector junction as a p^+-n^- step junction where $BV = 1.3\text{x}10^{17}/N_d$ even though the p-side doping is only about twenty times larger than the n-side. Thus $BV_{BD} = 1.3\text{x}10^{17}/2\text{x}10^{14} = 650$ V.

Check with drift region length formula $BV_{BD} = 10^{-5}\, L_d = (10^{-5})(.01) = 1000$ V

Two approaches disagree slightly. Use the more conservative estimate of 650 V for BV_{CBO}.

$BV_{CEO} = (10)^{-1/4}\,(650) = 650/1.78 = 365$

b. Depletion layer width L_{depl} approximately 100 microns. $L_{depl} = x_n + x_p$

Base width $W_B > x_p$; Charge balance requires $N_a\, x_p = N_d\, x_n$; Now x_n approximately equal to L_{depl} = 100 microns; Hence $x_p = L_{depl}\, N_d/N_a = (100)(2\text{x}10^{14})/(5\text{x}10^{15})$

$W_B > 4$ microns.

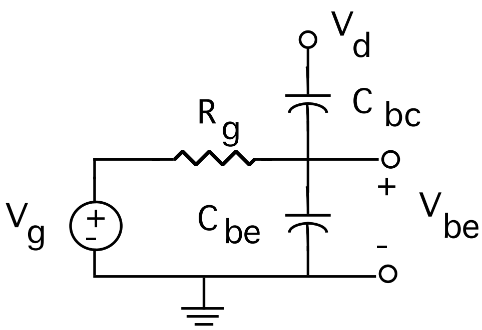
S21.3.

There are three basic reasons why NPN BJTs are more widely used than PNP BJTs.

a. Most power electronic converters use positive power supplies and NPN BJTs fit naturally into such circuits whereas PNP BJTs are more awkward to use.
b. NPN BJTs breakdown voltages are less sensitive to beta values compared to PNP BJTs.
c. Drift region of an NPN BJT is n-type and the ohmic resistance of the drift region is 3 times lower in an NPN BJT than in a comparable PNP BJT because of the larger mobility of electrons.

S21.4.

The turn-on delay is estimated using the equivalent circuit shown below. It is the time required for the base-emitter voltage to build up from -5 V to +0.7 V.



$$V_g = R_g\{C_{be} + C_{bc}\}\frac{dV_{be}}{dt} + V_{be}$$

$$V_{be}(0) = -5\text{ V},\ \ V_{be}(\infty) = 5\text{ V};$$

$$V_{be}(t_{d,on}) = 0.7\text{ V}$$

$$V_{be}(t) = 5 - 10\exp(-t/\tau)\ ;\ \tau = R_g\{C_{be} + C_{bc}\}$$

$$t_{d,on} = R_g\{C_{be} + C_{bc}\}\ln\{10/4.3)$$

Evaluation of $t_{d,jon}$ requires estimating the average value of the space charge capacitances C_{be} and C_{bc}.

$$\langle C_{be}\rangle = (1/2)\ \{C_{be}(-5V) + C_{be}(0V)\}\ \ ;$$

$$\langle C_{bc}\rangle = (1/2)\ \{C_{bc}(-100V) + C_{bc}(0.7V)\} \approx C_{bc}(-100V)$$

Estimate $\langle C_{be}\rangle$

$$C_{be}(V_{be}) = C_{be}(0)\frac{1}{\sqrt{1 - V_{be}/\phi_c}}\ \ ;\ \ \phi_c = \{kT/q\}\ln\left[\frac{N_a N_d}{n_i^2}\right]\ \ ;\ \ C_{be}(0) = \frac{\varepsilon A}{W_{depl}(0)}\ \ ;$$

$$W_{depl}(0) \approx \sqrt{\frac{2\,\varepsilon\,\phi_c}{q\,N_d}}\ \ ;\ \phi_c = 0.025\ln\left[\frac{(10^{19})(10^{16})}{10^{20}}\right] = [0.025)\ln(10^{15}) = 0.86\text{ V};$$

$$W_{depl}(0) \approx \sqrt{\frac{(2)(12)(9\text{x}10^{-14})(.86)}{(1.6\text{x}10^{-19})(10^{16})}} = 0.34\ \mu\text{m};$$

$$C_{be}(0) = \frac{(12)(9\text{x}10^{-14})(2)}{3.4\text{x}10^{-5}} = 64\text{ nF};\ C_{be}(-5V) = \frac{64\text{ nF}}{\sqrt{1 + 5/0.86}} = 25\text{ nF}$$

$$\langle C_{be}\rangle = (1/2)\ \{64\text{nF} + 25\text{nF}) = 45\text{ nF}$$

Estimate $\langle C_{bc}\rangle = C_{bc}(100V) = C_{bc}(0)\dfrac{1}{\sqrt{1 - V_{bc}/\phi_c}}$;

$$\phi_c = 0.025\ln\left[\frac{(10^{19})(10^{14})}{10^{20}}\right] = 0.75\text{ V};\ W_{depl}(0) \approx \sqrt{\frac{(2)(12)(9\text{x}10^{-14})(.86)}{(1.6\text{x}10^{-19})(10^{14})}} = 3.4\ \mu\text{m};$$

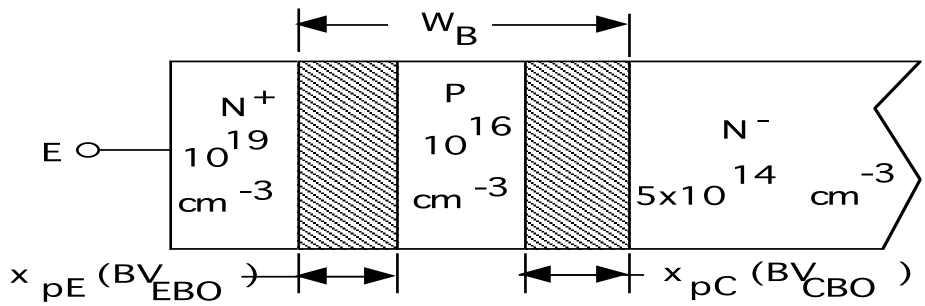
$C_{bc}(0) = \frac{(12)(9x10^{-14})(2)}{3.4x10^{-4}}$ = 6.4 nF ; $C_{bc}(-100V) = \frac{6.4 \text{ nF}}{\sqrt{1 + 100/0.75}}$ = 552 pF

$t_{d,on}$ = (4){4.5x10^{-8}+ 5.5x10$^{-10)}$}ln{2.32) ≈ 0.18 μsec

S21.5.

a. The blocking voltage rating of an NPN BJT is given by $BV_{CEO} = \frac{BV_{CBO}}{\beta^{1/4}}$; transistor must have low beta in order to not have severe reductions in BV_{CEO}. This requires relatively wide base widths. This will also help avoid reach-thru in the base.

b. Diagram of space charge layers in base region shown below.



Base-emitter junction is a one-sided step junction with the base having the lightly doped side.

$x_{pE}(BV_{EBO}) \approx [\varepsilon E_{BD}]/[q N_{aB}]$ = [(12)(9x10^{-14})(3x10^{5})]/[(1.6x10^{-19})(10^{16})] = 2 μm

Base-collector is approximately a one-sided step junction. Depletion layer thickness at breakdown on the collector side is given by $x_{nC}(BV_{CBO}) \approx [\varepsilon E_{BD}]/[q N_{dC}]$

$x_{nC}(BV_{CBO})$ = [(12)(9x10^{-14})(3x10^{5})]/(1.6x10^{-19})(5x10^{14}) = 40 μm

Charge neutrality at the base-collector junction requires

$x_{pC}(BV_{CBO})\, N_{aB} = x_{nC}(BV_{CBO}) N_{dC}$; $x_{pC}(BV_{CBO})$ = (40 μm)(5x10^{14})/(10^{16})

$x_{pC}(BV_{CBO})$ = 2μm

Required base width = 5(2μm + 2μm) = 20 μm = W_B

S21.6.

The drift region of the transistor appears to be a punch-thru geometry. Verify with following calculation. Depletion layer width in a long (non-punch-thru) drift region at breakdown is

$W_{depl} = [\varepsilon E_{BD}]/[q N_d]$ = [(12)(9x10^{-14})(3x10^{5})]/[(1.6x10^{-19})(10^{13})] = 2000 μm

>> 40 μm. Thus must use a punch-thru breakdown voltage estimate.

$BV_{CBO} = E_{BD}\, W_d - (qN_dW_d^{\,2})/(2\varepsilon)$

$BV_{CBO} = (3x10^{5})(5x10^{-3}) - (1.6x10^{-19})(10^{13})\,(4x10^{-3})^{2}[(2)(11.7)(8.9x10^{-14})]^{-1}$

$BV_{CBO} = 1500 - 12 \approx 1500$ V ; $BV_{CEO} = \dfrac{1500}{10^{0.25}} = 1500/1.73 \approx 870$ V

Inclusion of a 50% factor of safety yields a blocking voltage rating of (870)(1.5) = 1300 V

Chapter 22 - MOSFETs

S22.1.

The voltage and current rise and fall times are determined by how fast charge can be put on or removed from the gate-source and gate-drain capacitance. The drive circuit provides charge at about the same rate to these capacitances during all four time intervals (voltage rise, voltage fall, current rise, and current fall). During the current rise and fall times, the change in voltage across the capacitances is only 10-20 volts where as the change in voltage during the voltage rise and fall times, especially across the gate-drain capacitance is much larger. This means much more charge must be moved during the voltage transient than during the current transient. Since the rate of change of charge is the same in all intervals, the voltage rise and fall times must be longer than the current rise and fall times.

S22.2.

a. $$P_{mos} = I_o^2 r_{DS}(on)\, D + \frac{V_d I_o (t_{ri}+t_{fv}+t_{fi}+t_{rv})}{2} f_{sw}$$

$$P_{mos} = (50)^2(0.05)(0.5) + \frac{(250)(50)(25+175+25+175)(10^{-9})}{2} (10^4)$$

$$P_{mos} = 62.5 + 25 = 87.5 \text{ W}$$

b. $$P_{max} = \frac{T_{j,max} - T_a}{R_{\theta j\text{-}a}} = \frac{175 - 25}{0.5} = 300 \text{ watts}$$

$$f_{sw} = \frac{(2)(P_{max} - I_o^2 r_{DS}(on)\, D)}{V_d I_o (t_{ri}+t_{fv}+t_{fi}+t_{rv})}$$; worst case (lowest f_{sw}) occurs when D = maximum.

$$f_{sw} = \frac{(2)(300 - (50)^2(0.05)(0.9))}{(250)(50)(25+175+25+175)(10^{-9})} = 75 \text{ kHz}$$

S22.3.

a. $$P_{mos} = I_o^2 r_{DS}(on)\, D + \frac{V_d I_o (t_{ri}+t_{fv}+t_{fi}+t_{rv})}{2} f_{sw}$$

$$(t_{ri}+t_{fv}+t_{fi}+t_{rv}) = t_{ri} + 6t_{ri} + t_{ri} + 6t_{ri} = 14t_{ri}$$

$$t_{ri} = \frac{2(P_{mos} - I_o^2 r_{DS}(on)\, D)}{(14)(V_d I_o f_{sw})} = \frac{2(100 - (30)^2(0.1)(0.5))}{(14)(300)(30)(10^4)} = 87 \text{ nsec} = t_{fi}$$

$$t_{rv} = t_{fv} = 6t_{ri} = (6)(87\text{ns}) = 0.524\ \mu\text{sec}$$

$$T_{j,max} = P_{max}(R_{\theta j\text{-}c} + R_{\theta c\text{-}a}) + T_a$$

$R_{\theta c\text{-}a} = \frac{175 - 25}{200} - 0.25 = 0.75 - 0.25 = 0.5$ °C/Watt

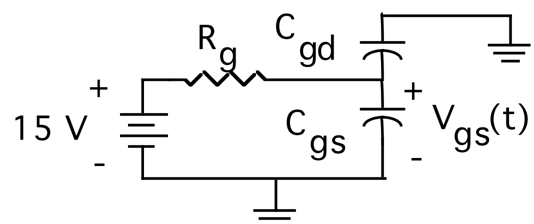
S22.4.

a. Threshold voltage = 5 V; Drain current begins to flow when $V_{GS} = 5$ V.

b. $I_D = g_m(V_{GS} - V_{th})$; From the waveforms shown $I_D = 100$ A when $V_{GS} = 10$ V; $g_m = (100\text{ A})/(10\text{V} - 5\text{V}) = 20$ A/V

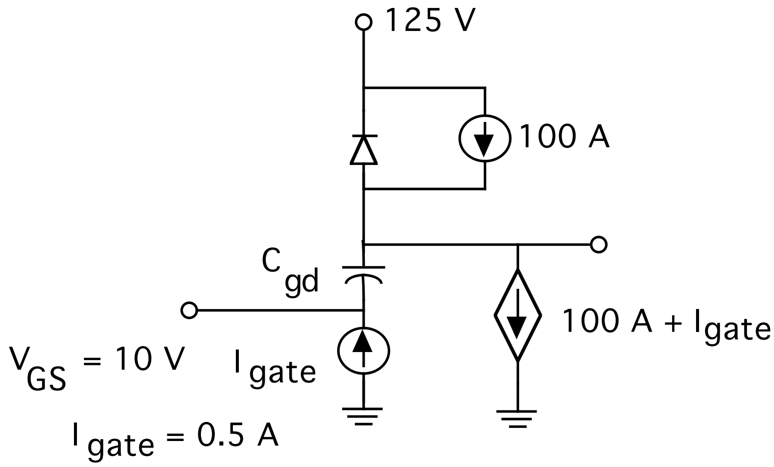
c. In on-state $I_D = 100$ A and $V_{DS(on)} = 3$ V; $R_{on} = 3\text{V}/100\text{A} = 0.03$ ohms

d. Estimate C_{gd} from the 250 nsec voltage fall time. During this interval equivalent circuit is as shown below.



$I_{gate} = (15\text{V} - 10\text{V})/(10\text{ ohms}) = 0.5$ A; $I_{gate} = - C_{gd}(dv_{DG}/dt)$; $dv_{DG}dt = dv_{DS}/dt$
$v_{DS} = 125\text{V} - [I_{gate}/C_{gd}]\, t$; At $t = t_{fv} = 250$ nsec, $v_{DS} = 0$; Solving for C_{gd} ;
$C_{gd} = (0.5\text{A})(250\text{nsec})/(125\text{V}) = 1$ nanofarad

e. Obtain C_{gs} from either turn-on delay time (33 nsec) or current risetime (21 nsec). Will use turn-on delay time. Equivalent circuit during $t_{d(on)} = 33$ nsec show below.
Governing equation : $(C_{gs} + C_{gd})\, dV_{gs}/dt = (15 - V_{gs})/R_g$
Boundary conditions: $V_{gs}(0) = -15$ V ; $V_{gs}(\infty) = 15$ V ;
$V_{gs}(t) = 15 - 30 \exp(-t/\tau)$; $\tau = R_g(C_{gs} + C_{gd})$; At t = 33 nsec, $V_{gs}(33\text{ nsec}) = 5$ V
$5 = 15 - 30 \exp(-33/\tau)$; $\tau = 33/\ln(3) = 33/1.1 = 30$ nsec $= (10)(C_{gs} + 1\text{nF})$
$C_{gs} = 3\text{nF} - 1\text{nF} = 2$ nF



S22.5.

$I_D = I_o = g_m(V_{Gsp} - V_{th})$; $V_{GSp} = I_o/g_m + V_{th}$

$Q_{g1} = (C_{gs} + C_{gd})V_{GSp} = (C_{gs} + C_{gd})[I_o/g_m + V_{th}]$

In going from Q_{g1} to Q_{g2}, the voltage, V_{GD} across C_{gd} goes from $V_{GSp} - V_d$ to V_{GSp}.

Thus $Q_{g2} = Q_{g1} + C_{gd}V_d = (C_{gs} + C_{gd})[I_o/g_m + V_{th}] + C_{gd}V_d$

Slope1 = $V_{GSp}/Q_{g1} = C_{gs} + C_{gd}$ = Slope2

S22.6.

Possible overstressing includes overvoltage at turn-off and excessive power dissipation.
Check for overvoltage at turn-off: $V_{DS} = V_d + L_\sigma di_D/dt =$

$V_{DS} =$ 100V + $(10^{-7}$H)(100A)/$(5x10^{-8}$sec) = 300 V > BV_{DSS} = 150 V. MOSFET is subjected to overvoltages at turn-off.

Check for excessive power dissipation.

$P_c = P_{cond} + E_{sw} f_s$; $P_{cond} = (100A)^2(0.01\Omega)(0.5)$ = 50 watts

E_{sw} = (0.5)100V)(100A)(200+200+50+50)$(10^{-9}$sec) = 0.0025 Joules

P_c = 50 +(.0025)$(3x10^4)$ = 125 watts;

T_j = (125W)(1°C/W) + 50°C = 175 °C > $T_{j,max}$ = 150 °C. MOSFET dissipating too much power.

S22.7.

During the turn-on delay period, MOSFET is off, and the gate-source voltage, $V_{gs}(t)$, is changing from - 10 V towards positive values. The gate-source capacitance and drain-source capacitance appear in parallel at the gate-source terminals. Equivalent circuit is shown below.

Governing equation : $(C_{gs} + C_{gd})\, dV_{gs}/dt = (10 - V_{gs})/R_g$

Boundary conditions: $V_{gs}(0) = -10$ V ; $V_{gs}(\infty) = 10$ V ;

$V_{gs}(t) = 10 - 20 \exp(-t/\tau)$;

$\tau = R_g(C_{gs} + C_{gd}) = (25)(1\text{nF} + .5\text{nF}) = (25)(1.5\text{x}10^{-9}) = 38$ nsec

At $t = t_{d(on)}$, $V_{gs}(t_{d(on)}) = 5$ V

$5 = 10 - 20 \exp(-t_{d(on)}/38)$; $t_{d(on)} = 38 \ln(20/5) = (38)(1.39) = 53$ nsec

Chapters 23 & 24 - SCRs and GTOs

S23.1.

a. $P_{SCR,max} = (T_{j,max} - T_c)/R_{\theta j\text{-}c} = (125 - 50)/0.05 = 75/0.05 = 1.5$ kW

b. Maximum load power and maximum P_{SCR} occurs at zero phase angle ($\alpha = 0$).

$$\langle P_{SCR} \rangle_{max} = \frac{V_s}{\pi R_L} + \left[\frac{V_s}{R_L}\right]^2 R_{on} = 1500 \text{ W}$$; Put in known values and solve for V_s.

$1500 = V_s/[(\pi)(1)] + (V_s)^2(5 \times 10^{-4})/(1)$;

Solve by successive approximation. $V_s = 1440$ V

$P_{L,max} = (V_s)^2/(2\, R_L) = (0.5)(1440)^2/1 = 1$ megawatt

S23.2.

a. Blocking voltage rating = (1.25)(440)(1.414) = 775 V

$P_L = 10^6$ watts $= V_{rms}\, I_{rms}$; sinewave waveform ; $I_{rms} = 10^6/440 = 2273$ A;

I_L(base-to-peak) = (1.414)(2273) = 3205 ; $I_{SCR}(t) = 3205 \sin(\omega t)$

$$\langle I_{SCR} \rangle = \frac{1}{2\pi}\int_0^{\pi} 3205\sin(\omega t)d(\omega t) = (2/\pi)(3205) = 2043 \text{ A;}$$

Inclusion of 25% factor of safety yields maximum forward current rating I_{rated}

$I_{rated} = (1.25)(2043) = 2555$ A

b. $\langle P_{SCR} \rangle = \langle V_{SCR} I_{SCR} \rangle = \langle (1 + R_{on} I_{SCR}) I_{SCR} \rangle = (1\text{V})\langle I_{SCR} \rangle + R_{on} \langle I_{SCR}^2 \rangle$

$$\langle I_{SCR}^2 \rangle = \frac{1}{2\pi}\int_0^{\pi} [3205\sin(\omega t)]^2\, d(\omega t) = \frac{(3205)^2}{2\pi}[\pi/2] = (3205)^2/4 = 2.56 \times 10^6$$

$\langle P_{SCR} \rangle = (1)(2043) + (2 \times 10^{-3})(2.6 \times 10^6) = 7043$ watts ; Inclusion of 25% factor of safety yields $P_{SCR,max} = 7043)(1.25) = 8800$ watts

S23.3.

Advantages - MOSFET controlled turn-off. Drive circuit does not have to accept large reverse gate currents. Faster turn-off.

Disadvantages - Need to find very high current zener diodes or put several in parallel. No simplification of drive circuit for GTO turn-on.

S23.4.

$V_{GTO} = L_{\sigma}\,(di/dt) + V_d = 1.5\,V_d$; $L_{\sigma} \leq (0.5V_d)/(di/dt)$

Chapter 25 - IGBTs

S25.1.

a. For the antisymmetric or punch-thru IGBT, breakdown voltage is given by:

$$BV_{DSS} = E_{BD}W_d - \frac{qN_dW_d^2}{2\varepsilon} = (3x10^5)(2.5x10^{-3}) - \frac{(1.6x10^{-19})(10^{14})\,(2.5x10^{-3})^2}{(2)(12)(9x10^{-14})}$$

$BV_{DSS} =$ 750 - (9.5x10^{-11})/(2.16x10^{-12}) = 750 - 44 = 706 V

For the symmetric or non-punch-thru IGBT, breakdown voltage given by

$$BV_{DSS} = \frac{qN_dW_d^2}{2\varepsilon} = \frac{(1.6x10^{-19})(10^{14})\,(2.5x10^{-3})^2}{(2)(12)(9x10^{-14})}$$ = 44 V; impact ionization is not occuring at this voltage but the depletion region has reached across the drift region and the high electric field in the depletion layer is in electrical contact with the heavily doped p-type regions on each side of the drift region. Large hole currents will flow, causing high power dissipation.

S25.2.

a. $BV_{CES} = \frac{E_{BD}\,W_d}{2}$ = (0.5)(3x10^5)(1.5x10^{-2}) = 2250 V ; To be conservative, the device manufacturer will reduce this voltage by 50 %, i.e. manufacturer's rating = (2250)(2/3) = 1500 V.

b. R_{on} = (1V)/(20A) = 0.05 ohms = $\frac{W_d}{q(\mu_n + \mu_p)\,n_b\,A}$;

$$A = \frac{1.5x10^{-2}}{(1.6x10^{-19})(2x10^3)(10^{16})(0.05)} \approx 0.1\ cm^2$$; This area represents on 25% of total silicon area. Total silicon area = 0.4 cm^2 ; width = length ≈ 0.64 cm

S25.3.

For new IGBT geometry with flat excess carrier distribution having a value of n_b;

$$R_{on,new} = \frac{W_d}{q(\mu_n+\mu_p)n_bA}$$

For older IGBTs, need to take into account both thermal equilibrium carrier densities as well as excess carrier densities.

$$dR_{on,old} = \frac{dx}{q(\mu_n + \mu_p)[n_bx/W_d + N_d]\,A}$$

$$dR_{on,old} = \frac{dx}{q(\mu_n + \mu_p)N_dA(1 + n_bx/(N_dW_d))}$$;

Integrate over drift region length to obtain $R_{on,old}$.

$$R_{on,old} = \int_0^{W_d} \frac{dx}{q(\mu_n + \mu_p)N_d A(1 + n_b x/(N_d W_d))} = \frac{W_d}{q(\mu_n + \mu_p)n_b A} \ln[1+ n_b/N_d]$$

$R_{on,old}/R_{on,new} = \ln[1 + n_b/N_d]$; If $n_b = 10^{17}$ cm^{-3} and $N_d = 10^{14}$ cm^{-3}, then

$R_{on,old}/R_{on,new} = \ln[1 + 10^3] \approx 7$

Chapter 26 - Emerging Devices

S26.1.

a. Voltage across R_{G2} probably should not exceed 100 V. Thus R_{G1} = 900 kΩ and R_{G2} = 100 kΩ.

b. MOSFET should be a low voltage (120-150 V= BV_{DSS}) high current (100-200 A or larger if possible) device.

S26.2.

a. Assume nonpunch-thru geometry. $W_d = \frac{2BV_{BD}}{E_{BD}}$

$$W_{d,Si} = \frac{(2)(125)}{3x10^5} = 8.3\ \mu m \ ;\ W_{d,GaAs} = \frac{(2)(125)}{4x10^5} = 6.3\ \mu m$$

b. $R_{on} = \frac{W_d}{q\mu_n N_d A}$; $N_d = \frac{\varepsilon E_{BD}{}^2}{2\ q\ BV_{BD}}$

$$N_{d,Si} = \frac{(12)(9x10^{-14})(3x10^5)^2}{(2)(1.6x10^{-19})(125)} = 2.4x10^{15}\ cm^{-3}$$

$$N_{d,GaAs} = \frac{(13)(9x10^{-14})(4x10^5)^2}{(2)(1.6x10^{-19})(125)} = 4.7x10^{15}\ cm^{-3}$$

$$R_{on,Si}A = \frac{8.3x10^{-4}}{(1.6x10^{-19})(1400)(\ 2.4x10^{15})} = 1.54x10^{-3}\ \text{ohms-cm}^2$$

$$R_{on,GaAs}A = \frac{6.3x10^{-4}}{(1.6x10^{-19})(8500)(\ 4.7x10^{15})} \approx 10^{-4}\ \text{ohms-cm}^2$$

S26.3.

Check drift region width: $W_d = 2BV_{BD}/E_{BD}$ = (2)(2000)/4x10^5 = 100 μm
Actual drift region length is only 50 μm which is too short for a non-punch-thru design. Correct by increasing length to 100 μm.

Check doping level: $N_d = \frac{\varepsilon E_{BD}}{qW_d} = \frac{(13)(9x10^{-14})(4x10^5)}{(1.6x10^{-19})(5x10^{-3})} = 4.7x10^{-7}/8x10^{-22}$

$N_d \approx 6x10^{14}\ cm^{-3}$; If W_d were 100 μm, N_d would have to be $3x10^{14}\ cm^{-3}$

Doping level is too high for 2000 V rating. Decrease doing to $3x10^{14}$ cm^{-3}

Check conduction area: J = (2000 A)/(1.25 cm^2) = 1600 A/cm^2 ; Current density exceeds allowable 250 A/cm^2. Increase area to 8 cm^2.

S26.4.

$$t_{rr} = \sqrt{\frac{2\,\tau\, I_F}{di_R/dt}} \;\; ; \; \tau = \frac{W_d^{\;2}}{(kT/q)(\mu_n + \mu_p)} \; ; W_d = \frac{2BV_{BD}}{E_{BD}} \; ; I_{rr} = \sqrt{2\,\tau\, I_F(di_R/dt)}$$

di_R/dt = (200 A)/(.5 μsec) = 4x108 A/sec

For GaAs, E_{BD} = 400 kV/cm, $\mu_n + \mu_p$ = 9000 cm^2/V-sec

BD_{BD} = 1.5 V_d = (1.5)(600) = 900 V; W_d = (2)(900)/(4x10^5) = 45 μm

$$\tau = \frac{(4.5x10^{-3})^2}{(.025)(9x10^3)} = 2x10^{-5}/225 = 90 \text{ nsec}$$

$$t_{rr} = \sqrt{\frac{(2)(9x10^{-8})(200)}{4x10^8}} = 300 \text{ nsec} \; ; I_{rr} = \sqrt{(2)(9x10^{-8})(200)(4x10^8)} = 120 \text{ A}$$

S26.5.

a. $$W_d = \frac{2BV_{BD}}{E_{BD}} = \frac{(2)(1500)}{2x10^6} = 1.5x10^{-3} \text{ cm} = 15\ \mu\text{m} \;;$$

$$N_d = \frac{\varepsilon E_{BD}^{\;2}}{2qBV_{BD}} = \frac{(10)(9x10^{-14})(4x10^6)^2}{(2)(1.6x10^{-19})(1500)} = 3x10^{16} \text{ cm}^{-3}$$

b. $$R_{on} = (2\text{ V})/(200\text{ A}) = 0.01\ \Omega = \frac{W_d}{q\,\mu_n\, N_d\, A} \;;$$

$$A = \frac{1.5x10^{-3}}{(1.6x10^{-19})(10^3)(3x10^{16})(0.01)} = 0.031 \text{ cm}^2$$

S26.6.

Figure	A	B	C	D	E	F
Type of Device	MOSFET	IGBT	PN junction diode	Schottky diode	JFET	FCT
High Power?	No	yes	yes	no	no	yes
Fast?	yes	yes	yes	yes	yes	no
$\frac{dv}{dt}$, $\frac{di}{dt}$ limits	no	yes	no	no	no	yes

S26.7.

a. $W_d = \frac{2BV_{BD}}{E_{BD}} = \frac{(2)(2000)}{10^7} = 4x10^{-4}$ cm = 4 μm ;

$$N_d = \frac{\varepsilon E_{BD}^2}{2qBV_{BD}} = \frac{(5.5)(9x10^{-14})(10^7)^2}{(2)(1.6x10^{-19})(2000)} = 8x10^{16} \text{ cm}^{-3}$$

b. Area A = (1000 A)/(800 A/cm^2) = 1.25 cm^2 ; $R_{on} = \frac{W_d}{q\,\mu_n\, N_d\, A}$;

$$R_{on} = \frac{4x10^{-4}}{(1.6x10^{-19})(2.2x10^3)(\,8x10^{16})(1.25)} = \frac{4x10^{-4}}{31} = 1.3x10^{-5} \text{ ohms;}$$

$V_{on} = I_{on}\, R_{on} = (10^3)(\,1.3x10^{-5}) = 0.013$ V = 13 mV

S26.8.

a. Assume nonpunch-thru geometry.

$W_d = \frac{2BV_{BD}}{E_{BD}} = \frac{(2)(3000)}{2x10^6} = 3x10^{-3}$ cm = 30 μm ;

$$N_d = \frac{\varepsilon E_{BD}^2}{2qBV_{BD}} = \frac{(10)(9x10^{-14})(2x10^6)^2}{(2)(1.6x10^{-19})(3000)} = 3.8x10^{15} \text{ cm}^{-3}$$

b. $P_{drift} = 500 \text{ watts} = I_{on}^2 \dfrac{W_d}{q\,\mu_n N_d A} = (1000\text{A})^2 \dfrac{(3\text{x}10^{-3})}{(1.6\text{x}10^{-19})(600)(3.8\text{x}10^{15})\text{A}}$

$500 = (1/\text{A})(8000)$; $\text{A} = 8000/500 = 16 \text{ cm}^2$

S26.9.

a. Assume that the structure is a non-punch-thru geometry.

$BV_{DSS} = E_{BD}W_d/2 = (10^7)(10^{-3})/2 = 5000$ V

Check to see if this estimate is consistent with the doping density;

$N_d = \dfrac{\varepsilon E_{BD}}{qW_d} = \dfrac{(5.5)(9\text{x}10^{-14})(10^7)}{(1.6\text{x}10^{-19})(10^{-3})} = 3\text{x}10^{16} \text{ cm}^{-3}$ > actual doping level of 10^{16} cm^{-3}.

The structure is a punch-thru geometry. Recalculate breakdown voltage.

$BV_{DSS} = E_{BD}\,W_d - (qN_dW_d^2)/(2\varepsilon) = (10^7)(10^{-3}) - \dfrac{(1.6\text{x}10^{-19})(10^{16})(10^{-3})^2}{(2)(5.5)(9\text{x}10^{-14})}$

$BV_{DSS} = 10^4 - 1600 = 8400$ V

b. $R_{on} = 1\text{V}/500\text{A} = 0.002\ \Omega = \dfrac{W_d}{q\mu_n N_d A}$; $A = \dfrac{10^{-3}}{(1.6\text{x}10^{-19})(2.2\text{x}10^3)(10^{16})(2\text{x}10^{-3})}$

$A \approx 0.14 \text{ cm}^2$

S26.10.

Specific examples of Schottky diodes in silicon, gallium arsenide, and silicon carbide shown below. Information about these components available on company websites.

Company	Part Number	Material	V_{max}	I_{max}	t_{rr}
IXYS	DSSK 30-018A	silicon	180 V	50 A	Not available
IXYS	DGSK 40-025A	Gallium arsenide	250 V	13 A	14 nsec
Infineon	SDP06S60	Silicon carbide	600	6 A	N.A.

S26.11.

a. $W_d = \dfrac{2BV_{BD}}{E_{BD}} = \dfrac{(2)(2000)}{2\text{x}10^6} = 2\text{x}10^{-3} \text{ cm} = 20\ \mu\text{m}$;

$N_d = \dfrac{\varepsilon E_{BD}^2}{2qBV_{BD}} = \dfrac{(10)(9\text{x}10^{-14})(2\text{x}10^6)^2}{(2)(1.6\text{x}10^{-19})(2000)} = 5.7\text{x}10^{15} \text{ cm}^{-3}$

b. Assume that V_{on} = 2V when I_{on} = 1000 A is dropped entirely across the drift region.

$$R_{on} = \frac{2V}{1000A} = 0.002 \text{ ohms} = \frac{W_d}{q\mu_n n_b A} = \frac{2x10^{-3}}{(1.6x10^{-19})(1000)(10^{17})(A)} = 1.3x10^{-4}/A$$

$$A = \frac{1.3x10^{-4}}{2x10^{-3}} = 0.065 \text{ cm}^2$$

S26.12.

a. $W_d = 2\ BV_{BD}/E_{BD} = (2)(600)/4x10^6 = 3\ \mu m$

$$N_d = \frac{\varepsilon\ E_{BD}}{q\ W_d} = \frac{(10)(9x10^{-14})(4x10^6)}{(1.6x10^{-19})(3x10^{-4})} = 7x10^{16} \text{ cm}^{-3}$$

b. Find R_{on} of diode using slope of 25 °C curve at higher currents.

$$R_{on} = \frac{2.1V - 1.2V}{12A - 2A} = 0.09\ \Omega = \frac{W_d}{q\ \mu_n N_d A}\ ;$$

$$A = \frac{3x10^{-4}}{(1.6x10^{-19})(10^3)(7x10^{16})(9x10^{-2})} = 3x10^{-4} \text{ cm}^{-2}$$

Chapter 27 - Snubbers

S27.1.

a. Three possibilities for overstresses. Overvoltage at turn-off, overcurrent at turn-on, and excessive power dissipation. Must check each possibility.

No overcurrent stress. I_C = 100 A < 125 A maximum allowed.

Overvoltage: $BV_{CEO} = BV_{CBO}/(\beta^{1/4}) = 300/(16)^{1/4} = 150$ V

At turn-off, collector-emitter voltage $= V_d + L_\sigma \frac{di}{dt} = V_d + L_\sigma \frac{I_o}{t_{fi}}$

$= 100 + (5\text{x}10^{-8})(100)/(2\text{x}10^{-7}) = 100 + 25 = 125 \text{ V} < 150 \text{ V} = BV_{CEO}$.

No overvoltage stress.

Power dissipation: allowable dissipation $P_{c,max} = \frac{T_{j,max} - T_a}{R_{\theta j\text{-}a}}$

$P_{c,max} = \frac{150 - 25}{2} = 62.5$ watts < 100 watts actual dissipation.

Overstressed by too much power dissipation.

b. Snubbers will reduce turn-on and turn-off losses. Need to estimate these losses without snubbers in order to determine if the use of snubbers is warranted.

$$P_{turn\text{-}on} = f_{sw} \frac{V_d I_{on}(t_{ri} + t_{fv})}{2} = (2.5\text{x}10^4) \frac{(100)(100)(10^{-7} + 10^{-7})}{2} = 25 \text{ watts}$$

$$P_{turn\text{-}off} = f_{sw} \frac{V_d I_{on}(t_{rv} + t_{fi})}{2} = (2.5\text{x}10^4) \frac{(100)(100)(2\text{x}10^{-7} + 2\text{x}10^{-7})}{2} = 50 \text{ watts}$$

Losses need to be reduced by 100 - 62.5 = 37.5 watts. This can be accomplished by using a turn-off snubber to reduce the turn-off losses. Turn-on snubber not needed.

S27.2.

a. Use turn-off snubber design formulas from Ch. 27 of Power Electronics, 2nd edition by Mohan, Undeland, and Robbins.

$$R_s = \frac{V_d}{0.2 I_o} = \frac{(5)(100\text{V})}{100\text{A}} = 5 \text{ ohms}; \; C_s = \frac{I_o t_{fi}}{2 V_d} = \frac{(100)(2\text{x}10^{-7})}{(2)(100)} = 0.1 \; \mu\text{F}$$

b. Power is dissipated in R_s at turn-on of the BJT. $P_{Rs} = (0.5) C_s V_d^2 f_{sw}$

$P_{Rs} = (0.5)(10^{-7})(100)^2(2.5x10^4) = 12.5$ watts

c. Collector current during turn-off $= I_o\{1 - \frac{t}{t_{fi}}\}$

Collector-emitter voltage during turn-off $= \frac{I_o t^2}{2C_s t_{fi}}$

$$P_c = W_c f_{sw} \;;\; W_c = \int_0^{t_{fi}} I_o(1 - \frac{t}{t_{fi}}) \frac{I_o}{2\,C_{s1}} \frac{t^2}{t_{fi}}\, d\,t = \frac{V_d\, I_o\, t_{fi}}{12}$$

$$P_c = \frac{(100)(100)(2x10^{-7})(2.5x10^4)}{12} = 4.2 \text{ watts}$$

Reduction in turn-off losses = 50 - 4.2 = 45.8 watts

S27.3.

a. The turn-off snubber design procedure for a BJT or MOSFET is based upon the build-up of the collector-emitter voltage to the power supply voltage V_d at the end of the current fall time t_{fi}. This means that the approximate rate of growth of the voltage is V_d/t_{fi} and takes no account of any limits on the growth rate that other physical mechanisms may impose. If the rated dv/dt is less than V_d/t_{fi}, then the snubber design procedure may lead to a value of dv/dt that is too large and may lead to unwanted turn-on of the GTO.

b. Choose C_s on the basis: $C_s \left|\frac{dv}{dt}\right|_{rated} = I_o/t_{fi}$: $C_s = \frac{1000}{(10^{-6})(7.5x10^8)} = 1.3$ F

Choose R_s so that maximum current out of the capacitor at turn-on about 0.2 I_o.

R_s = (1000V)/(200A) = 5 ohms.

S27.4.

a. Reverse recovery current of diode adds to total current going into switch at turn-on. If I_{rr}
. > 50 A, then the total current in S_w will exceed the 250 A limit.

$$I_{rr} = \sqrt{\frac{2\tau I_F(dI_R/dt)}{(1+S)}} = \sqrt{\frac{(2)(5x10^{-7})(200)(200/5x10^{-7})}{(1+1)}} = 200 \text{ A}$$

Need a turn-on snubber to reduce I_{rr} to 50 A. Do this by using turn-on snubber to control dI_R/dt.

b. Limit $dI_R/dt = \frac{(I_{rr})^2(1+S)}{2\tau I_F} = \frac{(50)^2(2)}{(2)(5x10^{-7})(200)} = 2.5x10^7$ A/sec

During switch turn-on, inductance is large so that entire voltage V_d is dropped across the inductor. Hence L_s di/dt = V_d. The di/dt = dI_R/dt. Hence $dI_R/dt = \frac{V_d}{L_s}$.

$$L_s = \frac{300V}{2.5x10^7 A/sec} = 1.2x10^{-5} H = 12\ \mu H$$

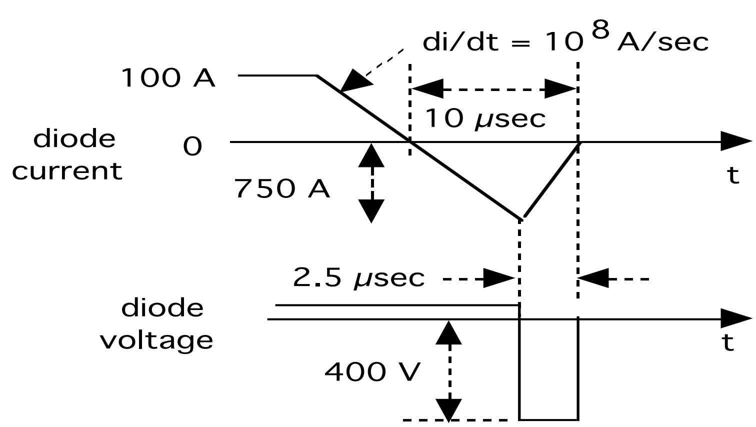
Snubber resistor R_s value set either by requirement that overvoltage developed across switch at turn-off limited to a safe value or the snubber recovery time 4 L_s/R_s be small compared to the switching period.

Overvoltage limit across R_s = 500 - 300 = 200 V. Choose a maximum of I_oR_s = 100 V. Then R_s = 100V/200A = 0.5 amps.

With this value of R_s, the recovery time would be $4(1.2x10^{-5})/(0.5)$ = 0.1 milliseconds. This is the same as the switching period. Thus need to increase R_s to approximately 1 ohm which would reduce the recovery time to 50 μsec.

S27.5.

a. When input voltage makes a transition (for example from -100 V to +100 V) all four diodes are forward biased as 100 A current changes from one pair of diodes to another. Full 00 V drop occurs across inductor. Thus rate of change of inductor current = $(100V)/(10^{-6}H) = 10^8$ A/sec. Diode current grows in reverse bias direction for 7.5 μsec and reaches a reverse recovery current of I_{rr} = 750 A. Current falls from 750 A to 0 in 2.5 μsec.Diode voltage and current waveforms during reverse recovery shown below. Results in reverse voltage of 300 V due to inductor and 100 V due to input source. Total overvoltage = 400V.



b. Use series RC circuit in parallel with diode bridge input. Use diode snubber design formulas from Power Electronics, 2 edition by Mohan, Undeland, and Robbins, p. 672-73.

$R_s = (1.3)V_d/I_{rr} = (1.3)(100)/(750) = 0.17$ ohms

$$C_s = \frac{L_\sigma (I_{rr})^2}{(V_d)^2} = \frac{(10^{-6})(750)^2}{(100)^2} = 5.6\text{x}10^{-5} = 56\ \mu\text{F}$$

c. During each input source transistion, energy equal to $W_R = 0.5\{L_\sigma I_{rr}^2 + C_s V_d^2\}$ dissipated in snubber resistor. Two transistions per period or $2\text{x}10^4$ per second.

$P_R = W_R f_{sw} = (2\text{x}10^4)(0.5)\{(10^{-6})(750)^2 + (5.6\text{x}10^{-5})(100)^2\} = 11.2$ kW

This value is unreasonably large because of the large value of t_{rr} which has been assumed. More typical values of t_{rr} for a 500 V rated diode would be a fraction of a μsec.

S27.6.

a. Two overstress possibilities, overvoltage at turn-off and excessive power dissipation due to combination of on-state losses and switching losses. No overcurrent stress since free-wheeling diode is ideal and thus has no reverse recovery current.

Check overvoltage first:

$$V_{CE,off} = V_d + L_\sigma \frac{di}{dt}\ ;\ \frac{di}{dt} \approx \frac{I_{on}}{t_{fi}}$$

$$V_{CE,off} = 500 + (10^{-7})\frac{40}{5\text{x}10^{-7}} = 508\ \text{V} < 700\ \text{V max. voltage rating.}$$

Check for excessive power dissipation:

$$P_d = (0.33)(I_o)(V_{DS,on}) + f_{sw}\frac{V_d\, I_{on}(t_{ri} + t_{fv} + t_{fi} + t_{rv})}{2}$$

Stray inductance L_σ will reduce voltage across IGBT at turn-on and increase it at turn-off. However the increase at turn-off is minimal as shown above and decrease at turn-on is also small. Hence formula for switching losses given above is valid.

$P_d = (0.33)(40)[0.8 +(0.01)(40)] + (0.5)(2\text{x}10^4)(500)(40)(1.4\text{x}10^{-6})$

$P_d = 16 + 280 = 296$ watts

Allowed power dissipation (assuming ambinet of 25 °C)

$P_{max} = (T_j - T_a)\ R_{\theta,j\text{-}a} = (150 - 25)(1) = 125$ watts < actual dissipation.

IGBT has excessive power dissipation.

b. Losses at turn-off = $f_{sw} \frac{V_d I_{on}(t_{fi} + t_{rv})}{2}$ = (0.5)(2x10^4)(500)(40)(10^{-6}) = 200 watts

Losses at turn-on = 280 - 200 = 80 watts.

Turn-off snubber will produce largest decrease in power dissipation.

S27.7.

a. $C_s = \frac{I_o t_{fi}}{2V_d} = \frac{(40)(5x10^{-7})}{(2)(500)}$ = 20 nF ; $R_s = \frac{5V_d}{I_o} = \frac{(5)(500)}{(40)}$ = 63 ohms

b. $E_{off} = \int_0^{t_{fi}} \frac{V_d t^2}{(t_{fi})^2} I_o(1 - t/t_{fi})\, dt = \frac{V_d I_o t_{fi}}{12} = \frac{(500)(40)(5x10^{-7})}{12} = 8.3x10^{-4}$

$P_{off} = E_{off} f_{sw}$ = (8.3x10^{-4})(2x10^4) = 16.7 watts

P_{off}(no snubber) - P_{off}(snubber) = 200 - 16 = 183 watts

S27.8.

a. Overvoltage at turn-off due to stray inductance L_σ.

Overvoltage = $L_\sigma I_o/t_{fi}$ = 400 V ; L_σ = (400)(2x10^{-7})/(40) = 2x10^{-6} Henries

b. Reverse recovery current = 100 amps

c. Both turn-on and overvoltage snubbers are needed.

d. $C_{ov} = \frac{L_\sigma I_o^2}{(\Delta V_{sw})^2} = \frac{(2x10^{-6})(200)^2}{(100)^2} = 8\ \mu F$; Recovery time ≈ 2 $R_{ov}C_{ov}$

2 $R_{ov}C_{ov} < T_{sw} = 1/f_{sw}$; $R_{ov} \approx \frac{(5x10^{-5})}{(2)(8x10^{-6})} \approx 3$ ohms

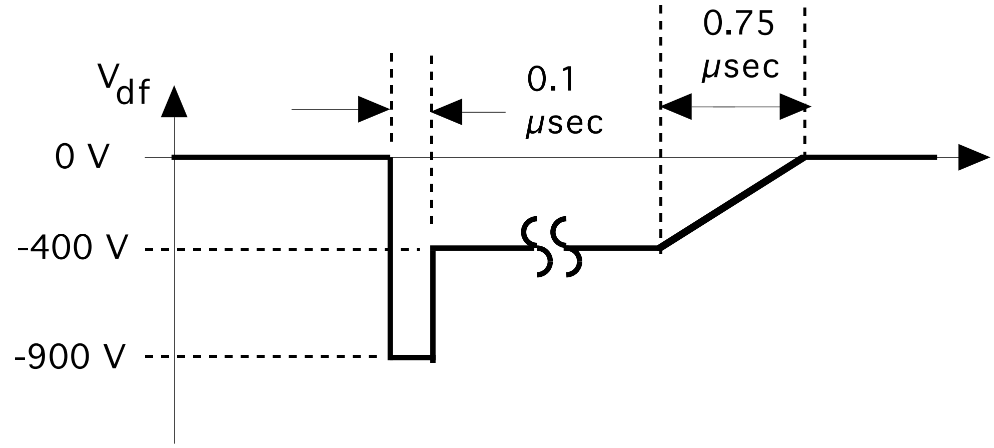
S27.9.

a. L_σ causes voltage reduction V_1 at turn-on and overvoltage V_2 at turn-off.

V_1 = (10^{-6})(100)/(6.6x10^{-7}) = 150 V

V_2 = (10^{-6})(100)/(10^{-6}) = 100 V

b. $V_{df} = V_{sw} - 400 + L_\sigma \frac{di_{sw}}{dt}$



c. The diode has an overvoltage of 700 V. Rated reverse voltage = 200 V. Applied reverse voltage = 900 V.
The dv/dt rating of the diode is exceeded. Rated dv/dt = 200 V/μsec. Applied dv/dt = 533 V/μsec.
The dv/dt rating of the switch is exceeded. Rated dv/dt = 100 v/μsec. Applied dv/dt = 533 V/μsec.

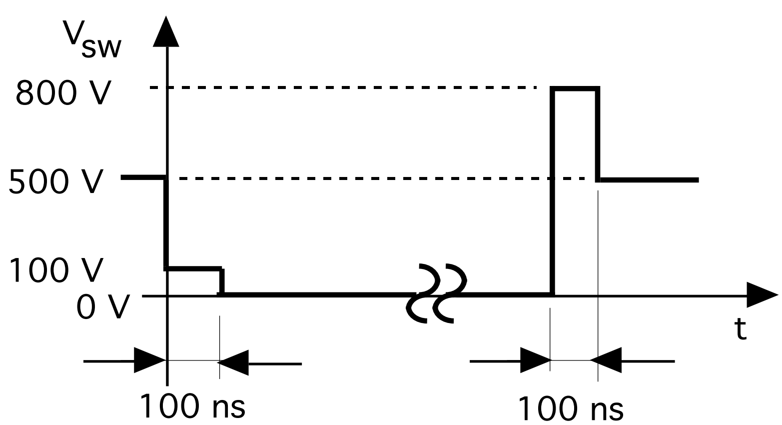
d. Diode snubber is a series R-C circuit in parallel with the diode.

$$C_s = L_\sigma \frac{I_{rr}^2}{V_d^2} = (10^{-6})(50)^2/(400)^2 \approx 16 \text{ nF}$$

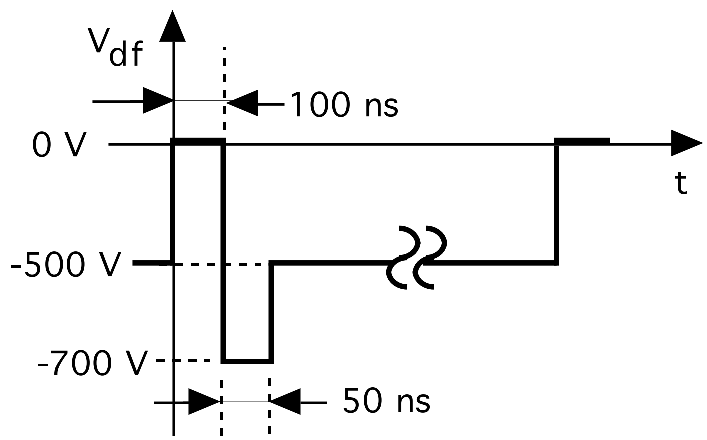
$$R_s = 1.3\ (V_d/I_{rr}) = (1.3)(400/50) \approx 10 \text{ ohms}$$

S27.10.

a. $V_{sw} = 500 - L_\sigma \frac{di_{sw}}{dt} + V_{df}$

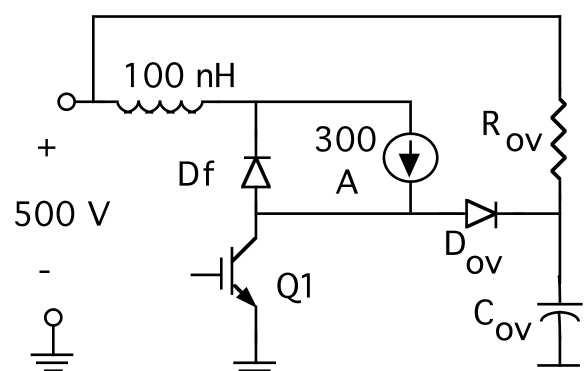


b. $V_{sw} - 500 + L_\sigma \frac{di_{sw}}{dt} = V_{df}$



c. IGBT is overstressed by 100 V at turn-off. Actual voltage = 800 V while the allowable voltage is 700 V.

d. Use overvoltage snubber circuit shown below and described in Power Electronics, 2nd Edition by Mohan, Undeland, and Robbins, chapter 27, p. 686-688.



$$C_{ov} = \frac{L_\sigma I_o^2}{(\Delta V_{sw})^2} = \frac{(10^{-7})(300)^2}{(50)^2} = 3.6\ \mu\text{F}$$

Choose R_{ov} on the basis that $3\ R_{ov}C_{ov} < t_{on}$ = on-time of IGBT.

$t_{on} = 0.5/f_{sw}$ in this problem. $t_{on} = 0.5/(2\text{x}10^4) = 2.5\text{x}10^{-5}$

$$R_{ov} = \frac{2.5\text{x}10^{-5}}{(3)(3.6\text{x}10^{-6})} = 2.3 \text{ ohms}$$

S27.11.

a. $V_1 = L_\sigma \frac{di_{sw}}{dt} = \frac{(10^{-7})(\ 200)}{2x10^{-7}} = 100$ V;

$$V_2 = \frac{L_\sigma I_o^{\ 2}}{C_{ov}} = \sqrt{\frac{(10^{-7})(200)^2}{10^{-5}}} = 20 \text{ V}$$

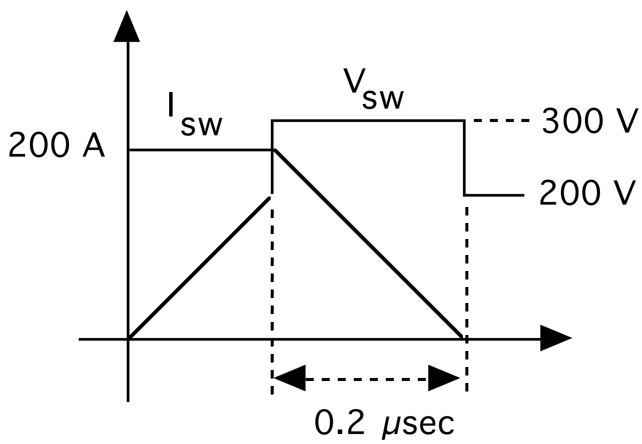
b. Time T_1 is a quarter of a cycle of the natural resonant frequency of the overvoltage snubber.

$T_1 = (0.5)(\pi)\sqrt{L_\sigma C_{ov}} = 1.6\ [(10^{-7})(10^{-5})]^{1/2} = 1.6\ \mu$sec

c. $3\ R_{ov}\ C_{ov} = T/2 = 25\ \mu$sec

$$R_{ov} = \frac{2.5x10^{-5}}{(3)(10^{-5})} \approx 1 \text{ ohm}$$

d. Overvoltage snubber only reduces turn-off losses, so only need to estimate turn-off loss reduction. Switch waveforms at turn-off without the snubber shown below.



No overvoltage snubber

Turn-off losses increased because of 100 nH inductance and equal P_{excess}.

$P_{excess} = (300)(200)(0.5)(2x10^{-7})(2x10^4) = 120$ watts.

Overvoltage snubber reduces overvoltage from 100 V to 20 V. Hence a conservative estimate of loss during the 0.2 μsec period is 120)(220/300) = 88 watts.
Loss reduction = 120 -88 = 32 watts.

S27.12.

a. $0 < t < t_{fi1}$ $I_{cap} = (1 - \beta)\, I_o\, (t/t_{fi1})$; $t_{fi1} < t < t_{fi2}$ $I_{cap} = (1 - \beta)\, I_o + \beta\, I_o\, (t/t_{fi2})$

$$V_{cap} = V_d = (1/C_s) \int_0^{t_{fi1}} (1-\beta) I_o (t/t_{fi1}) dt + (1/C_s) \int_{t_{fi1}}^{t_{fi2}} [(1-\beta) I_o + I_o (t/t_{fi2})] dt$$

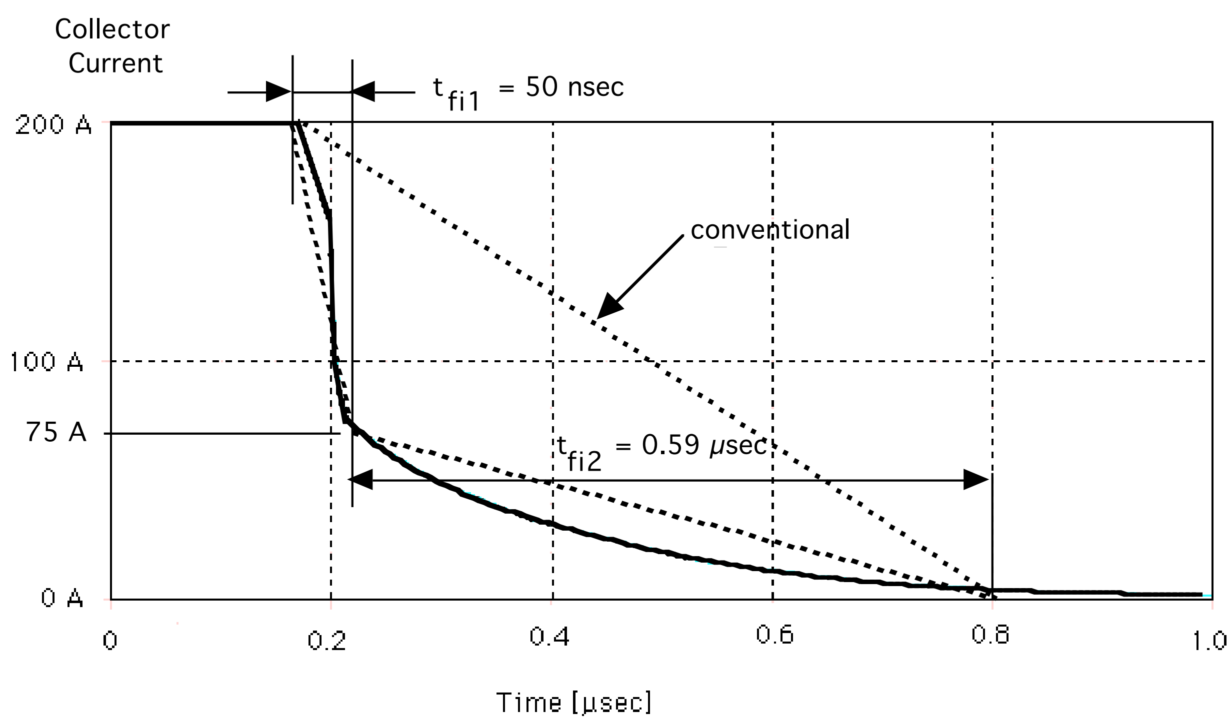
$$C_s = \frac{(1-\beta)\, I_o\, t_{fi1}}{2V_d} + \frac{(1-0.5\beta)\, I_o\, t_{fi2}}{V_d}$$

b. Collector current waveform shown below. From this waveform I_o = 200 A. t_{fi1} = 50 nsec. T_{fi2} = 0.59 μsec. β = 75/200 = 0.375

$$C_s = \frac{[1-.38](200)(5x10^{-8})}{(2)(400)} + \frac{(1 - 0.19)(200)(5.9x10^{-7})}{400}$$

$C_s \approx 0.28\ \mu F$

Conventional Approach: $C_s = I_o\, t_{fi}/(2V_d) = (200)(6x10^{-7})/800 = 0.15\ \mu F$



S27.13.

a. $C_s = \frac{I_o t_{fi}}{2V_d}$; I_o = 200 A, V_d = 200 V ; t_{fi} = 0.2 μsec

$$C_s = \frac{(200)(2x10^{-7})}{(2)(200)} = 0.1\ \mu F$$

b. Snubber resistance governed by two competing considerations: 1) keeping discharge current from C_s to reasonable values (say less than 0.2 I_o) when switch S_w turns on and 2) keeping R_s small enough so that capacitor has time to discharge to zero volts when S_w is on for the minimum time interval.

Consideration #1: $R_s = \frac{200V}{(0.2)(200A)}$ = 5 ohms.

Consideration #2: Assume minimum t_{on} = 10% of T_{sw} = (0.1)(20μsec) = 2 μsec

Conservative estimate of time required to discharge capacitor to zero = 4 R_sC_s

$$R_s = \frac{2\mu sec}{(4)(10^{-7}sec)} = 5 \text{ ohms.}$$

In this case both considerations yield same answer. Usually not the case.

c. $<P_{Rs}> = (0.5)C_sV_d^2 f_{sw} = (0.5)(10^{-7})(200)^2(5x10^4)$ = 100 watts

$<P_{sw}> = (E_{cond} + E_{on} + E_{off})f_{sw}$; $E_{cond} = (200)(1)(1.5x10^{-5}) = 3x10^{-3}$ joules

$E_{on} = 0.5\ (200)(200)(10^{-7}) = 2x10^{-3}$ joules ; Uneffected by turn-off snubber

$E_{off} = 0.5\ (200)(200)(4x10^{-7})/6 = 1.33x10^{-3}$ joules : Snubber reduces E_{off} by factor of six as indicated.

$<P_{sw}> = (3+2+1.3)x10^{-3}*5x10^4$ = 317 watts.

S27.14.

1) Diode oriented in wrong direction. Reverse orientation.
2) Diode breadkdown voltage too small. Increase to at least 500 V.
3) R_s should be in parallel with diode D_s, not in series as shown.
4) C_s too large. Should be = $\frac{(100A)(200nsec)}{(2)(500V)}$ = 20 nF
5) R_s too small. If goal is to keep $\frac{500V}{R_s}$ < 0.2)(100A) = 20A then R_s = 25 Ω.

   If goal is to keep $4R_sC_s = t_{on,min}$ = 4 μsec, then R_s = 50 ohms.
6) R_s power rating too small. $P_{Rs} = (0.5)(20nF)(500V)^2(10^5)$ = 250 watts

Chapter 29 - Heat Sinks and Component Temperature Control

S29.1.

a. $P_{sw} = (0.5)(100A)(1V) = 50$ watts

b. $T_{j,max} = \{R_{\theta,jc} + R_{\theta,sa}) P_{sw} + T_a$; $R_{\theta,sa} = \dfrac{T_{j,max} - T_a}{P_{sw}} - R_{\theta,jc}$

$$R_{\theta,sa} = \frac{150 - 30}{50} - 1 = 1.4 \text{ °C/W}$$

c. $R_{\theta,sa} = \dfrac{R_{\theta rad} R_{\theta conv}}{R_{\theta rad} + R_{\theta conv}} = 1.4$ °C/W ;

$T_s = 150$ °C = 423 °K ; $T_a = 30$ °C = 303 °K

$$R_{\theta rad} = \frac{T_s - T_a}{5.1A\{\{T_s/100\}^4 - \{T_a/100\}^4\}} = \frac{423 - 303}{5.1A\{423/100\}^4 - \{303/100\}^4\}} = 0.1/A$$

$$R_{\theta conv} = \frac{(d_{vert}/(T_s - T_a))^{1/4}}{1.3\ A} = \frac{(150 - 30)^{-1/4} A^{1/8}}{1.3\ A} \approx 0.226/A \text{ ; } A^{7/8} \approx A$$

$$1.4 = \frac{[0.1/A][0.226/A]}{0.1/A + 0.226/A} = \frac{0.0693}{A} \text{ ; } A = \frac{.0693}{1.4} = 0.0495 \text{ m}^2 = 495 \text{ cm}^2$$

S29.2.

a. Smallest heatsink has smallest area and thus largest $R_{\theta sa}$. $R_{\theta sa} = \dfrac{T_{jmax} - T_a}{P_{sw}} - R_{\theta jc}$

Device A: $R_{\theta sa} = (175 - 40)/20 - 2.5 = 4.25$ °C/W

Device B: $R_{\theta sa} = (150 - 40)/20 - 0.5 = 5$ °C/W

Device C: $R_{\theta sa} = (125 - 40)/20 - 0.4 = 3.85$ °C/W

Choose Device B

b. $T_s = (20)(5) + 40 = 140$ °C = 273 + 140 = 413 °K ; $T_a = 40$ °C = 313 °K

$$R_{\theta,sa} = \frac{R_{\theta rad} R_{\theta conv}}{R_{\theta rad} + R_{\theta conv}} = 5 \text{ °C/W ;}$$

$$R_{\theta rad} = \frac{T_s - T_a}{5.1A\{\{T_s/100\}^4 - \{T_a/100\}^4\}} = \frac{413 - 313}{5.1A\{413/100\}^4 - \{313/100\}^4\}}$$

$$R_{\theta rad} = 0.1/A$$

$$R_{\theta conv} = \frac{(d_{vert}/(T_s - T_a))^{1/4}}{1.3\ A} = \frac{A^{1/8}}{1.3\ A(140 - 40)^{1/4}} \approx 0.236/A\ ;\ A^{7/8} \approx A$$

$$5 = \frac{[0.1/A][0.236/A]}{0.1/A + 0.236/A} = \frac{0.0702}{A}\ ;\ A = \frac{.0702}{5} = 0.014\ m^2 = 140\ cm^2$$

S29.3.

a. $P_{diss} = 10 + 10^{-3} f_s = \frac{T_{jmax} - T_a}{R_{\theta ja}}$

$f_s = 10^3 \left[\frac{150 - 40}{5} - 10\right] = 12\ kHz$

b. $P_{diss} = 10 + 10^{-3} f_s = \frac{T_{jmax} - T_a}{R_{\theta jc} + R_{\theta sa}}$; $R_{\theta sa} = \frac{T_{jmax} - T_a}{10 + 10^{-3} f_s} - R_{\theta jc}$

$R_{\theta sa} = \frac{150 - 40}{10 + (10^{-3})(4x10^4)} - 1 = 1.2\ °C/W$

c. $R_{\theta sa} = 2.5\ °C/W = \frac{R_{\theta rad} R_{\theta conv}}{R_{\theta rad} + R_{\theta conv}}$; $R_{\theta conv} = \frac{(d_{vert}/(T_s - T_a))^{1/4}}{1.3\ A}$

$R_{\theta rad} = \frac{T_s - T_a}{5.1A\{\{T_s/100\}^4 - \{T_a/100\}^4\}}$;

For a cube of side d, the surface area $A = 6d^2$ or $d = \sqrt{A/6}$

$R_{\theta conv} = \frac{(\sqrt{A/6}/(T_s - T_a))^{1/4}}{1.3\ A} = \frac{1}{(150 - 40)^{1/4}(1.3)(6)^{1/8} A^{7/8}} \approx \frac{0.184}{A}$

$R_{\theta rad} = \frac{150 - 40}{5.1A\{\{423/100\}^4 - \{313/100\}^4\}} = \frac{0.096}{A}$

$2.5\ °C/W = \frac{(0.184/A)(0.096/A)}{(0.184/A) + (0.096/A)} = \frac{0.063}{A}$

$A = \frac{0.063}{2.5} = 0.025\ m^2 = 250\ cm^2$; $d = \sqrt{\frac{250}{6}} = 6.45\ cm$

S29.4.

a. $P_{sw} = \{E_{cond} + E_{on} + E_{off}\}f_{sw}$; $E_{cond} = (1V)(100A)(7.9x10^{-6}sec) = 7.9x10^{-4}$ Joules

$E_{on} = (100V)(100A)(1.5x10^{-7}sec)(0.5) = 7.5x10^{-4}$ Joules

$E_{off} =$ (100V)(100A)(1.5x10^{-7}sec)(0.5) = 7.5x10^{-4} Joules

P_{sw} = {7.5 + 7.5 + 7.9}{10^{-4})(5x10^4} = 115 watts

b. T_j = (2)(115) + 30 = 260 °C ; This temperature exceeds the rated junction temperature of 175 °C . Heat sink is needed.

$$R_{\theta sa} = \frac{175 - 30}{115} - 0.3 = 0.96 \text{ °C/W}$$

c. $$R_{\theta sa} = 1 \text{ °C/W} = \frac{R_{\theta rad} R_{\theta conv}}{R_{\theta rad} + R_{\theta conv}} \; ; R_{\theta conv} = \frac{(d_{vert}/(T_s - T_a))^{1/4}}{1.3 A}$$

$$R_{\theta rad} = \frac{T_s - T_a}{5.1A\{\{T_s/100\}^4 - \{T_a/100\}^4\}} \quad \text{; Assume } d_{vert} = \sqrt{A}$$

$$R_{\theta conv} = \frac{A^{1/8}}{(175 - 30)^{1/4}(1.3)\ A} \approx \frac{0.215}{A}$$

$$R_{\theta rad} = \frac{175 - 30}{5.1A\{\{448/100\}^4 - \{303/100\}^4\}} = \frac{0.069}{A}$$

$$R_{\theta sa} = 1 \text{ °C/W} = \frac{(0.069/A)(0.215/A)}{0.069/A + 0.215/A} = \frac{0.0522}{A} \; ; \; A = \frac{.0522}{1} \; 0.0522 \text{ m}^2 = 522 \text{ cm}^2$$

S29.5.

a. $P_{sw} = \{E_{cond} + E_{on} + E_{off}\}f_{sw}$; E_{cond} = (1V)(200A)(5x10^{-5}sec) = 10^{-2} Joules

$E_{on} =$ (200V)(200A)(4x10^{-7}sec)(0.5) = 8x10^{-3} Joules

$E_{off} =$ (200V)(200A)(4x10^{-7}sec)(0.5) = 8x10^{-3} Joules

P_{sw} = {8 + 8 + 10}{10^{-3})(10^4} = 260 watts

T_j = (260W)(0.1 °C/W + 0.3 °C/W) + 30 °C = 134 °C

b, At 22 kHz P_{sw} = (260W)(20/10) = 520 watts

$$R_{\theta sa} = \frac{120 - 30}{520} - 0.1 = 0.173 - .1 = 0.073 \text{ °C/W}$$

Using the provided chart, an airflow rate of about 1200 ft/min. is required from the fan.

S29.6.

a. Power dissipation = 0.5 $R_{DS(on)}(T_j=100°C)\ (300)^2$

From graph of normalized $R_{DS(on)}$ $(R_{DS(on)}(T_j)/\ R_{DS(on)}(25°C))$ we find

$R_{DS(on)}(T_j=100°C) = 1.6\ R_{DS(on)}(25°C) = (1.6)(.0042) = 0.0067$ ohms

Pwr dissipation = $(0.5)(.0067)(9x10^4) = 302$ watts

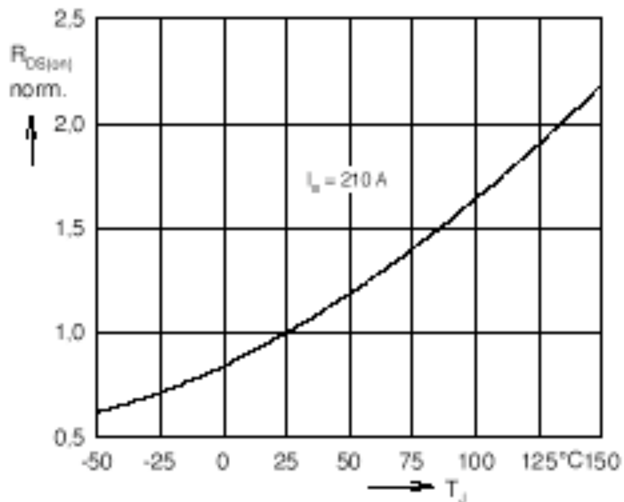


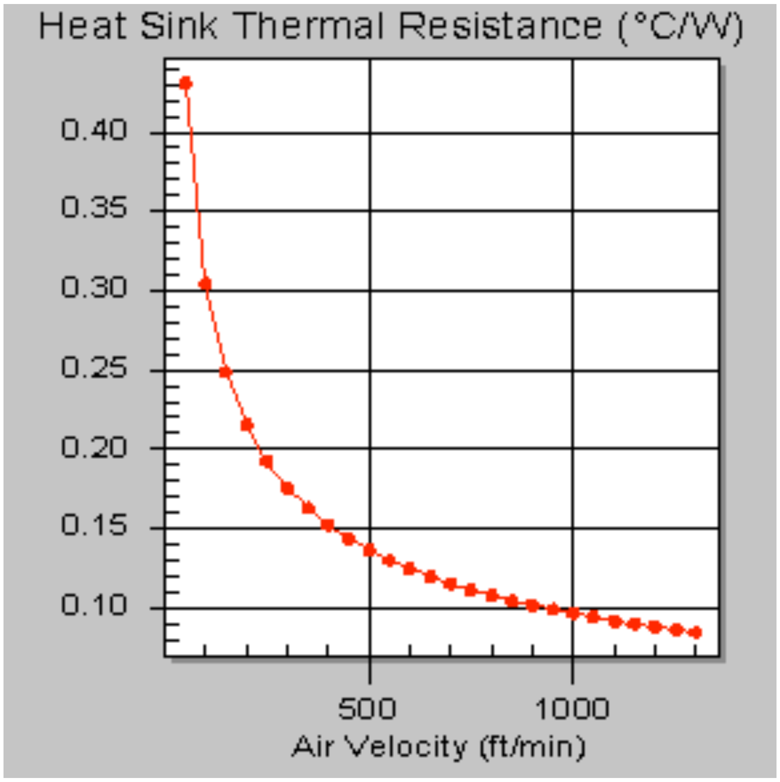
Fig. 4 $R_{DS(on)} = f(T_j)$, normalized

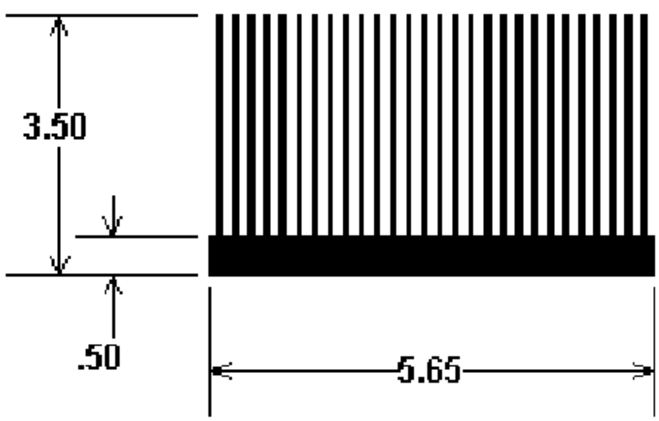
$R_{\theta,js}$ = thermal resistance from junction to surface (including a 30 micron thick layer of thermal grease) = 0.076 °C/W (from spec sheet for the transistor). Use of the smaller value of thermal resistance without heat transfer paste is unreasonably (even dangerously) optimistic.

$$100\ °C = (302\ W)(0.076\ °C/W + R_{\theta,sa}) + 35\ °C\ ;\ R_{\theta,sa} = 0.14\ °C/W$$

b. Number of heat sink vendors available. Will use an example from Aavid as per problem statement suggestion. VM400-02F has a footprint (on a heat sink) of about 5.5 inches by 2.25 inches. (From spec sheet of the MOSFET)

Use a bonded fin heat sink from Aavid. Part #: 420008 . This is heat sink has a width of 5.65 inches, a length of 3 inches (or what ever is requested) and 28 fins uniformly distributed along the width of 5.65 inches. The fins are 3.5 inches long. A drawing of the heat sink is shown below. The thermal resistance versus air flow rate over the fins is given below. A forced flow rate of about 500 ft per minute is needed to get to the required thermal resistance of 0.14 °C/W.


Heat Sink Thermal Resistance (°C/W)
0.40
0.35
0.30
0.25
0.20
0.15
0.10
500
1000
Air Velocity (ft/min)


3.50
.50
5.65

Chapter 30 - Magnetic Component Design

S30.1.

a . $R_{\theta sa} = \dfrac{R_{\theta rad}\, R_{\theta conv}}{R_{\theta rad} + R_{\theta conv}}$;

$R_{\theta conv} = \dfrac{(d_{vert}/(T_s - T_a))^{1/4}}{1.3\ A}$; $d_{vert} = 3a$: $A = 60a^2 = (60)(.02)^2 = 0.024\ m^2$

$R_{\theta conv} = \dfrac{((0.06)/(90 - 30))^{1/4}}{(1.3)(0.024)} = 5.48$ °C/W

$R_{\theta rad} = \dfrac{90 - 30}{(5.1)(0.024)\{\{363/100\}^4 - \{303/100\}^4\}} = 5.44$ °C/W

$R_{\theta sa} = \dfrac{(5.44)(5.48)}{5.44 + 5.48} = 2.73$ °C/W ; $P = \dfrac{90 - 30}{2.73} = 22$ watts

$P_{sp} = \dfrac{P}{Vol}$; Volume (Vol) of core = $25.8a^3 = (25.8)(8) = 206\ cm^3$

$P_{sp} = \dfrac{22W}{206\ cm^3} = 107\ mW/cm^3$

b. $P_{sp} = 107\ mW/cm^3 = k_{cu}\rho_{cu}J_{rms}{}^2 = (0.6)(22)\ \{J_{rms}(A/mm^2)\}^2$

$J_{rms} = \sqrt{\dfrac{107}{(0.6)(22)}} = 2.86\ A/mm^2$; Conductor Area $A_{cu} = \dfrac{I_{rms}}{J_{rms}} = \dfrac{2}{2.85} = 0.7\ mm^2$

$N = \dfrac{k_{cu}A_w}{A_{cu}}$; $A_w = 1.4a^2 = (1.4)(2)^2 = 560\ mm^2$; $N = \dfrac{(0.6)(560)}{0.7} = 480$

c. $L_{max} = \dfrac{N\ A_{core}B_{core}}{I}$; $A_{core} = 1.5a^2 = 6\ cm^2$

$B_{core} = B_{ac} = \left\{\dfrac{P_{sp}}{1.5x10^{-6} f^{1.3}}\right\}^{0.4}$ 3F3 ferrite

$B_{core} = \left\{\dfrac{107}{(1.5x10^{-6})(200)^{1.3}}\right\}^{0.4} = 88$ mT

$L_{max} = \dfrac{(480)(6x10^{-4})}{2.8} = 9$ mH > than 4 mH required. Reduce number of turns N.

$\dfrac{4}{9}(480) = 213 = N$

$$\Sigma g = \frac{A_{core}}{\frac{A_{core}B_{core}}{\mu_o NI} - \frac{a + d}{N_g}} \text{ ; } A_{core} = 1.5a^2 = (1.5)(2^2) = 6 \text{ cm}^2 \text{ ; } d = 1.5a = 3 \text{ cm}$$

$$\Sigma g = \frac{6x10^{-4}}{\frac{(6x10^{-4})(.088)}{(4\pi x10^{-7})(213)(2.8)} - \frac{.02 + .03}{4}} = 10 \text{ mm}$$

S30.2.

Check turns ratio: $N = \frac{A_{cu,sec}}{A_{cu,pri}} = \frac{10}{2.5} = 4$; Turns ratio adequate

Check power dissipation at required J_{rms} and B_{core} levels in the new application.

Allowable $P_{sp} = \frac{100°C - 40°C}{(2.7°C/W)(V_c + V_w)} = \frac{100°C - 40°C}{(2.7°C/W)(108 + 81)} = 117 \text{ mW/cm}^3$

$J_{rms} = \frac{9A}{2.5mm^2} = 3.6 \text{ A/mm}^2$; $P_{w,sp} = (0.3)(22)(3.6)^2 = 86 \text{ mW/mm}^3$

$P_{w,sp} < P_{sp,max}$;

$B_{ac} = \frac{1.414 V_{rms}}{\omega A_c N_{pri}}$; $V_{rms} = 500V$, $2\pi f = 628$ kRad/sec, $N_{pri} = 32$

$B_{ac} = \frac{(1.4)(500)}{(2\pi x10^5)(6x10^{-4})(32)} = 59 \text{ mT}$

$P_{c,sp} = (1.5x10^{-6})(100)^{1.3}(59)^{2.5} = 16 \text{ mW/cm}^3 < P_{sp,max}$

Transformer can be used in the the converter.

S30.3.

a. Use scaling factors to fill in table.

A_cA_w proportional to a^4 ; $R_{\theta sa}$ proportional to a^{-2} ; J_{rms} proportional to $a^{-1/2}$

B_{ac} proportional to $a^{-1/2}$; $k_{cu} A_cA_w J_{rms}B_{ac}$ proportional to a^3

Core size (a in cm)	A_cA_w (cm^4)	$R_{\theta sa}$ (°C/W)	J_{rms} (A/mm^2)	B_{ac} (mT)	$k_{cu}A_cA_w J_{rms} B_{ac}$ (joules)
0.5	0.13	39	8.4	336	0.00086
1	2.1	9.8	6	240	0.0069
1.5	10.6	4.4	4.9	196	0.0233
2	33.6	2.5	4.3	170	0.0552

b. Required $LII_{rms} = (10^{-3}H)(5A)((5A)(1.414) = 0.035$
From table, need core with a = 2 cm.

c. $P_{sp} = \frac{105°C - 35°C}{(2.5°C/W)(V_c + V_w)}$; $V_c + V_w = 26a^3 = 26(2)^3 = 208\ cm^3$

$P_{sp} = \frac{105 - 35}{(2.5)(208)} = 135\ mW/cm^3$; $J_{rms} = \sqrt{\frac{135}{(0.6)(22)}} = 3.2\ A/mm^2$

$A_{cu} = \frac{I_{rms}}{J_{rms}} = \frac{5}{3.2} = 1.56\ mm^2$; $N = \frac{k_{cu}A_w}{A_{cu}}$; $A_w = 1.4a^2 = (1.4)(4) = 5.6\ cm^2$

$N = \frac{(0.6)(5.6)}{0.0156} = 215$

S30.4.

a. $LII_{rms} = (10^{-3}H)(3A)(3\sqrt{3}\ A) = 0.0156$ Joules

From Power Electronics by Mohan et.al., p. 761 $LII_{rms} = \frac{C\ a^3 \sqrt{k_{cu}}}{\sqrt{f}}$

From Table 30-3, p. 762 of Mohan et.al., a core using 3F3 ferrite with a = 1 cm, f = 100 kHz, and similar temperature limits has $LII_{rms} = 0.0125 \sqrt{k_{cu}}$

Using $k_{cu} = 0.6$, $LII_{rms} = 0.0125 \sqrt{0.6} = 0.0097$

Solving for $C = \frac{(0.0097)\sqrt{10^5}}{(1^3)\sqrt{0.6}} \approx 4$

Each inductor section requires $(0.1mH)\ (3A)(3\sqrt{3}\ A) = 0.00156 = \frac{4\ a^3 \sqrt{0.6}}{\sqrt{10^5}}$

Solving for a yields: a = 0.54 cm

b. From Mohan et.al., p. 755, $J_{rms} = \frac{C_1}{\sqrt{k_{cu}\ a}}$; From Table 30-3, p. 762, with a = 1 cm, f = 100 kHz, and similar temperature limits, $J_{rms} = \frac{3.3}{\sqrt{k_{cu}}} = \frac{3.3}{\sqrt{0.6}} = 4.26\ A/mm^2$

Scaling to to a = 0.54 cm, $J_{rms} = 4.26 \sqrt{\frac{1}{0.54}} = 5.8\ A/mm^2$

$A_{cu} = \frac{3A}{5.8A/mm^2} = 0.52\ mm^2$

$N_w\ A_{cu} = k_{cu}\ A_w$; $A_w = 1.4a^2$; Table 30-1, p. 751 of Mohan et.al. ; $A_w = 0.41\ cm^2$

$$N_w = \frac{(0.6)(0.41\text{cm}^2)}{(0.0052\text{cm}^2)} = 43$$

c. $$\Sigma g = \frac{A_{core}}{\frac{A_{core}B_{core}}{\mu_o NI} - \frac{a+d}{N_g}}$$; $d = 1.5a = (1.5)(0.54) = 0.81$ cm ; $N_g = 4$;

$A_c = 1.5a^2 = (1.5)(0.54)^2 = 0.44\text{ cm}^2$; N = 43 ; $I = 3\sqrt{3} = 5.2$ A ;

$B_{ac} = B_{core} = \frac{C_2}{f^{0.52}\,a^{0.4}}$ for 3F3 ferrite; Eq. (30-16), p. 755 of Mohan et.al. $B_{core} = 170$ mT for a = 1 cm and f = 100 kHz. Scaling to a = 0.54 cm and f = 100 kHz gives

$$B_{core} = \frac{(170\text{mT})(1\text{cm})^{0.4}}{(0.54\text{cm})^{0.4}} = 218\text{ mT}$$

$$\Sigma g = \frac{4.45\text{x}10^{-5}\text{m}^2}{\frac{(4.45\text{x}10^{-5}\text{m}^2)(0.22\text{T})}{(4\pi\text{x}10^{-7}\text{H/m})(43)(5.2\text{A})} - \frac{(0.0054\text{m} + 0.0081\text{m})}{4}} = 1.4\text{ mm}$$

S30.5.

a. $LII_{rms} = (10^{-3}\text{H})(3\text{A})(3\sqrt{2}\text{ A}) = 0.0127$ Joules

Check: $k_{cu}A_wA_cB_{ac}J_{rms} > LII_{rms}$; $A_w = (1.4)(4) = 5.6\text{ cm}^2$; $A_c = (1.5)(4) = 6\text{ cm}^2$

$V_c = (13.5)(8) = 108\text{ cm}^3$; $V_w = 912.3)(8) = 98\text{ cm}^3$; $A_{sur} = (60)(4) = 238\text{ cm}^2$

$R_{\theta sa} = \frac{600}{238} = 2.5$ °C/W ; $P_{sp} = \frac{90 - 30}{(2.5)(108 + 98)} = 117\text{ mW/cm}^3$

$$J_{rms} = \sqrt{\frac{117}{(0.6)(22)}} = 3\text{ A/mm}^2$$

$$B_{ac} = \sqrt[0.4]{\frac{117}{(1.5\text{x}10^{-6})(200)^{1.3}}} = 91\text{ mT}$$

$k_{cu}A_wA_cB_{ac}J_{rms} = (0.6)(5.6\text{x}10^{-4})(6\text{x}10^{-4})(3\text{x}10^6)((.091) = 0.055$

0.055 > 0.0127 required value. Core can be used.

b. $L_{max}II_{rms} = L_{max}(3)(3\sqrt{2}) = 12.7\,L_{max} = 0.055$; $L_{max} = \frac{0.055}{12.7} = 4.2$ mH

Only need L = 1 mH

$$LI = NB_{core} A_c \ ; N = \frac{(0.001H)(3\sqrt{2}A)}{(0.091T)(0.0006m^2)} = 78$$

$$\% \text{ filled} = \frac{N A_{cu}}{k_{cu} A_w} (100) \ ; A_{cu} = \frac{3A}{3A/mm^2} = 1 \text{ mm}^2$$

$$\% \text{ filled} = \frac{(78)(1mm^2)}{(0.6)(560mm^2)} (100) = 23\%$$

S30.6.

a. Check to see if $V_{pri}I_{pri} + V_{sec}I_{sec} < 2.2k_{cu}f A_wA_cJ_{rms}B_{core}$

$V_{pri}I_{pri} + V_{sec}I_{sec} = (200)(5) + (2000)(0.5) = 2000$ watts

$A_wA_c = 2.1a^4 = 2.1 \text{ cm}^4 = 2.1\text{x}10^{-8} \text{ m}^4$

$P_{sp} = 240 \text{ mW/cm}^3$; see p. 766 of Mohan et. al.

$$B_{core} = \left[\frac{240}{(1.5\text{x}10^{-6})(300)^{1.3}}\right] = 0.1 \text{ T} \ ; \text{ from loss equation for 3F3 ferrite}$$

$$J_{rms} = \sqrt{\frac{240}{(22)(0.6)}} = 4.3 \text{ A/mm}^2 = 4.3\text{x}10^6 \text{ A/m}^2$$

$2.2k_{cu}f A_wA_cJ_{rms}B_{core} = (2.2)(0.6)(3\text{x}10^5)(2.1\text{x}10^{-8})(4.3\text{x}10^6)(0.1) = 3580$ watts

2000 W < 3580 W ; Core can be used for the transformer.

b. $A_c = 1.5a^2 = (1.5)(10^{-2}m)^2 = 1.5\text{x}10^{-4} \text{ m}^2$; see Mohan et. al. p. 751

$$N_{pri} \frac{V_{pri,rms}\sqrt{2}}{2\pi f B_{core}A_c} = \frac{(200)(1.414)}{(2\pi)(3\text{x}10^5)(0.1)(1.5\text{x}10^{-4})} = 10 \ ; \ N_{sec} = 100$$

$$A_{cu,pri} = \frac{I_{pri}}{J_{rms}} = \frac{5A}{4.3A/mm^2} = 1.2 \text{ mm}^2 \ ; \ A_{sec} = 0.12 \text{ mm}^2$$

c. Transformer has substantial overcurrent capability.Winding window only partially filled. Hence winding losses at I_{pri} = 5A much less than allowed max.

$$2\left[\frac{N_{pri}A_{cu,pri}}{0.6}\right] = \text{area of winding window occupied by winding.}$$

$$2\left[\frac{N_{pri}A_{cu,pri}}{0.6}\right] = \frac{(10)(1.2mm^2)}{0.6} = 0.4 \text{ cm}^2 \ ; A_w = 1.4 \text{ cm}^2$$

Winding window only 30% filled.

S30.7.

a. $A_c = (0.5)(OD\text{-}ID)(t) = (0.5)(0.5 - 0.3)(0.19)(2.54)^2 = 0.126\ cm^2$

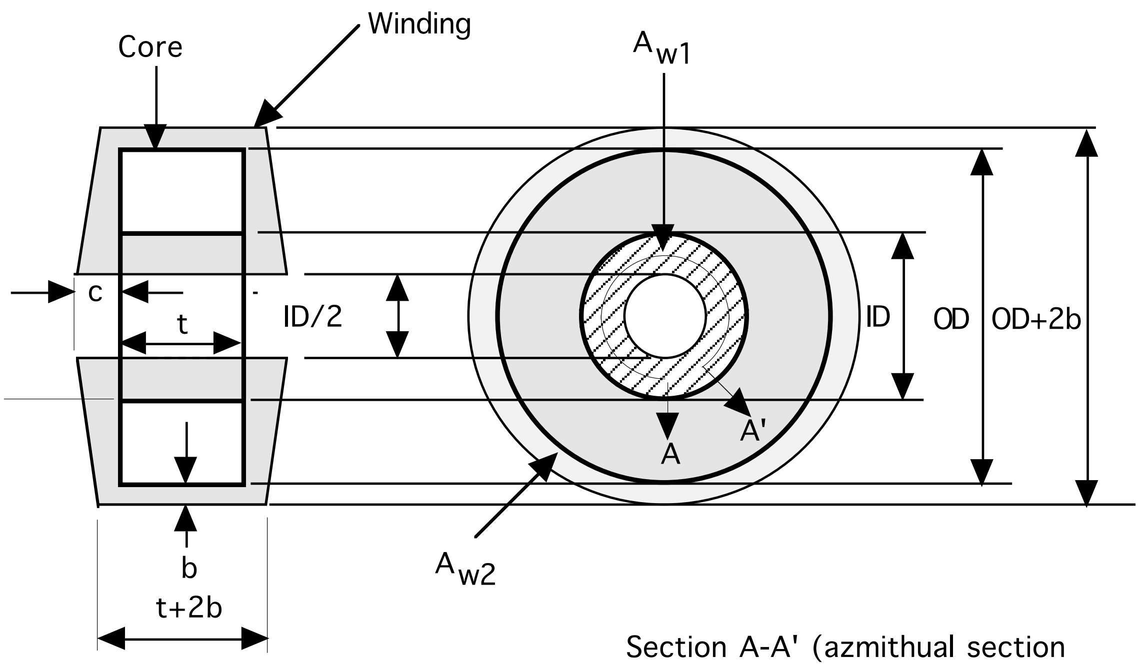
$A_w = A_{w1} = (0.25)\pi[ID^2 - 0.5ID^2]$; see figure of winding shown below.

$A_w = (0.25)\pi\ \{(0.3)^2 - (0.15)^2\}(2.54)^2 = 0.339\ cm^2$

b. $V_c = A_c l_m$; l_m = mean magnetic path length = $2\pi(OD + ID)/4 = 2\pi(0.5+0.3)/4*(2.54)$

$l_m = 3.19cm$; $V_c = (0.126)(3.19) = 0.402\ cm^3$

To estimate V_w refer to figure of winding shown below.



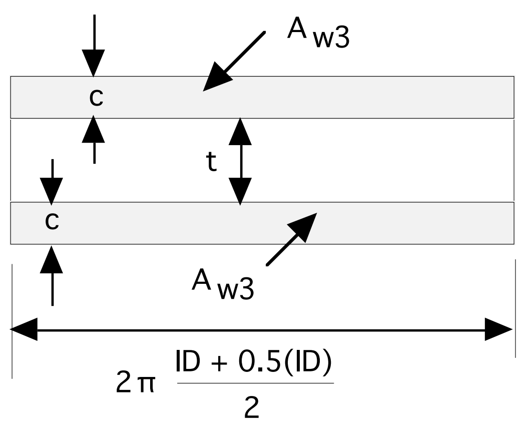
$$A_{w1} = \frac{\pi(ID)^2 - \pi(0.5ID)^2}{4}$$

$$A_{w2} = \frac{\pi(OD + 2b)^2 - \pi OD^2}{4}$$

$$A_{w3} \approx \frac{c2\pi(ID + 0.5ID)}{2}$$

A_{w3} = area of winding as it runs parallel to diameter of core at or near the inner diameter of core. See Section A-A'.

Section A-A' (azmithual section unwrapped at a diameter of {ID + 0.5ID}/2)



Now $A_w = A_{w1} = A_{w2} = A_{w3}$. Setting $A_w = A_{w2}$ gives

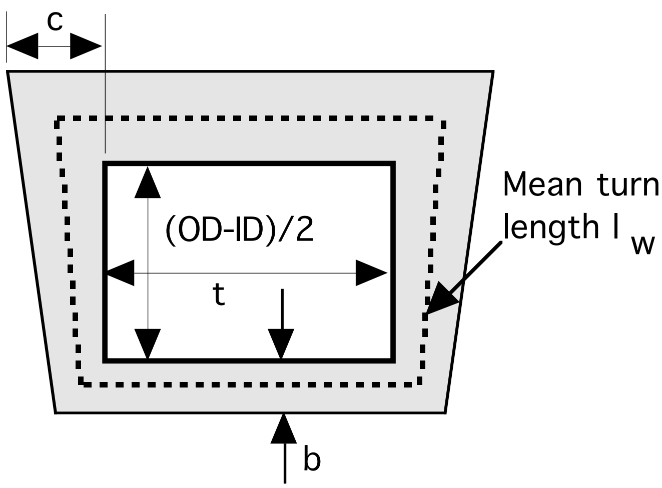
$b = -0.5(OD) + 0.5(OD^2 + 4A_w/\pi)^{0.5}$

$b = -(0.5)(.5)(2.54) + 0.5[\{0.5)(2.54)\}^2 + (4)(0.339)/\pi\}^{0.5} = -0.635 + .662 = .05$ cm

Setting $A_w = A_{w3}$ gives $c \approx 2A_w/\{3\pi ID\}$

$c = [(2)(0.339)]/[3\pi(0.3)(2.54)] = 0.095$ cm ;

$V_w = A_w l_w$ where l_w = mean turn length (see figure below).



$l_w = [\{t+c\} + \{t+b\} + 2\{(OD-ID)(0.5) + (0.5)(b+c)\}]$

$t = (0.19)(2.54) = 0.483$ cm

$l_w = [(0.483+0.095)+(0.483+0.05) + 2(0.5)(0.508+0.095+0.05)]$

$l_w = 1.76$ cm

$V_w = (0.339)(1.76) = 0.6\ cm^3$

$A_{surface} = A_{s1} + A_{s2} + A_{s3}$

$A_{s1} = 2\pi(ID/2)(t+2c) = 2\pi(0.15*2.54)[0.483+2(0.095)] = 1.61\ cm^2$

$A_{s2} = 2\pi(OD/2 + b)(t + 2b) = 2\pi(0.25*2.54 + 0.05)(0.483 + 2*0.05) = 2.51\ cm^2$

$A_{s3} \approx 2\{0.25\pi\}[\{OD+2b)^2 - (0.5*ID)^2] = 0.5\pi[(0.5*2.54+0.1)^2 - (0.5*.3*2.54)^2]$

$A_{s3} \approx 2.72\ cm^2$

$A_{surface} = 1.61 + 2.51 + 2.72 = 6.84\ cm^2$

c. $R_{\theta sa} = R_{\theta,rad} \| R_{\theta,conv}$

$$R_{\theta,rad} = \frac{[T_s - T_a]}{5.7\ EA\left[\left[\frac{T_s}{100}\right]^4 - \left[\frac{T_a}{100}\right]^4\right]} = \frac{90-35}{(5.7)(0.9)(6.84\times10^{-4})([3.63]^4 - [3.08]^4)}$$

$R_{\theta,rad} = 187$ °C/W

$$R_{\theta,conv} = \frac{[d_{vert}]^{0.25}}{1.34\ A\ [T_s - T_a]^{0.25}}$$; choose $d_{vert} = (A)^{0.5} = (6.84\ cm^2)^{0.5} = 2.62$ cm

$$R_{\theta,conv} = \frac{(0.0262)^{0.25}}{(1.34)(6.84x10^{-4})(90-35)^{0.25}} = 161\ °C/W$$

$R_{\theta sa} = (187)||(161) = 87$ °C/W

d. $P_{Diss} = [T_s - T_a]/ R_{\theta sa}$ = total allowable power dissipation in the inductor.

$P_{Diss} = (90 - 35)/87 = 630$ mW

$P_{sp} = P_{Diss}/[V_c + V_w] = 630\ mW/[(0.4\ cm^3) + (0.6\ cm^3)] = 600\ mW/cm^3$

$P_{sp} = k_{cu}\rho J^2 = (0.3)(2.2x10^{-8}\ ohm\text{-}m)J^2 = 6x10^5\ W/m^3$; assume Litz wire is used because of high frequency (100 kHz)

$J = [6x10^5/(0.3*2.2x10^{-8})]^{0.5} = [91x10^{12}]^{0.5} = 9.54x10^6\ A/m^2$

$J = 9.54\ A(rms)\ /mm^2$

$P_{sp,core} = 0.07f^{1.6}B^{2.3}$ W/lb : W/kg = (2.2 lbs/kg)(W/lbs) = 2.2 Watts/lb

$Watts/m^3 = (density\ in\ kg/m^3)(Watts/kg) = (2.2\ watts/lb)(8.5x10^3\ kg/m^3)$

$Watts/m^3 = 18.7x10^3$ watts/lb or $mW/cm^3 = 18.7$ watts/lb

$P_{sp,core} = (18.7)(0.07)f^{1.6}B^{2.3} = 1.31f^{1.6}B^{2.3}\ mW/cm^3$;

(f in kHz and B in kilogauss) ;

$P_{sp} = 600\ mW/cm^3 = P_{sp,core} = 1.31(100)^{1.6}B^{2.3}\ mW/cm^3$

$B^{2.3} = (600)/[(1.31)(100)^{1.6}] = 0.289$

$B = 0.58$ kilogauss (rms) $= 0.058\ weber/m^2$

e. $L_{max}\ I_{rated} = N_{max}B_{max}A_c$

$I_{rated} = 2$ A rms; $N_{max} = k_{cu}A_w/A_{cu}$; $A_{cu} = I_{rms}/J_{rms} = 2/9.54 = 0.21\ mm^2$

$A_{cu} = 2.1x10^{-3}\ cm^2$

$L_{max} = (0.3)(3.39x10^{-5}m^2)(0.058weber/m^2)(1.26x10^{-5}m^2)/[(2Arms)(2.1x10^{-7}m^2)]$

$L_{max} = 18$ microhenrys

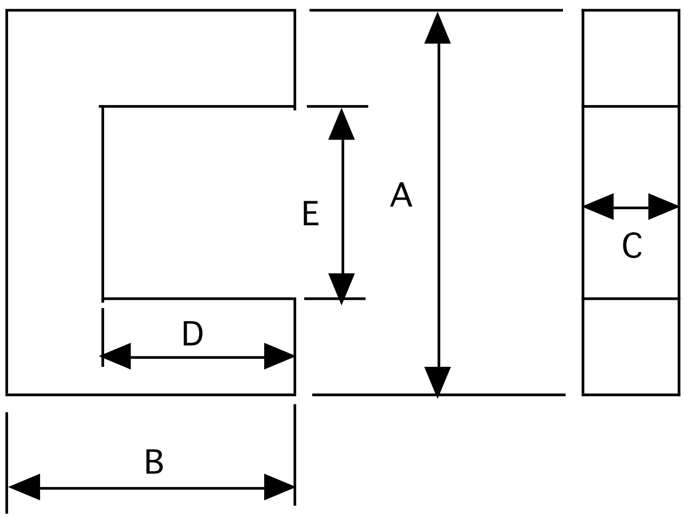
f. $Bl_m/\mu = NI$; thus $\mu = Bl_m/(NI)$; $NI = [k_{cu}A_w/A_{cu}][JA_{cu}] = k_{cu}A_wJ$

$\mu = Bl_m/[\,k_{cu}A_wJ] = (0.058)(0.032)/[(0.3)(3.4x10^{-5})(9.54x10^{6})]$

$\mu = 1.91x10^{-5}$ Henries/meter or $\mu/\mu_o = 1.91x10^{-5}/(4\pi x10^{-7}) = 15.2 \approx 15$

S30.8.

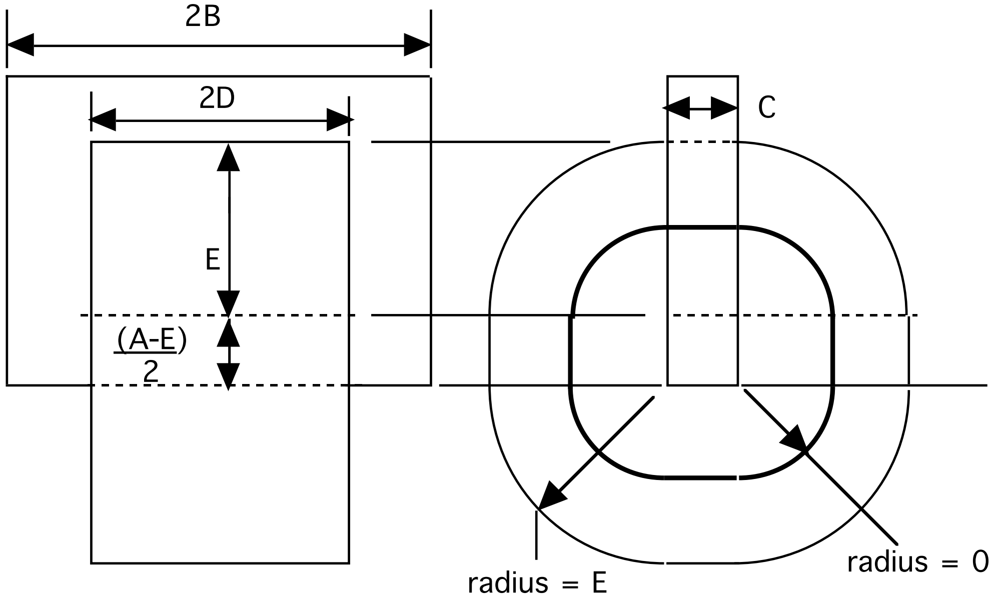
Dimensions of U-core shown below.



A = 33.3 mm
B = 22.2 mm
C = 9.4 mm
D = 12.7 mm
E = 14.3 mm

From catalog find, $V_{core} = 9490\ mm^3$; $A_{core} = 86.5\ mm^2$; $l_m = 110$ mm

Drawing of core+winding assembly shown below.



Mean turn length $l_w = \pi E + A - E + 2C = (3.14)(14.3) + 33.3 - 14.3 + (2)(9.4) = 82.7$ mm

Winding area $A_w = 2DE = (2)(12.7)(14.3) = 363\ mm^2$

Winding volume $V_w \approx l_w A_w = (82.7)(363) = 3.02x10^4\ mm^3$

<u>Surface area</u> A_s

Exposed core surface bottom and top = 2BC + 2(B-D)C = 4BC - 2DC
Exposed core surface left and right = 2AC
Exposed core surface front and back = 4(B-D)A+2D(A-E)/2*2
= 4A(B-D)+ 2D(A-E) = 4AB-2AD-2DE
Summing three terms together

$A_{s,c} = 2C(2B-D) + 2AC + 2A(2B-D) - 2DE = 2(2B-D)(A+C) + 2(AC-DE)$

$A_{s,c} = (2)((2)(22.2) - 12.7)(33.3+9.4) + (2)((33.3)(9.4) - (12.7)(14.3))$

$= (2)(31.7)(42.7) + (2)(313-182) = 2707 + 262 = 2969\ mm^2$

<u>Exposed surface area of winding</u> $A_{s,w}$

Top and bottom = $(2)(4)(0.25\pi)(E)^2 + (4)(0.5)(A-E)E + 2CE = E^2(6.3-2) + 2AE + 2CE$
$= 2.3E^2 + 2AE + 2CE$

Vertical surface = $2\pi E(2D) + 2(2D)(0.5)(A-E) + C(2D) = 4\pi DE + 2AD - 2DE + 2CD$
$= 10.6DE + 2AD + 2CD$

$A_{s,w} = 2.3E^2 + 2AE + 2CE + 10.6DE + 2AD + 2CD$

$= (2.3)(14.3)^2 + (2)(33.3)(14.3) + (2)(9.4)(14.3) + (10.6)(12.7)(14.3)$
$+ (2)(33.3)(12.7) + (2)(9.4)(12.7)$

$= 470+ 952+269+1925+846+239 = 4701 \text{ mm}^2$

$A_s = A_{s,c} + A_{s,w} = 2969 + 4701 = 7670 \text{ mm}^2 = 7.67\text{x}10^{-3} \text{ m}^2$

$R_\theta = 1/(hA_s) = 1/([10][7.67\text{x}10^{-3}]) = 13 \text{ °C/W}$

$$P_{sp} = \frac{\Delta T}{R_\theta(V_c + V_w)} = \frac{30}{(13)(9.5 + 30.2)} = 58 \text{ mW/cm}^3$$

$$J_{rms} = \sqrt{\frac{P_{sp}}{k_{cu}\rho_{cu}}} = \sqrt{\frac{0.058}{(0.3)(2.2\text{x}10^{-6})}} = 300 \text{ A/cm}^2$$

$$B_{ac} = \left\{\frac{Psp}{Kf^a}\right\}^{1/b} = \left\{\frac{58}{(1.5\text{x}10^{-6})(40)^{1.3}}\right\}^{1/2.5} = 160 \text{ mT}$$

Core V-I rating = $2.2k_{cu}fA_wA_cJ_{rms}B_{ac}$

Core V-I rating = $((2.2)(0.3)(4\text{x}10^4)(8.7\text{x}10^{-5})(3.5\text{x}10^{-4})(3\text{x}10^6)(0.16) = 386$ Watts
Required V-I rating = (50 V)(5 A) = 250 watts.
Hence this core is satisfactory for this application

S30.9.

a. $A_S = (59.6)(0.5)^2 = 14.9 \text{ cm}^2$; $R_\theta = \frac{10^3}{14.9} = 67 \text{ °C/W}$

$V_w = (12.3)(0.5)^3 = 1.54 \text{ cm}^3$; $V_c = (13.5)(0.5)^3 = 1.69 \text{ cm}^3$

$$P_{sp} = \frac{30°C}{(67 \text{ °C/W})(1.54 \text{ cm}^3 + 1.69 \text{ cm}^3)} = 139 \text{ mW/cm}^3$$

$$B_{ac} = \sqrt{\frac{139}{(10^{-6})(100)}} = 118 \text{ mT}$$

$$J_{rms} = \sqrt{\frac{139}{(0.3)(22)}} = 4.6 \text{ A/mm}^2$$

b. Core V-A rating = $2.2 \, k_{cu} \, f \, A_wA_cJ_{rms}B_{ac} \geq 2 \, V_{sec,min} \, I_{sec,max}$

$A_w = (1.4)(0.5)^2 = 0.35 \text{ cm}^2$; $A_c = (1.5)(0.5)^2 = 0.38 \text{ cm}^2$

Core rating = $(2.2)(0.3)(10^5)(3.5\text{x}10^{-5})(3.8\text{x}10^{-5})(4.6\text{x}10^6)(0.12) = 68$ watts

$V_{sec,min} = N_{sec,min} \frac{d\phi}{dt}$; $\frac{d\phi}{dt} = \omega \, B_{ac}A_c = (6.3)(10^5)((0.12)(3.8\text{x}10^{-5}) = 2.87$

$N_{sec,min} = 1$; $V_{sec,min} = (1)(2.87) = 2.87$ V base-to-peak or 2 V rms

$I_{sec,max} = \frac{68 \text{ watts}}{(2)(2V)} = 17$ A rms or 24 A base-to-peak

S30.10.

a. Each of the ten cores must have a rating equal to or exceeding
(5V)(100A) + (50V)(10A) = 1000 watts

Core rating = $2.2\, k_{cu}\, f\, A_w A_c J_{rms} B_{ac}$; $A_w A_c = 2.1a^4 = (2.1)(0.008)^4 = 8.6\text{x}10^{-9}\ m^3$

$R_{\theta sa} = \frac{1}{hA_s} = \frac{1}{(10)(59.6)(8\text{x}10^{-3})^2} = 26$ °C/W ; $P_{sp} = \frac{T_s - T_a}{R_{\theta sa}(V_c + V_w)}$

$V_c + V_w = 26a^3 = (26)(0.8)^3 = 13.3\ cm^3$

$$P_{sp} = \frac{100°C - 40°C}{(26°C/W)(\ 3.3\ cm^3)} = 174\ mW/cm^3$$

$$J_{rms} = \sqrt{\frac{174 mW/cm^3}{(0.3)(22)}} = 5.13\text{x}10^6\ A/m^2$$

$$B_{ac} = \sqrt[1/2.2]{\frac{174 mw/cm^3}{(1.5\text{x}10^{-6})(200)^{1.3}}} = 0.153T$$

Core rating = $(2.2)(0.3)(2\text{x}10^5 Hz)(8.6\text{x}10^{-9} m^4)(\ 5.13\text{x}10^6 A/m^2)(0.153T) = 890$ watts
Core's capability slightly less than the required 1000 watts. However it is 90% of the required value and if used would result in the surface temperature being slightly larger than 100 °C. Will go ahead and use the core.

b. $N_{pri} = \frac{\sqrt{2} V_{pri,rms}}{2\pi f B_{ac} A_c} = \frac{(1.414)(5)}{(6.3)(2\text{x}10^5)(0.153)(1.5)(0.008)^2} = 0.4$; round up to 0.5

$N_{sec} = 5$

c. $A_{cu,pri} = \frac{100A}{513 A/cm^2} = 0.195\ cm^2$; $A_{cu,sec} = (.195\ cm^2/10) = 0.0195\ cm^2$

S30.11.

Table 30-3 is reproduced below. The only thing that changes in the additional entries that are requested is the size of core specified by the value of scaling dimension "a". The various table entries will scale as indicated below.

$AP = 2.1(a/1)^4$; $R_\theta = 9.8(1/a)^2$; $P_{sp} = 237(1/a)$; $J_{rms} = 3.3\ (1/a)^{0.5} / \sqrt{k_{cu}}$;

$B_{ac} = 170(1/a)^{0.4}$: for 3F3 ferrite $P_{m,sp} = 1.5x10^{-6}\ f^{1.3}\ [B_{ac}]^{2.5} = P_{sp} = 237(a/1)$

$k_{cu}\ J_{rms}\ \hat{B}\ A_w\ A_{core} = .0125\sqrt{k_{cu}}\ (1/a)^{0.5}\ (1/a)^{0.4}(a/1)^4 = .0125\sqrt{k_{cu}}\ (a/1)^{3/1}$

Scaling dimension a [cm]	AP = $A_w A_{core}$ [cm^4]	R_θ ΔT=60 °C [°C/W]	P_{sp} @ ΔT=60 °C [mW/cm^3]	J_{rms} @ ΔT=60 °C & P_{sp} [A/mm^2]	B_{ac} @ ΔT=60 °C & 100 kHz [mT]	$k_{cu}\ J_{rms}\ \hat{B}$ •$A_w\ A_{core}$ [Joules]
•	•	•	•	•	•	•
1	2.1	9.8	237 mW/cm^3	$3.3/\sqrt{k_{cu}}$	170	$.0125\sqrt{k_{cu}}$
1.25	5,1	6.2	190	$3/\sqrt{k_{cu}}$	155	$.025\sqrt{k_{cu}}$
1.5	10.6	4.4	158	$2.7/\sqrt{k_{cu}}$	145	$.044\sqrt{k_{cu}}$
1.75	19.7	3.2	135	$2.5/\sqrt{k_{cu}}$	136	$.071\sqrt{k_{cu}}$
2	33.6	2.5	119	$2.3/\sqrt{k_{cu}}$	129	$0.11\sqrt{k_{cu}}$

S30.12.

a. The core material for this application should have the largest performance factor (performance factor $P = f\ B_{ac}$ where B_{ac} is the ac flux density which generates a specified core loss density). From the graph of performance factors for several different materials given in Fig. 30-3, page 747 of Power Electronics, Converters, Applications, and Design, 2nd Edition by Mohan, Undeland, and Robbins, 3F3 ferrite material has the largest performance factor at 100 kHz.

b. $L\ I\ I_{rms} = k_{cu}\ J_{rms}\ B\ A_w\ A_{core}$; $L\ I\ I_{rms} = (5x10^{-4})(8.5)(6) = 0.0255$
The temperature and frequency requirements for this inductor and the use of a double-E core geometry match those in the table of the previous problem (S30.11). Hence we can use the same relationship between core scaling dimension "a" here that was used in the previous problem.

$.0125\sqrt{k_{cu}}\ (a/1)^{3/1} = 0.0255$; $k_{cu} = 0.3$ as specified in problem statement.
Solving for a yields a = 1.53 cm

c. The core size of a = 1.53 cm closely matches the a = 1.5 cm entry in the table generated in problem S30.11. Hence we can use these entries to find the peak flux density B, J_{rms}, area of the copper winding A_{cu}, and the number of turns N.

$J_{rms} = 2.7/(0.3)^{0.5} = 4.93\ \text{A/mm}^2$; $A_{cu} = I_{rms}/J_{rms} = 6/4.93 = 1.22\ \text{mm}^2$

$B_{peak} = B_{ac}\frac{I_{peak}}{I_{ac}}$; From table in problem S30.11, $B_{ac} = 145$ mT; ;

$I_{peak} = I_{dc} + I_{ac} = 8.5$ A ; $(I_{rms})^2 = (6)^2 = 36 = (I_{dc})^2 + 0.5\,(I_{ac})^2$

Solving for I_{ac} yields $I_{ac} = 8.5$ A with $I_{dc} = 0$ or $I_{ac} = 2.8$ A with $I_{dc} = 5.7$ A.

Problem statement gives no way of deciding which is the correct situation. However if the dc current equals 5.7 A, the peak flux required of the ferrite would be 0.44 T which will exceed the saturation flux density of the 3F3 ferrite at 100 ºC. Hence we shall assume that $I_{ac} = 8.5$ A with $I_{dc} = 0$ and so $B_{peak} = B_{ac} = 145$ mT.

$N = \frac{k_{cu}\,A_w}{A_{cu}}$; $A_w = 1.4a^2 = (1.4)(1.5^2) = 3.15\ \text{cm}^2$;

N = (0.3)(3.15)/(0.0122) = 77 turns

d. $L_{max} = \frac{N\,A_{core}\,B_{ac}}{I_{ac}} = \frac{(77)(3.38\text{x}10^{-4})(0.145)}{8.5} = 0.444$ mH.

$A_{core} = 1.5a^2 = (1.5)(1.5^2) = 3.38\ \text{cm}^2$

The achievable inductance with a = 1.5 cm is just outside the minimum value of 4.5 mH. We could recalculate the parameters with core having a > 1.53 cm or live with inductance being slightly too small. For purposes of this problem we will stay with a = 1.5 cm.

Airgap length for double-E core is $\Sigma g = \dfrac{a}{\dfrac{B_{peak}\,A_c}{d\,\mu_o\,N\,I_{peak}} - \dfrac{a+d}{d\,N_g}}$

$$\Sigma g = \frac{0.015}{\dfrac{(.145)(3.38\text{x}10^{-4})}{(.0225)(4\pi\text{x}10^{-7})(77)(8.5)} - \dfrac{(.015+.0225)}{(.0225)(4)}} = 6.7\text{ mm}$$

e. The maximum achievable inductance is approximately equal to the desired inductance, so further adjustments are needed. If a larger core were used, then the maximum inductance would be significantly larger than the desired value of 0.5 mH. Some of the turns could be removed from the winding and the airgap length readjusted to give the desired inductance while making full use (maximum allowable ac flux) of the core. This would provide some cost and weight savings.

