# EE 740
# Economic Dispatch

Spring 2013

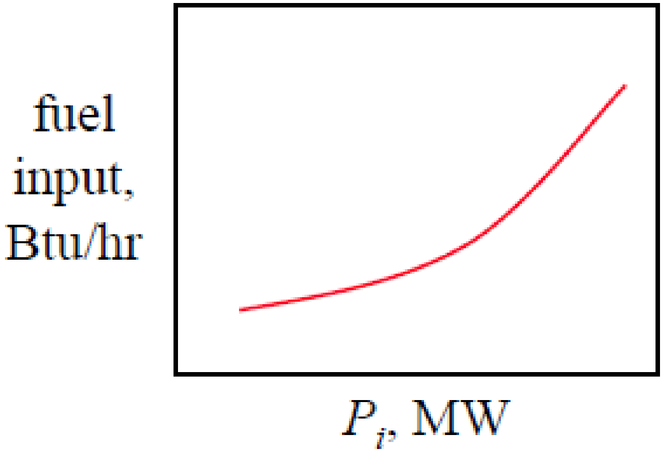
# Operating Costs

- **Factors influencing the minimum cost of power generation**
  - operating efficiency of prime mover and generator
  - fuel costs
  - transmission losses
- **The most efficient generator in the system does not guarantee minimum costs**
  - may be located in an area with high fuel costs
  - may be located far from the load centers and transmission losses are high
- **The problem is to determine generation at different plants to minimize the total operating costs**

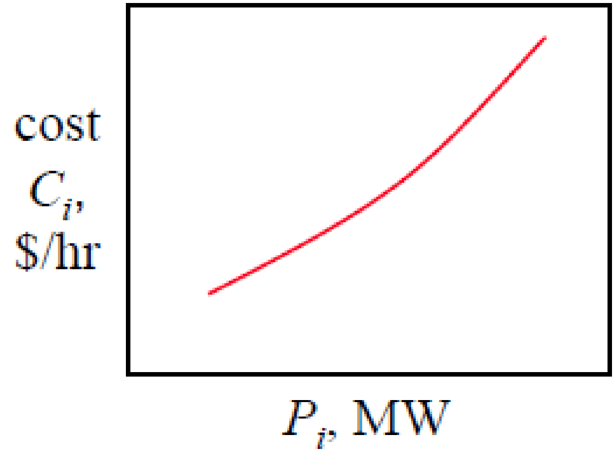
# Operating Costs

- **Generator heat rate curves lead to the fuel cost curves**

Heat-Rate Curve




Fuel-Cost Curve




  - The fuel cost is commonly express as a quadratic function

$$C_i = \alpha_i + \beta_i P_i + \gamma_i P_i^2$$

  - The derivative is known as the incremental fuel cost

$$dC_i / dP_i = \beta_i + 2\gamma_i P_i$$

# Economic Dispatch

- **The simplest problem is when system losses and generator limits are neglected**
  - minimize the objective or cost function over all plants
  - a quadratic cost function is used for each plant

$$C_{total} = \sum_{i=1}^{n_{gen}} C_i = \sum_{i=1}^{n_{gen}} \alpha_i + \beta_i P_i + \gamma_i P_i^2$$

  - the total demand is equal to the sum of the generators' output; the equality constrant

$$\sum_{i=1}^{n_{gen}} P_i = P_{Demand}$$

# Economic Dispatch

- **A typical approach using the Lagrange multipliers**

$$L = C_{total} + \lambda\left( P_{Demand} - \sum_{i=1}^{n_{gen}} P_i \right)$$

$$\frac{\partial L}{\partial P_i} = \frac{\partial C_{total}}{\partial P_i} + \lambda(0-1) = 0 \quad \rightarrow \quad \frac{\partial C_{total}}{\partial P_i} = \lambda$$

$$C_{total} = \sum_{i=1}^{n_{gen}} C_i \quad \rightarrow \quad \frac{\partial C_{total}}{\partial P_i} = \frac{dC_i}{dP_i} = \lambda \quad \forall i = 1, \ldots, n_g$$

$$\lambda = \frac{dC_i}{dP_i} = \beta_i + 2\gamma_i P_i$$

# Economic Dispatch

- the second condition for optimal dispatch

$$\frac{dL}{d\lambda} = \left( P_{Demand} - \sum_{i=1}^{n_{gen}} P_i \right) = 0 \quad \rightarrow \quad \sum_{i=1}^{n_{gen}} P_i = P_{Demand}$$

- rearranging and combining the equations to solve for λ

$$P_i = \frac{\lambda - \beta_i}{2\gamma_i}$$

$$\sum_{i=1}^{n_{gen}} \frac{\lambda - \beta_i}{2\gamma_i} = P_{Demand} \qquad \lambda = \frac{P_{Demand} + \sum_{i=1}^{n_{gen}} \frac{\beta_i}{2\gamma_i}}{\sum_{i=1}^{n_{gen}} \frac{1}{2\gamma_i}}$$

# Example

- **Neglecting system losses and generator limits, find the optimal dispatch and the total cost in $/hr for the three generators and the given load demand**

$$C_1 = 500 + 5.3P_1 + 0.004P_1^2 \; [\$ / MWhr]$$

$$C_2 = 400 + 5.5P_2 + 0.006P_2^2$$

$$C_3 = 200 + 5.8P_3 + 0.009P_3^2$$

$$P_{Demand} = 800 MW$$

# Example

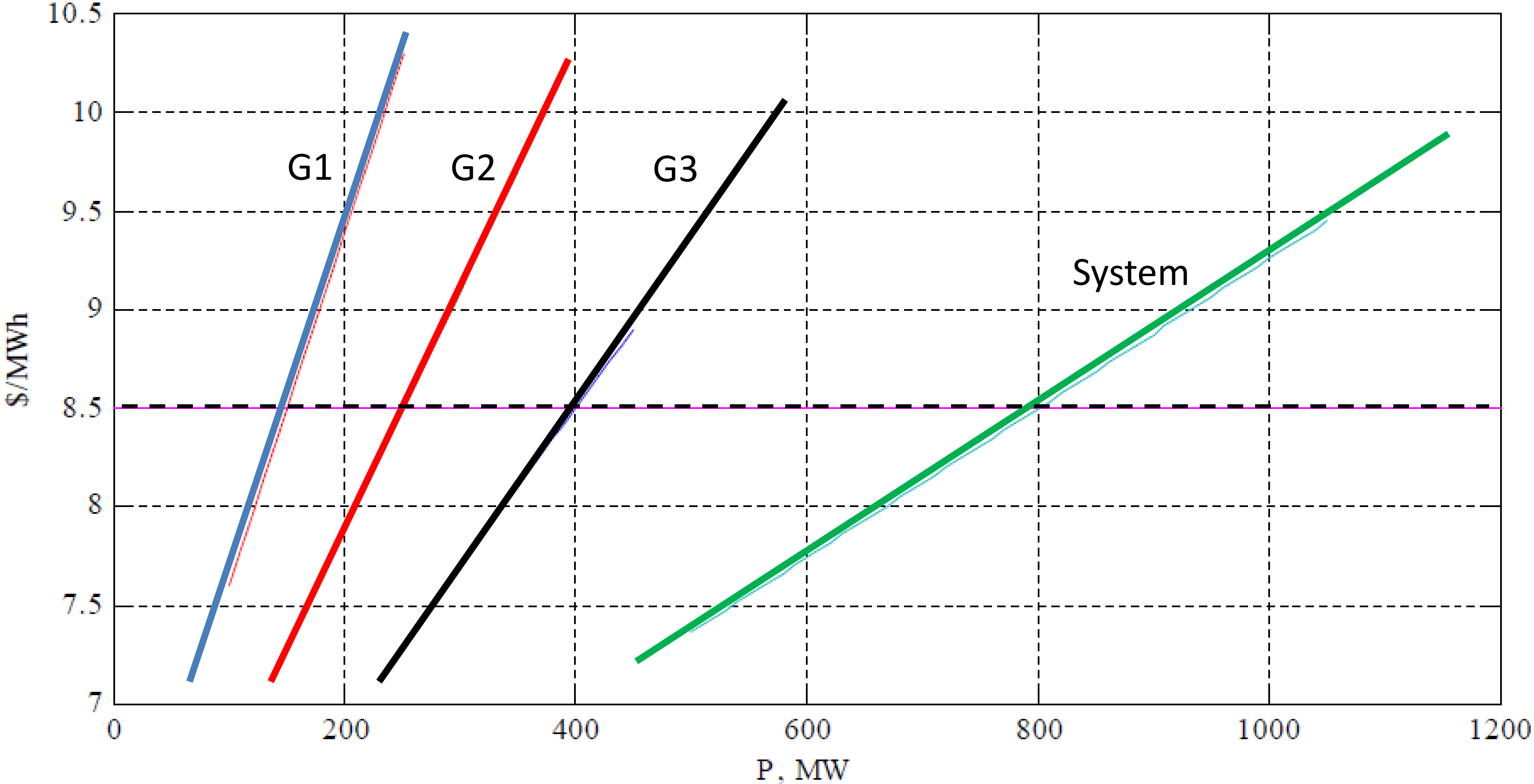
$$\lambda = \frac{P_{Demand} + \sum_{i=1}^{n_{gen}} \frac{\beta_i}{2\gamma_i}}{\sum_{i=1}^{n_{gen}} \frac{1}{2\gamma_i}} = \frac{800 + \frac{5.3}{0.008} + \frac{5.5}{0.012} + \frac{5.8}{0.018}}{\frac{1}{0.008} + \frac{1}{0.012} + \frac{1}{0.018}} = \$8.5 / \text{MWhr}$$

$$P_i = \frac{\lambda - \beta_i}{2\gamma_i} \quad \Rightarrow \quad
\begin{aligned}
P_1 &= \frac{8.5 - 5.3}{2(0.004)} = 400 \text{ MW} \\
P_2 &= \frac{8.5 - 5.5_i}{2(0.006)} = 250 \text{ MW} \\
P_3 &= \frac{8.5 - 5.8}{2(0.009)} = 150 \text{ MW}
\end{aligned}$$

$$P_{Demand} = 800 \text{ MW} = 400 + 250 + 150 \text{ MW}$$

# Example

Incremental cost curves



# Economic Dispatch with Generator Limits

- **The power output of any generator should not exceed its rating nor be below the value for stable boiler operation**
  - Generators have a minimum and maximum real power output limits
- **The problem is to find the real power generation for each plant such that cost are minimized, subject to:**
  - Meeting load demand - equality constraints
  - Constrained by the generator limits - inequality constraints
- **The Kuhn-Tucker conditions**

$$dC_i / dP_i = \lambda \quad \leftarrow \quad P_{i(\min)} < P_i < P_{i(\max)}$$

$$dC_i / dP_i \leq \lambda \quad \leftarrow \quad P_i = P_{i(\max)}$$

$$dC_i / dP_i \geq \lambda \quad \leftarrow \quad P_i = P_{i(\min)}$$

# Example

- **Neglecting system losses, find the optimal dispatch and the total cost in $/hr for the three generators and the given load demand and generation limits**

$$C_1 = 500 + 5.3P_1 + 0.004P_1^2 \quad [\$ / \text{MWhr}]$$

$$C_2 = 400 + 5.5P_2 + 0.006P_2^2$$

$$C_3 = 200 + 5.8P_3 + 0.009P_3^2$$

$$200 \le P_1 \le 450$$

$$150 \le P_2 \le 350$$

$$100 \le P_3 \le 225$$

$$P_{Demand} = 975 \text{ MW}$$

# Example

$$\lambda = \frac{P_{Demand} + \sum_{i=1}^{n_{gen}} \frac{\beta_i}{2\gamma_i}}{\sum_{i=1}^{n_{gen}} \frac{1}{2\gamma_i}} = \frac{975 + \frac{5.3}{0.008} + \frac{5.5}{0.012} + \frac{5.8}{0.018}}{\frac{1}{0.008} + \frac{1}{0.012} + \frac{1}{0.018}} = \$9.163/\text{MWh}$$

$$P_i = \frac{\lambda - \beta_i}{2\gamma_i}$$

$$P_1 = \frac{9.16 - 5.3}{2(0.004)} = 483 \text{ MW}$$

$$P_2 = \frac{9.16 - 5.5_i}{2(0.006)} = 305 \text{ MW}$$

$$P_3 = \frac{9.16 - 5.8}{2(0.009)} = 187 \text{ MW}$$

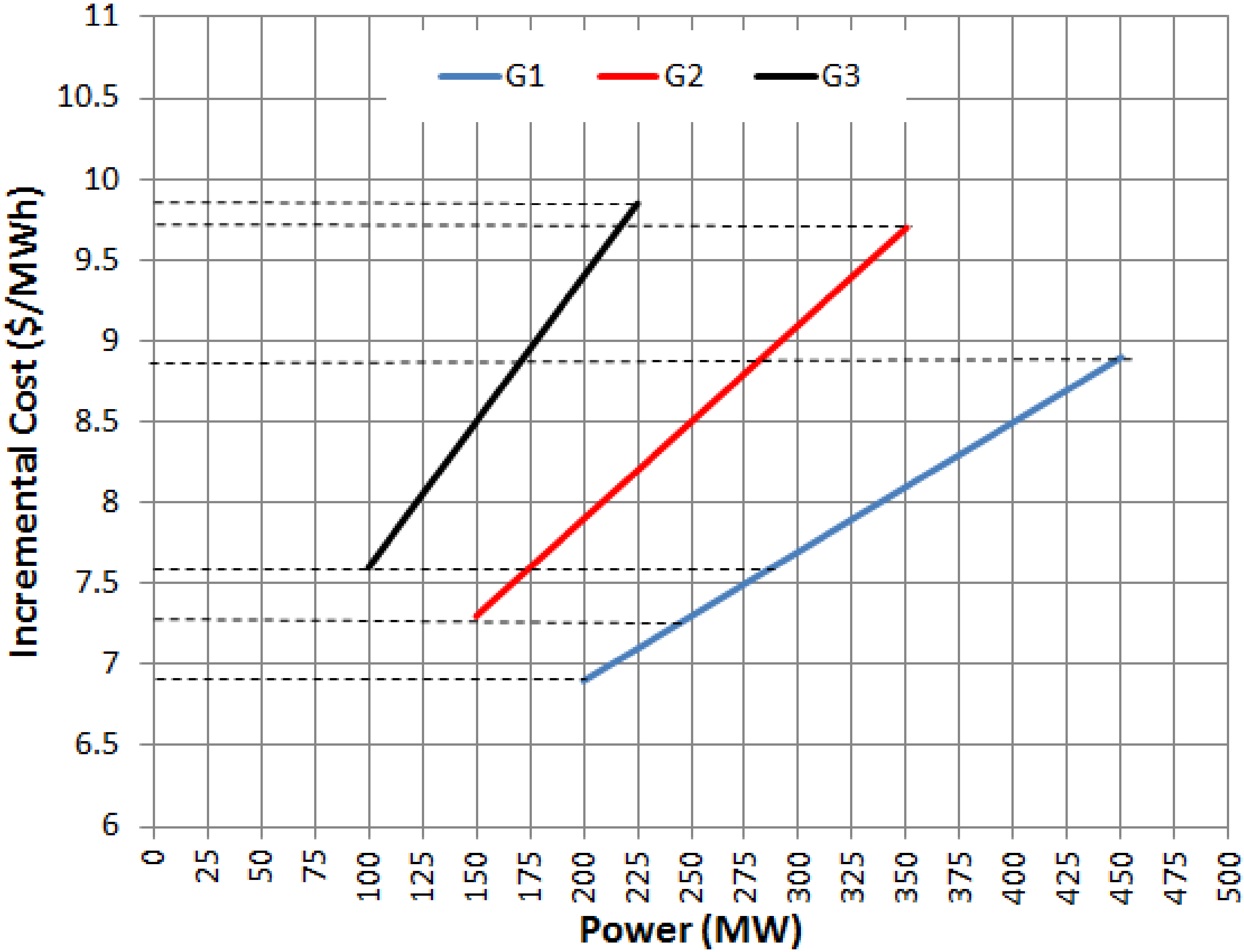
$$P_{Demand} = 975 = 450 + 315 + 210$$

Upper limit violated:
→ P1= 450 MW

→ solve the dispatch problem with two generators:
P2 + P3 = 525 MW

→ λ = \$9.4/MWh
→ P2 = 315 MW
→ P3 = 210 MW

# Example


G1
G2
G3
Incremental Cost ($/MWh)
11
10.5
10
9.5
9
8.5
8
7.5
7
6.5
6
0
25
50
75
100
125
150
175
200
225
250
275
300
325
350
375
400
425
450
475
500
Power (MW)

# Economic Dispatch including Losses

- For large interconnected system where power is transmitted over long distances with low load density areas
  - transmission line losses are a major factor
  - losses affect the optimum dispatch of generation
- One common practice for including the effect of transmission losses is to express the total transmission loss as a quadratic function of the generator power outputs
  - simplest form: $$P_L = \sum_{i=1}^{n_{gen}} \sum_{j=1}^{n_{gen}} P_i B_{ij} P_j$$
  - Kron's loss formula: $$P_L = \sum_{i=1}^{n_{gen}} \sum_{j=1}^{n_{gen}} P_i B_{ij} P_j + \sum_{j=1}^{n_{gen}} B_{0j} P_j + B_{00}$$

# Economic Dispatch including Losses

- $B_{ij}$ **are called the loss coefficients**
  - they are assumed to be constant
  - reasonable accuracy is expected when actual operating conditions are close to the base case conditions used to compute the coefficients
- **The economic dispatch problem is to minimize the overall generation cost,** $C$**, which is a function of plant output**
- **Constraints:**
  - the generation equals the total load demand plus transmission losses
  - each plant output is within the upper and lower generation limits - inequality constraints

# Economic Dispatch including Losses

$$f: \quad C_{total} = \sum_{i=1}^{n_{gen}} C_i = \sum_{i=1}^{n_{gen}} \alpha_i + \beta_i P_i + \gamma_i P_i^2$$

$$g: \quad \sum_{i=1}^{n_{gen}} P_i = P_{demand} + P_{losses}$$

$$u: \quad P_{i(\min)} \le P_i \le P_{i(\max)} \qquad i = 1, \cdots, n_{gen}$$

**The resulting optimization equation**

$$L = C_{total} + \lambda \left( P_{demand} + P_{losses} - \sum_{i=1}^{n_{gen}} P_i \right) + \sum_{i=1}^{n_{gen}} \mu_{i(\max)} \left( P_{i(\max)} - P_i \right)$$

$$+ \sum_{i=1}^{n_{gen}} \mu_{i(\min)} \left( P_i - P_{i(\min)} \right)$$

$$P_i < P_{i(\max)}: \quad \mu_{i(\max)} = 0 \qquad\qquad P_i > P_{i(\min)}: \quad \mu_{i(\min)} = 0$$

# Economic Dispatch including Losses

- **When generator limits are not violated:**

$$\frac{\partial L}{\partial P_i} = 0 = \frac{\partial C_{total}}{\partial P_i} + \lambda\left(0 + \frac{\partial P_L}{\partial P_i} - 1\right)$$

$$\frac{\partial C_{total}}{\partial P_i} = \frac{\partial}{\partial P_i}\left(C_1 + C_2 + \cdots + C_{n_{gen}}\right) = \frac{dC_i}{dP_i}$$

$$\therefore \quad \lambda = \frac{dC_i}{dP_i} + \lambda\frac{\partial P_L}{\partial P_i} = \left(\frac{1}{1 - \partial P_L / \partial P_i}\right)\frac{dC_i}{dP_i} = L_i \frac{dC_i}{dP_i}$$

- The effect of transmission losses introduces a penalty factor that depends on the location of the plant
- The minimum cost is obtained when the incremental cost of each plant multiplied by its penalty factor is the same for all plants

# Example

- **Find the optimal dispatch and the total cost in $/hr**
  - fuel costs and plant output limits

$$C_1 = 200 + 7.0P_1 + 0.008P_1^2 \ [\$/\mathrm{hr}] \quad 10 \le P_1 \le 85 \ \mathrm{MW}$$

$$C_2 = 180 + 6.3P_2 + 0.009P_2^2 \quad 10 \le P_2 \le 80$$

$$C_3 = 140 + 6.8P_3 + 0.007P_3^2 \quad 10 \le P_3 \le 70$$

  - real power loss and total load demand

$$P_{loss} = 0.000218\,P_1^2 + 0.000228\,P_2^2 + 0.000179\,P_3^2$$

$$P_{Demand} = 150 \ \mathrm{MW}$$

# Example

$$\left(\frac{1}{1-.000436P_1}\right)(7+.016P_1) = \left(\frac{1}{1-.000456P_2}\right)(6.3+.018P_2)$$

$$= \left(\frac{1}{1-.000358P_3}\right)(6.8+.014P_3),$$

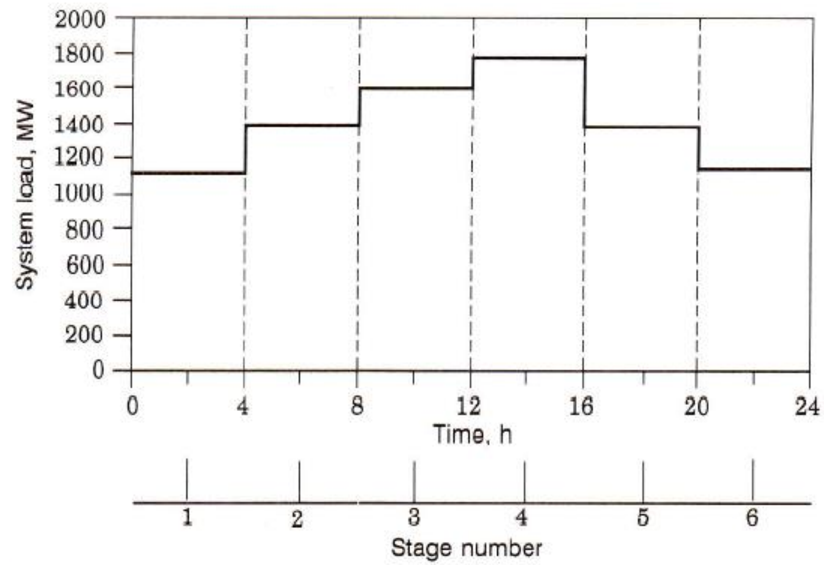
$$P_1 + P_2 + P_3 - 150 = P_{loss}$$

Results (obtained numerically):

- $P_1$ = 35.1 MW
- $P_2$ = 64.1 MW
- $P_3$ = 52.5 MW
- $P_{loss}$ = 1.7 MW
- $P_{demand}$ = 150 MW

# Unit Commitment

- Given a utility with k generators and an hourly load profile: determine which generators to commit, along with the start-up and shut-down times of each generator.
- Example:
  - 24 hr load profile (6 intervals with 4 hours/interval)
  - 4 units ($2^4 - 1 = 15$ possible combinations).
  - Number of inter-stage transitions: $15^6 = 11.4 \times 10^6$




Combinations

| Unit | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | $x_7$ | $x_8$ | $x_9$ | $x_{10}$ | $x_{11}$ | $x_{12}$ | $x_{13}$ | $x_{14}$ | $x_{15}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 0 |
| 2 | 1 | 1 | 1 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 0 |
| 3 | 1 | 1 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 1 | 0 |
| 4 | 1 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |

# Unit Commitment (UC)

- In reality the size of the problem is significantly reduced due to the following:
  - Some unit combinations are not feasible (i.e., do not meet the demand)
  - Practical operating requirements (e.g., some units must run at all time).
  - During each stage, only a subset of combinations is used for the next stage (i.e., those resulting in lower cost).

- The UC problem consists of mixed variable (discrete and continuous). Dynamic programming (using backward and forward sweep) is generally used to solve the UC problem.