

**EE 340**  
**Economic Dispatch**

Spring 2013

# Operating Costs

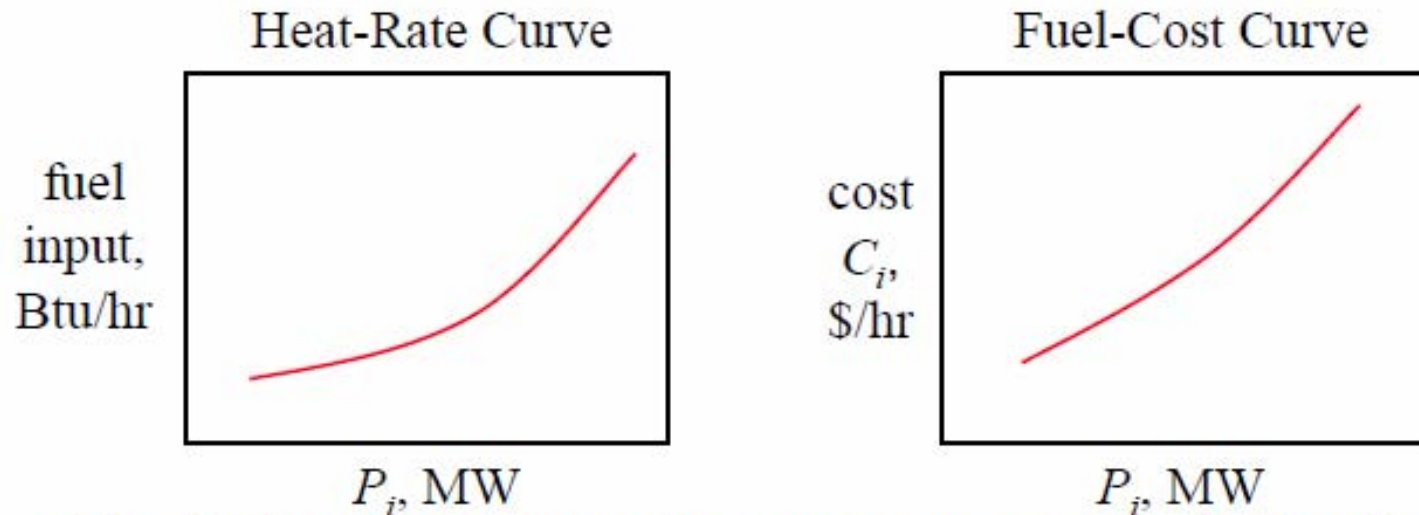
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- **Factors influencing the minimum cost of power generation**
  - ◆ operating efficiency of prime mover and generator
  - ◆ fuel costs
  - ◆ transmission losses
- **The most efficient generator in the system does not guarantee minimum costs**
  - ◆ may be located in an area with high fuel costs
  - ◆ may be located far from the load centers and transmission losses are high
- **The problem is to determine generation at different plants to minimize the total operating costs**

# Operating Costs

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- Generator heat rate curves lead to the fuel cost curves



- ◆ The fuel cost is commonly express as a quadratic function

$$C_i = \alpha_i + \beta_i P_i + \gamma_i P_i^2$$

- ◆ The derivative is known as the incremental fuel cost

$$\frac{dC_i}{dP_i} = \beta_i + 2\gamma_i P_i$$

# Economic Dispatch

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- **The simplest problem is when system losses and generator limits are neglected**
  - ◆ minimize the objective or cost function over all plants
  - ◆ a quadratic cost function is used for each plant

$$C_{total} = \sum_{i=1}^{n_{gen}} C_i = \sum_{i=1}^{n_{gen}} \alpha_i + \beta_i P_i + \gamma_i P_i^2$$

- ◆ the total demand is equal to the sum of the generators' output; the equality constraint

$$\sum_{i=1}^{n_{gen}} P_i = P_{Demand}$$

# Economic Dispatch

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- A typical approach using the Lagrange multipliers

$$L = C_{total} + \lambda \left( P_{Demand} - \sum_{i=1}^{n_{gen}} P_i \right)$$

$$\frac{\partial L}{\partial P_i} = \frac{\partial C_{total}}{\partial P_i} + \lambda(0 - 1) = 0 \quad \rightarrow \quad \frac{\partial C_{total}}{\partial P_i} = \lambda$$

$$C_{total} = \sum_{i=1}^{n_{gen}} C_i \quad \rightarrow \quad \frac{\partial C_{total}}{\partial P_i} = \frac{dC_i}{dP_i} = \lambda \quad \forall i = 1, \dots, n_g$$

$$\lambda = \frac{dC_i}{dP_i} = \beta_i + 2\gamma_i P_i$$

# Economic Dispatch

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- ◆ the second condition for optimal dispatch

$$\frac{dL}{d\lambda} = \left( P_{Demand} - \sum_{i=1}^{n_{gen}} P_i \right) = 0 \quad \rightarrow \quad \sum_{i=1}^{n_{gen}} P_i = P_{Demand}$$

- ◆ rearranging and combining the equations to solve for  $\lambda$

$$P_i = \frac{\lambda - \beta_i}{2\gamma_i}$$

$$\sum_{i=1}^{n_{gen}} \frac{\lambda - \beta_i}{2\gamma_i} = P_{Demand} \quad \lambda = \frac{P_{Demand} + \sum_{i=1}^{n_{gen}} \frac{\beta_i}{2\gamma_i}}{\sum_{i=1}^{n_{gen}} \frac{1}{2\gamma_i}}$$

## Example

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- Neglecting system losses and generator limits, find the optimal dispatch and the total cost in \$/hr for the three generators and the given load demand

$$C_1 = 500 + 5.3P_1 + 0.004P_1^2 \text{ [\$ / } MW\text{hr]}$$

$$C_2 = 400 + 5.5P_2 + 0.006P_2^2$$

$$C_3 = 200 + 5.8P_3 + 0.009P_3^2$$

$$P_{Demand} = 800MW$$

# Example

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$$\lambda = \frac{P_{Demand} + \sum_{i=1}^{n_{gen}} \frac{\beta_i}{2\gamma_i}}{\sum_{i=1}^{n_{gen}} \frac{1}{2\gamma_i}} = \frac{800 + \frac{5.3}{0.008} + \frac{5.5}{0.012} + \frac{5.8}{0.018}}{\frac{1}{0.008} + \frac{1}{0.012} + \frac{1}{0.018}} = \$8.5 / \text{MWhr}$$

$$P_1 = \frac{8.5 - 5.3}{2(0.004)} = 400 \text{ MW}$$

$$P_i = \frac{\lambda - \beta_i}{2\gamma_i} \Rightarrow P_2 = \frac{8.5 - 5.5}{2(0.006)} = 250 \text{ MW}$$

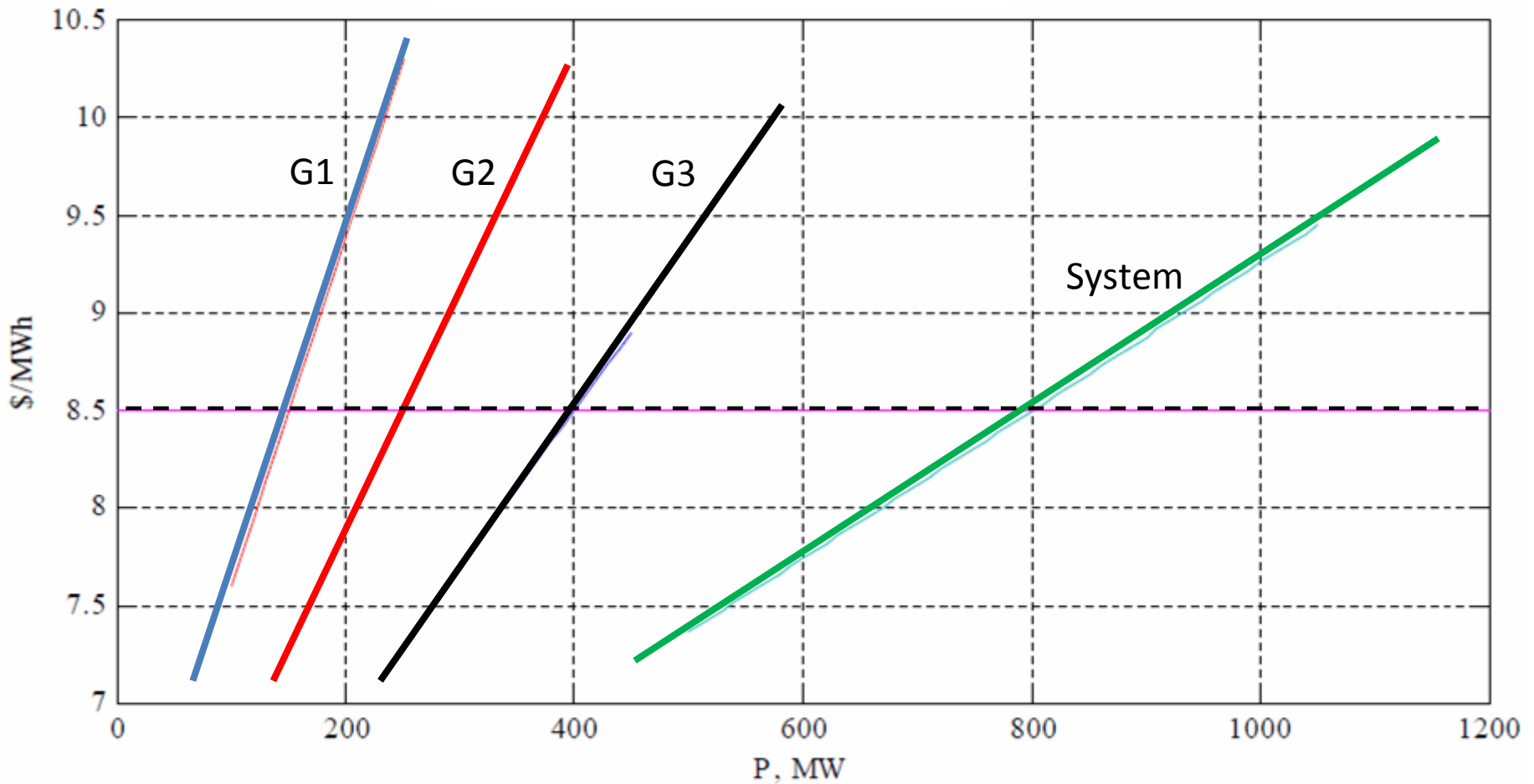
$$P_3 = \frac{8.5 - 5.8}{2(0.009)} = 150 \text{ MW}$$

$$P_{Demand} = 800 \text{ MW} = 400 + 250 + 150 \text{ MW}$$



# Example

Incremental cost curves



# Economic Dispatch with Generator Limits

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- **The power output of any generator should not exceed its rating nor be below the value for stable boiler operation**
  - ◆ Generators have a minimum and maximum real power output limits
- **The problem is to find the real power generation for each plant such that cost are minimized, subject to:**
  - ◆ Meeting load demand - equality constraints
  - ◆ Constrained by the generator limits - inequality constraints

- **The Kuhn-Tucker conditions**

$$dC_i/dP_i = \lambda \quad \leftarrow \quad P_{i(\min)} < P_i < P_{i(\max)}$$

$$dC_i/dP_i \leq \lambda \quad \leftarrow \quad P_i = P_{i(\max)}$$

$$dC_i/dP_i \geq \lambda \quad \leftarrow \quad P_i = P_{i(\min)}$$

## Example

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- Neglecting system losses, find the optimal dispatch and the total cost in \$/hr for the three generators and the given load demand and generation limits

$$C_1 = 500 + 5.3P_1 + 0.004P_1^2 \quad [\$ / \text{MWhr}]$$

$$C_2 = 400 + 5.5P_2 + 0.006P_2^2$$

$$C_3 = 200 + 5.8P_3 + 0.009P_3^2$$

$$200 \leq P_1 \leq 450$$

$$150 \leq P_2 \leq 350$$

$$100 \leq P_3 \leq 225$$

$$P_{\text{Demand}} = 975 \text{ MW}$$

# Example

$$\lambda = \frac{P_{Demand} + \sum_{i=1}^{n_{gen}} \frac{\beta_i}{2\gamma_i}}{\sum_{i=1}^{n_{gen}} \frac{1}{2\gamma_i}} = \frac{975 + \frac{5.3}{0.008} + \frac{5.5}{0.012} + \frac{5.8}{0.018}}{\frac{1}{0.008} + \frac{1}{0.012} + \frac{1}{0.018}} = \$9.163/\text{MWh}$$

$$P_1 = \frac{9.16 - 5.3}{2(0.004)} = 483 \text{ MW}$$

Upper limit violated:  
→ P1= 450 MW

$$P_i = \frac{\lambda - \beta_i}{2\gamma_i}$$

$$P_2 = \frac{9.16 - 5.5}{2(0.006)} = 305 \text{ MW}$$

→ solve the dispatch problem with two generators:

$$P_3 = \frac{9.16 - 5.8}{2(0.009)} = 187 \text{ MW}$$

P2 + P3 = 525 MW

→ λ = \$9.4/MWh

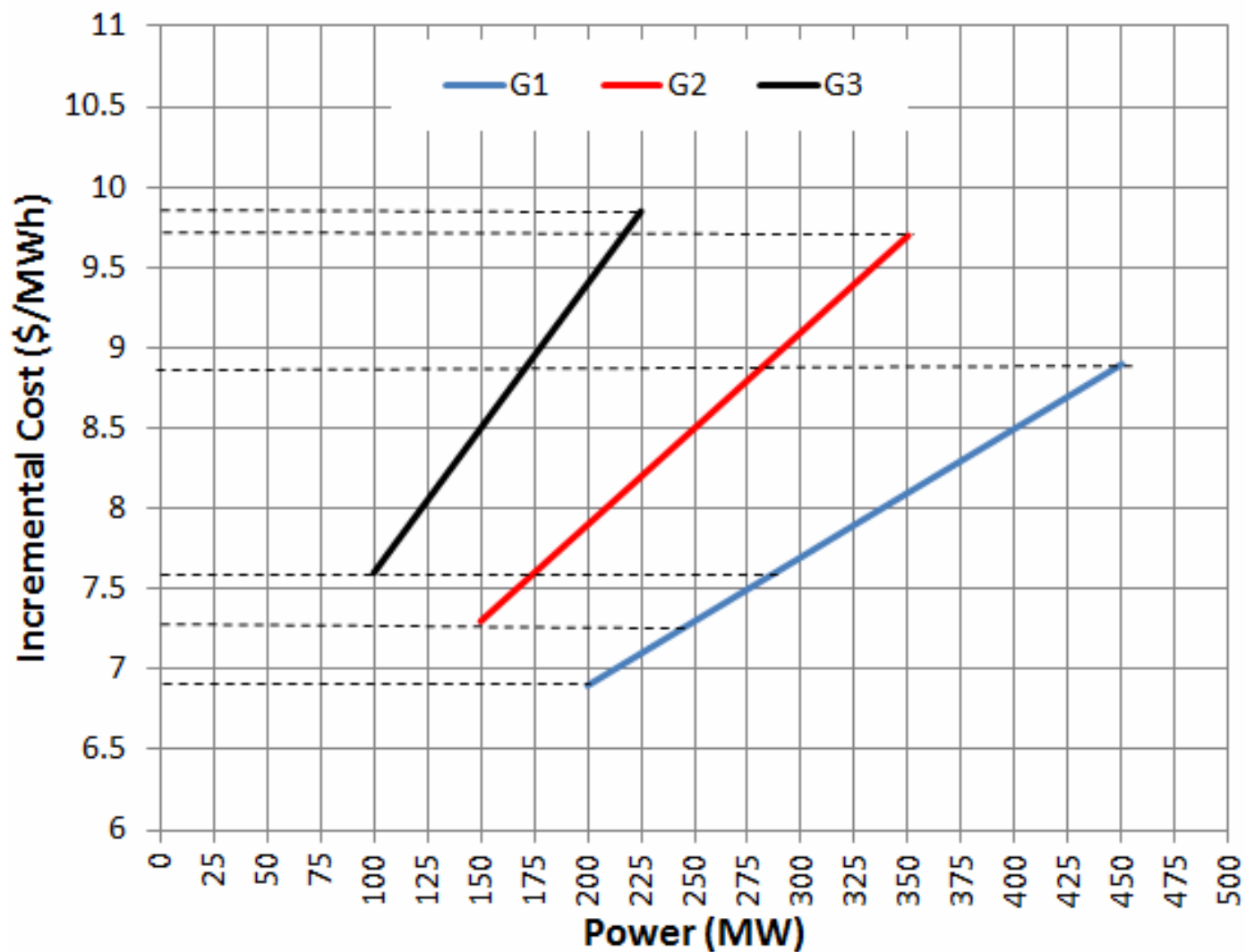
→ P2 = 315 MW

→ P3 = 210 MW

$$P_{Demand} = 975 = 450 + 315 + 210$$

# Example

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# Economic Dispatch including Losses

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- For large interconnected system where power is transmitted over long distances with low load density areas
  - ◆ transmission line losses are a major factor
  - ◆ losses affect the optimum dispatch of generation
- One common practice for including the effect of transmission losses is to express the total transmission loss as a quadratic function of the generator power outputs

- ◆ simplest form: 
$$P_L = \sum_{i=1}^{n_{gen}} \sum_{j=1}^{n_{gen}} P_i B_{ij} P_j$$

- ◆ Kron's loss formula: 
$$P_L = \sum_{i=1}^{n_{gen}} \sum_{j=1}^{n_{gen}} P_i B_{ij} P_j + \sum_{j=1}^{n_{gen}} B_{0j} P_j + B_{00}$$

# Economic Dispatch including Losses

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- $B_{ij}$  are called the loss coefficients
  - ◆ they are assumed to be constant
  - ◆ reasonable accuracy is expected when actual operating conditions are close to the base case conditions used to compute the coefficients
- The economic dispatch problem is to minimize the overall generation cost,  $C$ , which is a function of plant output
- Constraints:
  - ◆ the generation equals the total load demand plus transmission losses
  - ◆ each plant output is within the upper and lower generation limits - inequality constraints

# Economic Dispatch including Losses

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$$f: C_{total} = \sum_{i=1}^{n_{gen}} C_i = \sum_{i=1}^{n_{gen}} \alpha_i + \beta_i P_i + \gamma_i P_i^2$$

$$g: \sum_{i=1}^{n_{gen}} P_i = P_{demand} + P_{losses}$$

$$u: P_{i(\min)} \leq P_i \leq P_{i(\max)} \quad i = 1, \dots, n_{gen}$$

The resulting optimization equation

$$L = C_{total} + \lambda \left( P_{demand} + P_{losses} - \sum_{i=1}^{n_{gen}} P_i \right) + \sum_{i=1}^{n_{gen}} \mu_{i(\max)} (P_{i(\max)} - P_i) + \sum_{i=1}^{n_{gen}} \mu_{i(\min)} (P_i - P_{i(\min)})$$

$$P_i < P_{i(\max)}: \mu_{i(\max)} = 0$$

$$P_i > P_{i(\min)}: \mu_{i(\min)} = 0$$



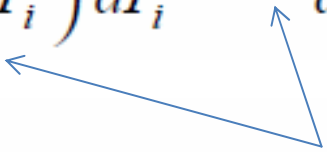
# Economic Dispatch including Losses

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- When generator limits are not violated:

$$\frac{\partial L}{\partial P_i} = 0 = \frac{\partial C_{total}}{\partial P_i} + \lambda \left( 0 + \frac{\partial P_L}{\partial P_i} - 1 \right)$$

$$\frac{\partial C_{total}}{\partial P_i} = \frac{\partial}{\partial P_i} (C_1 + C_2 + \dots + C_{n_{gen}}) = \frac{dC_i}{dP_i}$$

$$\therefore \lambda = \frac{dC_i}{dP_i} + \lambda \frac{\partial P_L}{\partial P_i} = \left( \frac{1}{1 - \partial P_L / \partial P_i} \right) \frac{dC_i}{dP_i} = L_i \frac{dC_i}{dP_i}$$


- ◆ The effect of transmission losses introduces a penalty factor that depends on the location of the plant
- ◆ The minimum cost is obtained when the incremental cost of each plant multiplied by its penalty factor is the same for all plants

## Example

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- Find the optimal dispatch and the total cost in \$/hr
  - ◆ fuel costs and plant output limits

$$C_1 = 200 + 7.0P_1 + 0.008P_1^2 \text{ [\$ / hr]} \quad 10 \leq P_1 \leq 85 \text{ MW}$$

$$C_2 = 180 + 6.3P_2 + 0.009P_2^2 \quad 10 \leq P_2 \leq 80$$

$$C_3 = 140 + 6.8P_3 + 0.007P_3^2 \quad 10 \leq P_3 \leq 70$$

- ◆ real power loss and total load demand

$$P_{loss} = 0.000218 P_1^2 + 0.000228 P_2^2 + 0.000179 P_3^2$$

$$P_{Demand} = 150 \text{ MW}$$

## Example

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$$\left(\frac{1}{1-.000436P_1}\right)(7+.016P_1) = \left(\frac{1}{1-.000456P_2}\right)(6.3+.018P_2)$$

$$= \left(\frac{1}{1-.000358P_3}\right)(6.8+.014P_3),$$

$$P_1 + P_2 + P_3 - 150 = P_{loss}$$

Results (obtained numerically):

- $P_1 = 35.1$  MW
- $P_2 = 64.1$  MW
- $P_3 = 52.5$  MW
- $P_{loss} = 1.7$  MW
- $P_{demand} = 150$  MW