Power Flow Analysis

EE 340

Introduction

- **A power flow study** (**load-flow study**) is a steady-state analysis whose target is to determine the voltages, currents, and real and reactive power flows in a system under a given load conditions.
- The purpose of power flow studies is to plan ahead and account for various hypothetical situations. For example, if a transmission line is be taken off line for maintenance, can the remaining lines in the system handle the required loads without exceeding their rated values.

Power-flow analysis equations

The basic equation for power-flow analysis is derived from the nodal analysis equations for the power system: For example, for a 4-bus system,

$$\begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix}$$

where Y_{ij} are the elements of the bus admittance matrix, V_i are the bus voltages, and I_i are the currents injected at each node. The node equation at bus i can be written as

$$I_i = \sum_{j=1}^{n} Y_{ij} V_j$$

Power-flow analysis equations

Relationship between per-unit real and reactive power supplied to the system at bus i and the per-unit current injected into the system at that bus:

$$S_i = V_i I_i^* = P_i + jQ_i$$

where V_i is the per-unit voltage at the bus; I_i* - complex conjugate of the per-unit current injected at the bus; P_i and Q_i are per-unit real and reactive powers. Therefore,

$$I_i^* = (P_i + jQ_i)/V_i \quad \Rightarrow I_i = (P_i - jQ_i)/V_i^*$$

$$\Rightarrow P_i - jQ_i = V_i^* \sum_{j=1}^{n} Y_{ij} V_j = \sum_{j=1}^{n} Y_{ij} V_j V_i^*$$

Power flow equations

Let $Y_{ij} = |Y_{ij}| \angle \theta_{ij} \quad and \quad V_i = |V_i| \angle \delta_i$

Then $P_i - jQ_i = \sum_{j=1}^{n} |Y_{ij}||V_j||V_i| \angle(\theta_{ij} + \delta_j - \delta_i)$

Hence, $P_i = \sum_{j=1}^{n} |Y_{ij}||V_j||V_i| \cos(\theta_{ij} + \delta_j - \delta_i)$

and $Q_i = -\sum_{j=1}^{n} |Y_{ij}||V_j||V_i| \sin(\theta_{ij} + \delta_j - \delta_i)$

Formulation of power-flow study

- There are 4 variables that are associated with each bus:
  - P,
  - Q,
  - V,
  - δ.
- Meanwhile, there are two power flow equations associated with each bus.
- In a power flow study, two of the four variables are defined an the other two are unknown. That way, we have the same number of equations as the number of unknown.
- The known and unknown variables depend on the type of bus.

Formulation of power-flow study

Each bus in a power system can be classified as one of three types:

1. **Load bus (P-Q bus)** – a buss at which the real and reactive power are specified, and for which the bus voltage will be calculated. All busses having no generators are load busses. In here, **V and δ are unknown**.
2. **Generator bus (P-V bus)** – a bus at which the magnitude of the voltage is defined and is kept constant by adjusting the field current of a synchronous generator. We also assign real power generation for each generator according to the economic dispatch. In here, **Q and δ are unknown**

3. **Slack bus (swing bus)** – a special generator bus serving as the reference bus. Its voltage is assumed to be fixed in both magnitude and phase (for instance, $1\angle 0^{\circ}$ pu). In here, **P and Q are unknown**.

Formulation of power-flow study

- Note that the power flow equations are non-linear, thus cannot be solved analytically. A numerical iterative algorithm is required to solve such equations. A standard procedure follows:
  1. Create a bus admittance matrix Y_{bus} for the power system;
  2. Make an initial estimate for the voltages (both magnitude and phase angle) at each bus in the system;
  3. Substitute in the power flow equations and determine the deviations from the solution.
  4. Update the estimated voltages based on some commonly known numerical algorithms (e.g., Newton-Raphson or Gauss-Seidel).
  5. Repeat the above process until the deviations from the solution are minimal.

Example

Consider a 4-bus power system below. Assume that

- bus 1 is the slack bus and that it has a voltage $V1 = 1.0\angle 0°$ pu.
- The generator at bus 3 is supplying a real power $P3$ = 0.3 pu to the system with a voltage magnitude 1 pu.
- The per-unit real and reactive power loads at busses 2 and 4 are $P2$ = 0.3 pu, $Q2$ = 0.2 pu, $P4$ = 0.2 pu, $Q4$ = 0.15 pu.

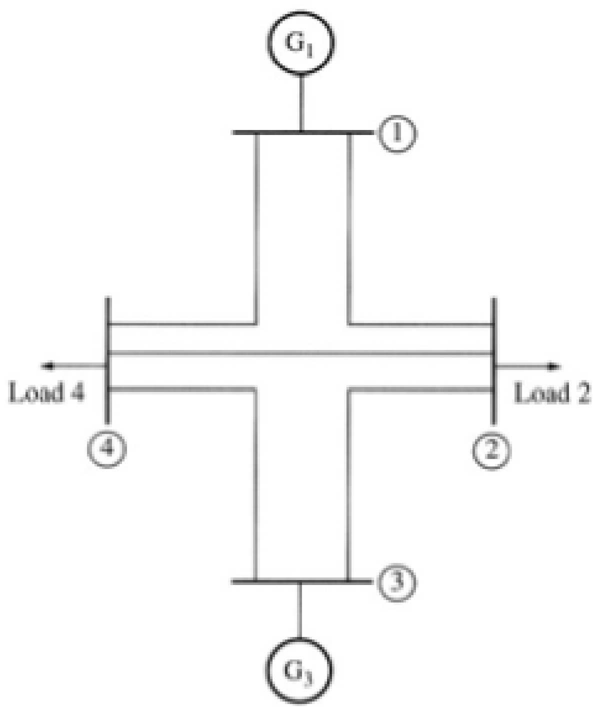

Table of Buses:

Bus 1	Slack bus
Bus 2	Load bus
Bus 3	Generator bus
Bus 4	Load bus

Example (cont.)

- Y-bus matrix (refer to example in book)

$$Y_{bus} = \begin{bmatrix} 1.7647 - j7.0588 & -0.5882 + j2.3529 & 0 & -1.1765 + j4.7059 \\ -0.5882 + j2.3529 & 1.5611 - j6.6290 & -0.3846 + j1.9231 & -0.5882 + j2.3529 \\ 0 & -0.3846 + j1.9231 & 1.5611 - j6.6290 & -1.1765 + j4.7059 \\ -1.1765 + j4.7059 & -0.5882 + j2.3529 & -1.1765 + j4.7059 & 2.9412 - j11.7647 \end{bmatrix}$$

- Power flow solution:

$$V_1 = 1.0\angle 0^\circ \; pu$$
$$V_2 = 0.964\angle -0.97^\circ \; pu$$
$$V_3 = 1.0\angle 1.84^\circ \; pu$$
$$V_4 = 0.98\angle -0.27^\circ \; pu$$

- By knowing the node voltages, the power flow (both active and reactive) in each branch of the circuit can easily be calculated.