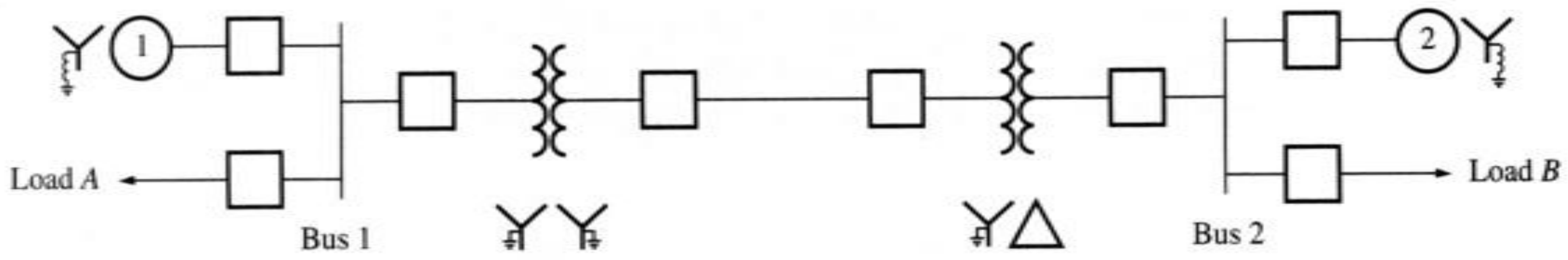
Power System Representation: Voltage-Current Relations, and Power Flow Analysis

EE 340

One-line diagram
(simple power system)



- Typical information included:
  - Device ratings and impedances
  - Load demand in terms of real and reactive powers.

Per-unit equivalent circuit

- Real power systems are convenient to analyze using their **per-phase** (since its is a balanced three-phase) **per-unit** (since there are many transformers) equivalent circuits.
- Recall: given the base apparent power (3—phase) and base voltage (line-to-line), the base current and base impedance are given by

$$I_{base} = \frac{S_{3\phi,base}}{\sqrt{3}V_{LL,base}}$$

$$Z_{base} = \frac{V_{LL,base}}{\sqrt{3}I_{base}} = \frac{\left(V_{LL,base}\right)^2}{S_{3\phi,base}}$$

Per-unit system

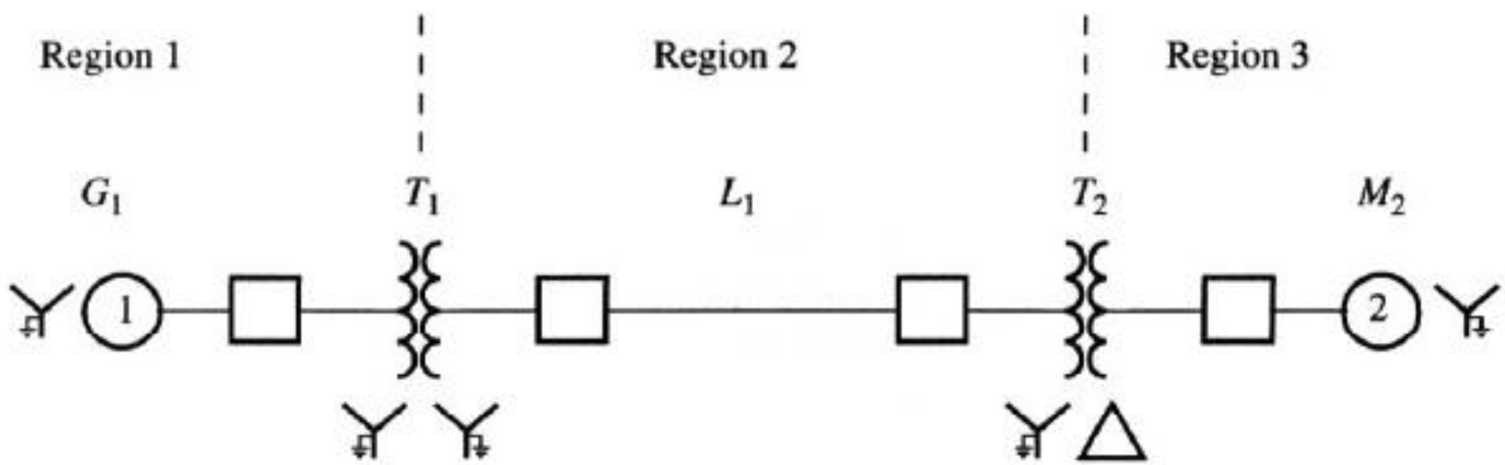
- The base apparent power and base voltage are specified at a point in the circuit, and the other values are calculated from them.
- The base voltage varies by the voltage ratio of each transformer in the circuit but the base apparent power stays the same through the circuit.
- The per-unit impedance may be transformed from one base to another as

$$Per-unit\ Z_{new} = per-unit\ Z_{old}\left(\frac{V_{old}}{V_{new}}\right)^2\left(\frac{S_{new}}{S_{old}}\right)$$

Example

Example 10.2: a power system consists of one synchronous generator and one synchronous motor connected by two transformers and a transmission line. Create a per-phase, per-unit equivalent circuit of this power system using a base apparent power of 100 MVA and a base line voltage of the generator G_1 of 13.8 kV. Given that:

G_1 ratings: 100 MVA, 13.8 kV, R = 0.1 pu, X_s = 0.9 pu;
T_1 ratings: 100 MVA, 13.8/110 kV, R = 0.01 pu, X_s = 0.05 pu;
T_2 ratings: 50 MVA, 120/14.4 kV, R = 0.01 pu, X_s = 0.05 pu;
M ratings: 50 MVA, 13.8 kV, R = 0.1 pu, X_s = 1.1 pu;
L_1 impedance: R = 15 Ω, X = 75 Ω.



$$V_{base,1} = 13.8\,kV \qquad V_{base,2} = V_{base,1}\frac{110}{13.8} = 110\,kV \qquad V_{base,3} = V_{base,2}\frac{14.4}{120} = 13.2\,kV$$

Example (cont.)

$$Z_{base,1} = \frac{V_{LL,base}^{\;2}}{S_{3\phi,base}} = \frac{(13.8\,kV)^2}{100\ MVA} = 1.904\,\Omega$$

$$Z_{base,2} = \frac{V_{LL,base}^{\;2}}{S_{3\phi,base}} = \frac{(110\,kV)^2}{100\ MVA} = 121\,\Omega$$

$$Z_{base,3} = \frac{V_{LL,base}^{\;2}}{S_{3\phi,base}} = \frac{(13.2\,kV)^2}{100\ MVA} = 1.743\,\Omega$$

$R_{G1,pu}$ = 0.1 per unit
$X_{G1,pu}$ = 0.9 per unit
$R_{T1,pu}$ = 0.01 per unit
$X_{T1,pu}$ = 0.05 per unit

$$R_{line,system} = \frac{15}{121} = 0.124\ per\ unit$$

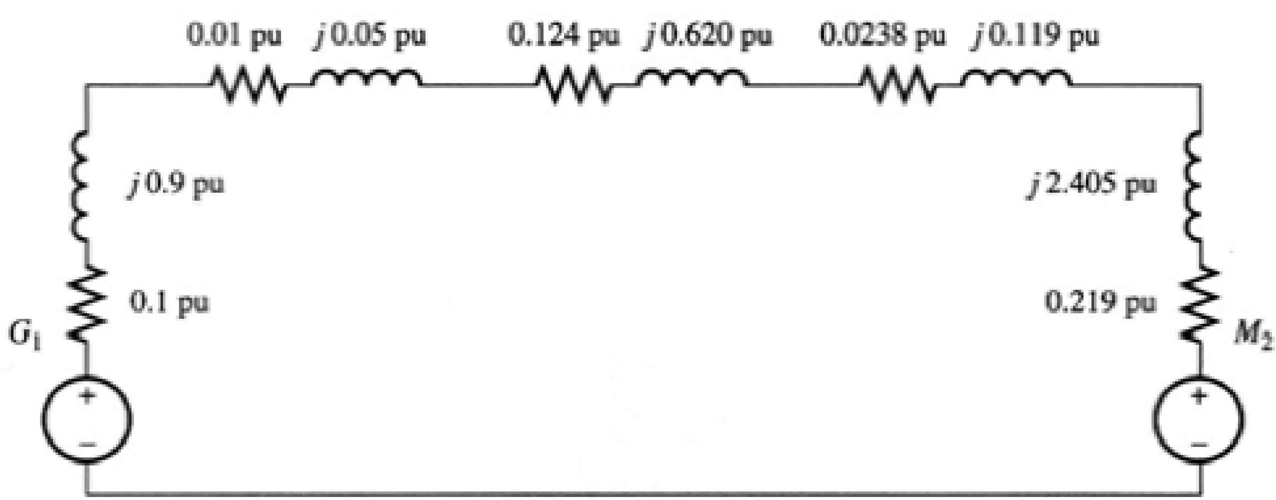
$$X_{line,system} = \frac{75}{121} = 0.620\ per\ unit$$

$$R_{M2,pu} = 0.1(14.8/13.2)^2(100/50) = 0.219\ per\ unit$$

$$X_{M2,pu} = 1.1(14.8/13.2)^2(100/50) = 2.405\ per\ unit$$

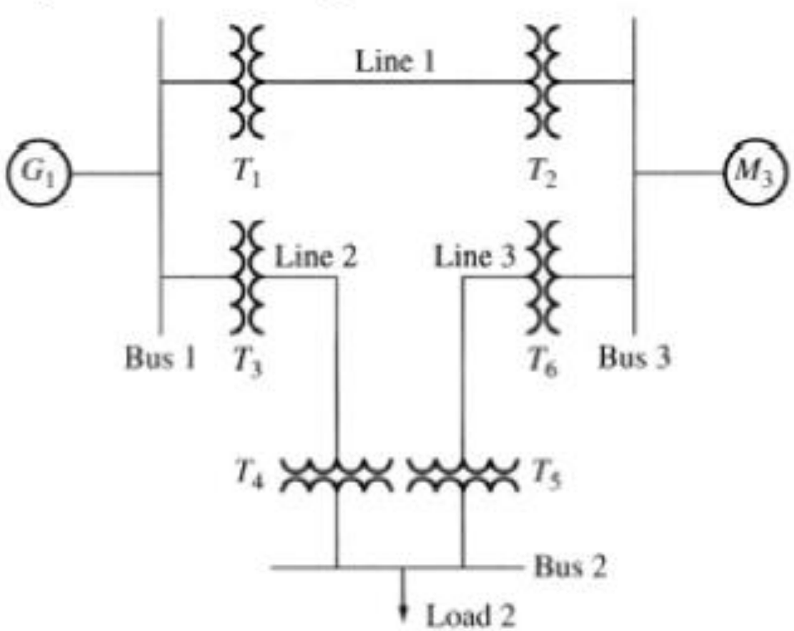
$$R_{T2,pu} = 0.01(14.4/13.2)^2(100/50) = 0.238\ per\ unit$$

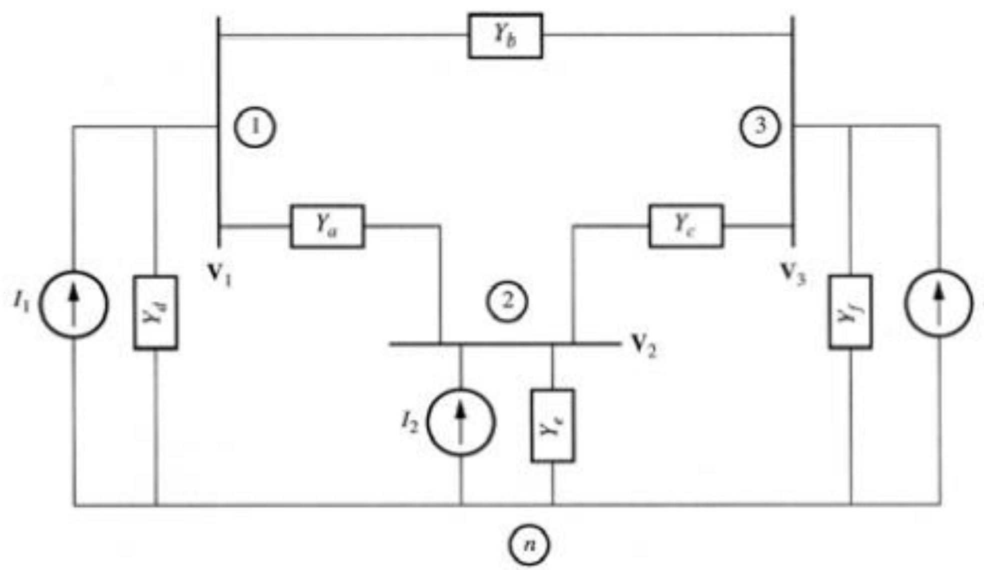
$$X_{T2,pu} = 0.05(14.4/13.2)^2(100/50) = 0.119\ per\ unit$$



Node equations

- Once the per-unit equivalent circuit is created, it can be used to determine the voltages, currents, and powers at various points .
- The most common technique used to solve such circuits is nodal analysis. To simplify the equations,
  - Replace the generators by their Norton equivalent circuits
  - Replace the impedances by their equivalent admittances
  - Represent the loads by the current they draw (for now)





Node equations

- According to Kirchhoff's current flow law (KCL), **the sum of all currents entering any node equals to the sum of all currents leaving the node**.
- KCL can be used to establish and solve a system of simultaneous equations with the unknown node voltages.
- Assuming that the current from the current sources are entering each node, and that all other currents are leaving the node, applying the KCL to the 3 nodes yields

$$(V_1 - V_2)Y_a + (V_1 - V_3)Y_b + V_1 Y_d = I_1$$
$$(V_2 - V_1)Y_a + (V_2 - V_3)Y_c + V_2 Y_e = I_2$$
$$(V_3 - V_1)Y_b + (V_3 - V_2)Y_c + V_3 Y_f = I_3$$

Node equations – the $\mathbf{Y_{bus}}$ matrix

- In matrix from,

$$\begin{bmatrix} Y_a+Y_b+Y_d & -Y_a & -Y_b \\ -Y_a & Y_a+Y_c+Y_e & -Y_c \\ -Y_b & -Y_c & Y_b+Y_c+Y_f \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

Which is an equation of the form:

$$Y_{bus}V = I$$

where Y_{bus} is the bus admittance matrix of a system, which has the form:

$$Y_{bus} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix}$$

Y_{bus} has a regular form that is easy to calculate:

1) The diagonal elements Y_{ii} equal the sum of all admittances connected to node *i*.
2) Other elements Y_{ij} equal to the negative admittances connected to nodes *I* and *j*.

The diagonal elements of Y_{bus} are called the self-admittance or driving-point admittances of the nodes; the off-diagonal elements are called the mutual admittances or transfer admittances of the nodes.

Y_{bus} and Z_{bus} matrices of a power network

Inverting the bus admittance matrix Y_{bus} yields the bus impedance matrix:

$$Z_{bus} = Y_{bus}^{-1}$$

Simple technique for constructing Y_{bus} is only applicable for components that are not mutually coupled. The technique applicable to mutually coupled components can be found elsewhere.

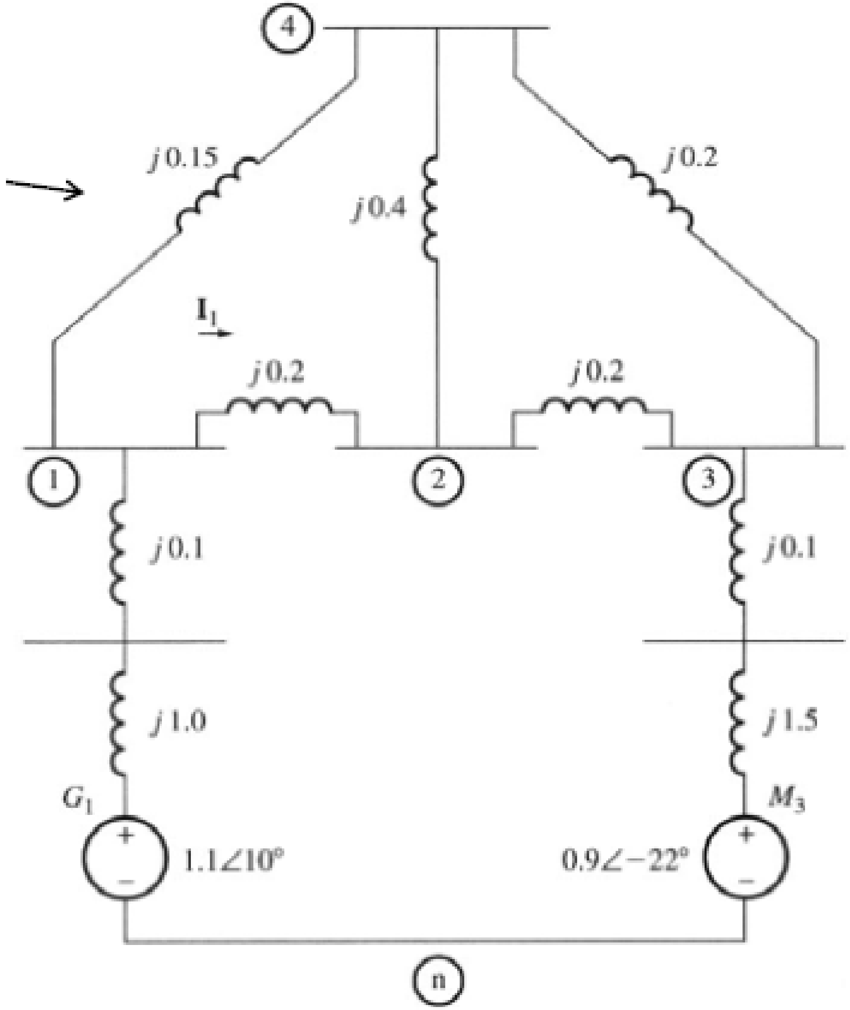
Once Y_{bus} is calculated, the solution

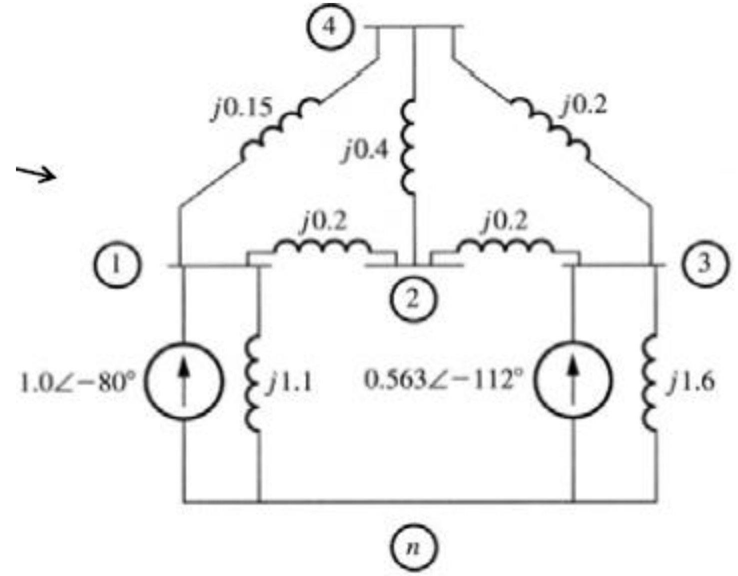
$$V = Y_{bus}^{-1} I$$

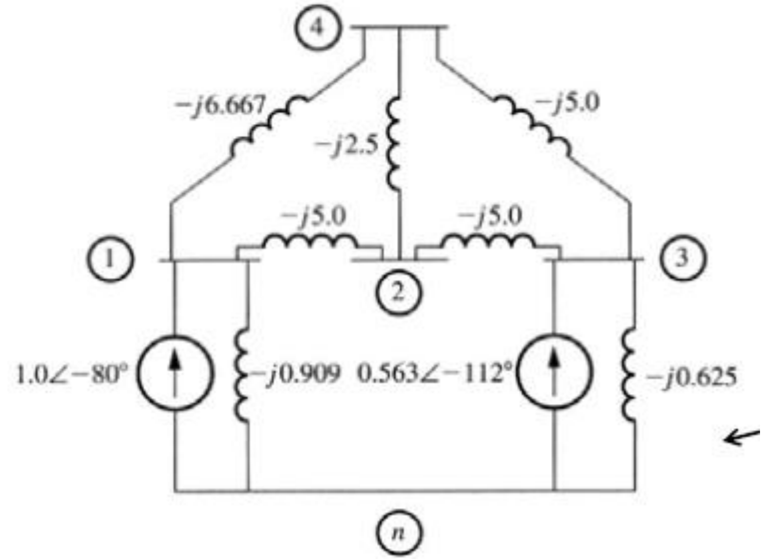
or

$$V = Z_{bus} I$$

Example


4
j 0.15
j 0.4
j 0.2
I₁
j 0.2
j 0.2
1
2
3
j 0.1
j 0.1
j 1.0
j 1.5
G₁
M₃
1.1∠10°
0.9∠−22°
n


4
j0.15
j0.4
j0.2
j0.2
j0.2
1
2
3
1.0∠−80°
j1.1
0.563∠−112°
j1.6
n


4
−j6.667
−j2.5
−j5.0
−j5.0
−j5.0
1
2
3
1.0∠−80°
−j0.909
0.563∠−112°
−j0.625
n

Example (cont.)

The resulting admittance matrix is:

$$Y_{bus} = \begin{bmatrix} -j12.576 & j5.0 & 0 & j6.667 \\ j5.0 & -j12.5 & j5.0 & j2.5 \\ 0 & j5.0 & -10.625 & j5.0 \\ j6.667 & j2.5 & j5.0 & -j14.167 \end{bmatrix}$$

The current vector for this circuit is:

$$I = \begin{bmatrix} 1.0\angle -80^\circ \\ 0 \\ 0.563\angle -112^\circ \\ 0 \end{bmatrix}$$

The solution to the system of equations will be

$$V = Y_{bus}^{-1} I = \begin{bmatrix} 0.989\angle -0.60^\circ \\ 0.981\angle -1.58^\circ \\ 0.974\angle -2.62^\circ \\ 0.982\angle -1.48^\circ \end{bmatrix} V$$

Problems

- 10.3
- 10.5
- 10.7, 10.8, 10.9

Power-flow analysis equations

The basic equation for power-flow analysis is derived from the nodal analysis equations for the power system: For example, for a 4-bus system,

$$\begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix}$$

where Y_{ij} are the elements of the bus admittance matrix, V_i are the bus voltages, and I_i are the currents injected at each node. The node equation at bus i can be written as

$$I_i = \sum_{j=1}^{n} Y_{ij} V_j$$

Power-flow analysis equations

Relationship between per-unit real and reactive power supplied to the system at bus i and the per-unit current injected into the system at that bus:

$$S_i = V_i I_i^* = P_i + jQ_i$$

where V_i is the per-unit voltage at the bus; I_i* - complex conjugate of the per-unit current injected at the bus; P_i and Q_i are per-unit real and reactive powers. Therefore,

$$I_i^* = (P_i + jQ_i)/V_i \quad \Rightarrow I_i = (P_i - jQ_i)/V_i^*$$

$$\Rightarrow P_i - jQ_i = V_i^* \sum_{j=1}^{n} Y_{ij} V_j = \sum_{j=1}^{n} Y_{ij} V_j V_i^*$$

Power flow equations

Let $$Y_{ij} = |Y_{ij}| \angle \theta_{ij} \quad and \quad V_i = |V_i| \angle \delta_i$$

Then $$P_i - jQ_i = \sum_{j=1}^{n} |Y_{ij}||V_j||V_i| \angle (\theta_{ij} + \delta_j - \delta_i)$$

Hence, $$\boxed{P_i = \sum_{j=1}^{n} |Y_{ij}||V_j||V_i| \cos(\theta_{ij} + \delta_j - \delta_i)}$$

and $$\boxed{Q_i = -\sum_{j=1}^{n} |Y_{ij}||V_j||V_i| \sin(\theta_{ij} + \delta_j - \delta_i)}$$

Formulation of power-flow study

- There are 4 variables that are associated with each bus:
  - P,
  - Q,
  - V,
  - δ.
- Meanwhile, there are two power flow equations associated with each bus.
- In a power flow study, two of the four variables are defined an the other two are unknown. That way, we have the same number of equations as the number of unknown.
- The known and unknown variables depend on the type of bus.

Formulation of power-flow study

Each bus in a power system can be classified as one of three types:

1. **Load bus (P-Q bus)** – a buss at which the real and reactive power are specified, and for which the bus voltage will be calculated. All busses having no generators are load busses. In here, **V and δ are unknown**.
2. **Generator bus (P-V bus)** – a bus at which the magnitude of the voltage is defined and is kept constant by adjusting the field current of a synchronous generator. We also assign real power generation for each generator according to the economic dispatch. In here, **Q and δ are unknown**
3. **Slack bus (swing bus)** – a special generator bus serving as the reference bus. Its voltage is assumed to be fixed in both magnitude and phase (for instance, $1\angle 0^{\circ}$ pu). In here, **P and Q are unknown**.

Formulation of power-flow study

- Note that the power flow equations are non-linear, thus cannot be solved analytically. A numerical iterative algorithm is required to solve such equations. A standard procedure follows:
  1. Create a bus admittance matrix Y_{bus} for the power system;
  2. Make an initial estimate for the voltages (both magnitude and phase angle) at each bus in the system;
  3. Substitute in the power flow equations and determine the deviations from the solution.
  4. Update the estimated voltages based on some commonly known numerical algorithms (e.g., Newton-Raphson or Gauss-Seidel).
  5. Repeat the above process until the deviations from the solution are minimal.

Example

Consider a 4-bus power system below. Assume that

- bus 1 is the slack bus and that it has a voltage $V1 = 1.0\angle 0°$ pu.
- The generator at bus 3 is supplying a real power $P3 = 0.3$ pu to the system with a voltage magnitude 1 pu.
- The per-unit real and reactive power loads at busses 2 and 4 are $P2 = 0.3$ pu, $Q2 = 0.2$ pu, $P4 = 0.2$ pu, $Q4 = 0.15$ pu.

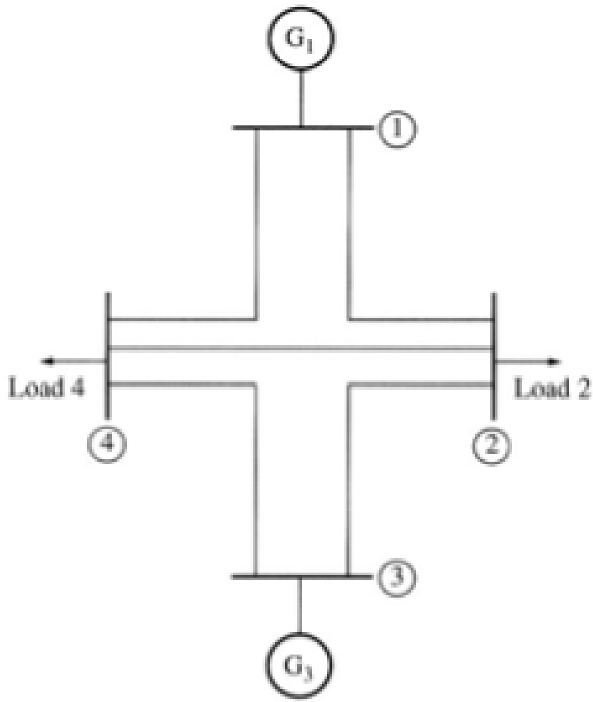


Table of Buses:

Bus 1	Slack bus
Bus 2	Load bus
Bus 3	Generator bus
Bus 4	Load bus

Example (cont.)

- Y-bus matrix (refer to example in book)

$$Y_{bus} = \begin{bmatrix} 1.7647 - j7.0588 & -0.5882 + j2.3529 & 0 & -1.1765 + j4.7059 \\ -0.5882 + j2.3529 & 1.5611 - j6.6290 & -0.3846 + j1.9231 & -0.5882 + j2.3529 \\ 0 & -0.3846 + j1.9231 & 1.5611 - j6.6290 & -1.1765 + j4.7059 \\ -1.1765 + j4.7059 & -0.5882 + j2.3529 & -1.1765 + j4.7059 & 2.9412 - j11.7647 \end{bmatrix}$$

- Power flow solution:

$$V_1 = 1.0\angle 0^\circ \ pu$$
$$V_2 = 0.964\angle -0.97^\circ \ pu$$
$$V_3 = 1.0\angle 1.84^\circ \ pu$$
$$V_4 = 0.98\angle -0.27^\circ \ pu$$

- By knowing the node voltages, the power flow (both active and reactive) in each branch of the circuit can easily be calculated.