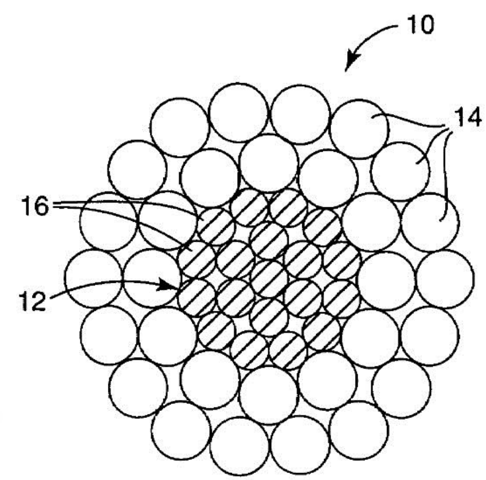
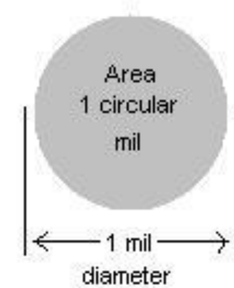
EE 340
Power Transmission Lines

Physical Characteristics – Overhead lines

- An overhead transmission line usually consists of three conductors or bundles of conductors containing the three phases of the power system.
- In overhead transmission lines, the bare conductors are suspended from a pole or a tower via insulators. The conductors are usually made of aluminum cable steel reinforced (ACSR).



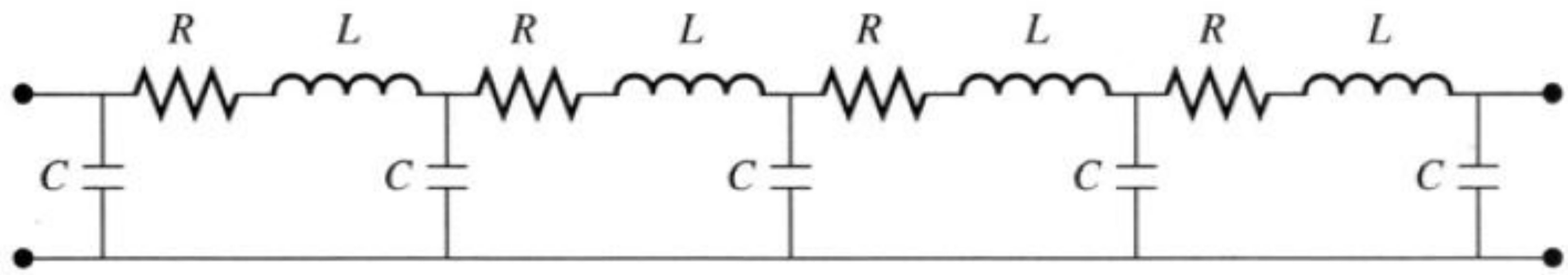


Physical Characteristics – underground cables

- Cable lines are designed to be placed underground or under water. The conductors are insulated from one another and surrounded by protective sheath.
- Cable lines are more expensive and harder to maintain. They also have capacitance problem – not suitable for long distance.

Electrical Characteristics

- Transmission lines are characterized by a **series resistance**, **inductance**, and **shunt capacitance** per unit length.
- These values determine the power-carrying capacity of the transmission line and the voltage drop across it at full load.



- The DC resistance of a conductor is expressed in terms of resistively, length and cross sectional area as follows:

$$R_{DC} = \frac{\rho l}{A}$$

Cable resistance

- The resistively increases linearly with temperature over normal range of temperatures.
- If the resistively at one temperature and material temperature constant are known, the resistively at another temperature can be found by

$$\rho_{T2} = \frac{M + T_2}{M + T_1} \rho_{T1}$$

Material	Resistivity at 20°C [Ω·m]	Temperature constant [°C]
Annealed copper	$1.72 \cdot 10^{-8}$	234.5
Hard-drawn copper	$1.77 \cdot 10^{-8}$	241.5
Aluminum	$2.83 \cdot 10^{-8}$	228.1
Iron	$10.00 \cdot 10^{-8}$	180.0
Silver	$1.59 \cdot 10^{-8}$	243.0

Cable Resistance

- AC resistance of a conductor is always higher than its DC resistance due to the skin effect forcing more current flow near the outer surface of the conductor. The higher the frequency of current, the more noticeable skin effect would be.
- Wire manufacturers usually supply tables of resistance per unit length at common frequencies (50 and 60 Hz). Therefore, the resistance can be determined from such tables.

Aluminum Conductor Steel Reinforced

Electrical Properties

	SIZE & STRANDING		RESISTANCE				60 HZ REACTANCE 1 FOOT EQUIVALENT SPACING		
			DC	AC-60-HZ(Ohms/1000 Ft.)				Inductive (Ohms/1000 Ft.)	
CODE WORD	AWG or kcmil	Aluminum/ Steel	(Ohms/1000 Ft.) @20°	@25° C	@50° C	@75° C	Capacitive (Megohms-1000 Ft.)	@25° C	
								Inductive (Ohms/1000 Ft.)	GMR (Ft.)
WAXWING	266.8	18/1	0.0644	0.0657	0.0723	0.0788	0.576	0.0934	0.0197
PARTRIDGE	266.8	26/7	0.0637	0.0652	0.0714	0.0778	0.565	0.0881	0.0217
MERLIN	336.4	18/1	0.0510	0.0523	0.0574	0.0625	0.560	0.0877	0.0221
LINNET	336.4	26/7	0.0506	0.0517	0.0568	0.0619	0.549	0.0854	0.0244
ORIOLE	336.4	30/7	0.0502	0.0513	0.0563	0.0614	0.544	0.0843	0.0255
CHICKADEE	397.5	18/1	0.0432	0.0443	0.0487	0.0528	0.544	0.0856	0.0240
IBIS	397.5	26/7	0.0428	0.0438	0.0481	0.0525	0.539	0.0835	0.0265
LARK	397.5	30/7	0.0425	0.0434	0.0477	0.0519	0.533	0.0824	0.0277
PELICAN	477.0	18/1.	0.0360	0.0369	0.0405	0.0441	0.528	0.0835	0.0263
FLICKER	477.0	24/7	0.0358	0.0367	0.0403	0.0439	0.524	0.0818	0.0283
HAWK	477.0	26/7	0.0357	0.0366	0.0402	0.0438	0.522	0.0814	0.0290
HEN	477.0	30/7	0.0354	0.0362	0.0389	0.0434	0.517	0.0803	0.0304
OSPREY	556.5	18/1	0.0309	0.0318	0.0348	0.0379	0.518	0.0818	0.0284
PARAKEET	556.5	24/7	0.0307	0.0314	0.0347	0.0377	0.512	0.0801	0.0306

Line inductance

- The series inductance of a transmission line consists of two components: internal and external inductances, which are due the magnetic flux inside and outside the conductor respectively.
- The inductance of a transmission line is defined as the number of flux linkages [Wb-turns] produced per ampere of current flowing through the line:

$$L = \frac{\lambda}{I}$$

- The inductance of a single-phase transmission line is given by (see derivation in the book): (*r* : conductor radius - assumed solid, *D*: distance between cables, $\mu = 4\pi \times 10^{-7}$ H/m, $r' = r\, e^{-1/4} = .7788\, r$)

$$l = \frac{\mu}{\pi}\left(\frac{1}{4} + \ln\frac{D}{r}\right) \quad [H/m]$$

$$L = 4x10^{-7}\ln\left(\frac{D}{r'}\right) \text{ H/m}$$

Remarks on line inductance

- **The greater the spacing between the phases of a transmission line, the greater the inductance of the line.**
  - Since the phases of a high-voltage overhead transmission line must be spaced further apart to ensure proper insulation, a high-voltage line will have a higher inductance than a low-voltage line.
  - Since the spacing between lines in buried cables is very small, series inductance of cables is much smaller than the inductance of overhead lines
- **The greater the radius of the conductors in a transmission line, the lower the inductance of the line.** In practical transmission lines, instead of using heavy and inflexible conductors of large radii, two and more conductors are bundled together to approximate a large diameter conductor, and reduce corona loss.

Per-Phase Inductance of 3-phase transmission line

$$L = 2x10^{-7}\ln(\frac{GMD}{GMR}) \text{ H/m}$$

where the Geometric Mean Distance (GMD) is defined by

$$GMD = \sqrt[3]{D_1 D_2 D_3}$$

where D1, D2, and D3 are the distances between the 3 conductors. The Geometric Mean Radius (GMR) is supplied by the manufacturer (takes into account the cable strands). For a solid conductor, GMR = 0.7788 r.

For a 60 Hz system, the reactance of the line is

$$X_L = 0.754\text{x}10^{-4} \ln\left(\frac{GMD}{GMR}\right) \frac{\text{Ohms}}{\text{m}}$$

$$X_L = 0.1213 \ln\left(\frac{GMD}{GMR}\right) \frac{\text{Ohms}}{\text{mi}}$$

Shunt capacitance

- Since a voltage *V* is applied to a pair of conductors separated by a dielectric (air), charges *q* of equal magnitude but opposite sign will accumulate on the conductors. Capacitance C between the two conductors is defined by

$$C = \frac{q}{V}$$

- The capacitance of a single-phase transmission line is given by (see derivation in the book): (ε = 8.85 x 10^{-12} F/m)

$$C = \frac{2\pi\epsilon}{\ln(\frac{D}{r})} \text{ F/m}$$

Capacitance of 3-phase transmission line

- The capacitance per phase is computed by

$$C = \frac{2\pi\epsilon}{\ln(\frac{GMD}{r})} \text{ F/m}$$

- The shunt admittance per phase at 60 Hz is given by

$$y = 2\pi f C = \frac{2.1 \; x10^{-8}}{\ln(\frac{GMD}{r})} \text{ S.m}$$

- The shunt capacitive reactance per phase at 60 Hz is given by

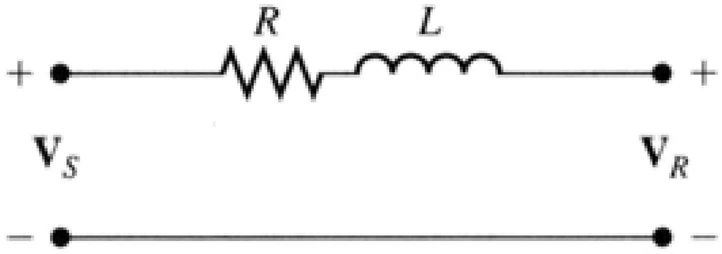
$$X_c = 47.7 \text{ x}10^6 \ln\left(\frac{GMD}{r}\right) \text{ Ohm.m}$$

Remarks on line capacitance

1. **The greater the spacing between the phases of a transmission line, the lower the capacitance of the line**.
   - Since the phases of a high-voltage overhead transmission line must be spaced further apart to ensure proper insulation, a high-voltage line will have a lower capacitance than a low-voltage line.
   - Since the spacing between lines in buried cables is very small, shunt capacitance of cables is much larger than the capacitance of overhead lines.
2. **The greater the radius of the conductors in a transmission line, the higher the capacitance of the line**. Therefore, bundling increases the capacitance.

Short line model

- Overhead transmission lines shorter than 50 miles can be modeled as a series resistance and inductance, since the shunt capacitance can be neglected over short distances.



- The total series resistance and series reactance can be calculated as

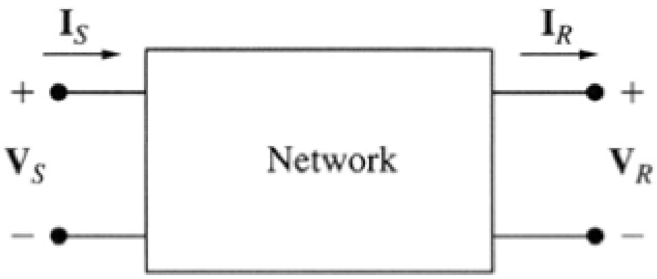
$$R = rd$$

$$X = xd$$

- where *r*, *x* are resistance and reactance per unit length and *d* is the length of the transmission line.

Short line model

- Two-port network model:



$$V_S = AV_R + BI_R$$
$$I_S = CV_R + DI_R$$

$$A = 1$$
$$B = Z$$
$$C = 0$$
$$D = 1$$

$$I_S = I_R$$

$$V_R = V_S - RI - jX_L I$$

- The equation is similar to that of a transformer (w/o shunt impedance)

Short line

Voltage Regulation: $$VR = \frac{V_{nl} - V_{fl}}{V_{fl}} \cdot 100\%$$

1. If unity-PF (resistive) loads are added at the end of a line, the voltage at the end of the transmission line **decreases slightly** – small positive VR.
2. If leading (capacitive) loads are added at the end of a line, the voltage at the end of the transmission line **increases** – negative VR.
3. If lagging (inductive) loads are added at the end of a line, the voltage at the end of the transmission line **decreases significantly** – large positive VR.

Power flow and efficiency

- Input powers

$$P_{in} = 3V_S I_S \cos\theta_S = \sqrt{3} V_{LL,S} I_S \cos\theta_S$$

$$Q_{in} = 3V_S I_S \sin\theta_S = \sqrt{3} V_{LL,S} I_S \sin\theta_S$$

$$S_{in} = 3V_S I_S = \sqrt{3} V_{LL,S} I_S$$

- Output powers

$$P_{out} = 3V_R I_R \cos\theta_R = \sqrt{3} V_{LL,R} I_R \cos\theta_R$$

$$Q_{out} = 3V_R I_R \sin\theta_R = \sqrt{3} V_{LL,R} I_R \sin\theta_R$$

$$S_{out} = 3V_R I_R = \sqrt{3} V_{LL,R} I_R$$

- Efficiency

$$\eta = \frac{P_{out}}{P_{in}} \cdot 100\%$$

Short line – simplified

- If the resistance of the line is ignored, then

$$I\cos\theta = \frac{V_S \sin\delta}{X_L} \qquad \boxed{P = \frac{3V_S V_R \sin\delta}{X_L}}$$

- Where δ is the angle between the two end voltages. Therefore, for fixed voltages, the power flow through a transmission line depends on the angle between the input and output voltages.
- Maximum power flow occurs when $\delta = 90^o$.

$$\boxed{P_{\max} = \frac{3V_S V_R}{X_L}}$$

- Notes:
  - The maximum power handling capability of a transmission line is a function of the **square of its voltage.**
  - The maximum power handling capability of a transmission line is inversely proportional to its series reactance (some very long lines include series capacitors to reduce the total series reactance).
  - The angle δ controls the power flow through the line. Hence, it is possible to control power flow by placing a phase-shifting transformer.

Example

- A line with reactance X and negligible resistance supplies a pure resistive load from a fixed source V_S. Determine the maximum power transfer, and the load voltage V_R at which this occurs. *(Hint: recall the maximum power transfer theorem from your basic circuits course)*

- *Ans:* $P_{\max} = \frac{V_S^2}{2X}, \qquad V_R = \frac{V_S}{\sqrt{2}}$

Line Characteristics

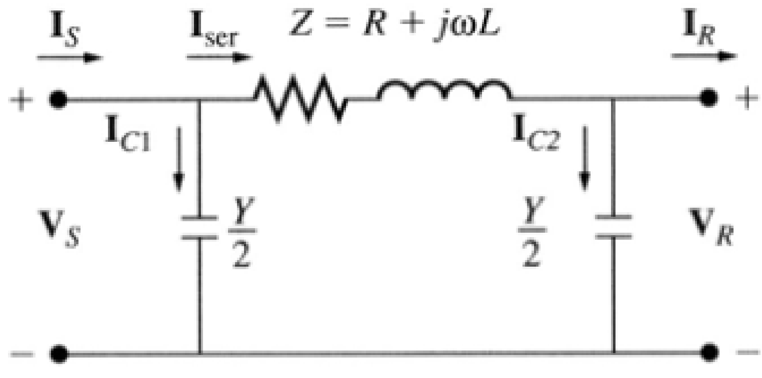
- To prevents excessive voltage variations in a power system, the ratio of the magnitude of the receiving end voltage to the magnitude of the ending end voltage is generally within

$$0.95 \leq V_S/V_R \leq 1.05$$

- The angle δ in a transmission line should typically be ≤ 30° to ensure that the power flow in the transmission line is well below the static stability limit.
- Any of these limits can be more or less important in different circumstances.
  - In short lines, where series reactance *X* is relatively small, the **resistive heating** usually limits the power that the line can supply.
  - In longer lines operating at lagging power factors, the **voltage drop** across the line is usually the limiting factor.
  - In longer lines operating at leading power factors, the **maximum angle δ** can be the limiting f actor.

Medium Line (50-150 mi)

- the shunt admittance must be included in calculations. However, the total admittance is usually modeled (π model) as two capacitors of equal values (each corresponding to a half of total admittance) placed at the sending and receiving ends.



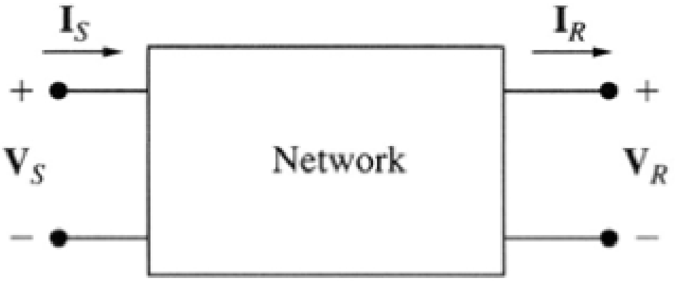
- The total series resistance and series reactance are calculated as before. Similarly, the total shunt admittance is given by

$$Y = yd$$

- where *y* is the shunt admittance per unit length and *d* is the length of the transmission line.

Medium Line

- Two-port network:



$$V_S = AV_R + BI_R$$
$$I_S = CV_R + DI_R$$

$$A = \frac{ZY}{2} + 1$$

$$B = Z$$

$$C = Y\left(\frac{ZY}{4} + 1\right)$$

$$D = \frac{ZY}{2} + 1$$

Determining the Line Circuit Parameters

- Note: Typically, $|R + jX_L| << |1/Y|$
- Under Open-Circuit Test (phase quantities):

$$V_S = AV_R$$

$$I_S = CV_R \approx YV_S$$

$$P \approx 0, \quad Q \approx -YV_S^2$$

- Under Short-Circuit Test:

$$V_S = BI_R = ZI_R, \quad I_S = DI_R$$

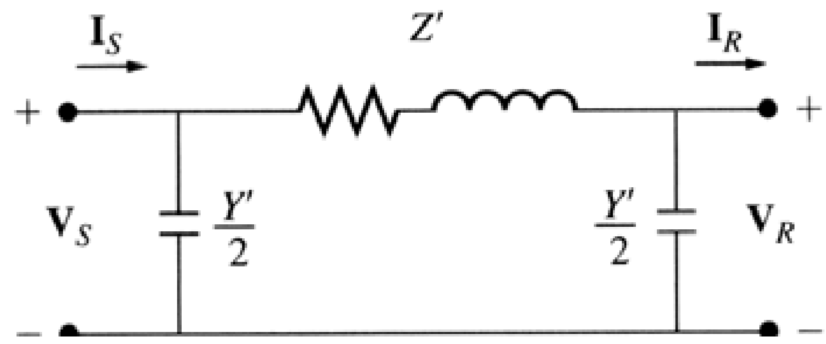
$$P \approx RI_R^2, \quad Q = X_L I_R^2 - (Y/2)V_S^2,$$

Long Lines (> 150 mi)

- For long lines, both the shunt capacitance and the series impedance must be treated as distributed quantities. The voltages and currents on the line are found by solving the differential equations of the line.
- However, it is possible to model a long transmission line by a π-model but with *modified* series impedance *Z'* and shunt admittance *Y'* . Hence one can perform calculations on that model using ABCD constants. These modified values are

$$Z' = Z\frac{\sinh \gamma d}{\gamma d}$$

$$Y' = Y\frac{\tanh(\gamma d/2)}{\gamma d/2}$$



where the propagation constant is defined by $\gamma = \sqrt{yz}$

Long line series and shunt compensation

- **Shunt reactors** are used to compensate the line shunt capacitance under light load or no load to regulate voltage.
- **Series capacitors** are often used to compensate the line inductive reactance in order to transfer more power.

Homework # 3

Consider a 3-phase, 60 Hz, 345 kV transmission line with a total series impedance $Z = 200 \angle 80^\circ\ \Omega$ and shunt admittance $Y = j\ 0.001$ S. Assume the sending end is connected to a stiff 345 kV supply, while the receiving end is supplying a load with a per-phase impedance $Z_{load} \angle \theta^\circ\ \Omega$.

1) Plot the receiving end voltage as a function of the real power supplied to the load as the load impedance varies infinity down to zero for the following 3 cases:
   a. $Z_{load} \angle 0^\circ$
   b. $Z_{load} \angle +30^\circ$
   c. $Z_{load} \angle -30^\circ$

Practice Problems

- 9.11 – 9.17