# Sunlight and its Properties
# Part I

EE 446/646

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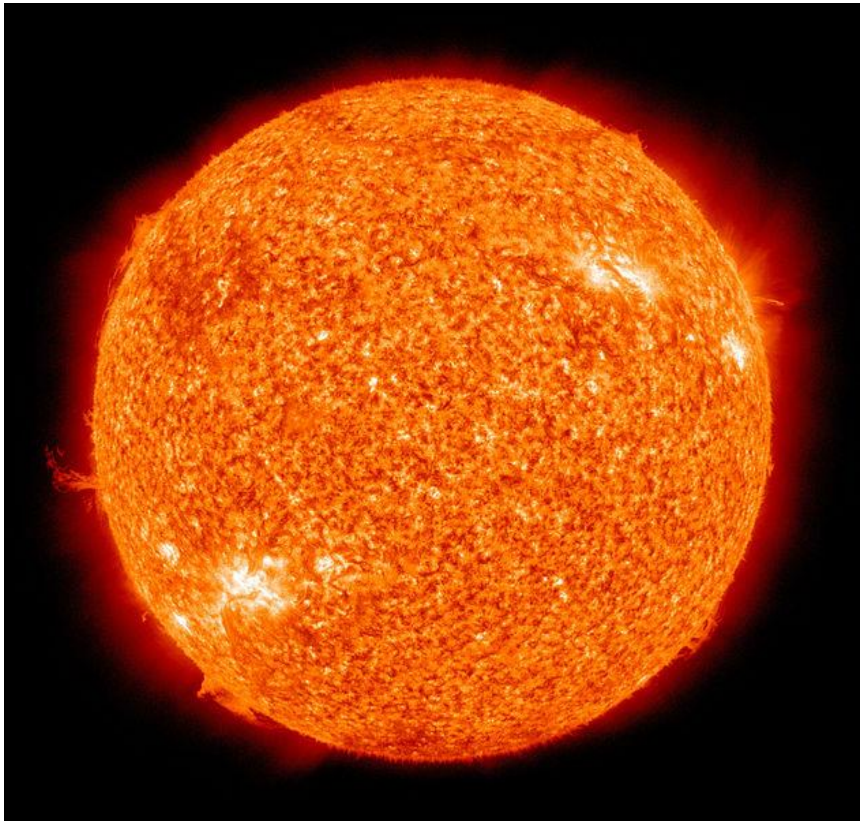
# The Sun – a Thermonuclear Furnace

- The sun is a hot sphere of gas whose internal temperatures reach over 20 million deg. K.
- Nuclear fusion reaction at the sun's core converts hydrogen to helium.
- A nuclear fusion releases a tremendous amount of thermal energy according to Einstein's formula:

$$E = mC^2 \approx 3.8x10^{20} MJ$$

where *m* is the mass lost/s and *C* is the speed of light (3 x $10^8$ m/s).

- This energy is radiated outward from the sun's surface into space.



# Plank’s Blackbody Formula

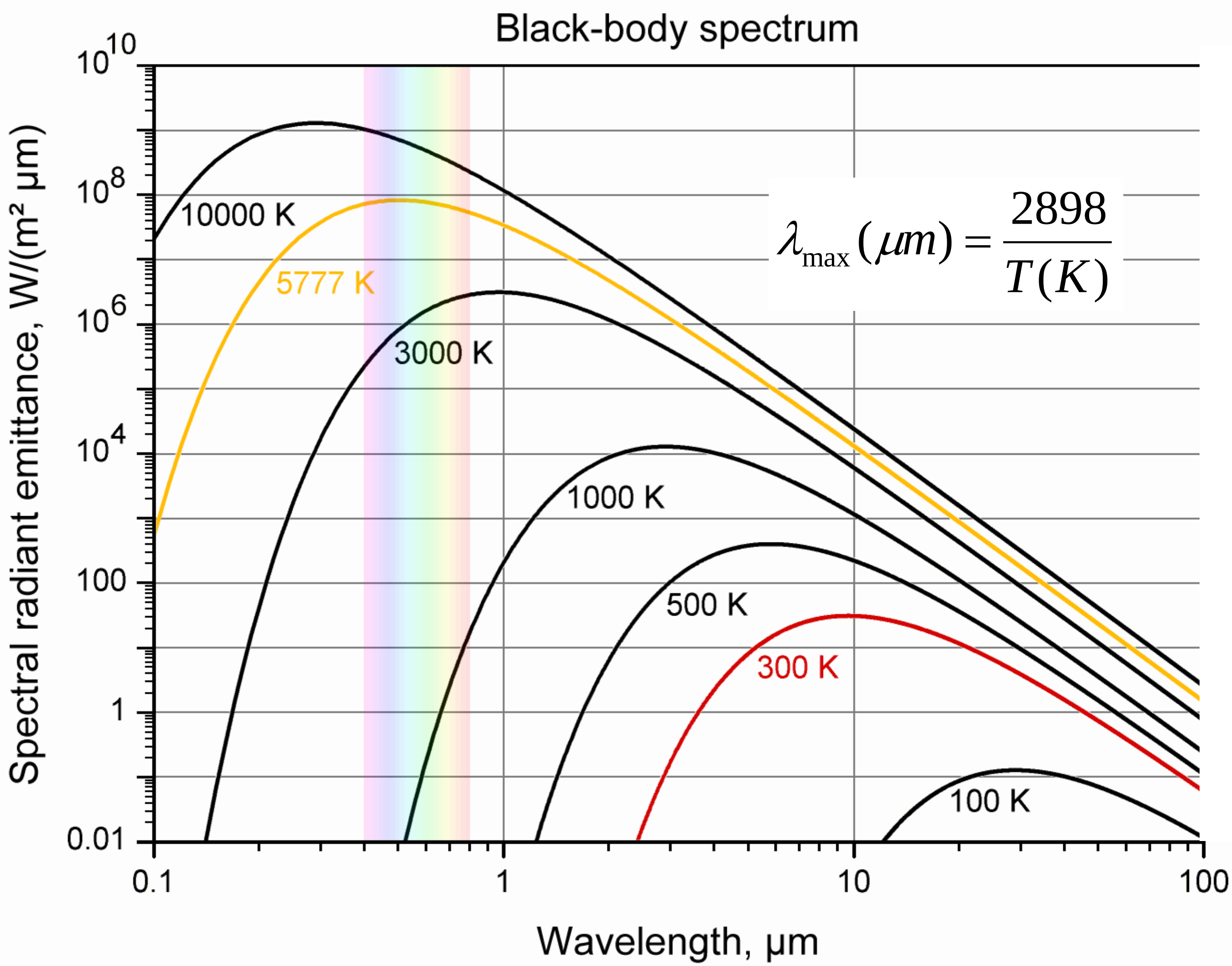
- A **black body** is one that absorbs all the EM radiation that strikes it. To stay in thermal equilibrium, it must emit radiation at the same rate as it absorbs it.
- The radiant energy emitted by any object is estimated according to Plank’s formula:

$$E = \frac{3.74x10^{8}}{\lambda^{5}[e^{\frac{14,400}{\lambda T}} - 1]}$$

- *E*: Energy density at the surface (W/m$^2$- μm)
- λ: wavelength (μm)
- T: temperature of the body (K)
- Temperature conversion:

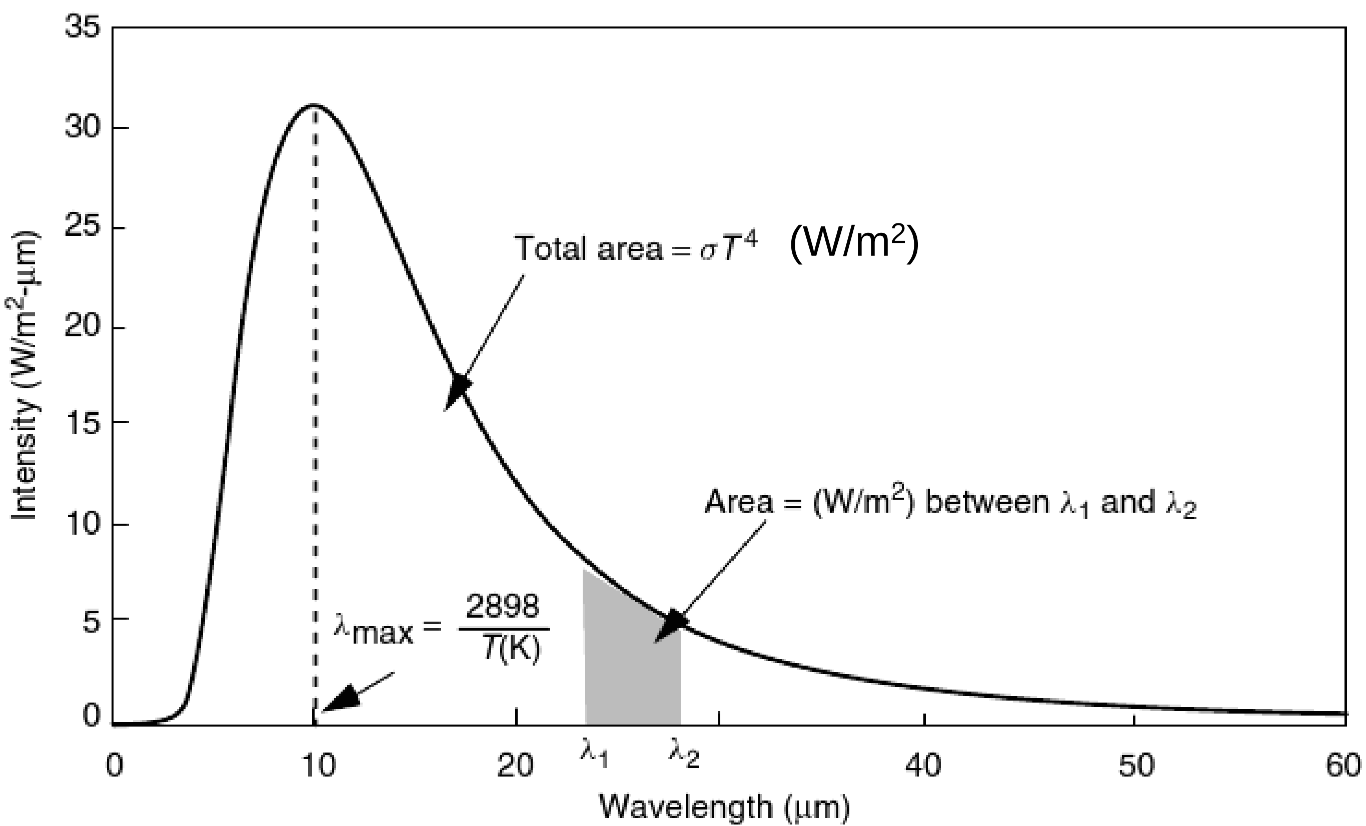
° F = 1.8° C + 32
° K = ° C + 273.

# Black-Body Spectrum at Different Temperatures



# Spectral Emissive Power of the Earth

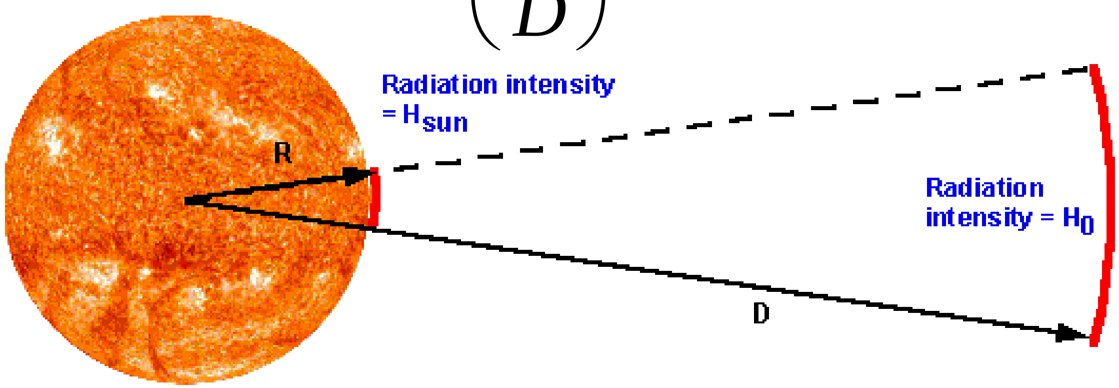
- Earth Model: a blackbody @15$^o$C (288$^o$K), radius: 6.37 x10$^6$m
- Maximum power is emitted: 32 W/m$^2$- µm at λ = 10.1 µm
- Total power emitted (= total area under curve) = 390 W/m$^2$
- Stephan-Boltzmann constant: σ = 5.67x10$^{-8}$W/m$^2$-K$^4$



# Spectral Emissive Power of the Sun

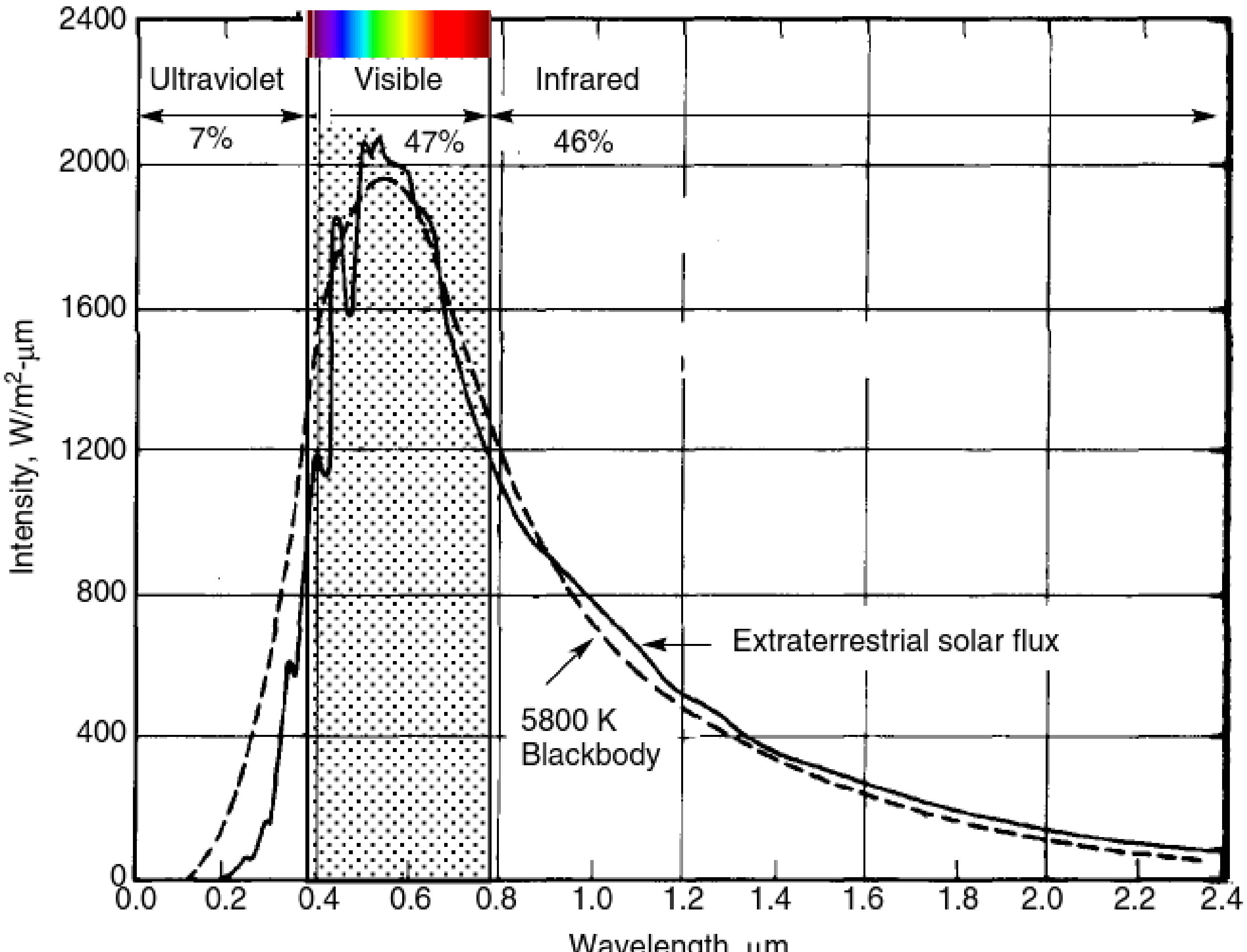
- Sun Model: a blackbody @5527°C (5800°K), radius of the sun: R = 6.955x$10^8$ m
- Total power radiated at the surface: 6.4 x $10^7$ W/m$^2$
- Maximum power emitted: 84 x$10^6$ W/m$^2$- µm at $\lambda$ = 0.5 µm
- The energy radiated on an object in space decreases as the object moves further away from the sun. The energy density $E_D$ on an object some distance D from the sun is found by

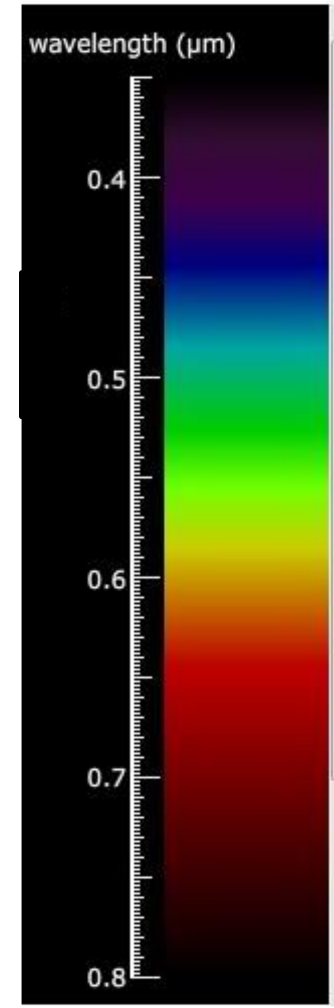
$$E_D = \left(\frac{R}{D}\right)^2 E$$



# Extraterrestrial Solar Spectrum

- Distance of earth outer atmosphere to sun: D = $1.5 \times 10^{11}$ m
- Scale factor: $(R/D)^2 = 21.5 \times 10^{-6}$
- Total area under blackbody curve: 1.37 $kW/m^2$



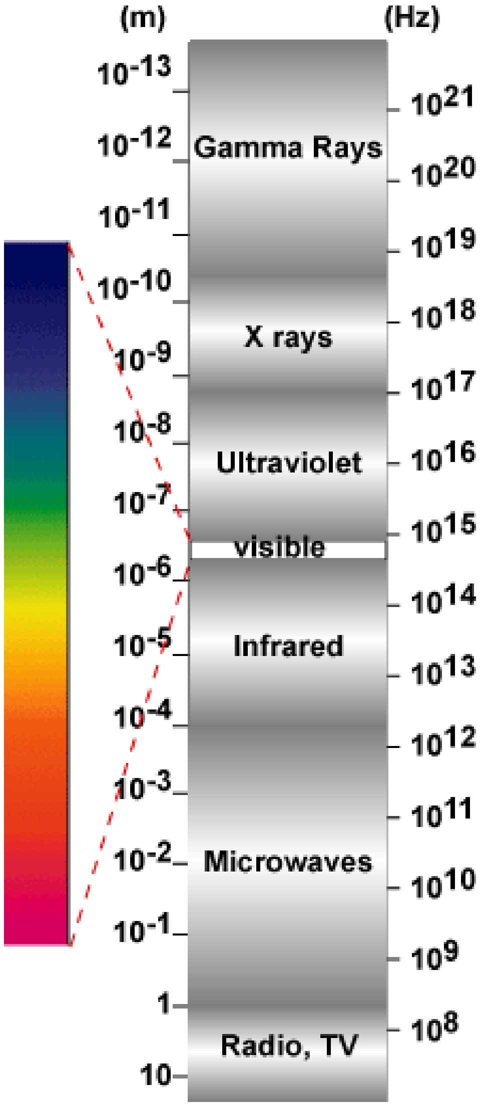



# Properties of Light

- Visible light was viewed as a small subset of the electromagnetic spectrum since the late 1860s.
- The electromagnetic spectrum describes light as continuous number of waves with different wavelengths.
- Relation between wavelength λ (m), and frequency *f* (Hz) :

$$f = \frac{c}{\lambda}$$

where c is the speed of light: c = 3 x $10^8$ m/sec.

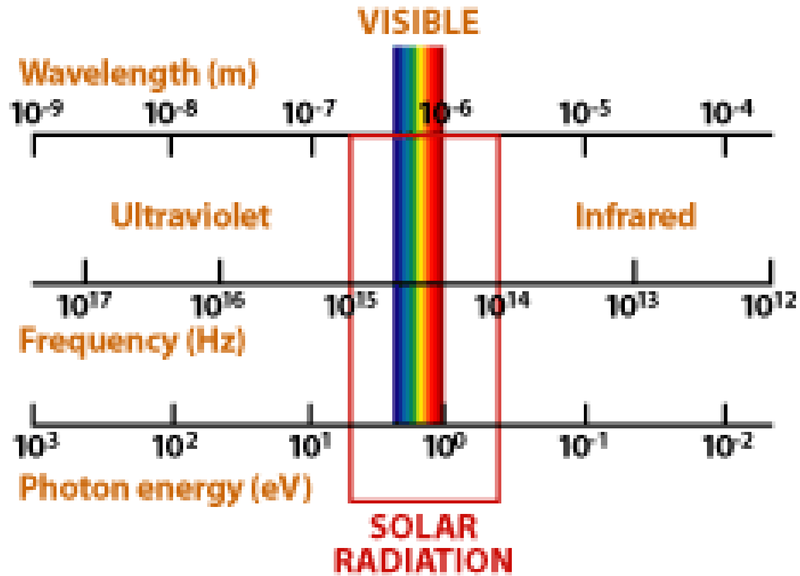


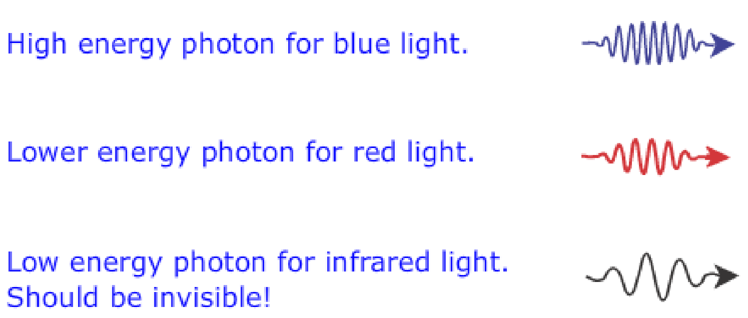
# Properties of Solar Spectrum

- The sun emits almost all of its energy in a range of wavelengths from about $2x10^{-7}$ to $2x10^{-6}$ meters.
- Nearly half of this energy is in the visible light region.
- Each wavelength corresponds to a frequency: the shorter the wavelength, the higher the frequency (refer to previous slide)
- Each wavelength corresponds to an energy: the shorter the wavelength, the greater the energy (expressed in eV).
- Relation between the energy *E* of a photon and the wavelength:

$$E = \frac{hc}{\lambda}$$

where *h* (= $6.626 \times 10^{-34}$ Joule·sec) is Planck's constant



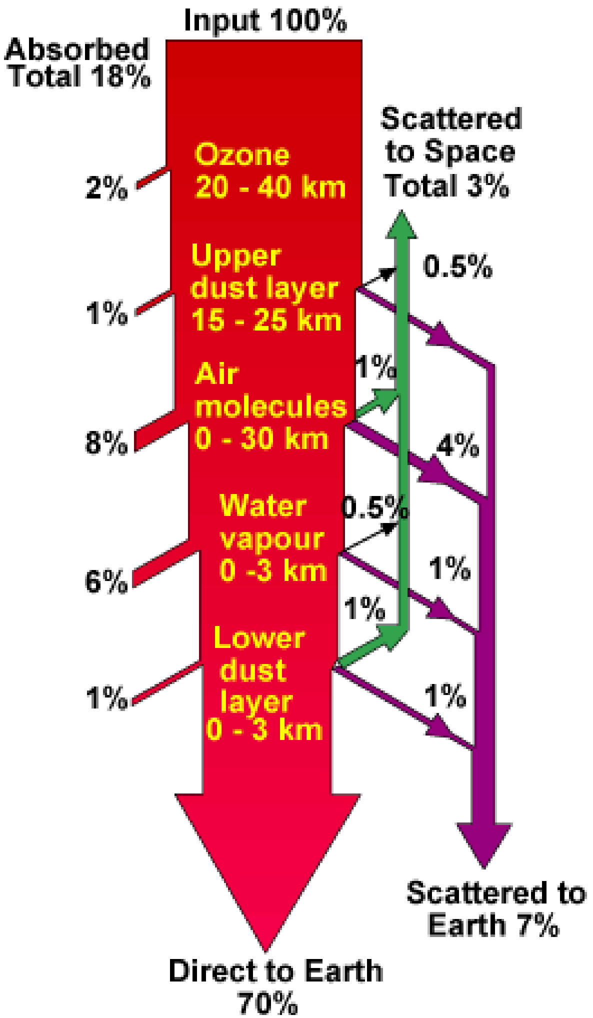



# Solar radiation intensity of other planets

| Planet | Distance D ($\times 10^9$ m) | Solar Radiation ($W/m^2$) |
|---|---|---|
| Mercury | 57 | 9228 |
| Venus | 108 | 2586 |
| Earth | 150 | 1367 |
| Mars | 227 | 586 |
| Jupiter | 778 | 50 |
| Saturn | 1426 | 15 |
| Uranus | 2868 | 4 |
| Neptune | 4497 | 2 |
| Pluto | 5806 | 1 |

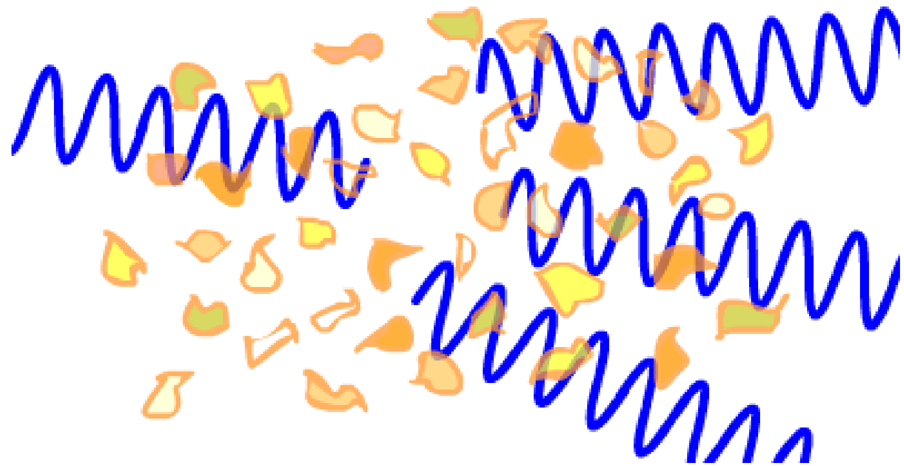
# Effect of Atmosphere on Direct Sunlight

- As sunlight enters the earth's atmosphere, some of it gets absorbed, some is scattered and some passes through and reaches the earth's surface.
- Different molecules in the atmosphere absorb significant amounts of sunlight at certain wavelengths
  - Water vapor and carbon dioxide absorb sunlight in the visible and infrared region
  - Ozone absorbs sunlight in the UV region

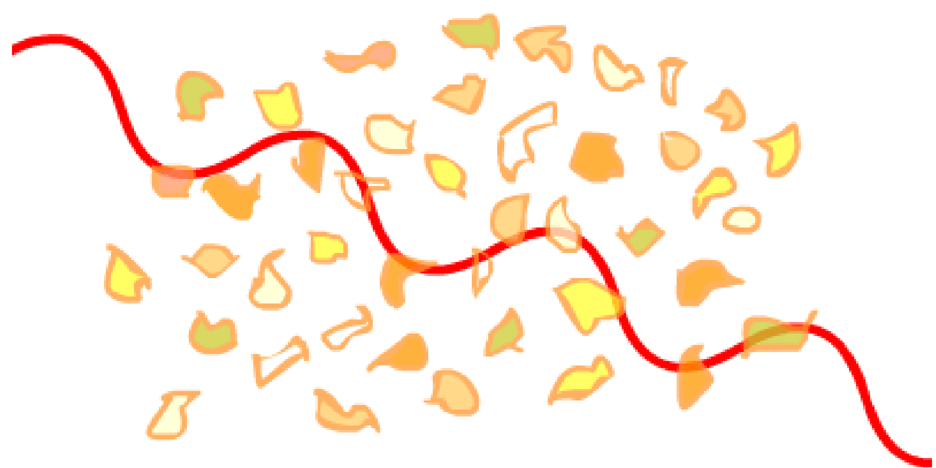


# Scattering of incident light

- Blue light has a wavelength similar to the size of particles in the atmosphere – thus scattered.

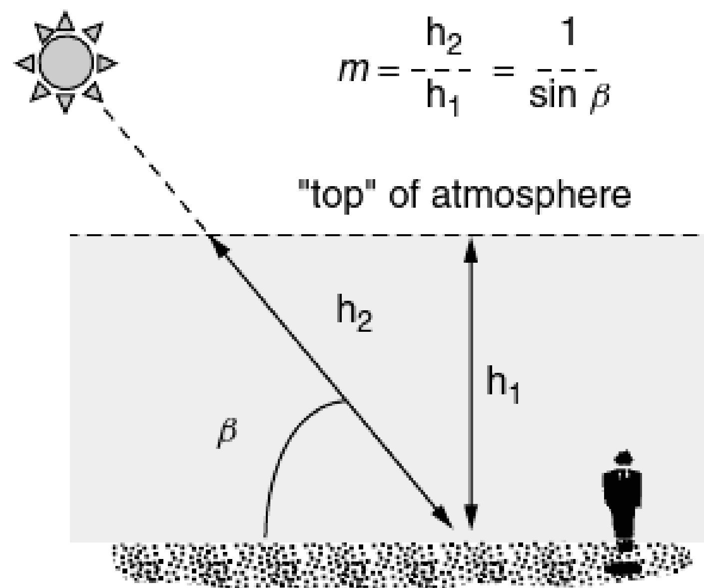


- Red light has a wavelength that is larger than most particles – thus unaffected



# Air Mass Ratio

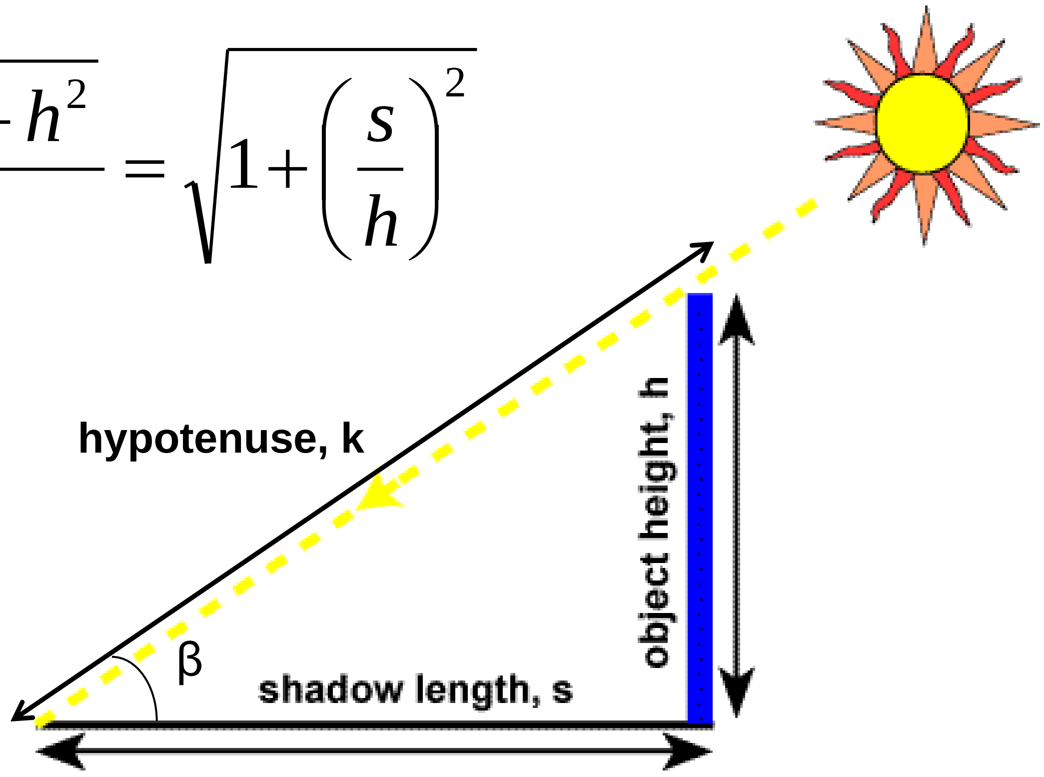
- The terrestrial spectrum also depends on how much atmosphere the radiation has to pass through to reach the surface.
- Air Mass (AM) ratio is the optical path length through earth's atmosphere relative to the minimum path. This ratio is approximately equal to the inverse of the sine of the sun's altitude angle β.
  - AM0 means no atmosphere
  - AM1 means the sun is directly overhead.
  - AM1.5 (an air mass ratio of 1.5) is often assumed as the average spectrum at the earth's surface. AM1.5 is chosen as the standard calibration spectrum for PV Cells.



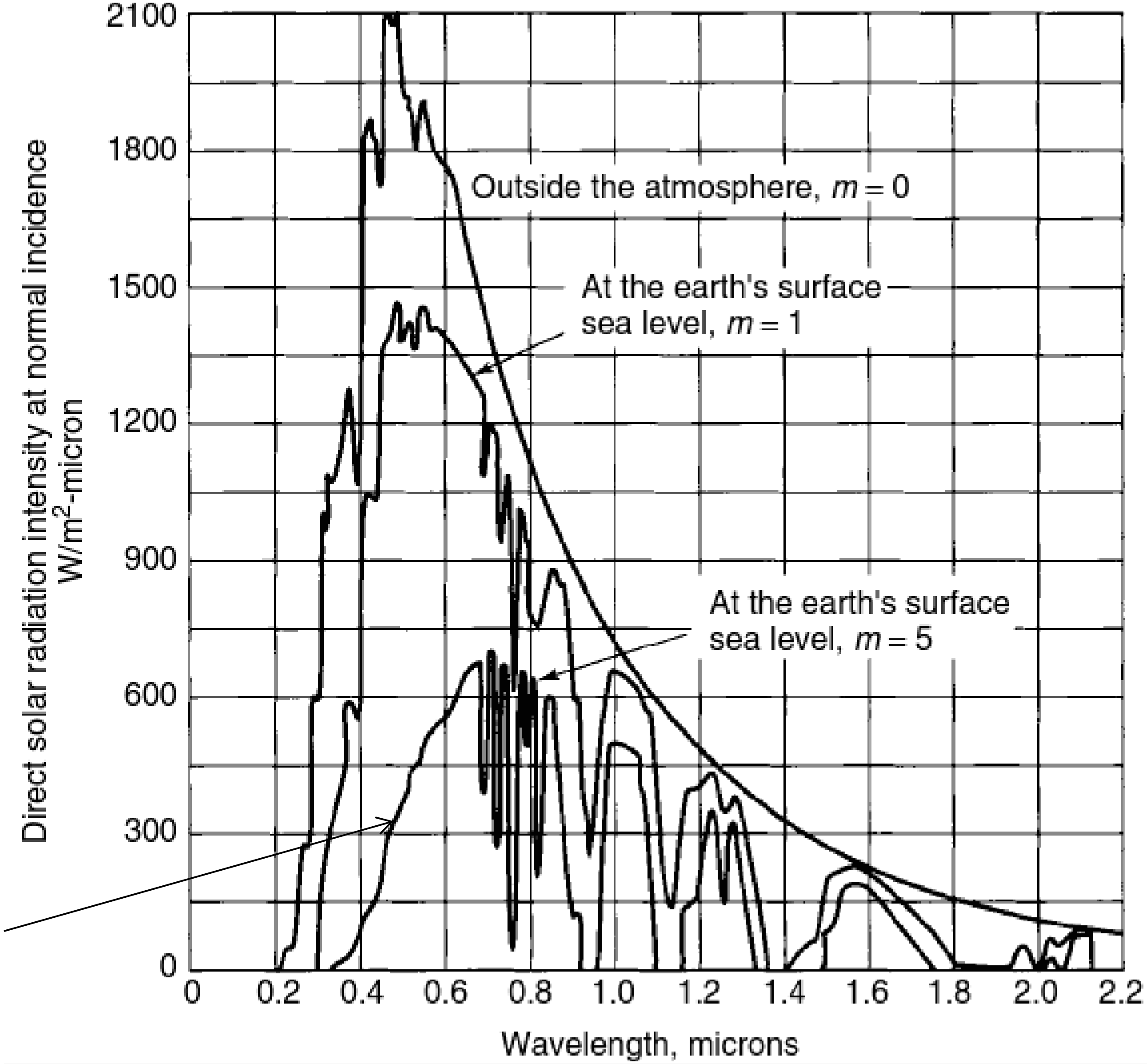
# Air Mass Ratio

- An easy method to determine the AM is from the shadow *s* of a vertical object. Air mass is the length of the hypotenuse *k* divided by the object height *h*:

$$AM \approx \frac{\sqrt{s^2 + h^2}}{h} = \sqrt{1 + \left(\frac{s}{h}\right)^2}$$

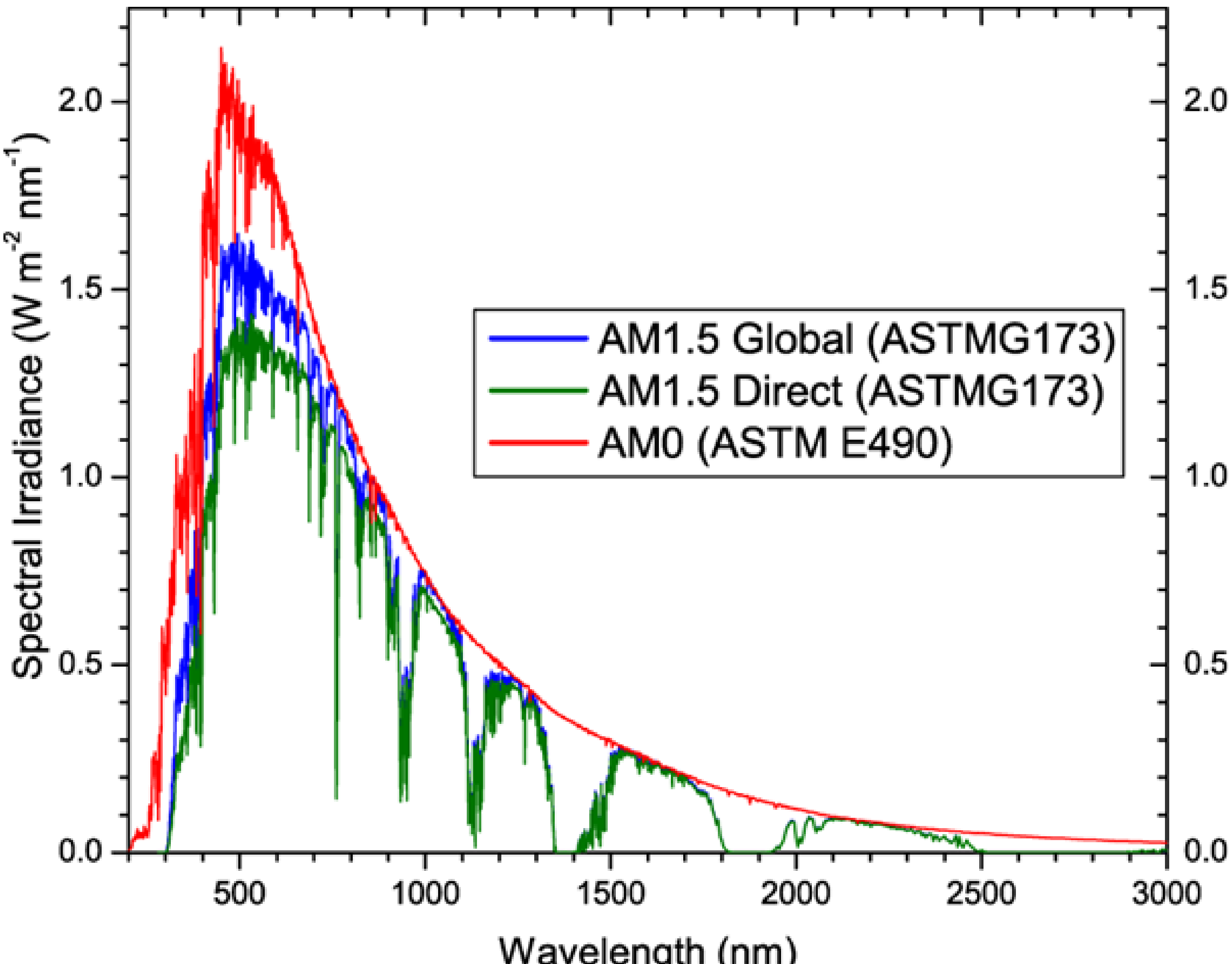


# Solar Spectrum for Various Air Mass Ratios




Spectrum shifts towards longer wavelengths

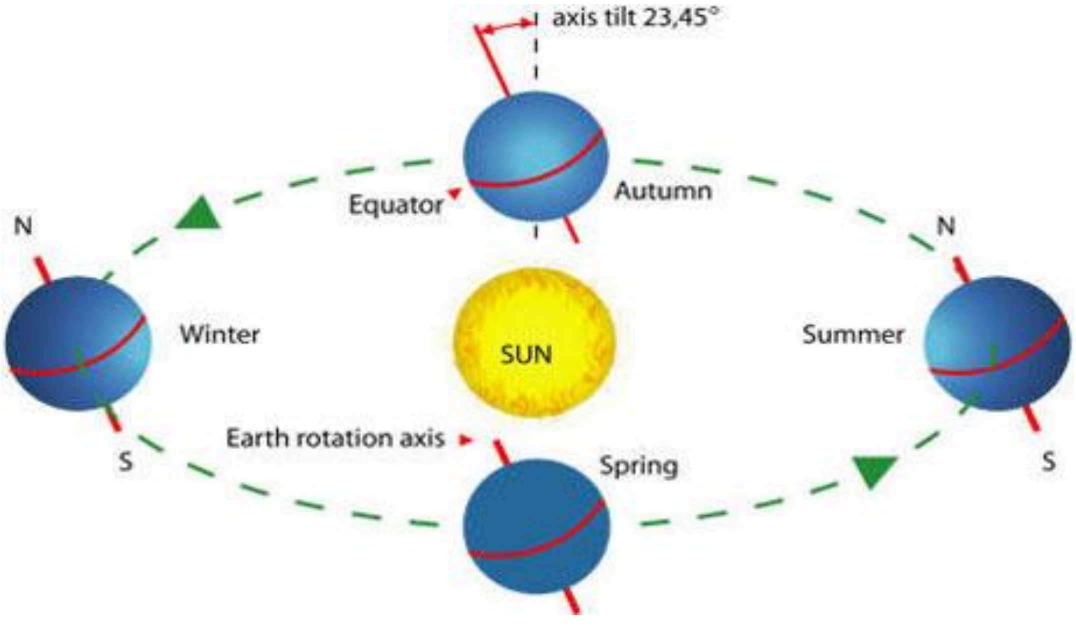
# Standard Solar Spectrum for Testing PV Cells
(Excel file available)



# The Earth's orbit

- The earth revolves around the sun in an elliptical orbit once per year.
- The earth is farthest from the sun ($152x10^6$km) on July 3. It is closest to the sun ($147\ x10^6$km) on January 2.
- The variation in distance between the sun and earth is written in terms of *day number n* as follows:

$$d = 1.5x10^8\{1 + 0.017\sin[\frac{360(n-93)}{365}]\} \quad km$$

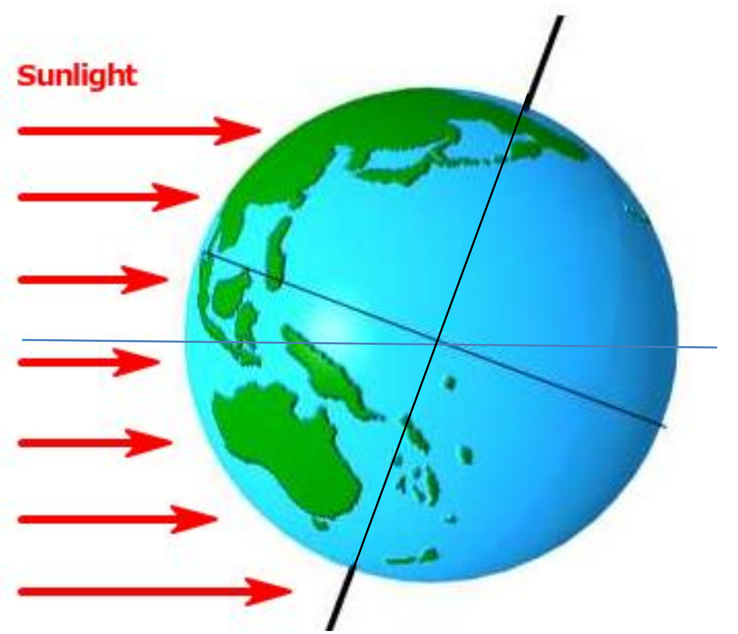


# Day Numbers of First Day of Each Month

| | | | |
|---|---|---|---|
| January | $n = 1$ | July | $n = 182$ |
| February | $n = 32$ | August | $n = 213$ |
| March | $n = 60$ | September | $n = 244$ |
| April | $n = 91$ | October | $n = 274$ |
| May | $n = 121$ | November | $n = 305$ |
| June | $n = 152$ | December | $n = 335$ |

# Earth Rotation

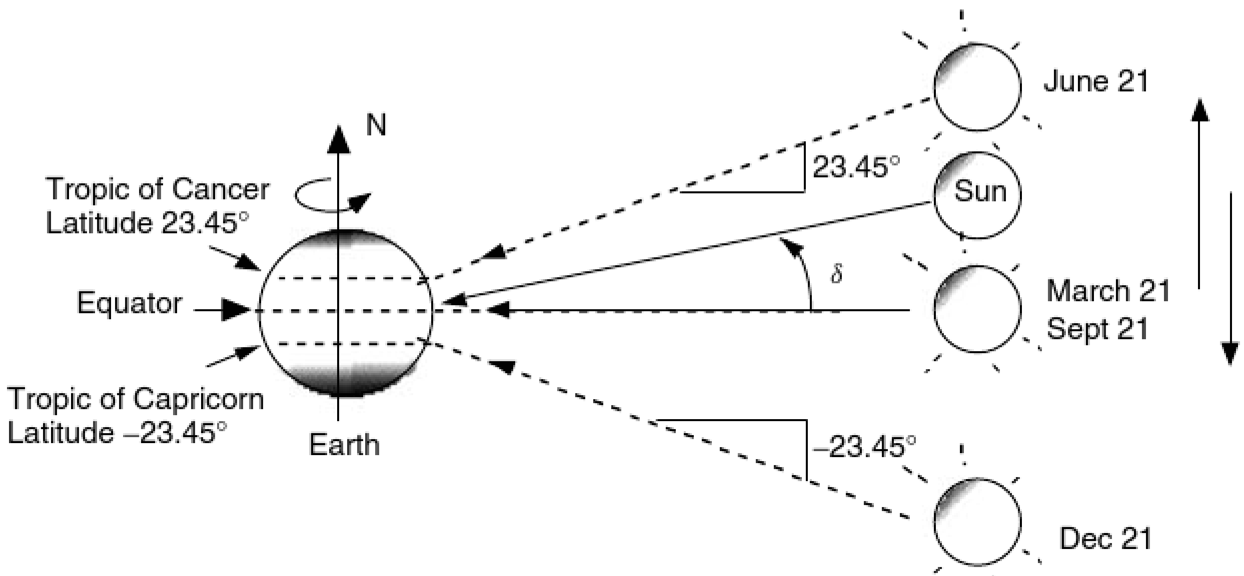
- The earth also rotates about its own axis once per day.
- The polar axis of the earth is inclined by an angle of 23.45$^{o}$ to the plane of the earth's orbit – results in seasons.
- On March 21 and September 21, the sun rays are perpendicular to the equator (equinox) -12 hours of daytime everywhere on earth.
- On December 21, the sun reaches its lowest point in the sky in the northern hemisphere i.e., Winter Solstice.
- On June 21, the sun reaches its highest point in the sky in the northern hemisphere, i.e., Summer Solstice.




**Winter Solstice**

# Convenient Reference: Fixed earth spinning around its north-south axis

- This causes the sun to move up and down as the seasons progress (higher in the sky during the summer, and lower during the winter).
- On summer solstice, the sun appears vertically above the Tropic of Cancer (which is at 23.45° latitude above the equator)
- On winter solstice, the sun appears vertically above the Tropic of Capricorn (which is at 23.45° latitude below the equator)
- At the two equinoxes, the sun appears vertically above the equator (which is at 0° latitude)



# Declination Angle

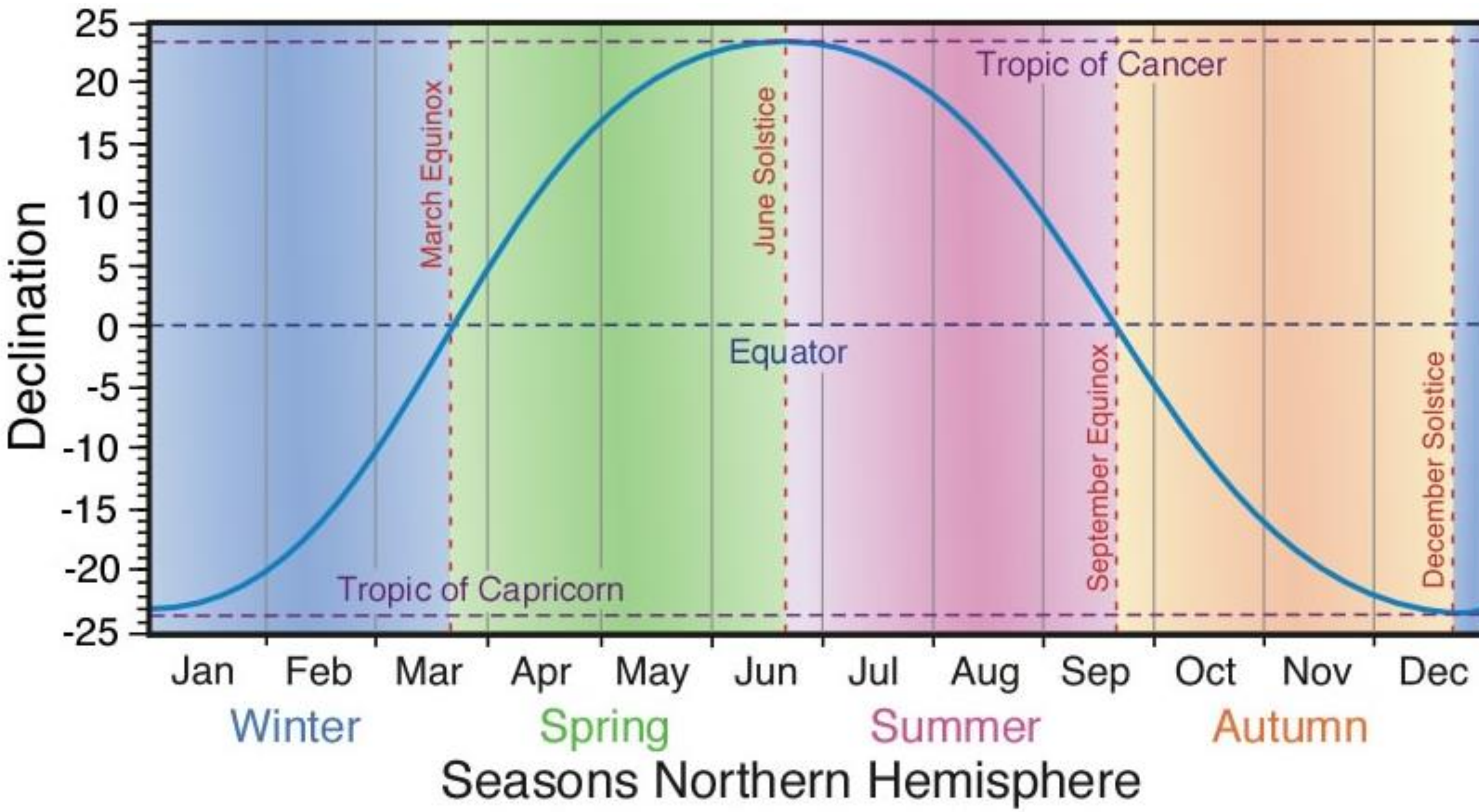
- The declination angle δ is the angle of deviation of the sun from directly above the equator.
- At any given $n^{th}$ day of the year, this angle can be calculated by the following formula.

$$\delta = 23.45^o \sin[\frac{360(n-81)}{365}]$$

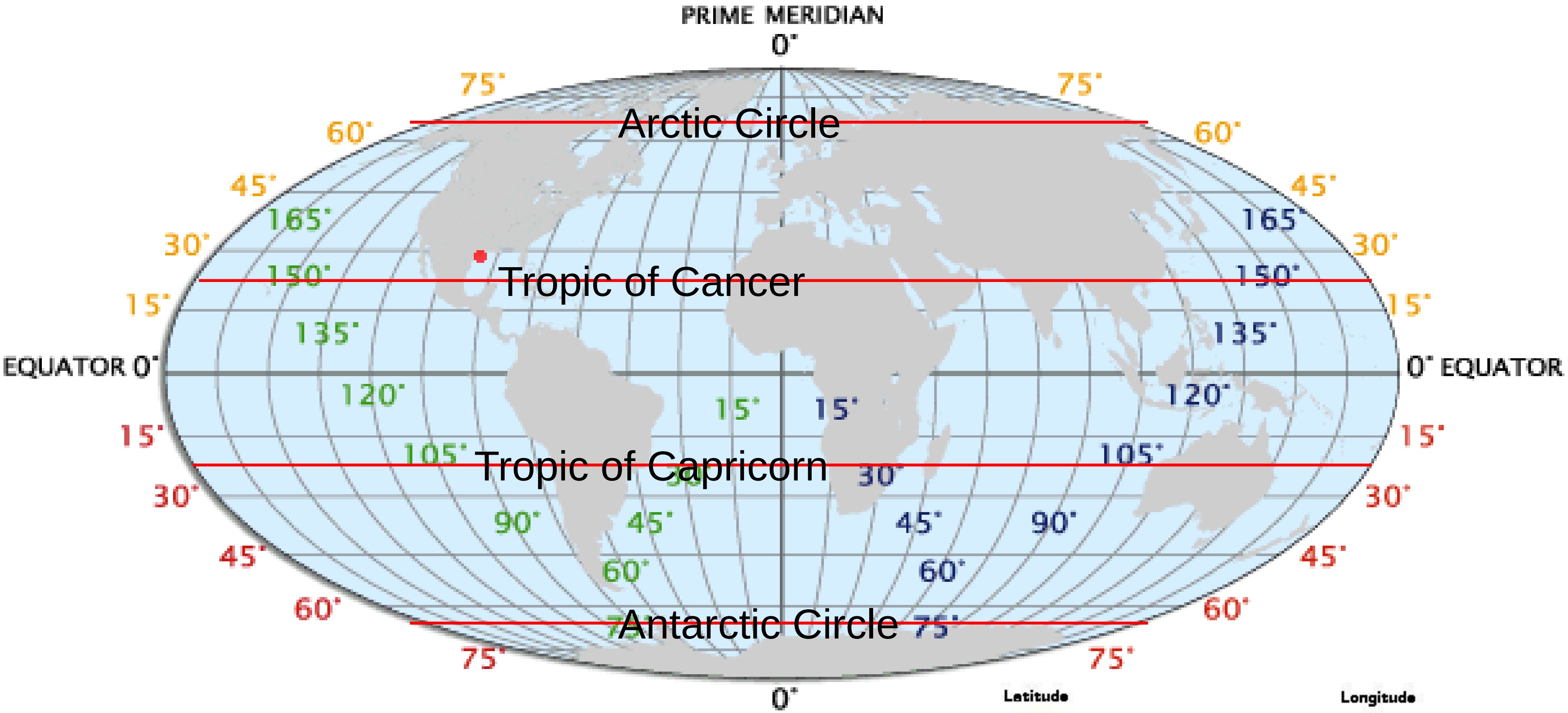
**TABLE 7.2 Solar Declination $\delta$ for the 21st Day of Each Month (degrees)**

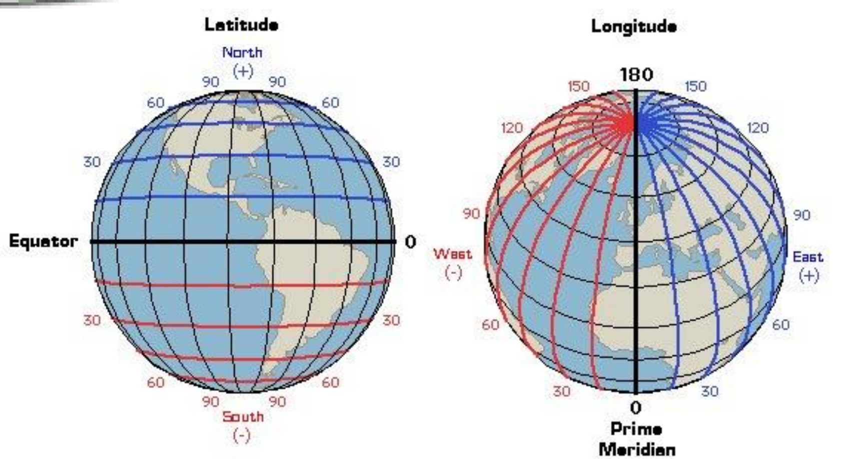
| Month: | Jan | Feb | Mar | Apr | May | Jun | July | Aug | Sept | Oct | Nov | Dec |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\delta$: | −20.1 | −11.2 | 0.0 | 11.6 | 20.1 | 23.4 | 20.4 | 11.8 | 0.0 | −11.8 | −20.4 | −23.4 |

# Declination Angle



# Longitude and Latitude Angles



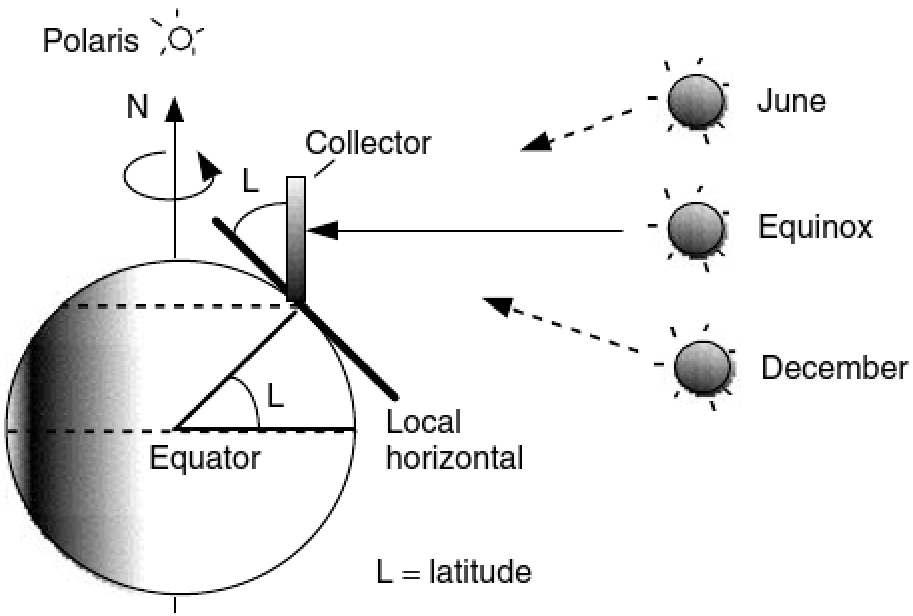




Arctic and Antarctic Circles: ± 66.55°

Tropics of Cancer and Capricorn: ± 23.45°

# Solar Noon:
# An important reference point for solar calculations

- Solar noon is the moment when the sun appears the highest in the sky, compared to its positions during the rest of the day.  It occurs when the sun is directly over the local meridian (line of longitude).
- **Rule-of-thumb:** for optimal annual performance of a solar collector facing south, the tilt angle should equal the local latitude angle *L*.



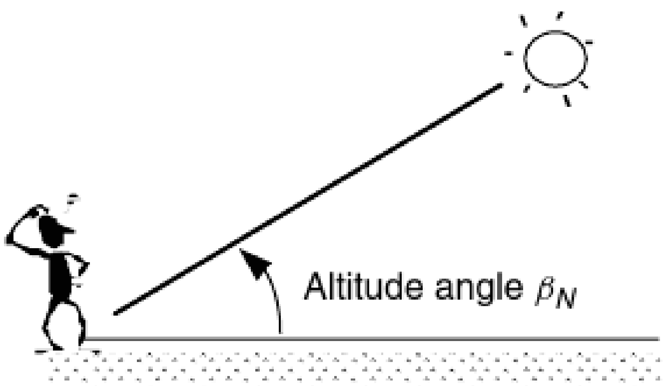
# Sun Altitude Angle at Solar Noon

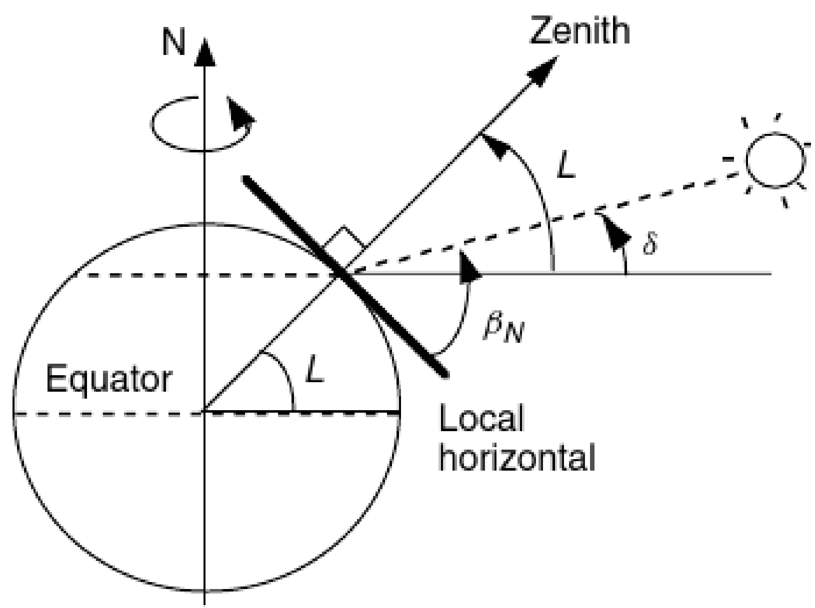
- The altitude angle at solar noon at a particular location and day is computed by

$$\beta_N = 90 - L + \delta$$

- The tilt angle that will produce maximum power at solar noon is equal to

$$optimal\ tilt\ angle = 90^o - \beta_N = L - \delta$$



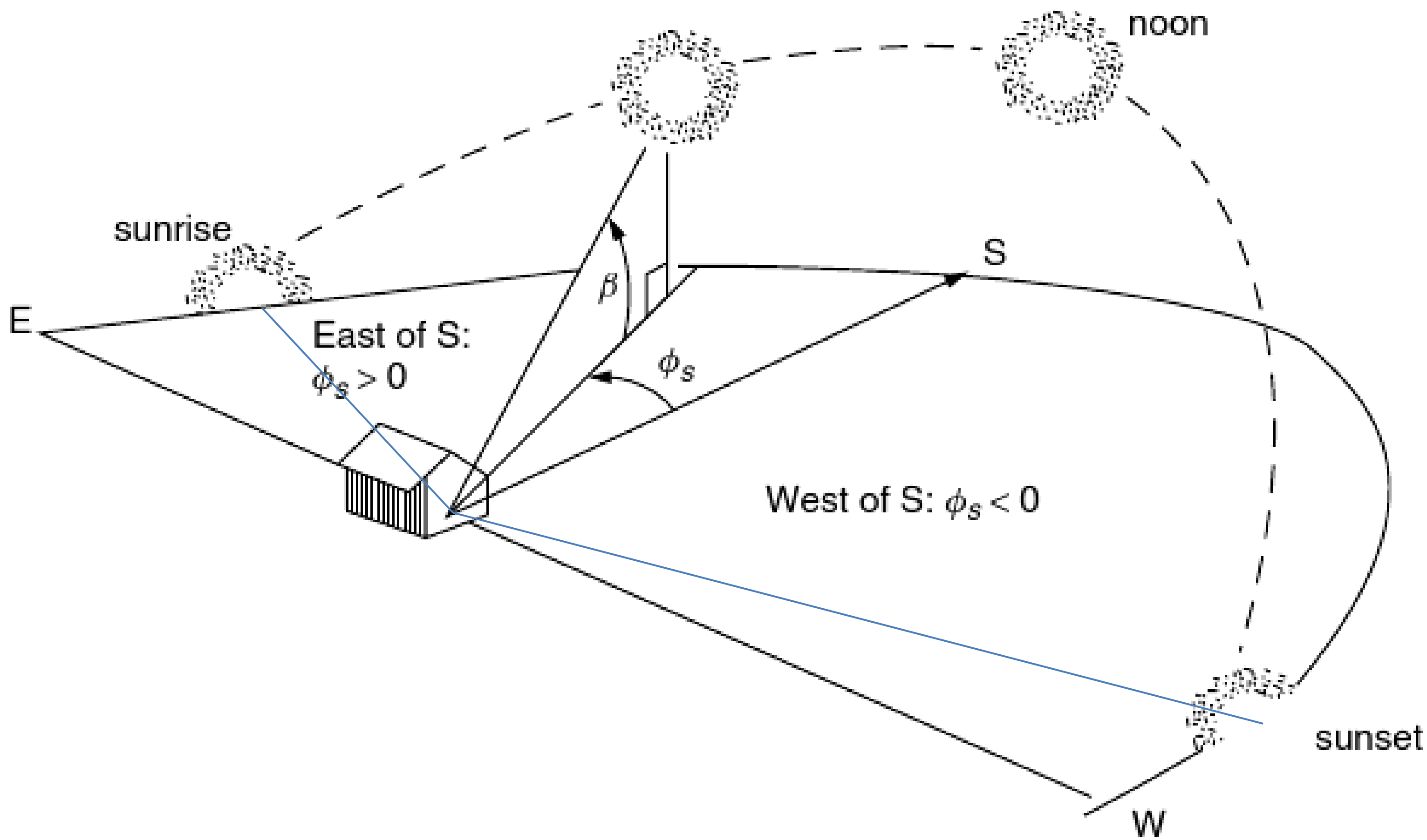
# Practice Problems

1. At what angle should a South-facing collector at 36 latitude be tipped up to in order to have it be normal to the sun's rays at solar noon on the following dates: a. March 21, b. January 21, c. April 1. *(Answer: ……….)*
2. What should be the tilt angle of a solar collector in Quito, Ecuador (latitude = $0^o$) , in order to capture maximum energy on an annual basis? *(Answer: ………..)*
3. Determine the maximum and minimum altitude (or elevation) angles of the sun at solar noon, in Las Vegas, NV. Then determine the corresponding air mass ratio. *(Answer:………………)*
4. The capital city of Iceland (Reykjavik) is situated at a latitude angle of 64 deg. Determine the sun elevation angle in this location on Christmas day at solar noon. *(Answer: ……….)*

# Sun Position (other than solar noon)

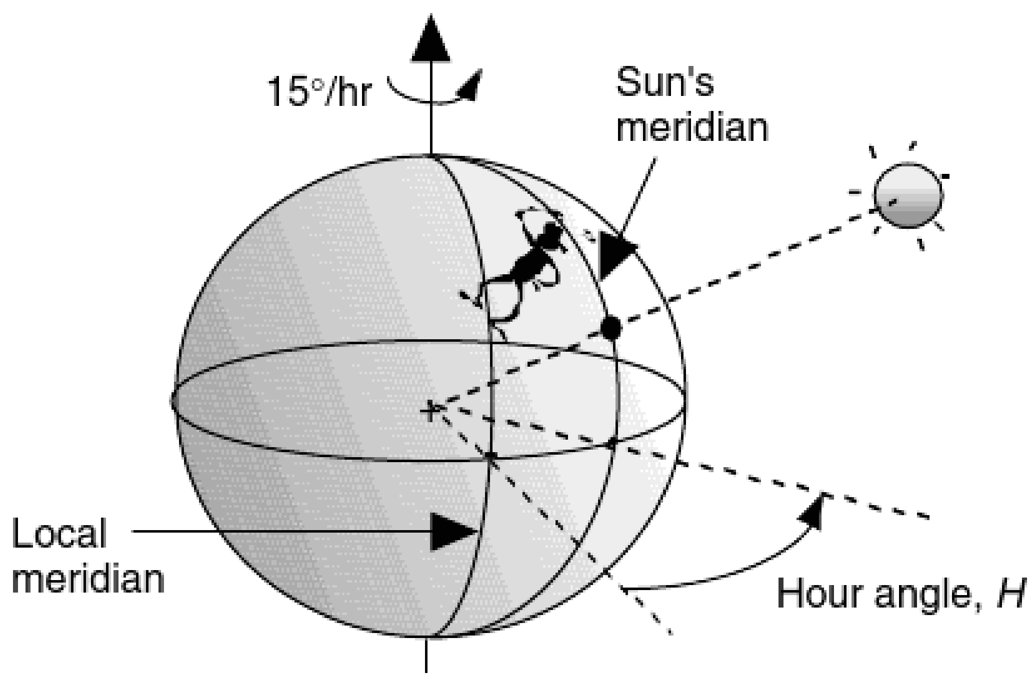
- The sun position at any time of day is described in terms of its altitude angle $\beta$ and azimuth angle $\phi$ (which measures that sun's angular position east or west of south).
- These angles depend on the latitude, day number, and most importantly, time of day.
- For the Northern Hemisphere, the azimuth angle is
  - Positive in the morning with the sun in the east
  - Zero at solar noon
  - Negative win the afternoon with the sun in the west
- For solar work in the Southern Hemisphere, azimuth angles are measured relative to the north.

# Sun Position



# Hour Angle

- The hour angle is the number of degrees that the earth must rotate before the sun is directly above the local solar meridian (line of longitude).
- The hour angle is the difference between the local meridian and the sun's meridian, with positive values occurring in the morning and negative values in the afternoon.



# Hour Angle and Solar Time

- Since the earth rotates 360$^o$ in 24 hours, or 15$^o$ per hour, the hour angle can be described by

$$H = \left( \frac{15^o}{hour} \right) . (12 - ST)$$

where ST is the solar time, the time before the sun crosses the local meridian. For example, 11:00 am solar time means 1 hour before the sun reaches the local meridian.

- Examples:
  - At 11:00 am solar time (ST = 11), the hour angle is 15$^o$.
  - At 2:18 pm solar time (ST=14.3), the hour angle is -34.5$^o$
  - At midnight solar time (ST = 0 or 24), the hour angle is ± 180$^o$.

# Sun Location Formulas

- The sun's altitude and azimuth angles can be determined in terms of the hour angle H, declination angle $\delta$, and latitude angle $L$ by the following formulas:

$$\sin\beta = \sin\delta \sin L + \cos\delta \cos L \cos H$$

$$\sin\phi = \frac{\cos\delta \sin H}{\cos\beta}$$

# Note about the azimuth angle

- During the equinoxes, the sun rises directly from the east (+90°) and sets directly to the west (-90°) in every place on earth.
- During spring and summer in the early morning and late afternoon, the sun's azimuth angle is liable to be more than 90° from south (this never happens in the fall and winter).
- Since the inverse of a sine function is ambiguous, i.e., sin(x) = sin(180° – x), we need a test to determine whether the azimuth angle is greater or less than 90°

$$Test: \quad if \quad \cos H \geq \frac{\tan \delta}{\tan L}, \quad then \quad |\phi| \leq 90^o \quad otherwise, \quad |\phi| > 90^o$$

# Sunrise, Sunset and Daylight Hours

- The hour angle at sunrise is determined by setting the altitude angle *L* to zero:

$$H = \cos^{-1}\{-\tan\delta \tan L\}$$

- The suns azimuth angle at sunrise is then computed by:

$$\phi = \sin^{-1}(\sin\delta \sin H)$$

- The suns azimuth angle at sunset is equal to the negative value of $\phi$.

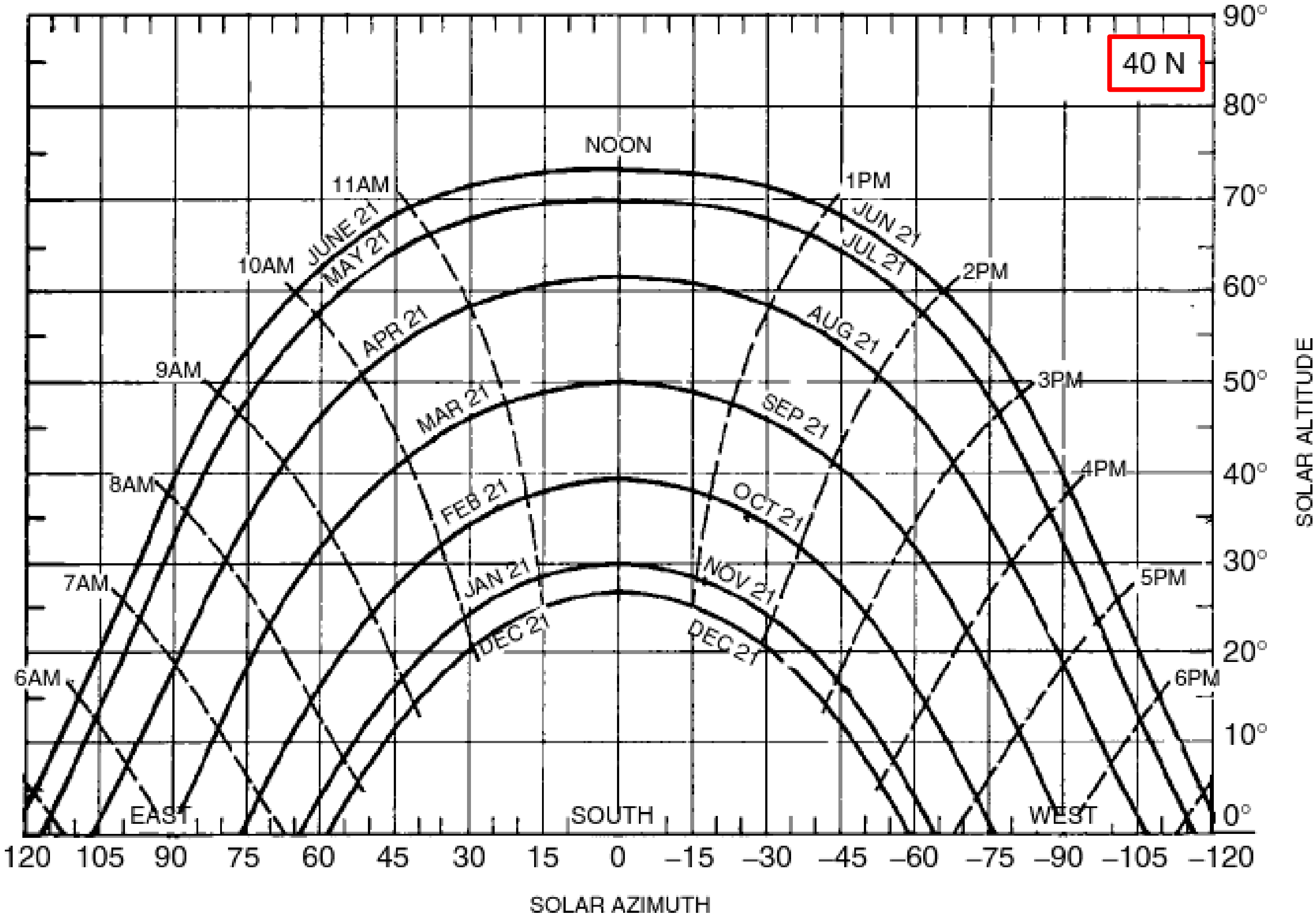
- Solar time at sunrise: $ST = 12 - \dfrac{H}{15}$

- The Daylight Hours (DH): $DH = \dfrac{2}{15} H \ hours$

# Example: Sun Location

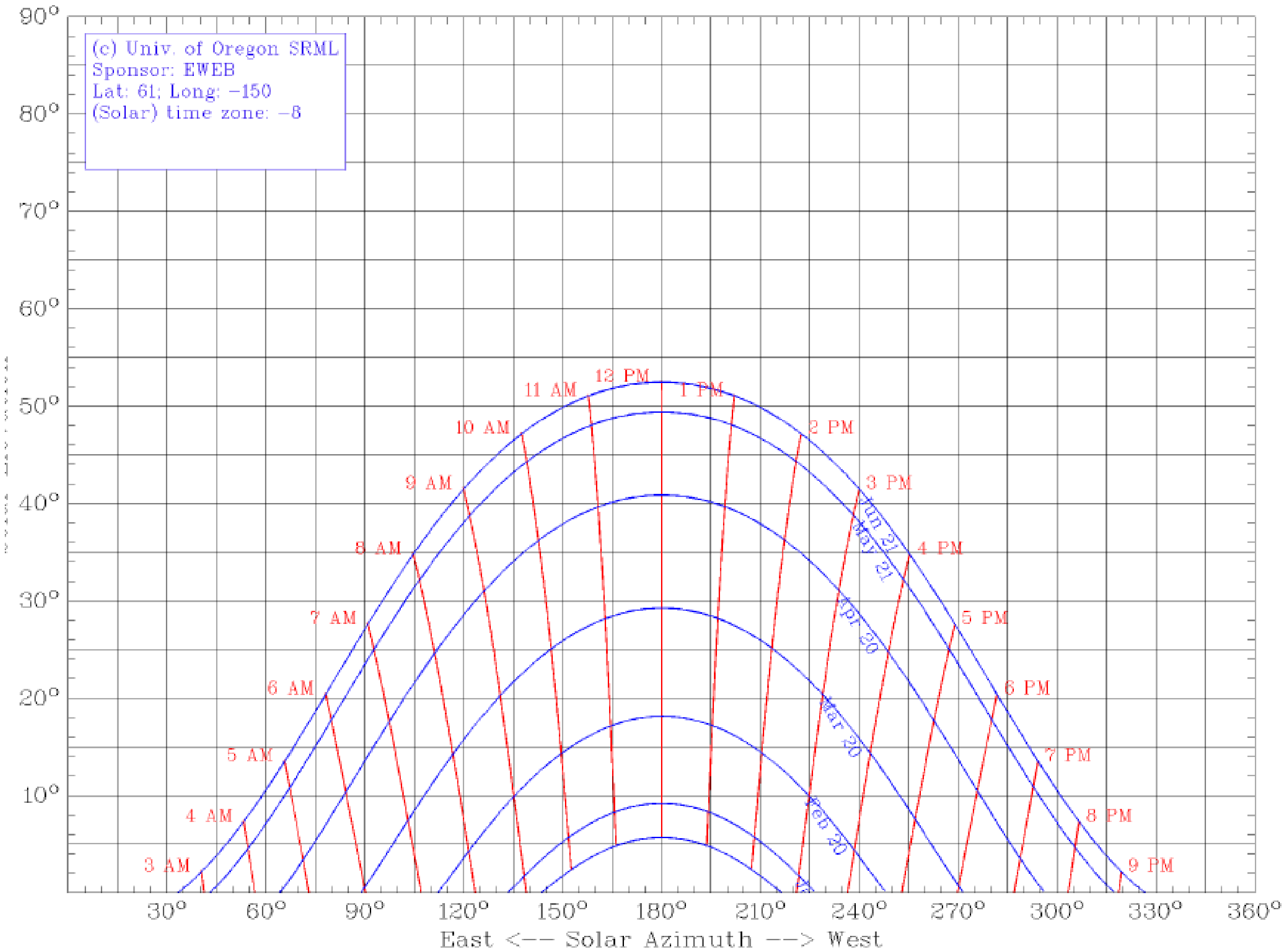
- Find the sun altitude and azimuth angles of the sun at 3:00 pm solar time (ST) in Boulder, CO (latitude: $40^o$) on the summer solstice.
- Answer:
  - the declination angle: $\delta = 23.45^o$
  - Solar time: ST = 15,
  - the hour angle $H = - 45^o$.
  - The altitude angle $\beta = 48.8^o$.
  - Sin $\phi = -0.985$, the $\phi = -80^o$ ($80^o$ west of south) or $260^o$ ($110^o$ west of south)
  - Test: $\cos H = 0.707$, $\tan\delta/\tan L = 0.517$, then $\phi = - 80^o$ (i.e., $80^o$ west of south)

# Sun Path Diagram (plot of sun altitude versus azimuth for specific latitude and days of the year)



# Another Resource: http://solardat.uoregon.edu/

Anchorage, Alaska



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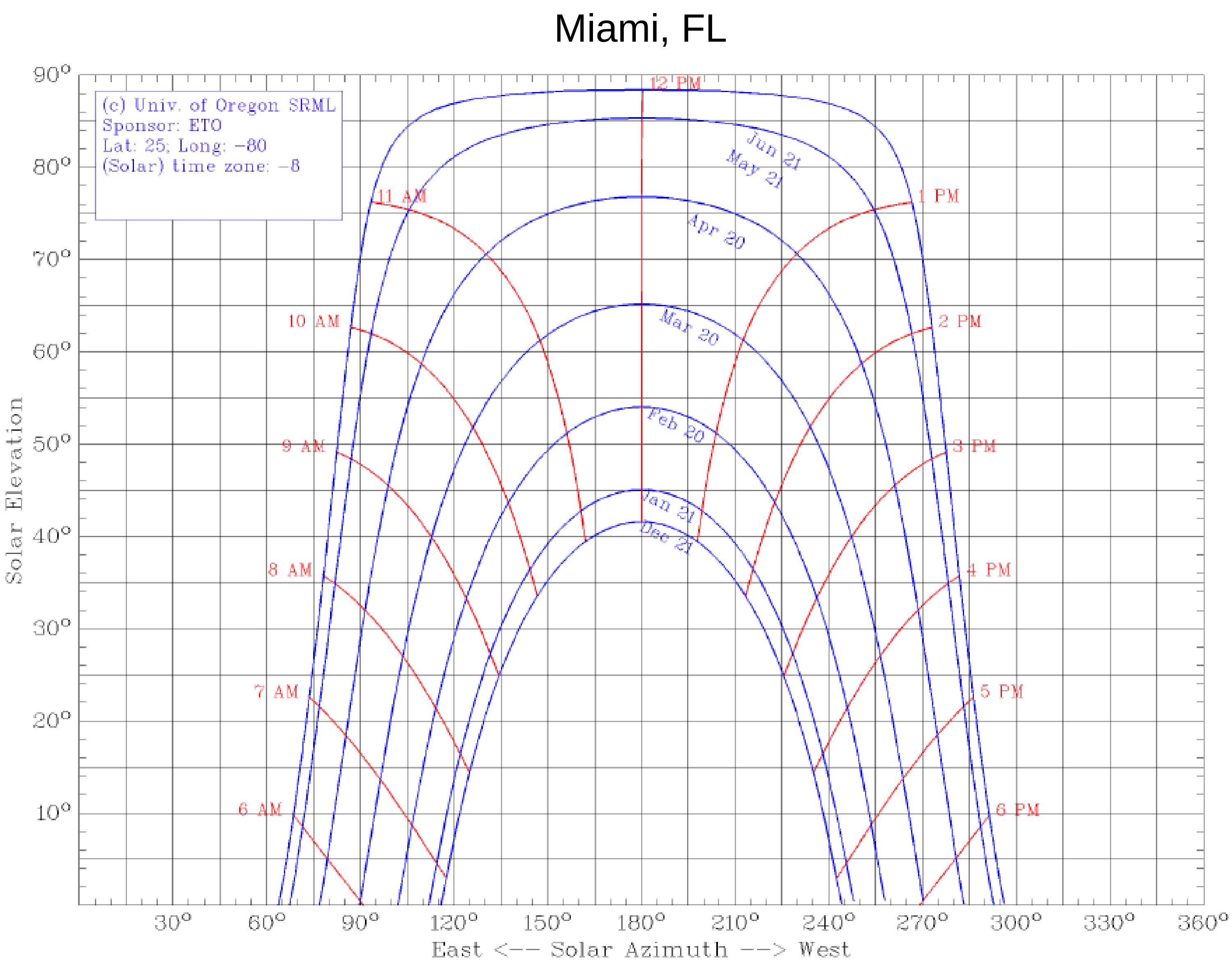
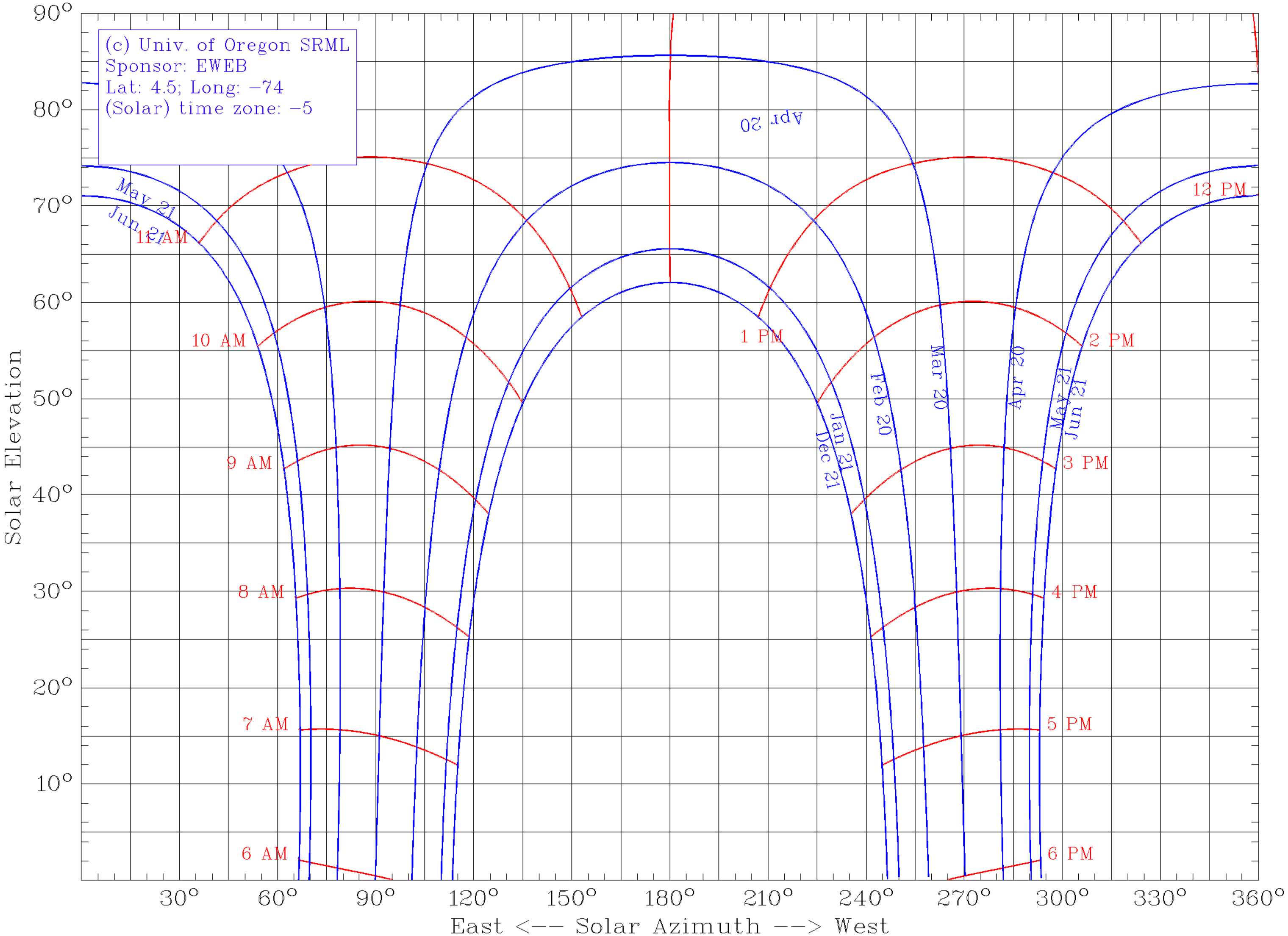


# http://solardat.uoregon.edu/SunChartProgram.html

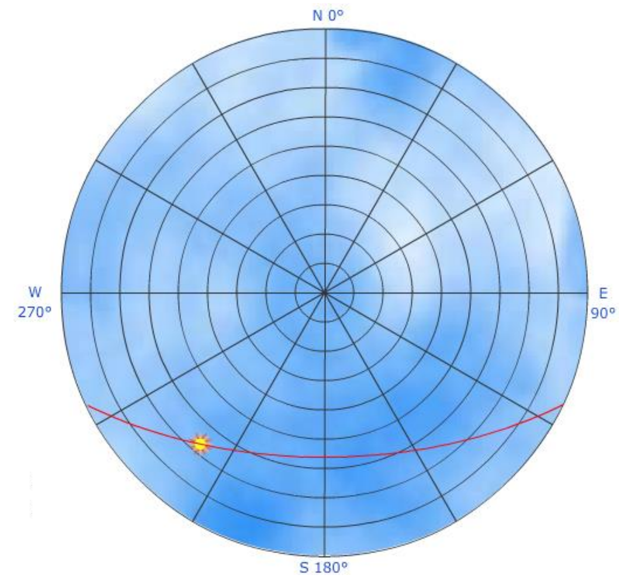
## Bogota, Columbia



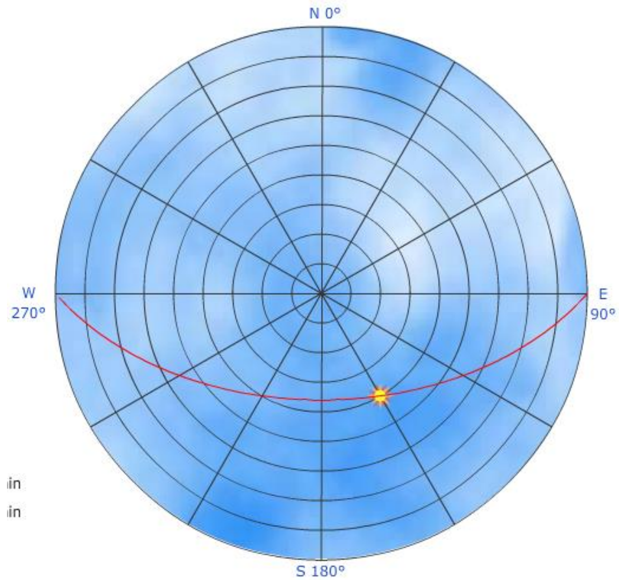
# Sun position on polar plot: (radial = azimuth, concentric: altitude)

Las Vegas NV:
longitude (115$^0$ 10’ W), latitude (36$^o$ 10’ N)

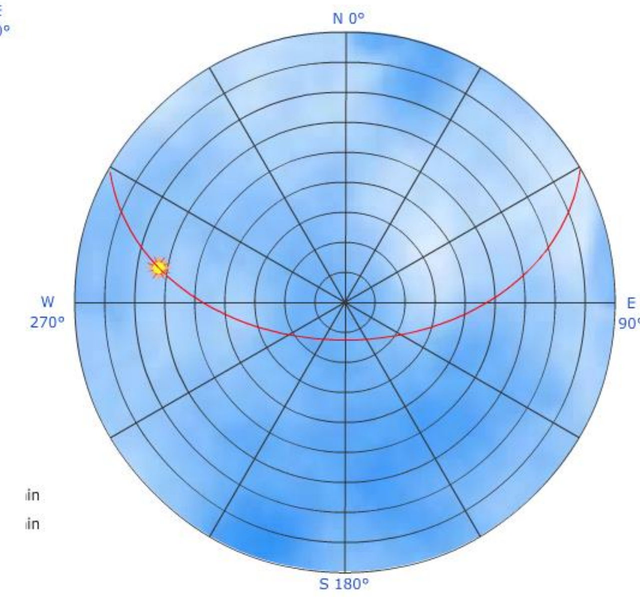
- n = 21, ST: 15:05
- →Altitude: 23$^o$
- →Azimuth: -40$^o$




- n = 81, ST: 11:13
- →Altitude: 50$^o$
- →Azimuth: 30$^o$



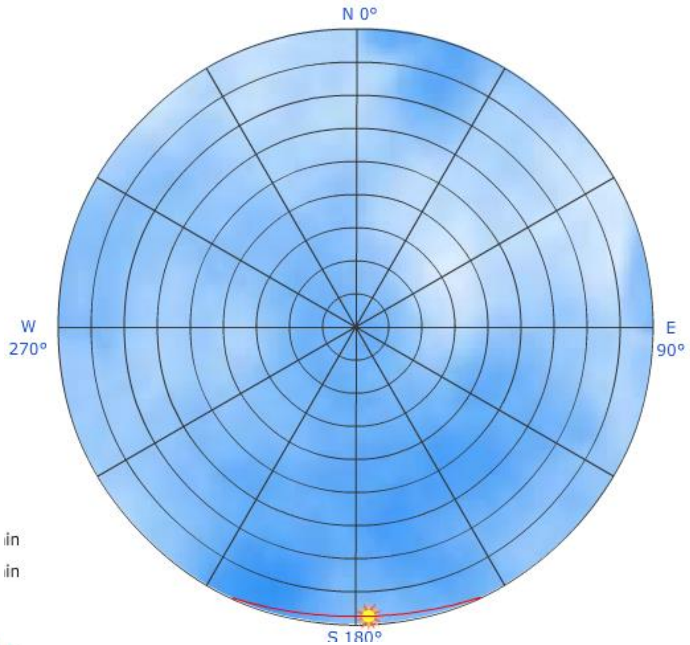

- n = 173, ST: 17:13
- →Altitude: 27$^o$
- →Azimuth: -100$^o$

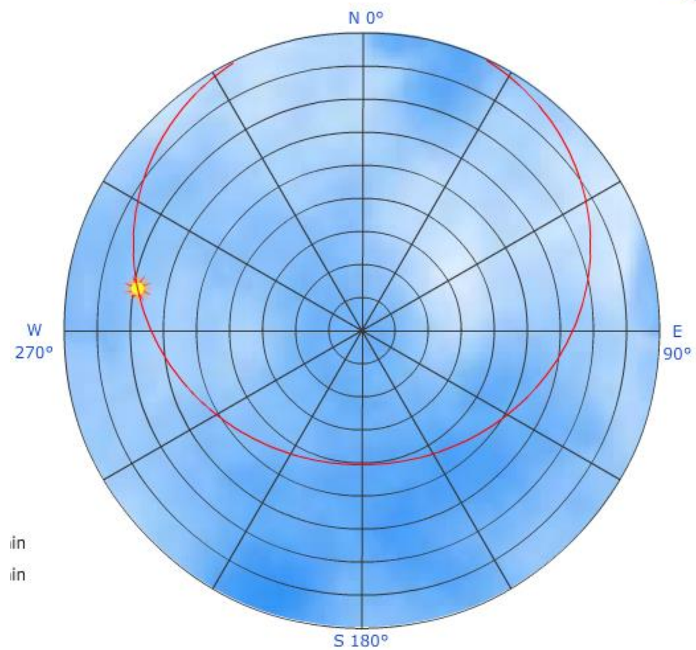


# Sun position on polar plot: (radial = azimuth, concentric: elevation)

Fairbanks, AK:
longitude ($147^0$ 43' W), latitude ($64^o$ 50' N)

- n = 356, ST: 11:00
- →Altitude: $2.5^o$
- →Azimuth: $2.5^o$



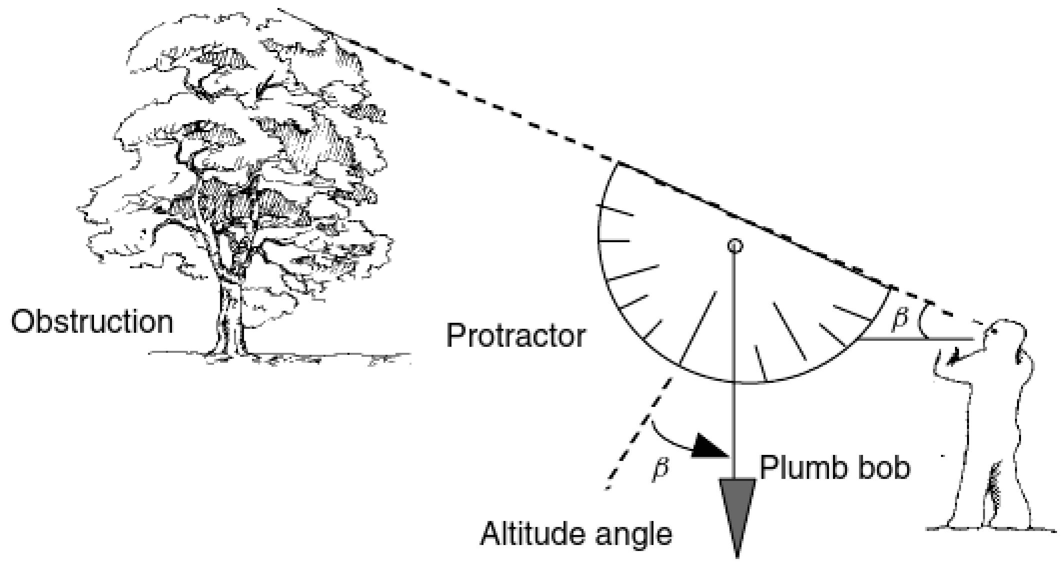



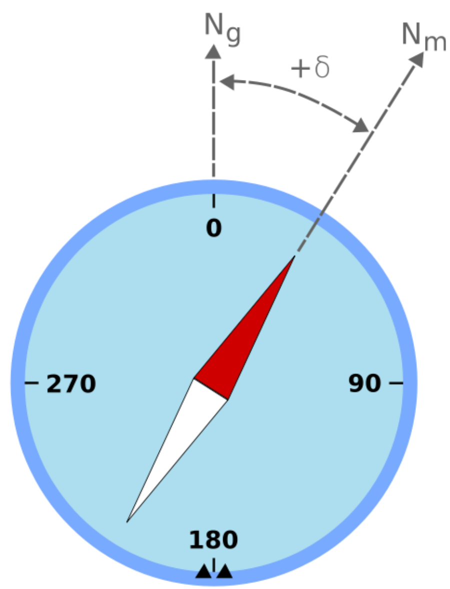

- d = 173, ST: 17:13
- →Altitude: $21^o$
- →Azimuth: $-100^o$

# Method to Determine the Impact of Shading Using Sun Path Diagrams

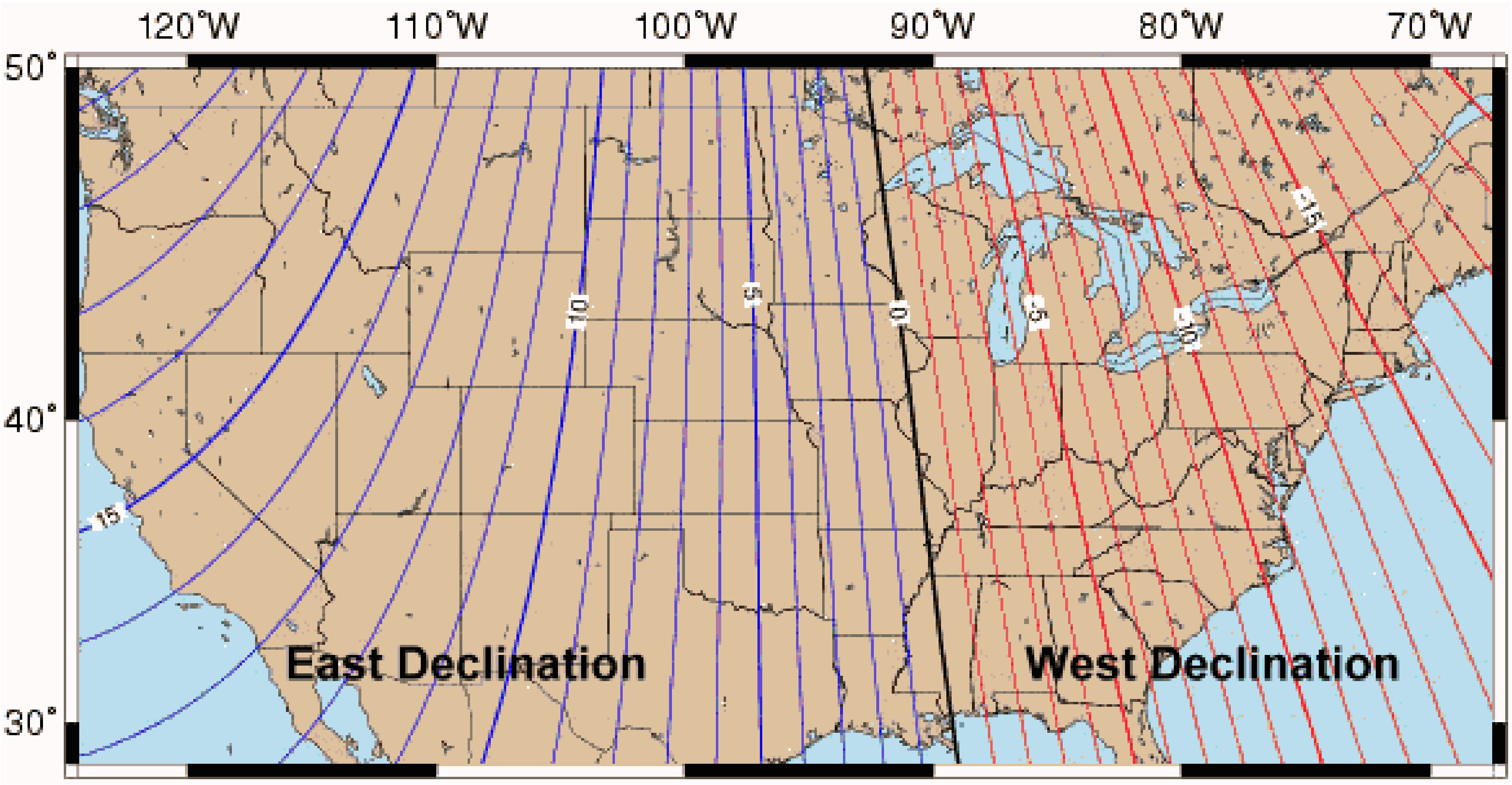
- Measure altitude angles of southerly obstructions using a protractor and a plummet.

- Measure the azimuth angles of these obstructions using a compass (make sure to adjust these angles to the true south).

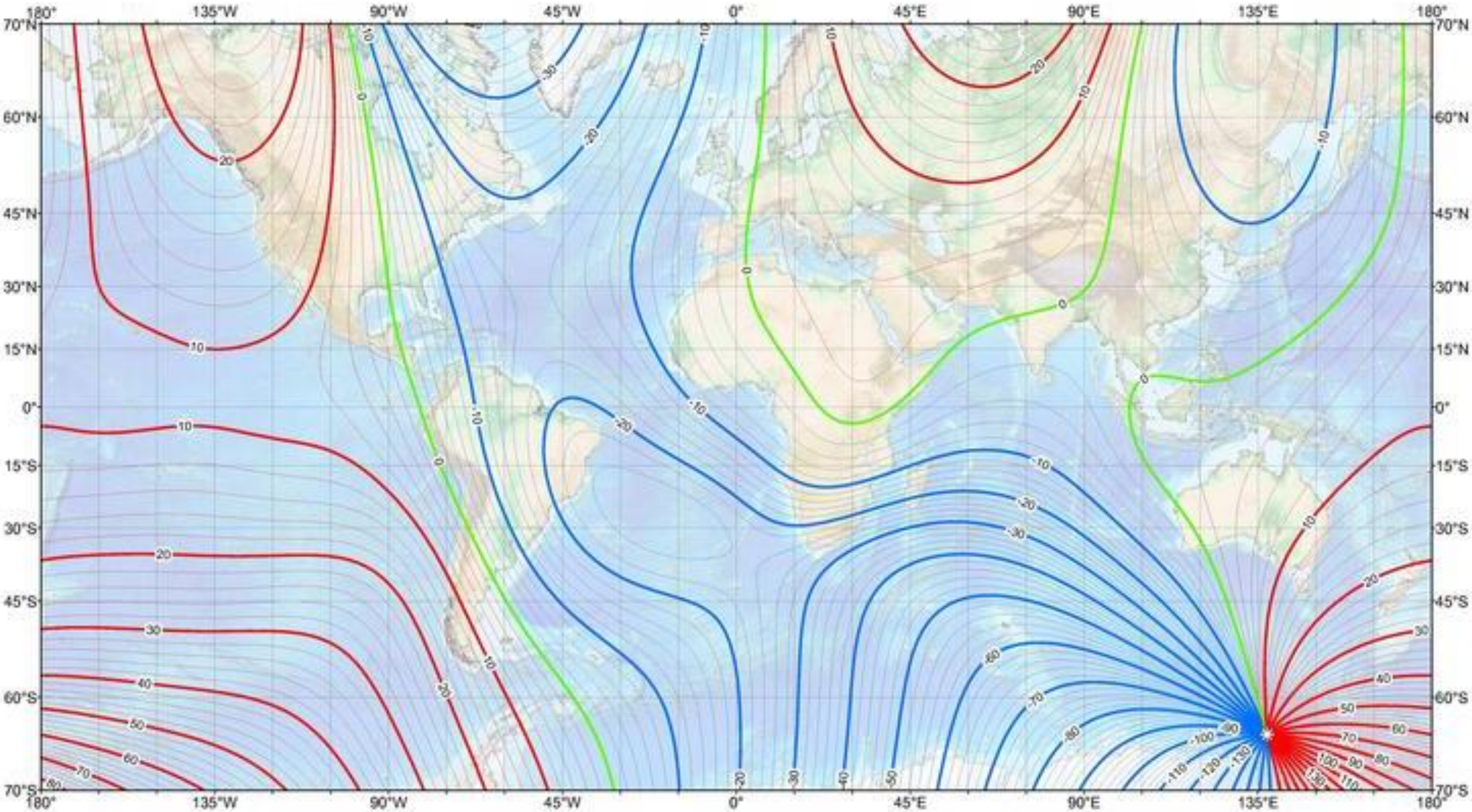


# Lines of Equal Magnetic Declination Angles

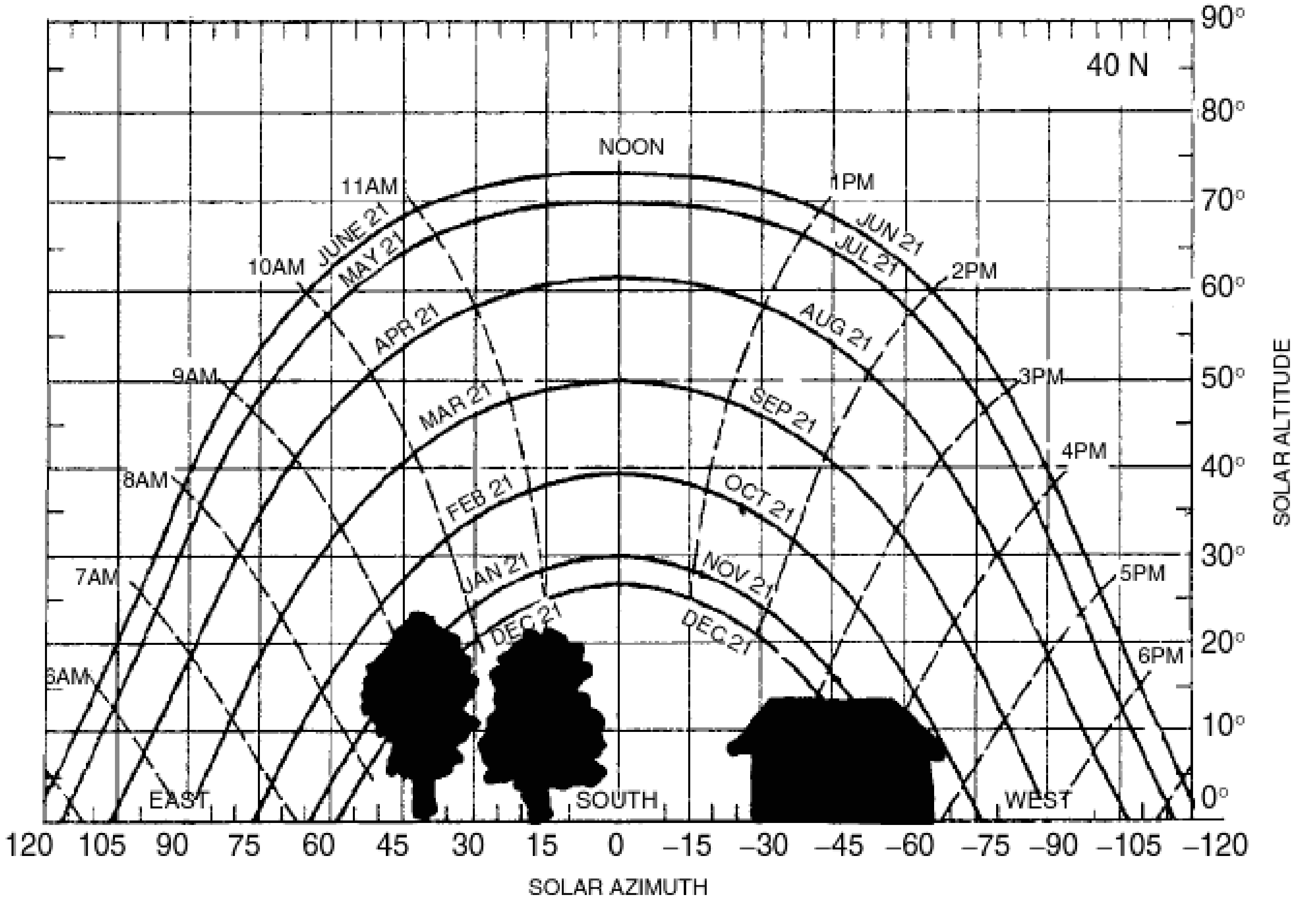


# World Magnetic Field Declination Angles (2010)

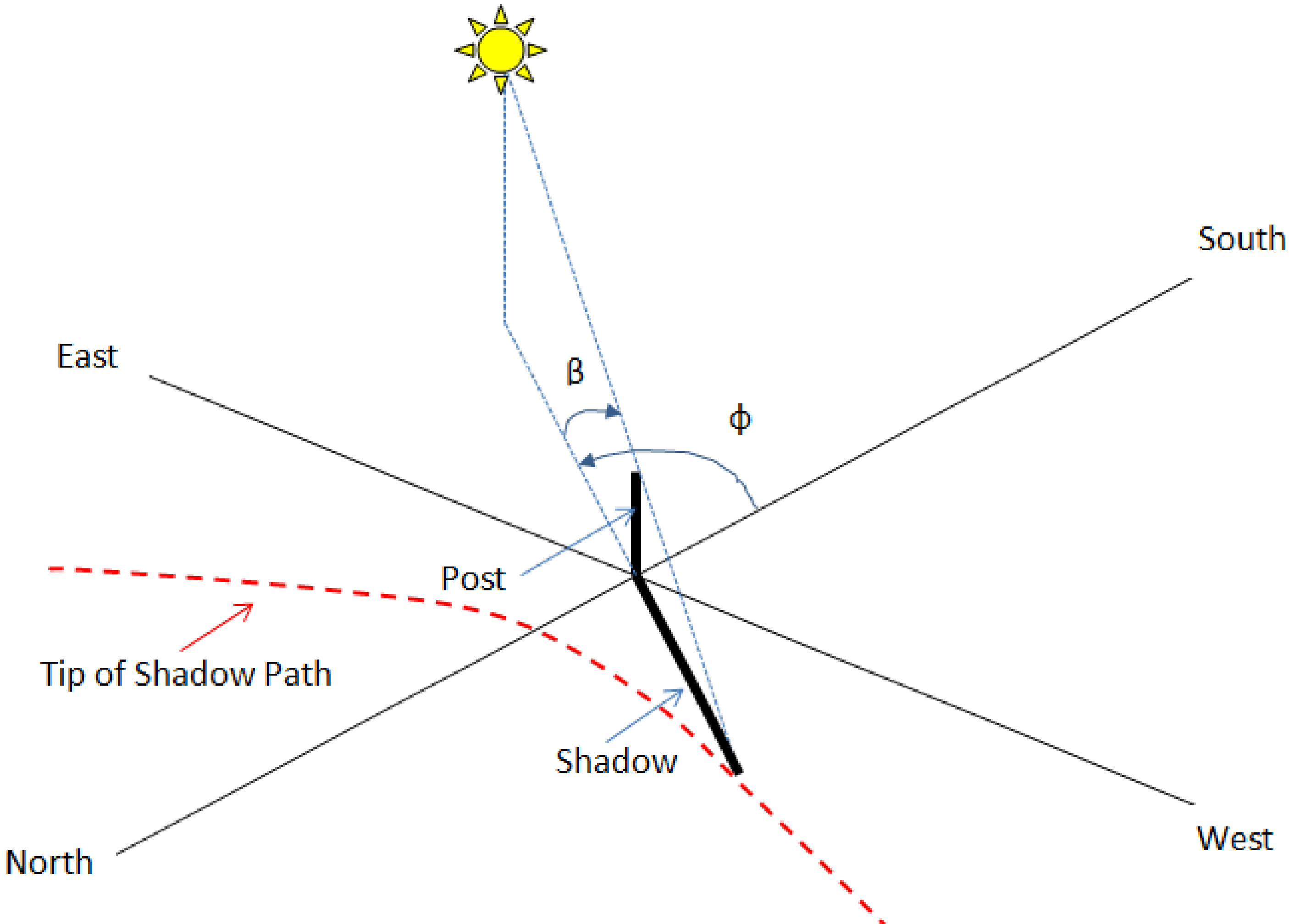


# Sun Path Diagrams and Shading

- Draw a sketch of the obstructions using the measured angles and superimpose on the sun path diagram.
- Estimate the amount of sunlight hours lost.



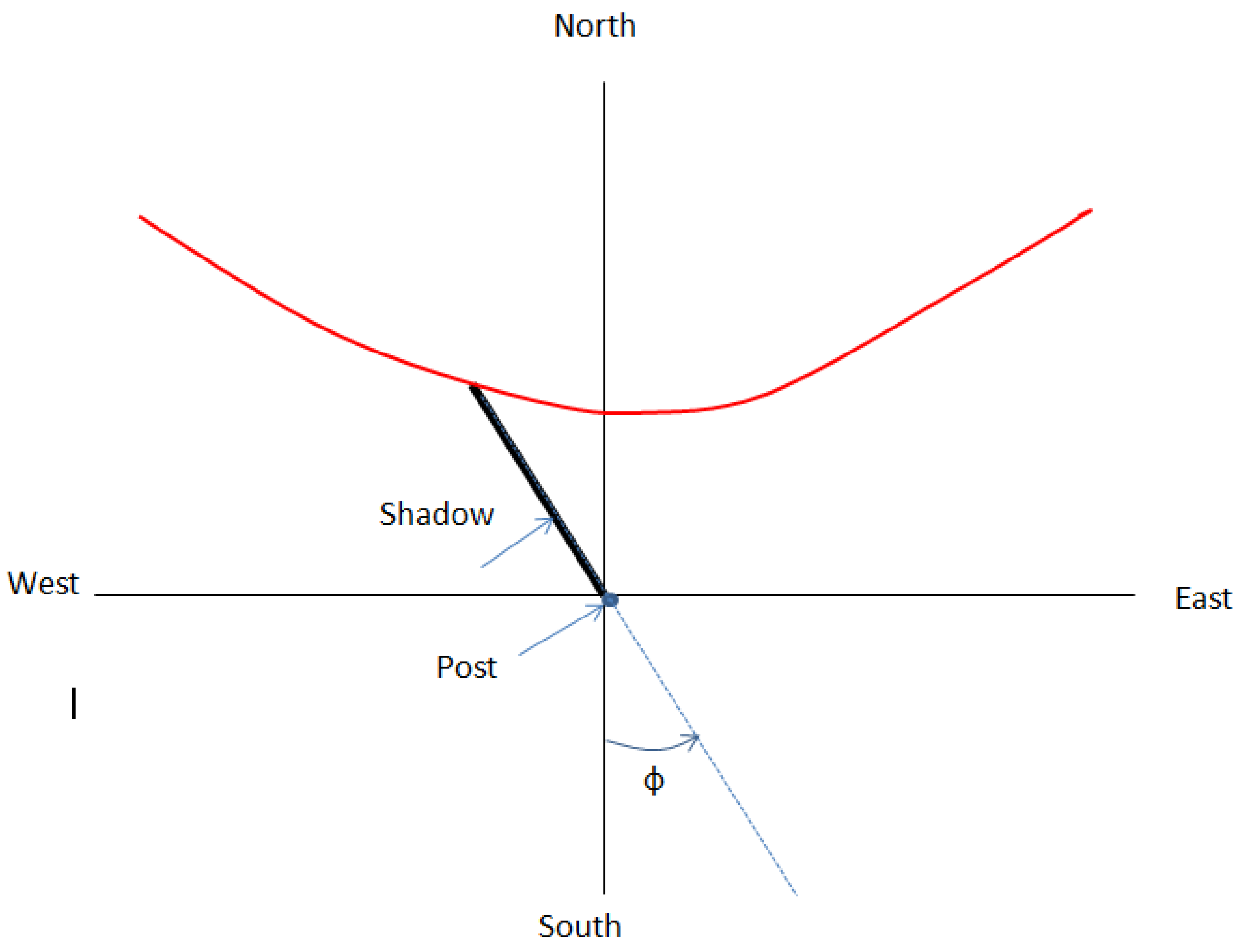
# Characterizing the Shadow of a Post




Shadow length: Post Height/ tanβ

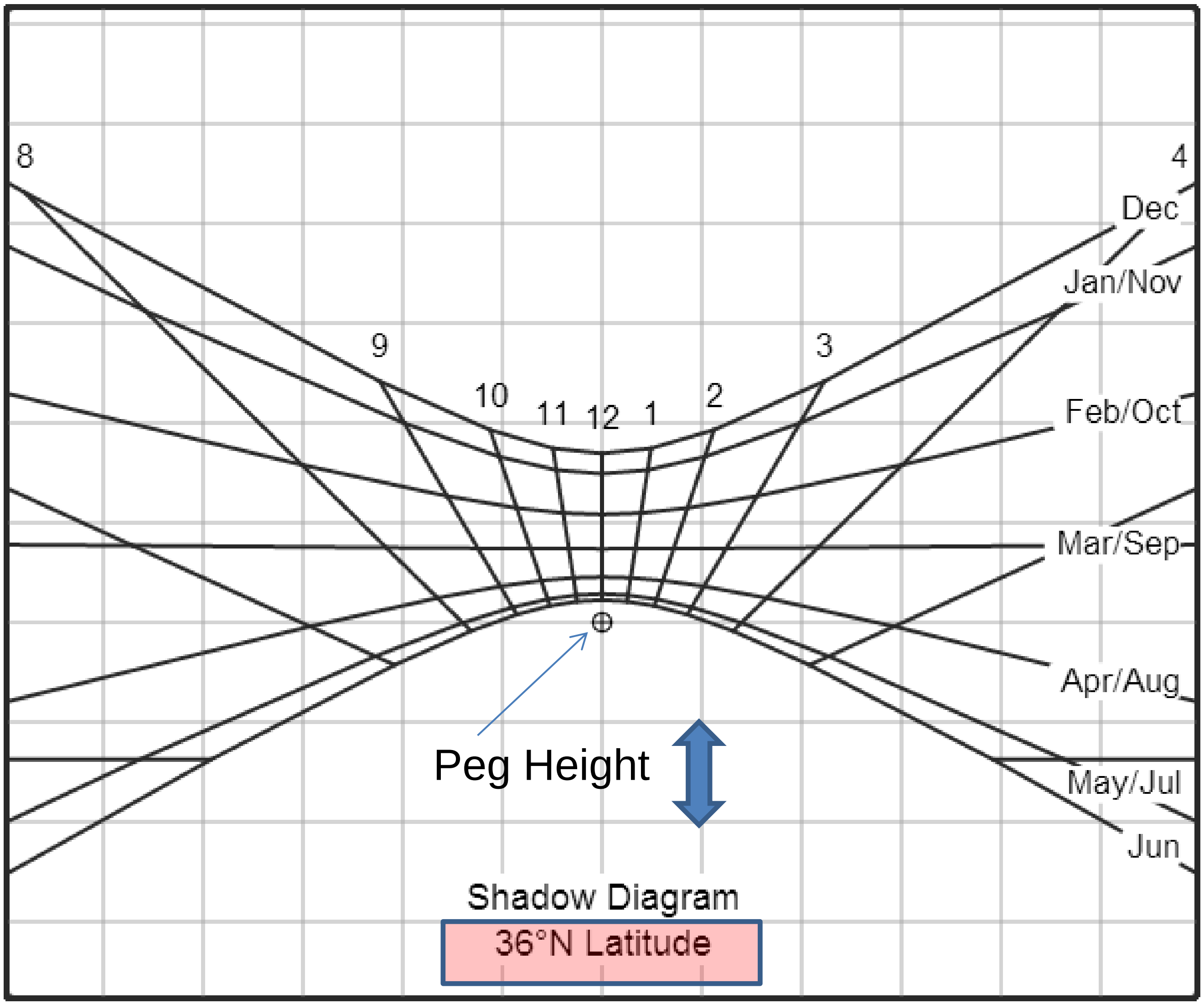
Shadow direction: (90° – ϕ) North of West

# Top View of Post and its Shadow Path

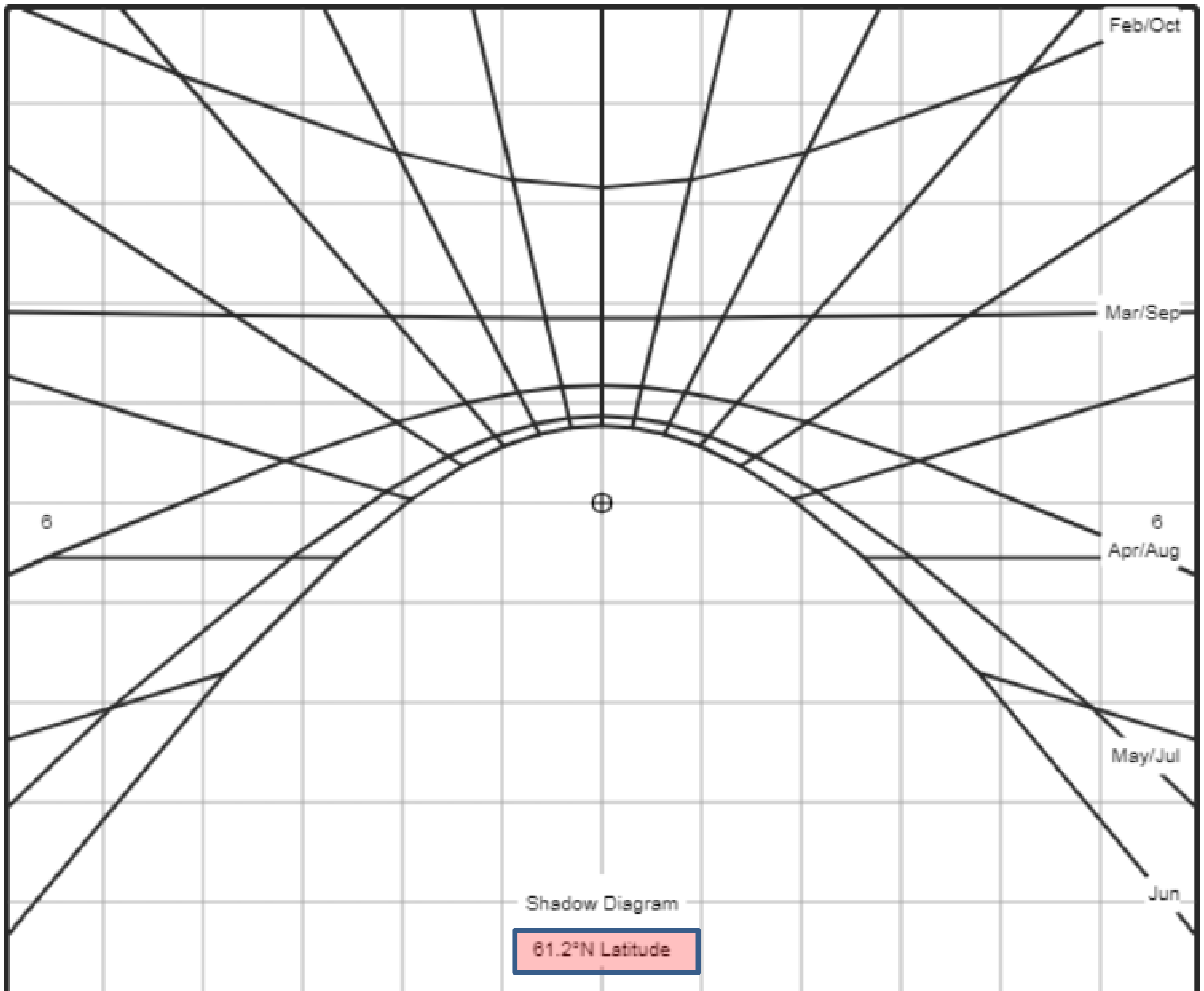


# Shadow diagram creator

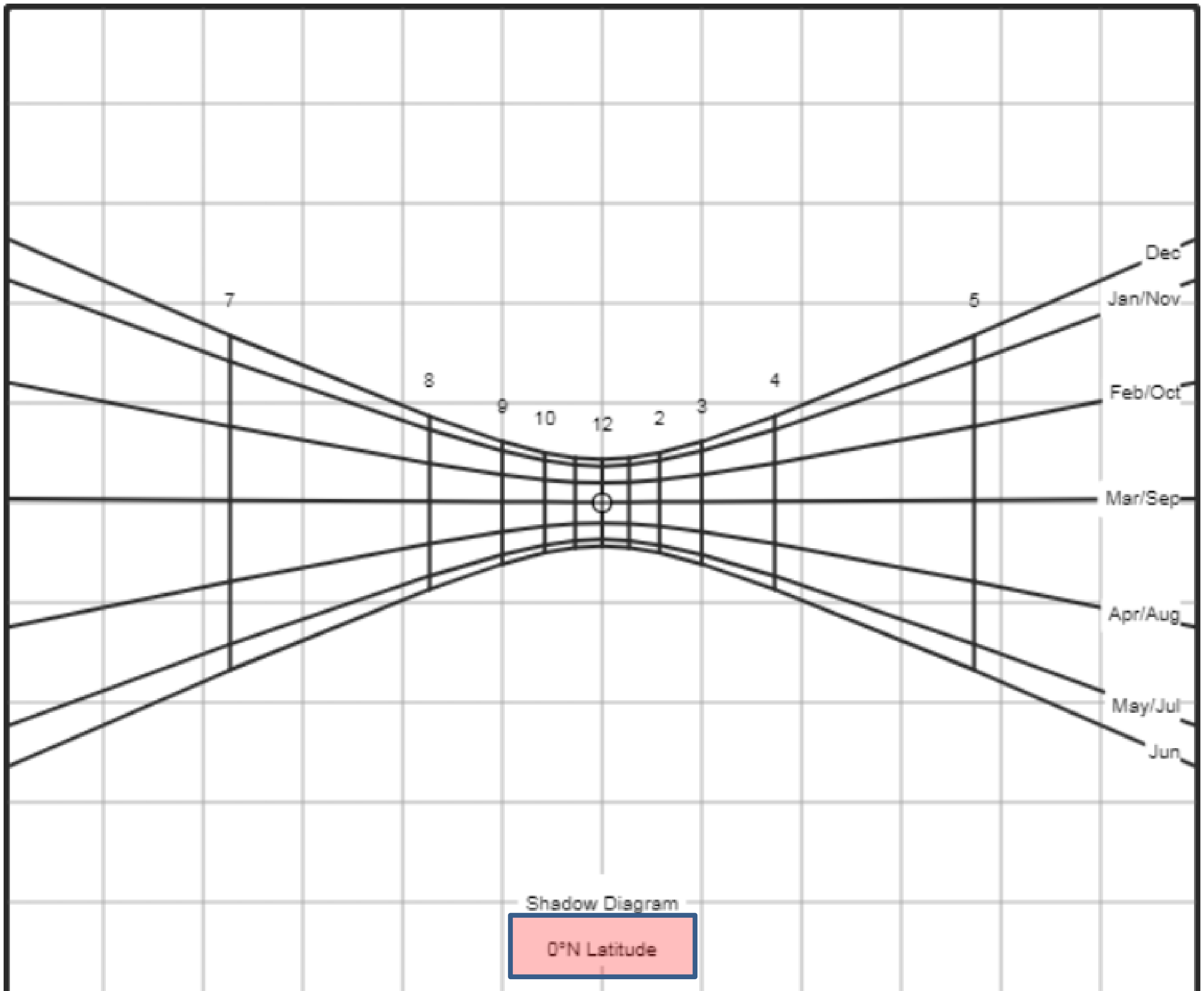
http://stanford.edu/group/shadowdiagram



# Shadow Diagram



# Shadow Diagram



# Practice Problems

- Determine the sun position in Las Vegas, NV (Latitude = 36 deg.) at 3:30pm today (Sept. 10).

  Ans: ……………
- Compute today's sunrise and sunset times and daylight hours (all times are in terms of solar time).

  Ans:……………….
- Find the sun position at 10:00am (solar time) in San Francisco, CA (latitude = 38 deg.) on July 21$^{st}$.

  Ans:…………….
- Determine the shadow length and orientation of a utility pole that is 35 feet high in Las Vegas, NV, at 8:00 am on October 21.

  Ans: …………..
- Solve the first 5 problems in chapter 4 of your book.