Chapter 4: Problems

4.1 Consider the design of a "light shelf" for the south side of an office building located at a site with latitude 30°. The idea is that the light shelf should help keep direct sunlight from entering the office. It also bounces light up onto the ceiling to distribute natural daylight more uniformly into the office.

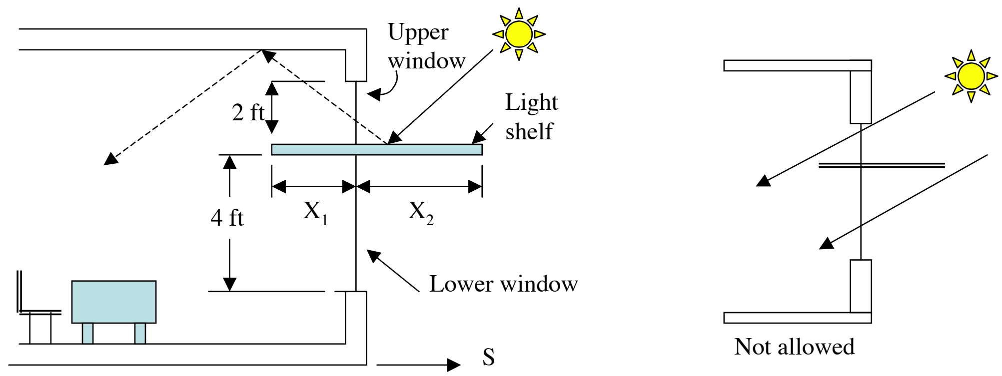


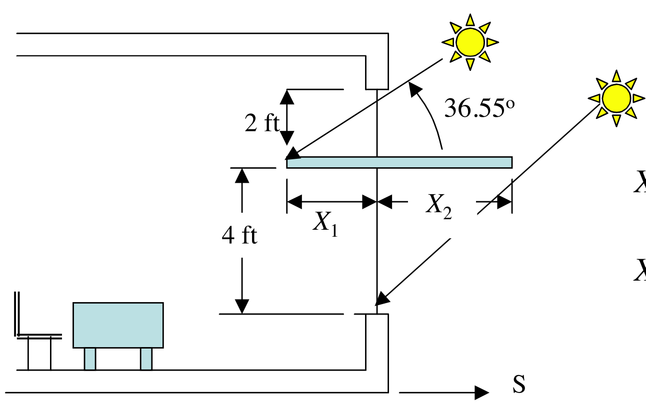
Figure P4.X

As shown, the window directly above the light shelf is 2-feet high and the window directly below is 4-ft. What should the dimensions X_1 and X_2 be to be sure that no direct sunlight ever enters the space at solar noon?

SOLN: The worst day will be at the winter solstice when the sun is lowest in the sky, at which point the declination is -23.45°.

From (4.7) the noon altitude angle of the sun is

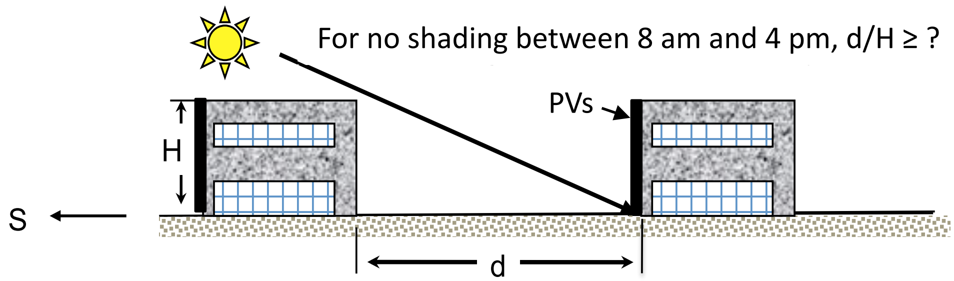
$$\beta_N = 90 - L + \delta = 90 - 30 - 23.45 = 36.55^o$$



$$X_1 = \frac{2}{\tan 36.55^o} = 2.7 \text{ ft}$$

$$X_2 = \frac{4}{\tan 36.55^o} = 5.4 \text{ ft}$$

4.2 Rows of buildings with photovoltaics covering vertical south-facing walls need to have adequate spacing to assure one building doesn't shade the collectors on another.

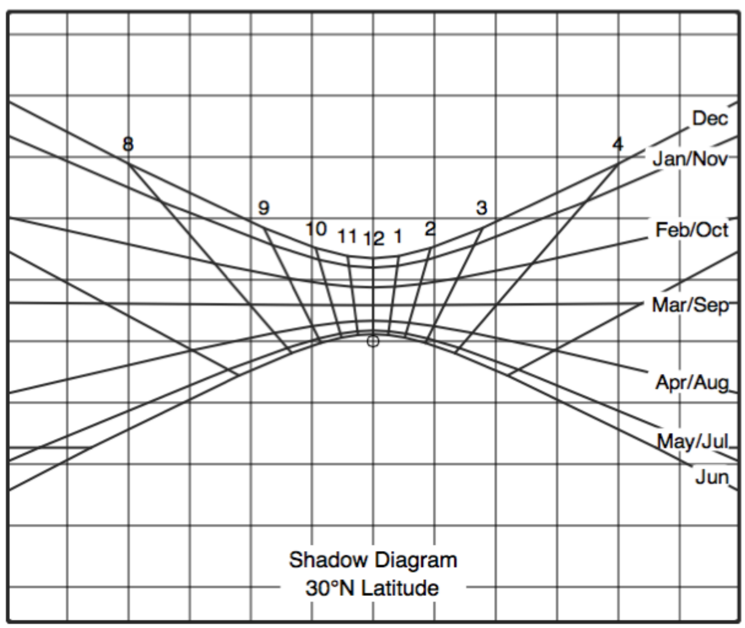


a. Using the shadow diagram for 30ºN in Appendix F, roughly what ratio of separation distance (d) to building height (H) would assure no shading anytime between 8 am and 4 pm?

b. If the spacing is such that d = H, during what months will the rear building receive full solar exposure.

a. SOLN: 8 am and 4 pm are just about 3 squares back, so with the peg being H, that means the separation d is about 3 H… d/H ≥ 3. (or, more accurately, d/H ≥ 2.8)

b. SOLN: Looks like full sun for all of spring and summer, but shading begins in October and runs into February .



4.3 Consider the challenge of designing an overhang to help shade a south-facing, sliding-glass patio door. You would like to shade the glass in the summer to help control air-conditioning loads, and you would also like the glass to get full sun in the winter to help provide passive solar heating of the home.

Suppose the slider has a height of 6.5 ft, the interior ceiling height is 8 ft, and the local latitude is 40º.

a. What should be the overhang projection *P* to shade the entire window at solar noon during the solstice in June?

b. With that overhang, where would the shade line, *Y*, be at solar noon on the winter solstice?

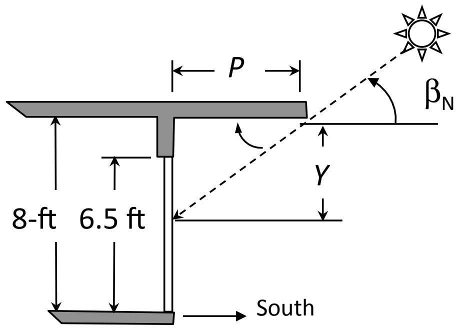


Figure P4.X

a. SOLN: To shade the window in June

$$\text{Summer solstice } \beta_N = 90 - L + \delta = 90 - 40 + 23.45 = 73.45^o$$

$$P = \frac{Y}{\tan\beta_N} = \frac{8}{\tan 73.45^o} = 2.377 \text{ ft}$$

b. SOLN: In winter, when you want the sun, the shade line will be at

$$\text{Winter solstice } \beta_N = 90 - 40 - 23.45 = 26.55^o$$

$$Y = P\tan\beta_N = 2.377\tan 26.55^o = 1.188 \text{ ft}$$

So, the shade line is 8 ft - 1.188 ft = 6.8 ft above the floor, so the entire window is in the sun.

c. The shadow distance *Y* for a south-facing window when it is not solar noon is given by

$$Y = \frac{P\tan\beta}{\cos\phi_S}$$

Will the bottom of the shadow line designed for solar noon still shade the window at 10 am?

SOLN: 10:00 am is 2 h before solar noon, so H = 2h x 15°/h = 30°. From (4.8)

$$\sin\beta = \cos L\cos\delta\cos H + \sin L\sin\delta$$

$$= \cos 40\cdot\cos 23.45\cdot\cos 30 + \sin 40\cdot\sin 23.45$$

$$\beta = 59.82^o$$

From (4.9)

$$\sin\phi_S = \frac{\cos\delta\sin H}{\cos\beta} = \frac{\cos 23.45\cdot\sin 30}{\cos 59.82} = 0.912$$

$$\phi_S = 65.846^o$$

From the above equation

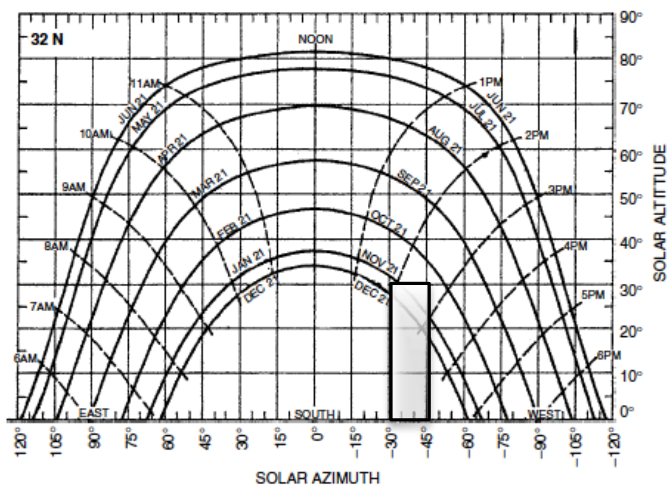
$$Y = \frac{P \tan\beta}{\cos\phi_S} = \frac{2.377 \tan 59.82}{\cos 65.846} = 9.99 \text{ ft}$$

So, the shadow still covers the window (ignoring edge effects). In fact, it will be shaded all day long so designing for solar noon is a good strategy.

4.4 Suppose you are concerned about how much shading a tree will cause for a proposed photovoltaic system. Standing at the site with your compass and plumb bob, you estimate the altitude angle of the top of the tree to be about 30° and the width of the tree to have azimuth angles that range from about 30° to 45° west of south. Your site is at latitude 32°.

Using a sun path diagram (Appendix C), describe the shading problem the tree will pose (approximate shaded times each month).

SOLN: The tree shades the site from roughly 2 pm to 3 pm from roughly mid-November through mid-January.



4.5 Suppose you are concerned about a tall thin tree located 100 ft from a proposed PV site. You don't have a compass or protractor and plumb bob, but you do notice that an hour before solar noon on June 21 it casts a 30 ft shadow directly toward your site. Your latitude is 32° N.

a. How tall is the tree?

b. What is its azimuth angle with respect to your site?

c. Using an appropriate sun path diagram from Appendix C, roughly what are the first and last days in the year when the shadow will land on the site?

SOLN:

a. It is the summer solstice so $\delta = 23.45°$ and it is an hour before solar noon so the hour angle is 15°. From (4.8) the altitude angle of the sun is

$$\sin\beta = \cos L \cos\delta \cos H + \sin L \sin\delta$$
$$= \cos 32 \cos 23.45 \cos 15 + \sin 32 \sin 23.45 = 0.962$$
$$\beta = 74.23^o$$

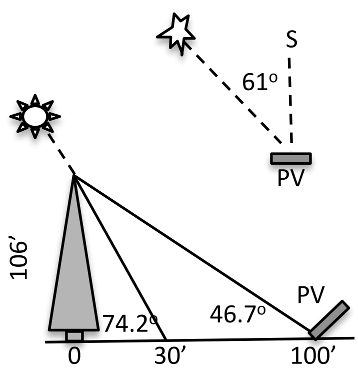
From h/30 = tan 74.23° the tree height is h = 30 tan 74.23° = 106 ft

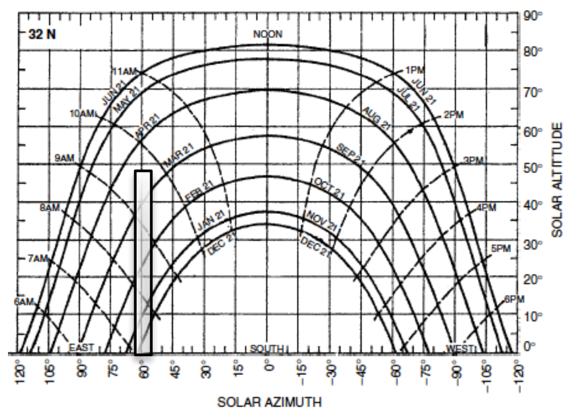
b. From (4.9) the azimuth angle of the tree is

$$\phi_S = A\sin\left(\frac{\cos\delta \sin H}{\cos\beta}\right) = A\sin\left(\frac{\cos 23.45 \sin 15}{\cos 74.23}\right) = 60.9^o$$

c. The altitude angle of the tree relative to the site is β = Atan(106/100) = 46.7°

From the sun path diagram for 32° and using the altitude of the tree as 46.7° and the azimuth 60.6° it looks like the site is shaded briefly each day from early September to early April.





4.6 Using Figure 4.18, what is the greatest difference between local standard time and solar time for the following locations? At approximately what date would that occur?

a. San Francisco, CA (longitude 122°, Pacific Time Zone)

b. Boston, MA (longitude 71.1°, Eastern Time Zone)

c. Boulder, CO (longitude 105.3°, Mountain Time Zone)

d. Greenwich, England (longitude 0°, Local time meridian 0°)

SOLN:

Using (4.14) ST – CT = 4(Local Meridian – Local Long) + E

The extremes of the equation of time Emax ≈ +16.5 min around n = 303, or about October 30, and Emin ≈ - 14.6 min around n = 44, or about February 13. Using time zones from Table 4.3:

a. San Francisco: MAX = 4(120 – 122) + E = -8 –14.6 = -22.6 minutes around February 13

b. Boston: MAX = 4(75 – 71.1) + E = 15.6 + 16.5 = 32.1 minutes around October 30

c. Boulder: 4(105 – 105.3) + E = - 1.2 - 14.6 = -15.8 minutes around February 13

(check -1.2 + 16.5 = 15.3 around October 30… not quite as bad)

d. Greenwich: MAX = 4(0 – 0) + E = 16.5 minutes around October 30

4.7 Calculate the following for (geometric) sunrise in Seattle, latitude 47.63°, longitude 122.33° W (in the Pacific Time Zone), on the summer solstice (June 21st).

a. Find the azimuth angle of sunrise relative to due south.

b. Find the time of sunrise expressed in solar time.

c. Find the local time of sunrise. Compare it to the website http://aa.usno.navy.mil/data/docs/RS_OneDay.html. Why do they differ?

SOLN:

a. From (4.16) and (4.9) realizing β = 0 at sunrise:

$$(4.16)\quad \cos H = -\tan L \tan\delta = -\tan 47.63 \tan 23.45 = -0.4755 \quad \text{so } H = 118.39^o$$

$$(4.9)\quad \sin\phi_S = \frac{\cos\delta \sin H}{\cos\beta} = \frac{\cos 23.45 \sin 118.39}{\cos 0} = 0.807$$

so ϕ_S could be either 53.81° or 180-53.81 = 126.19°

Test this using (4.11)

$$\cos H = \cos 118.39 = -0.475 \quad \text{and} \quad \frac{\tan\delta}{\tan L} = \frac{\tan 23.45}{\tan 47.63} = 0.396$$

$$\text{since } \cos H < \frac{\tan\delta}{\tan L} \quad \text{then} \quad |\phi_S| > 90$$

So the azimuth angle at sunrise is 126.19° toward the north-east (which is obvious from any sun path diagram).

b. Sunrise can be found from the hour angle

$$Sunrise = \frac{H}{15^o/h} = \frac{118.39^o}{15^o{}_{/h}} = 7.893h = 7h\ 53.56\,\text{min before solar noon}$$

That is, in solar time, Sunrise = 4:06.44 am.

c. For local time, we need to use the equation of time. From Table 4.1, June 1st is day number n = 152. So June 21st is n = 172.

$$(4.13)\qquad \beta = \frac{360}{364}(n-81) = \frac{360}{364}(172-81) = 90^o$$

$$(4.12)\quad \begin{aligned} E &= 9.87 \sin 2B - 7.53 \cos B - 1.5 \sin B \\ &= 9.87 \sin(2\cdot 90) - 7.53 \cos 90 - 1.5 \sin 90 = -1.5\,\text{min} \end{aligned}$$

For Seattle. longitude 122.33° in the Pacific Time Zone, with local time meridian 120°

$$CT = ST - 4\ (min/^o)(Local\ time\ meridian\ -\ Local\ longitude) - E(min)$$

CT = 4:06.44 am – 4 (120 – 122.33) –(-1.5) = 4:06.44 + 10.94 minutes = 4:17 am

(The navy website shows it to be 4:12 am in 2012… the 5 min difference is due to their use of the top rim of the sun instead of the center of the sun and they don't ignore refraction.. both features lead to their sunrise being about 5 minutes earlier than our geometric sunrise).

Daylight Savings Time for geometric sunrise is an hour later 5:17 am.

4.8 Suppose it is the summer solstice, June 21 (n = 172) and weather service says sunrise is 4:11 am pacific standard time and sunset is at 8:11 pm (PST). If we ignore the differences between geometric and weather service sunrise/sunset times we can use our equations to provide a rough estimate of local latitude and longitude.

a. Estimating solar noon as the midway point between sunrise and sunset, at what time will it be solar noon?

b. Use (4.12 - 4.14) to estimate your local longitude.

c. Use (4.17) to estimate your local latitude.

SOLN:

a. The total day length is 8:11 pm - 4:11 am = 16 h, so solar noon would be at about 4:11 + 8 h = 12:11 pm

That is, clock time is 11 minutes later than solar time.

b. From (4.12) and (4.13)

$$B=\frac{360}{364}(n-81)=\frac{360}{364}(172-81)=90^{o}$$

$$E=9.87\sin 2B-7.53\cos B-1.5\sin B$$

$$=9.87\sin 2\cdot(90)-7.53\cos(90)-1.5\sin(90)=-1.5\ minutes$$

Using the local time meridian for Pacific Standard Time of 120° in (4.14)

$$ST=CT+4\left(\text{min/}^{o}\right)\left(\text{Local time meridian-Local longitude}\right)+E(\text{min})$$

$$12:00=12:11+4\left(120-Longitude\right)-1.5$$

$$\text{Longitude}=\frac{11+480-1.5}{4}=122.4^{o}$$

c. With sunrise being 8 h before noon, the sunrise hour angle is

$$\text{Sunrise hour angle H}_{SR}=\left(\frac{15^{\circ}}{\text{hr}}\right)\left(\text{hours before solar noon}\right)$$

$$=\left(\frac{15^{\circ}}{\text{hr}}\right)\text{ x }\left(8\right)hrs=120^{o}$$

from (4.17) $\tan L=-\dfrac{\cos H}{\tan\delta}=-\dfrac{\cos 120^{o}}{\tan 23.45^{o}}=1.15$ Lat = 49°

(Actually, this is Seattle, Lat 47.6° and Long 122.3° so we're a bit off, as could be expected).

4.9 A south-facing collector at latitude 40° is tipped up at an angle equal to its latitude. Compute the following insolations for January 1st at solar noon:

a. The direct beam insolation normal to the sun's rays

b. Beam insolation on the collector

c. Diffuse radiation on the collector

d. Reflected radiation with ground reflectivity 0.2.

SOLN:

a. Beam normal to rays

$$A = 1160 + 75\sin\left[\frac{360}{365}(n-275)\right] = 1160 + 75\sin\left[\frac{360}{365}(1-275)\right] = 1235$$

$$k = 0.174 + 0.035\sin\left[\frac{360}{365}(1-100)\right] = 0.1393$$

$$\delta = 23.45\sin\left[\frac{360}{365}(1-81)\right] = -23.0^o$$

$$\beta_N = 90 - L + \delta = 90 - 40 - 23 = 27^o$$

$$m = \sqrt{(708\sin\beta)^2 + 1417} - 708\sin\beta = \sqrt{(708\sin 27)^2 + 1417} - 708\sin 27 = 2.197$$

$$I_B = Ae^{-km} = 1235e^{-0.1393x2.197} = 909.4W/m^2$$

b. Beam on collector

$$\cos\theta = \cos\beta\cos(\phi_S - \phi_C)\sin\Sigma + \sin\beta\cos\Sigma$$
$$= \cos 27\cos(0-0)\sin 40 + \sin 27\cos 40 = 0.9205$$

$$I_{BC} = I_B\cos\theta = 909.4x0.9205 = 837.1\ W/m^2$$

$$\cos\theta = \cos\beta\cos(\phi_S - \phi_C)\sin\Sigma + \sin\beta\cos\Sigma$$

$$= \cos 27^o\cos(0 - 0^o)\sin 40^o + \sin 27^o\cos 40^o = 0.9205$$

$$I_{BC} = I_B\cos\theta = 908.7\ W/m^2 \cdot 0.9205 = 836.5\ W/m^2$$

c. Diffuse

$$C = 0.095 + 0.04\sin\left[\frac{360}{365}(n-100)\right] = 0.095 + 0.04\sin\left[\frac{360}{365}(1-100)\right] = 0.05535$$

$$I_{DC} = C \cdot I_B\left(\frac{1+\cos\Sigma}{2}\right) = 0.05535x909.4\left(\frac{1+\cos 40}{2}\right) = 44.4\ W/m^2$$

d. Reflected

$$I_{RC} = I_B \rho \left(C + \sin\beta\right)\left(\frac{1-\cos\Sigma}{2}\right)$$

$$= 0.2 \cdot 909.4\left(0.05535 + \sin 27\right)\left(\frac{1-\cos 40}{2}\right) = 10.8\ W/m^2$$

e. Total insolation on the collector

$$I_C = I_{BC} + I_{DC} + I_{RC} = 836.5 + 44.4 + 10.8 = 892\ W/m^2$$

4.10 Create a "Clear Sky Insolation Calculator" for direct, diffuse and reflected radiation using the spreadsheet shown in Figure 4.26 as a guide. Confirm that it gives you 859 W/m^2 under the following conditions: August 1, 30^o latitude, tilt 40^o, southwest facing (-45^o), 3 pm solar time, reflectance 0.2.

Use the calculator to compute clear sky insolation under the following conditions (times are solar times):

a. Jan. 1, latitude 40^o, horizontal insolation, solar noon, reflectance =0

b. Mar. 15, latitude 20^o, south-facing collector, tilt 20^o, 11:00 am, ρ = 0.2.

c. July 1, latitude 48^o, south-east collector (azimuth 45^o), tilt 20^o, 2 pm, ρ = 0.3.

SOLN:

a. 463 W/m^2

b. 1035 W/m^2

c. 744 W/m^2

4.11 The following table shows TMY data (W/m^2) for Denver (latitude 39.8^o) on July 1 (n = 182, δ = 23.12^o). Calculate the expected irradiation on the following collector surfaces. Notice answers are given for some of them to help check your work.

TMY Time	TMY GHI	TMY DNI	TMY DHI
5.00	-	-	-
6.00	92	334	40
7.00	273	608	68
8.00	476	744	93
9.00	667	820	114
10.00	825	864	128
11.00	938	889	138
12.00	998	901	143
13.00	1,000	901	143
14.00	944	890	138
15.00	835	866	129
16.00	637	677	172
17.00	455	608	134
18.00	172	51	154
19.00	67	104	49

a. South-facing, fixed 40° tilt, reflectance 0.2, solar noon.

SOLN:

$$\delta = 23.45\sin\left[\frac{360}{365}(n-81)\right] = 23.45\sin\left[\frac{360}{365}(182-81)\right] = 23.12^o$$

$$\beta_N = 90 - L + \delta = 90 - 39.8 + 23.12 = 73.32$$

$$\begin{aligned}\cos\theta &= \cos\beta\cos(\phi_S-\phi_C)\sin\Sigma + \sin\beta\cos\Sigma \\ &= \cos 73.32^o\cos(0-0)\sin 40 + \sin 73.32\cos 40 \\ &= 0.9183\end{aligned}$$

$$I_{BC} = DNI\cos\theta = 901x0.9183 = 827.4 \text{ W/m}^2$$

$$I_{DC} = I_{DH}\left(\frac{1+\cos\Sigma}{2}\right) = 143\left(\frac{1+\cos 40}{2}\right) = 126.3 \text{ W/m}^2$$

$$I_{RC} = \rho I_H\left(\frac{1-\cos\Sigma}{2}\right) = 0.2x998\left(\frac{1-\cos 40}{2}\right) = 23.3 \text{ W/m}^2$$

$$I_C = I_{BC} + I_{DC} + I_{RC} = 827.4 + 126.3 + 23.3 = 977 \text{ W/m}^2$$

b. South-facing, fixed 30° tilt, reflectance 0.2 solar noon. (ans: 1024 W/m²)

c. Horizontal, north-south axis, tracking collector (HNS), reflectance 0.2, at 11:00 am solar time.

SOLN:

$$\delta = 23.45\sin\left[\frac{360}{365}(n-81)\right] = 23.45\sin\left[\frac{360}{365}(182-81)\right] = 23.12^o$$

$$\sin\beta = \cos L\cos\delta\cos H + \sin L\sin\delta$$
$$= \cos 39.8\cos 23.12\cos 15 + \sin 39.8\sin 23.12 = 0.8729$$
$$\beta = 69.04^o$$

$$\sin\phi_S = \frac{\cos\delta\sin H}{\cos\beta} = \frac{\cos 23.12\sin 15}{\cos 69.04} \qquad \phi_S = 41.72^o$$

$$\cos\theta = \sqrt{1-(\cos\beta\cos\phi_S)^2} = \sqrt{1-(\cos 69.04\cos 41.72)^2} = 0.964$$

$$I_{BC} = DNI\cos\theta = 889x0.964 = 857 \text{ W/m}^2$$

$$I_{DC} = I_{DHI}\left[\frac{1+\sin\beta/\cos\theta}{2}\right] = 138\left[\frac{1+\sin 69.04/0.964}{2}\right] = 136 \text{ W/m}^2$$

$$I_{RC} = \rho I_H\left[\frac{1-\sin\beta/\cos\theta}{2}\right] = 0.2x938\left[\frac{1-\sin 69.04/0.964}{2}\right] = 3 \text{ W/m}^2$$

$$I_C = I_{BC} + I_{DC} + I_{RC} = 857 + 136 + 3 = 996 \text{ W/m}^2$$

d. Horizontal north-south axis, tracking collector (HNS), reflectance 0.2, at solar noon (ans: 1006 W/m^2).

e. Two-axis tracker, reflectance 0.2, at solar noon.

SOLN:

$$\cos\theta = 1$$

$$I_{BC} = DNI\cos\theta = 901x1 = 901 \text{ W/m}^2$$

$$I_{DC} = I_{DH}\left(\frac{1+\sin\beta}{2}\right) = 143\left(\frac{1+\sin 73.32}{2}\right) = 140 \text{ W/m}^2$$

$$I_{RC} = \rho I_{GHI}\left(\frac{1-\sin\beta}{2}\right) = 0.2x998\left(\frac{1-\sin 73.32}{2}\right) = 4 \text{ W/m}^2$$

$$I_C = I_{BC} + I_{DC} + I_{RC} = 901 + 140 + 4 = 1045 \text{ W/m}^2$$

f. Two-axis tracker, reflectance 0.2, at 11:00 am solar time. (Ans. 1029 W/m^2)

g. One-axis tracker, vertical mount (VERT), tilt = 30^o at 11:00 am solar time, reflectance 0.2. (Ans: 1019 W/m^2)

h. One-axis tracker, vertical mount (VERT), tilt = 30^o at solar noon, reflectance 0.2.

SOLN:

$$I_{BC} = I_{DNI}\sin(\beta + \Sigma) = 901\sin(73.32 + 30) = 876.7 \text{ W/m}^2$$

$$I_{DC} = I_{DHI}\left(\frac{1+\cos\Sigma}{2}\right) = 143\left(\frac{1+\cos 30}{2}\right) = 133.4W/m^2$$

$$I_{RC} = \rho GHI\left(\frac{1+\cos\Sigma}{2}\right) = 0.2x998\left(\frac{1-\cos 30}{2}\right) = 13.4W/m^2$$

$$I_C = I_{BC} + I_{DC} + I_{RC} = 876.7 + 133.4 + 13.4 = 1024 \text{ W/m}^2$$

4.12 Download TMY3 data for Mountain View (Moffett Field, latitude 37.4), CA from the NREL website (http://rredc.nrel.gov/solar/old_data/nsrdb/1991-2005/tmy3). The raw data is in CSV format. By opening the CSV file in Excel, you can convert it to normal rows and columns of data. Be sure to save it then as an Excel file.

a. Use TMY3 to find the irradiation on a south-facing photovoltaic module with a fixed 18° tilt angle on September 21 (equinox) at solar noon. Assume 0.2 reflectance.

b. Compare that to clear-sky irradiation on that module.

SOLN:

a. Here's a portion of the data page

	A	B	C	D	E	F	G	H	I	J	K	
2	_ Webl	MOUNTAIN VIEW MOFFETT FLD NAS	CA	-8	37.4	-122	12					
3	Date (MM/DD /YYYY)	Time (HH:MM)	ETR (W/m^2)	ETRN (W/m^2)	GHI (W/m^2)	GHI source	GHI uncert (%)	DNI (W/m^2)	DNI source	DNI uncert (%)	DHI (W/m^2)	DH so
4	1/1/05	1:00	0	0	0	2	0	0	2	0	0	
871	9/2/97	12:00	1149	1342	879	1	10	876	1	16	129	
872	9/2/97	13:00	1158	1342	888	1	10	878	1	16	130	
873	9/2/97	14:00	1095	1342	832	1	24	865	1	27	125	
874	9/2/97	15:00	966	1342	716	1	10	835	1	16	115	

It's an equinox, so δ = 0 and β = 90 - Lat + δ = 90-37.4 = 52.6°, $\Phi_S = 0 = \Phi_C$.

$$\cos\theta = \cos\beta\cos(\phi_S - \phi_C)\sin\Sigma + \sin\beta\cos\Sigma$$

$$= \cos 52.6\cos(0-0)\sin 18 + \sin 52.6\cos 18 = 0.9432$$

$$I_{BC} = I_{DNI}\cos\theta = 865x0.9433 = 815.9 \; W/m^2$$

$$I_{DC} = I_{DHI}\left(\frac{1+\cos\Sigma}{2}\right) = 125\left(\frac{1+\cos 18}{2}\right) = 121.9 \text{ W/m}^2$$

$$I_{RC} = \rho I_{GHI}\left(\frac{1-\cos\Sigma}{2}\right) = 0.2x832\left(\frac{1-\cos 18}{2}\right) = 4.1 \text{ W/m}^2$$

$$I_C = I_{BC} + I_{DC} + I_{RC} = 815.9 + 121.9 + 4.1 = 942 \text{ W/m}^2$$

b. Clear sky insolation from our clear-sky spreadsheet is

I_C = 855 + 95 + 4 = 954 W/m² versus 942 for a "typical" Sept 21.

4.13 In Example 4.13 the average irradiation in September on a 30º fixed-tilt, south-facing collector in Oakland (latitude 37.73, horizontal insolation 7.32 kWh/m²-day, reflectivity 0.2) was estimated to be 6.7 kWh/m²-day. Repeat that calculation if the collector tilt angle is only 10º.

SOLN: From the example, the fraction of irradiation that is diffuse was found to be 0.259. That doesn't change, so the diffuse horizontal radiation is still

$$\overline{I}_{DH} = 0.259 \cdot 7.32 = 1.90 \text{ kWh/m}^2\text{-day}$$

The diffuse radiation on the collector is given by (4.55)

$$\overline{I}_{DC} = \overline{I}_{DH}\left(\frac{1+\cos\Sigma}{2}\right) = 1.90\left(\frac{1+\cos 10^o}{2}\right) = 1.886 \text{ kWh/m}^2\text{-day}$$

The reflected radiation on the collector is given by (4.56)

$$\overline{I}_{RC} = \rho\,\overline{I}_H\left(\frac{1-\cos\Sigma}{2}\right) = 0.2\cdot 7.32\left(\frac{1-\cos 10^o}{2}\right) = 0.01\; kWh/m^2 - day$$

From (4.51), the beam radiation on the horizontal surface is still

$$\overline{I}_{BH} = \overline{I}_H - \overline{I}_{DH} = 7.32 - 1.90 = 5.42 \text{ kWh/m}^2\text{-day}$$

To adjust this for the collector tilt, first find the sunrise hour angle on the collector from (4.59)

$$H_{SRC} = \min\left\{\cos^{-1}(-\tan L\ \tan\delta),\ \ \cos^{-1}\left[-\tan(L-\Sigma)\tan\delta\right]\right\}$$

$$= \min\left\{\cos^{-1}\left(-\tan 37.73^o\ \tan 21.35^o\right),\ \ \cos^{-1}\left[-\tan\left(37.73-10\right)^o \tan 21.35^o\right]\right\}$$

$$= \min\left\{107.6^o, 101.9^o\right\} = 101.9^o = 1.778\ radians$$

The beam tilt factor (4.58) is thus

$$\overline{R}_B = \frac{\cos(L-\Sigma)\cos\delta\sin H_{SRC} + H_{SRC}\sin(L-\Sigma)\sin\delta}{\cos L\cos\delta\sin H_{SR} + H_{SR}\sin L\sin\delta}$$

$$= \frac{\cos\left(37.73-10\right)^o\cos 21.35^o\sin 101.9^o + 1.778\sin\left(37.73-10\right)^o\sin 21.35^o}{\cos 37.73^o\cos 21.35^o\sin 107.6^o + 1.878\sin 37.73^o\sin 21.35^o}$$

$$= 0.988$$

So the beam insolation on the collector is

$$\overline{I}_{BC} = \overline{I}_{BH}\ \overline{R}_B = 5.42 \cdot 0.988 = 5.36 \text{ kWh/m}^2\text{-day}$$

Total insolation on the collector is thus

$$\overline{I}_C = \overline{I}_{BC} + \overline{I}_{DC} + \overline{I}_{RC} = 5.36 + 1.89 + 0.01 = 7.26 \text{ kWh/m}^2\text{-day}$$

That's a 7.26 / 6.7 = 1.08 .. an 8% increase (it will be a decrease in the winter, though).