PROBLEMS CHAPTER 5

5.1 The experience curve shown in Fig 5.1 is based on the following equation:

$$C(t) = C(0)\left[\frac{N(t)}{N(0)}\right]^K \qquad \text{where } K = \frac{\ln(1-LR)}{\ln 2}$$

where C(t) is the unit cost at time t, C(0) is the unit cost at time t = 0, N(t) is the cumulative production by time t, and LR is the learning ratio, which is the fractional decrease in cost per doubling of cumulative production.

a. From Fig. 5.1, let t=0 be at which PV production costs were about $30/W, cumulative production to that point was about 10 MW, and the subsequent learning curve showed a 24.3% decline in costs per doubling of production. What would a continuation of this learning curve suggest the c-Si module price would be after 1 million MW of cumulative production would have been achieved?

SOLN:

$$K = \frac{\ln(1-LR)}{\ln 2} = \frac{\ln(1-0.243)}{\ln 2} = -0.40163$$

$$C(t) = C(0)\left[\frac{N(t)}{N(0)}\right]^K = \$30\left[\frac{10^6\,MW}{10\,MW}\right]^{-0.40163} = \$0.29/W$$

Seems to agree with the figure projection.

b. With 2012 c-Si modules costing $$0.90/W and cumulative production to that point having been 100,000 MW, what would have been the learning rate over the period between 10 MW and 100,000 MW?

SOLN:

$$C(t) = C(0)\left[\frac{N(t)}{N(0)}\right]^K = \$30\left[\frac{10^5\,MW}{10\,MW}\right]^K = \$0.90/W$$

$$(10{,}000)^K = 0.90/30 = 0.03$$

$$K\ln(10{,}000) = 9.21K = \ln(0.03) = -3.5065$$

$$K = -0.38 = \frac{\ln(1-LR)}{\ln 2}$$

$$(-0.38)\ln 2 = -0.2634 = \ln(1-LR)$$

$$e^{-0.2634} = 0.768 = 1 - LR$$

$$LR = 1 - 0.768 = 0.232 = 23.3\% \text{ per doubling}$$

(which agrees with the figure).

c. An easy way to determine the learning ratio LR from real data is to start with the log of both sides of the basic equation

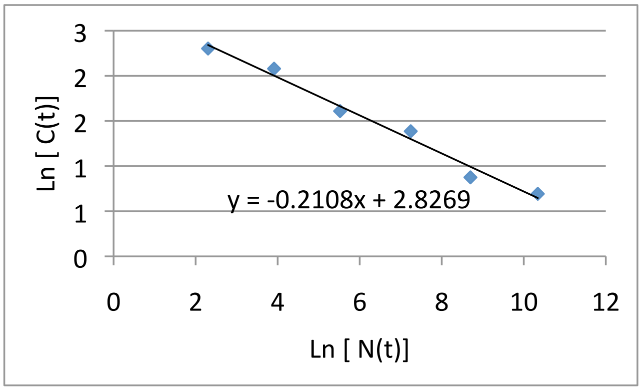
$$\ln C(t) = \ln\left[C(0)\right] + K \ln\left[\frac{N(t)}{N(0)}\right]$$

Using the following data, plot ln(C(t) vs ln(N(t)/N(0)], fit a straight line. The slope of that line is K. Use it to find LR.

N(t)	C(t)
10	10.0
50	8.0
250	5.0
1400	4.0
6000	2.4
31000	2.0

SOLN:

Log N(t)	Log C(t)
1	1
1.699	0.903
2.398	0.699
3.146	0.602
3.778	0.380
4.491	0.301



So, K = -0.2108 (interesting… it doesn't depend on ln or Log10..)

$$\text{LR} = 1 - e^{K \ln 2} = 1 - \exp(-0.2108 \ln 2) = 0.136 = 13.6\%/\text{doubling}$$

5.2 For the simple equivalent circuit of a 0.017 m^2 photovoltaic cell shown below, the reverse saturation current is I_0 = 4 x 10^{-11} A and at an insolation of 1-sun the short-circuit current is I_{SC} = 6.4 A,. At 25°C, find the following:

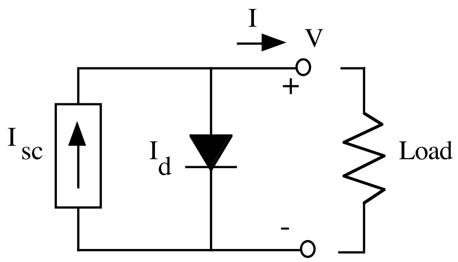


Figure P5.2

a. The open-circuit voltage.

b. The load current and output power when the output voltage is V = 0.55 V.

c. The efficiency of the cell at V = 0.55V.

SOLN:

a. Open circuit voltage from (5.11) is

$$V_{OC} = 0.0257\ln\left(\frac{I_{SC}}{I_0}+1\right) = 0.0257\ln\left(\frac{6.4}{4x10^{-11}}+1\right) = 0.663V$$

b. When the output voltage is 0.57 V, the load current will be

$$I_L = I_{SC} - I_d = I_{SC} - I_0\left(e^{38.9V}-1\right)$$
$$= 6.4 - 4x10^{-11}\left(e^{38.9x0.57}-1\right) = 6.23A$$
$$P = IV = 6.23A \cdot 0.57V = 3.55W$$

c. Cell efficiency

$$\eta = \frac{output}{input} = \frac{3.55W}{0.017m^2 x1000W/m^2} = 0.209 = 20.9\%$$

5.3 Suppose the equivalent circuit for the PV cell in Problem 5.2 includes a parallel resistance of R_P = 10 Ω. At 25°C, with an output voltage of 0.57V, find the following:

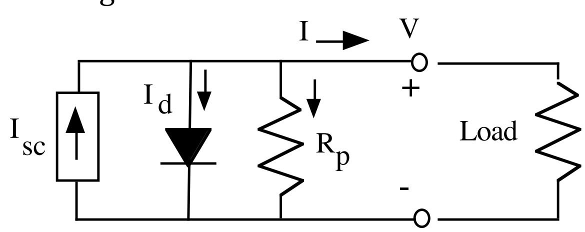


Figure P5.3

a. The load current and the power delivered to the load.

b. The efficiency of the cell.

SOLN:

a. Current and power when the output voltage is 0.57V

$$I_L = I_{SC} - I_d - \frac{V_d}{R_P} = I_{SC} - I_0\left(e^{38.9V}-1\right) - \frac{V_d}{R_P}$$
$$= 6.4 - 4x10^{-11}\left(e^{38.9x0.57}-1\right) - \frac{0.57}{10} = 6.173A$$
$$P = IV = 6.173A \cdot 0.57V = 3.519W$$

b. Cell efficiency

$$\eta = \frac{output}{input} = \frac{3.519W}{0.017m^2 x1000W/m^2} = 0.207 = 20.7\%$$

5.4 The following figure shows two I-V curves. Both have zero series resistance. One is for a PV cell with an equivalent circuit having an infinite parallel resistance. For the other, what is the parallel resistance in its equivalent circuit?

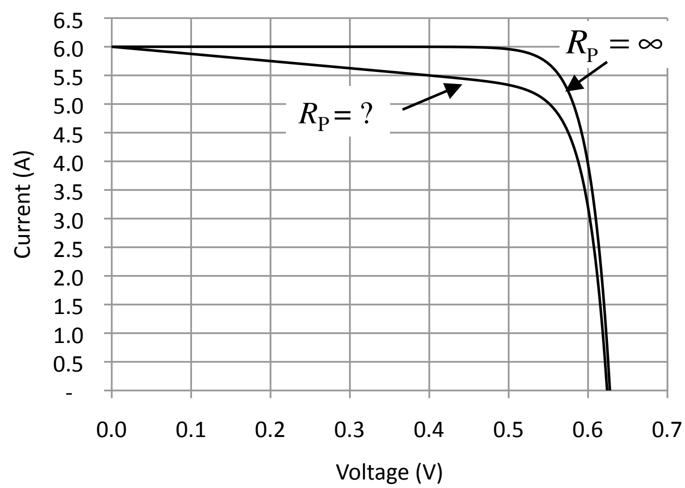


Figure P5.4

SOLN:

The slope of the drop off in current can be estimated from the point where V = 0.4V and ΔI = 6.0 – 5.5 = 0.5 A:

$$R_P = \frac{\Delta V}{\Delta I} = \frac{0.4}{0.5} = 0.8\Omega$$

5.5 The following figure shows two I-V curves. Both have equivalent circuits with infinite parallel resistances. One circuit includes a series resistance while the other one does not. What is the series resistance for the cell that has one?

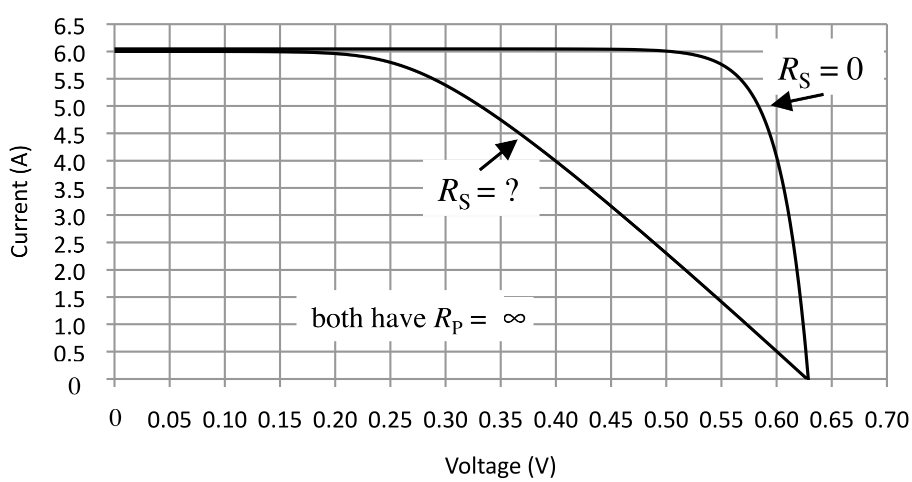


Figure P5.5

SOLN: The drop in voltage is due to series resistance, so using for example I = 4A, the drop in voltage is from 0.60 A to – 0.40A so

$$R_S = \frac{\Delta V}{I} = \frac{0.6 - 0.4V}{4A} = 0.05\Omega$$

5.6 Recreate the spreadsheet that was started in Example 5.4 for a 72-cell, 233-W PV module for which the equivalent circuit of each cell has both series (0.001Ω) and parallel resistances (10.0Ω).

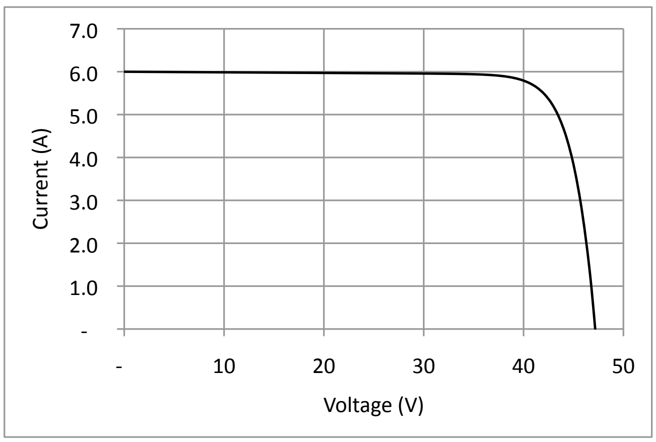
a. From your spreadsheet, what is the current, voltage and power delivered when the diode voltage V_d is 0.4 V ?

b. Plot the entire I-V curve for this module.

SOLN:

a. V_d = 0.4 V: I = 5.96 A, V = 28.4 V, P = 169.1 W

b.



5.7 Consider how you might make a simple, cheap pyranometer out of a single small (e.g. 1 cm²) PV cell along with a precision load resistor. If the PV cell has the following *I-V* curve and the goal is for the digital multimeter (DMM, with infinite input resistance), when set on its millivolt dc- scale, to give you direct readings of insolation.

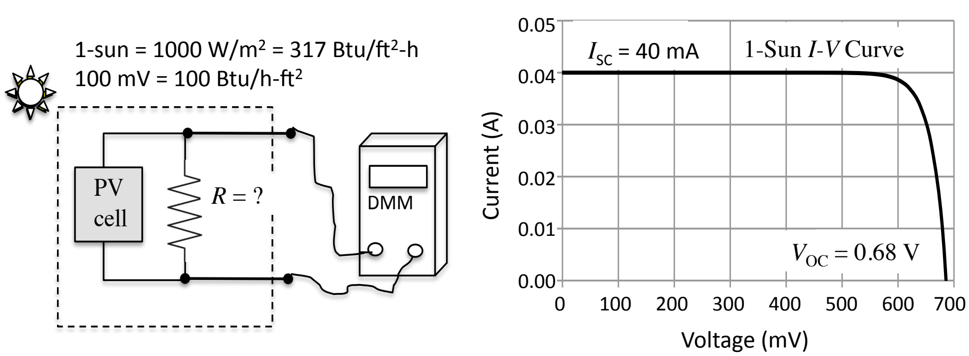
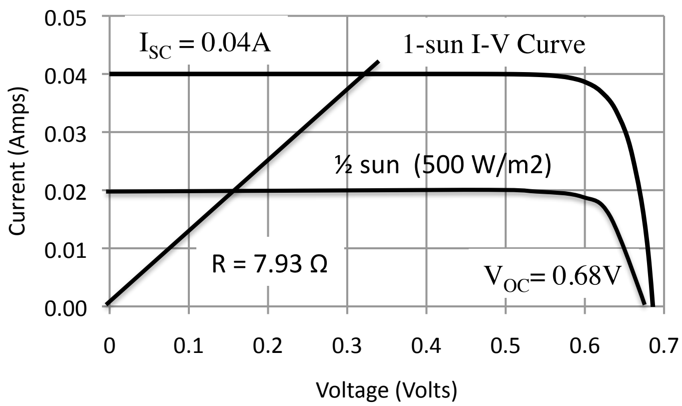


Figure P5.7

a. Find the load resistance that the pyranometer needs if the goal is to have the output of the DMM on a millivolt (mV) scale provide insolation readings directly in Btu/ft²-h (Full sun = 1 kW/m² = 317 Btu/ft²h = 317 mV).

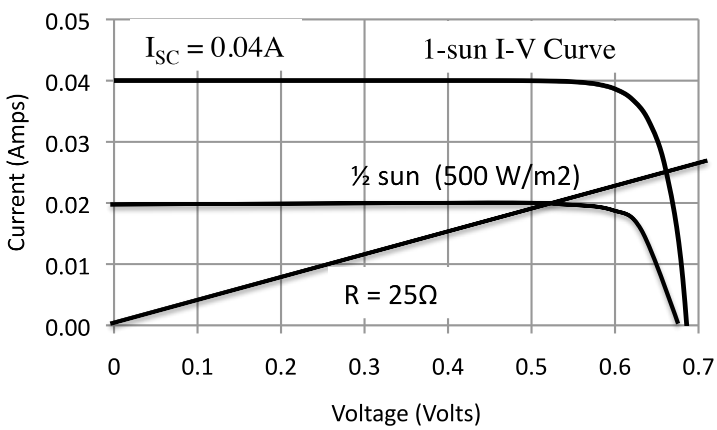
Sketch the I-V curve with your load resistance superimposed onto it. Show the PV-curve at both 1-sun and 1/2-sun insolation.

SOLN: $$R = \frac{V}{I} = \frac{0.317V}{0.04A} = 7.93\Omega$$



b. Suppose you want the mV reading to be W/m². What resistance would work (but only for modest values of insolation). Draw an *I-V* curve with this resistor on it and make a crude estimate of the range of insolations for which it would be relatively accurate.

SOLN: $$R = \frac{V}{I} = \frac{1.0V}{0.04A} = 25\Omega$$



Looks like it is pretty good at least up to about 500 W/m², above that it's getting too close to the rounded knee of the curve.

5.8 A 4-module array has two south-facing modules in series exposed to 1000 W/m² of insolation, and two west-facing modules exposed to 500 W/m². The 1-sun *I-V* curve for a single module with its maximum power point at 4A, 40V is shown below.

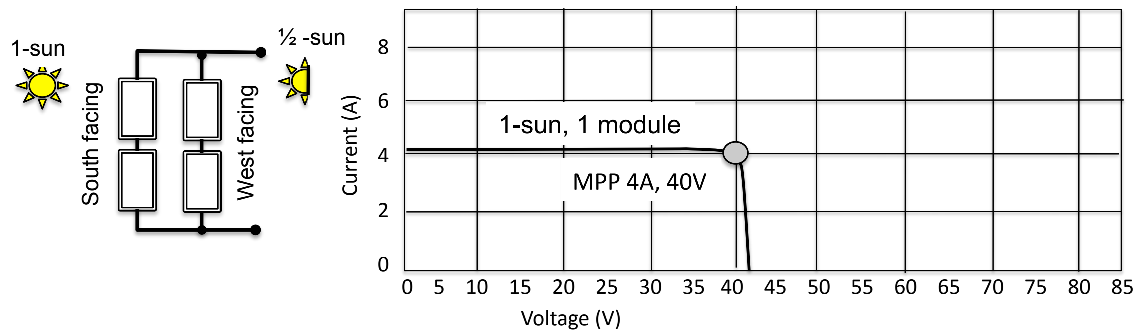
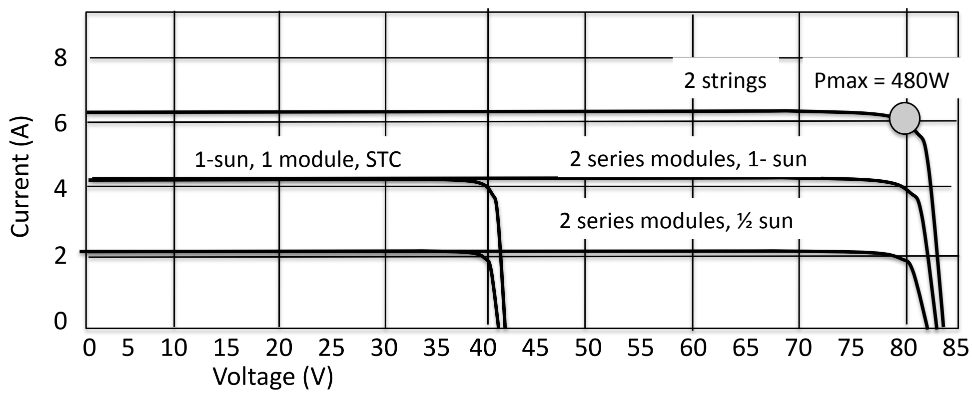


Figure P5.8

Draw the I-V curve for the 4-module array under these conditions. What is the output power (W) at the array's MPP?

SOLN: 480 W



5.9 A 200-W c-Si PV module has NOCT = 45°C and a temperature coefficient for rated power of -0.5%/°C.

a. At 1-sun of irradiation while the ambient is 25°C, estimate the cell temperature and output power.

SOLN: Using (5.23),

$$T_{\text{cell}} = T_{\text{amb}} + \left(\frac{NOCT - 20^{\circ}}{0.8}\right) \cdot S = 25 + \left(\frac{45-20}{0.8}\right) \cdot 1 = 56.25^{o}C$$

$$P_{\max} = 200\text{W}\left[1\text{-}0.5\%/^{\circ}\text{C}\left(56.25\text{-}25\right)^{\circ}\text{C}\right] = 168.8\text{ W} \quad \text{... a drop of 15.6\%}$$

b. Suppose the module is rigged with a heat exchanger that can cool the module while simultaneously providing solar water heating. How much power would be delivered if the module temperature is now 35°C? What % improvement is that?

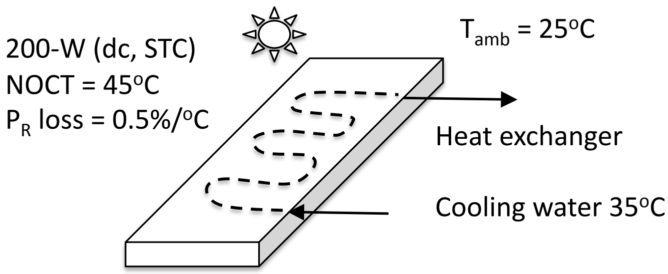


Figure P 5.9

SOLN:

$$P_{\max} = 200\text{W}\left[1\text{-}0.5\%/^\circ\text{C}\left(35\text{-}25\right)^\circ\text{C}\right]=190\text{ W}$$

$$\text{Improvement} = \frac{190-168.8}{168.8} = 12.56\%$$

c. Suppose ambient is the same temperature, but now insolation drops to 0.8 kW/m². What % improvement in power output would the heat exchanger provide if it still maintains the cell temperature at 35°C?

SOLN: Without cooling

$$T_{\text{cell}} = T_{\text{amb}} + \left(\frac{NOCT-20^\circ}{0.8}\right)\cdot S = 25 + \left(\frac{45-20}{0.8}\right)\cdot 0.8 = 50^o C$$

$$P_{\max} = 200\text{W x }0.8\left[1\text{-}0.5\%/^\circ\text{C}\left(50\text{-}25\right)^\circ\text{C}\right]=140\text{ W}$$

With cooling:

$$T_{\text{cell}} = T_{\text{amb}} + \left(\frac{NOCT-20^\circ}{0.8}\right)\cdot S = 25 + \left(\frac{35-20}{0.8}\right)\cdot 0.8 = 40^o C$$

$$P_{\max} = 200\text{W x }0.8\left[1\text{-}0.5\%/^\circ\text{C}\left(40\text{-}25\right)^\circ\text{C}\right]=148\text{ W}$$

Improvement = 148/140 = 1.057 = 5.7% improvement (much less than at 1-sun).

5.10 Consider this very simple model for cells wired in series within a PV module. Those cells that are exposed to full sun deliver 0.5 V; those that are completely shaded act like 5-Ω resistors. For a module containing 40 such cells, an idealized *I-V* curve with all cells in full sun is as follows.

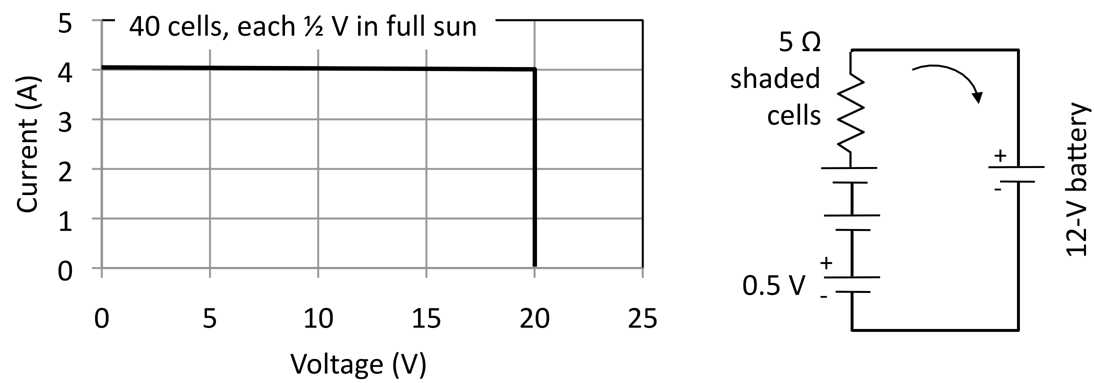
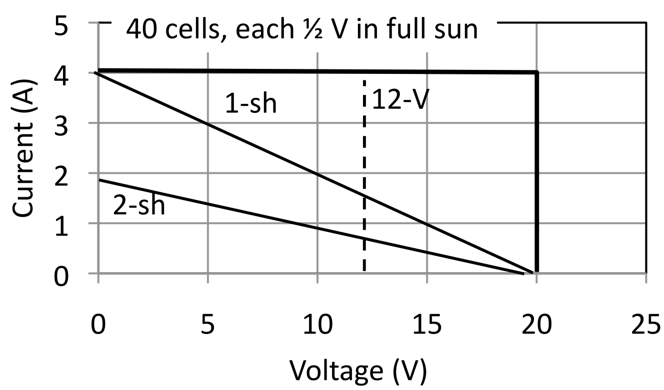


Figure P 5.10

a. Draw the PV *I-V* curves that will result when one cell is shaded and when two cells are shaded (no battery load).

b. If you are charging an idealized 12-V battery (vertical *I-V* curve), compare the current delivered under these three circumstances (full sun and both shaded circumstances).

SOLN: a.



b. Battery charging:

Full sun: $I = 4$ A

1-cell shaded: $I = (19.5 - 12)/5 = 1.5$ A

2-cells shaded $I = (19 - 12)/10 = 0.7$A

5.11 An idealized 1-sun *I-V* curve for a single 80-W module is shown below. For two such modules wired in series, draw the resulting *I-V* curve if the modules are exposed to only 1/2 sun, and one cell, in one of the modules, is shaded. Assume the shaded cell has an equivalent parallel resistance of 10 Ω.

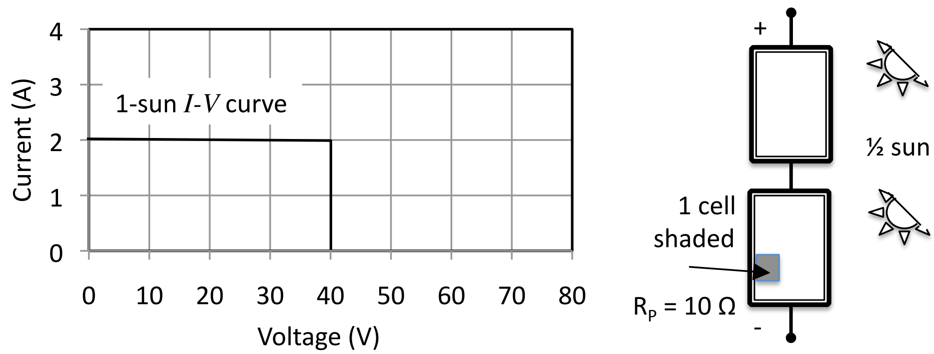
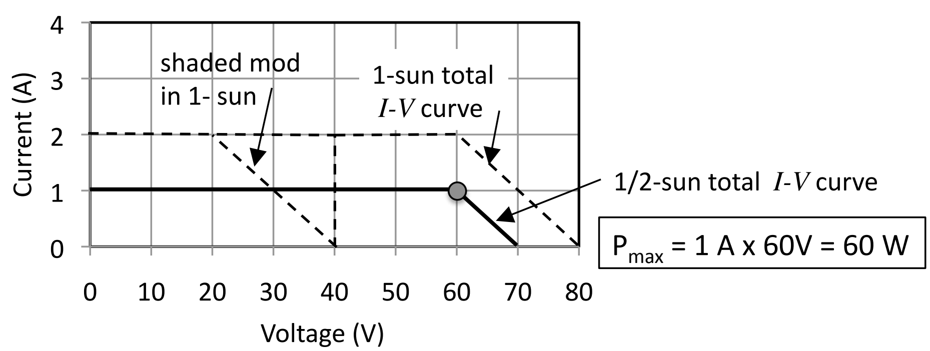


Figure P 5.11

Sketch the resulting I-V curve. How much power would be generated at the maximum power point?

SOLN:



5.12 The 1-sun *I-V* curve for a 40-cell PV module in full sun is shown below along with an equivalent circuit for a single cell (including its 10Ω parallel resistance).

An array with two such modules in series has one fully shaded cell in one of the modules. Consider the potential impact of bypass diodes around each of the modules.

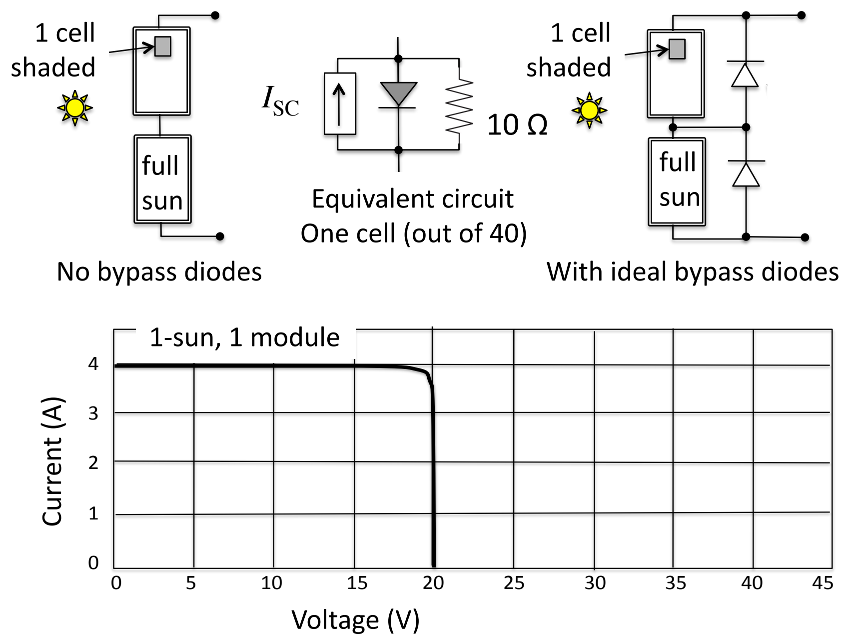
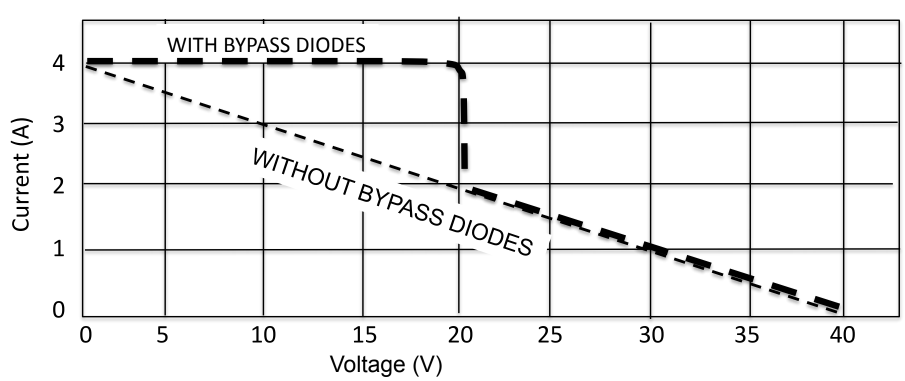


Figure P 5.12

a. Sketch the 1-sun *I-V* curve for the series combination of modules with one cell shaded but no bypass diodes. Find the power output at the maximum power point. Compare it to the output when there is no shading.

b. Sketch the 1-sun *I-V* curve when the bypass diodes are included. Estimate the maximum power output now (close is good enough).

SOLN:



a. MPP without diodes is at 2A x 20V = 40W could guess, which is fine or prove it by

$$I = 4 - 0.1V \quad \text{so } P = VI = 4V - 0.1V^2$$

$$dP/dV = 4 - 0.1 \times 2V = 0$$

$$\text{So } V = 4/0.2 = 20V, \quad I = 2A, \quad P_{max} = 2 \times 20 = 40W$$

Without diodes, the output went from 160 W down to 40 W when 1 cell is shaded !

b. MPP with diodes is at about 4A x 20V = 80 W. Still lost half of the 160 W output when there is no shading.

5.13 Consider a single 87.5 W, First Solar CdTe module (Table 5.3) used to charge a 12-V battery.

a. What duty cycle should be provided to a maximum-power-point, buck-boost converter to deliver 14-V to the battery when the module is working at standard test conditions (STC)? How many amps will it deliver to the battery under those conditions?

SOLN:

a. The First Solar module has an STC MPP of 1.78 A, 49.2 V. So the converter needs to drop the 49.2 V down to 14 V. From (5.36)

$$\frac{D}{1-D} = \frac{14V}{49.2V}$$

$$14 - 14D = 49.2D \quad .... \quad \text{D} = \frac{14}{14+49.2} = 0.2215$$

Since we assume power in equals power out,

$$49.2x1.78 = 14xI_B$$

$$I_B = \frac{49.2x1.78}{14} = 6.255A$$

b. Suppose ambient temperature is 25°C with 1-sun of insolation. Recalculate the amps delivered to the battery.

SOLN:

b. First find cell temperature. From the table, NOCT = 45°C so from (5.23):

$$T_{cell} = T_{amb} + \left(\frac{NOCT-20}{0.8}\right)S = 25 + \left(\frac{45-20}{0.8}\right)1 = 56.25^o$$

These cells have a - 0.25%/°C power degradation, so at 25°C:

Power loss = 0.25%/°C x (56.25-25)°C = 7.8125%

Power @ 25°C = 87.5 (1 - 0.078125) = 80.39 W

Again, with power in equal power out:

80.39 = 14V x I A Charging current = 80.39/14 = 5.74 A

That's a loss of 8.2% due to temperature.