

PROBLEMS CHAPTER 6

6.1 A clean, 1 m², 15% efficient module (STC), has its own 90% efficient inverter. Its NOCT is 45°C and its rated power degrades by 0.5%/°C above the 25°C STC.

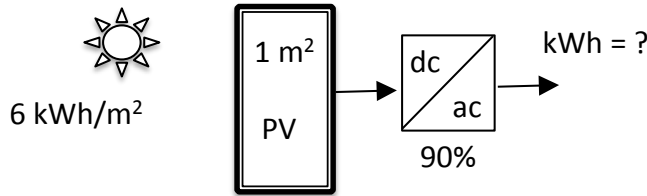


Figure P 6.1

- a. What is its standard test condition (STC) rated power of the module?
- b. For a day with 6 kWh/m² of insolation, find the kWh that it would deliver if it operates at its NOCT temperature. Assume the only deratings are due to temperature and inverter efficiency.

SOLN:

- a. The STC output of this module $P_{dc,STC} = 0.15 \times 1 \text{ m}^2 \times 1 \text{ kW/m}^2 = 0.15 \text{ kW}$
- b. Operating at its NOCT temperature,
 - Temp derating = $[1 - 0.5\%/^{\circ}\text{C} \times (45-25)^{\circ}\text{C}] = 0.90$
 - Inverter derating = 0.90
 - Total derating = $0.90 \times 0.90 = 0.81$
 - Energy = $0.15 \text{ kW} \times 0.81 \times 6 \text{ h/d} = 0.729 \text{ kWh/day}$

6.2 NREL's PVWATTS website predicts that 5.56 kWh/m²-day of insolation on a south-facing, 40° tilt array in Boulder, CO, will deliver 1459 kWh/yr of ac energy per kW_{dc,STC} of PV modules.

- a. Using the "peak-hours" approach to performance estimation, what overall derate factor (including temperature effects) would yield the same annual energy delivered?

SOLN:

$$\text{kWh/yr} = P_{dc,STC} \times \text{Overall Derate} \times (\text{h/daypeaksun}) \times 365\text{d/yr}$$

$$\text{Overall Derate} = \frac{1459 \text{ kWh/yr}}{1 \text{ kW} \times 5.56 \text{ h/day} \times 365\text{d/yr}} = 0.7189$$

- b. Since PVWATTS' derate value of 0.77 includes everything but temperature impacts, what temperature induced derating needs to be included to make the peak-hours approach predict the same annual energy?

(Overall Derate = PVWATTS Derate x Temperature Derate).

SOLN:

$$\text{kWh/yr} = P_{\text{dc,STC}} \times \text{Overall Derate} \times (\text{h/daypeaksun}) \times 365\text{d/yr}$$

$$\text{Boulder Overall Derate} = \frac{1459 \text{ kWh/yr}}{1 \text{ kW} \times 5.56 \text{ h/day} \times 365\text{d/yr}} = 0.7189$$

$$\text{Temperature Derate} = \frac{\text{Overall Derate}}{\text{Default } 0.77} = \frac{0.7189}{0.77} = 0.9337$$

which is a $1 - 0.9337 = 0.066 = 6.6\%$ loss due to temperature.

- c. Use the PVWATTS website to find the overall annual temperature derate factors for a cold place (Bismarck, ND) and a hot place (Houston, TX). Use the same south-facing, 40° tilt array.

SOLN:

Bismarck: PVWATTS predicts 1352 kWh/yr per kW dc, STC from 5.00 kWh/m²-day insolation:

$$\text{kWh/yr} = P_{\text{dc,STC}} \times \text{Overall Derate} \times (\text{h/daypeaksun}) \times 365\text{d/yr}$$

$$\text{Bismarck Overall Derate} = \frac{1352 \text{ kWh/yr}}{1 \text{ kW} \times 5.00 \text{ h/day} \times 365\text{d/yr}} = 0.7408$$

$$\text{Temperature Derate} = \frac{\text{Overall Derate}}{\text{Default } 0.77} = \frac{0.7408}{0.77} = 0.962$$

Houston: PVWATTS predicts 1193 kWh/yr from 4.69 kWh/m²-day:

$$\text{Houston Overall Derate} = \frac{1193 \text{ kWh/yr}}{1 \text{ kW} \times 4.69 \text{ h/day} \times 365\text{d/yr}} = 0.6969$$

$$\text{Temperature Derate} = \frac{\text{Overall Derate}}{\text{Default } 0.77} = \frac{0.6969}{0.77} = 0.9051$$

So performance is cut by 3.8% in cold Bismarck and 9.5% in hot Houston.

- 6.3** You are to size a grid-connected PV system to deliver 4000 kWh/yr in a location characterized by 5.5 kWh/m²-day of insolation on the array.
- a. Find the dc, STC rated power of the modules assuming a 0.72 derate factor.

SOLN:

$$\text{kWh/yr} = P_{\text{R}} (\text{kW}) \times (\text{h/day full sun}) \times 365 \text{ day/yr} \times \text{Derate}$$

$$P_{\text{R}} (\text{kW}) = \frac{4000 \text{ kWh/yr}}{5.5 \text{ h/day} \times 365 \text{ day/yr} \times 0.72} = 2.77 \text{ kW}$$

- b. Find the PV collector area required if, under standard test conditions, these are 18%-efficient modules.

SOLN:

$$P_{dc,STC} = A(m^2) \cdot \frac{1 \text{ kW}}{m^2} \cdot \eta$$

$$A = \frac{2.77 \text{ kW}}{1 \text{ kW/m}^2 \times 0.18} = 15.39 \text{ m}^2$$

- c. Find the first-year net cost of electricity (\$/kWh) if the system costs \$4 per peak watt (\$4/W_{dc,STC}), it is paid for with a 5%, 30-yr loan, interest on the loan is tax deductible, and the owner is in a 29% marginal tax bracket.

SOLN:

$$CRF(5\%, 30yr) = \frac{0.05(1.05)^{30}}{(1.05)^{30} - 1} = 0.06505 / yr$$

System Cost = \$4/W x 2770 W = \$11,080

Annual payments = \$11,080 x 0.06505/yr = \$720.8

1st year interest = 0.05 x \$11,080 = \$554

Reduced taxes = \$554 x 0.29 = \$160.7

1st year cash flow = \$720.8 - \$160.7 = \$560

1st year PV cost = \$560/4000 kWh = \$0.14 = 14 ¢/kWh

- 6.4** The beginning of a financial spreadsheet for a PV system is shown below. Fill in the row for year 2.

year	payment	interest	delta bal	loan bal	tax savs	net cost	\$/kWh
0				\$8,000.00			
1	\$500.00	\$400.00	\$100.00	\$7,900.00	\$120.00	\$380.00	\$0.19000
2	?	?	?	?	?	?	?

Table P 6.4

SOLN:

Payment the same \$500/y

Interest rate x \$8000 = \$400, therefore $i = 5\%/yr$, so $5\% \times \$7900 = \395

Delta Bal = \$500 - \$395 = \$105

Loan Bal = 7900 - 105 = \$7795

Tax savs (yr-1) = \$120 = MTB x \$400 so MTB = 25%

Tax savs (yr 2) = 25% x \$395 = \$118.50

Net Cost = \$500 - \$118.50 = \$381.50

Year 1: \$/kWh = \$0.19 = \$380/kWh, so kWh = 2000/yr

Year 2: \$/kWh = \$381.50/2000 kWh = \$0.19075/kWh

year	payment	interest	delta bal	loan bal	tax savs	net cost	\$/kWh
0				\$8,000.00			
1	\$500.00	\$400.00	\$100.00	\$7,900.00	\$120.00	\$380.00	\$0.19000
2	?	?	?	?	?	?	?
2	\$ 500.00	\$ 395.00	\$ 105.00	\$ 7,795.00	\$ 118.50	\$ 381.50	\$ 0.19075

6.5 Recreate the cash flow spreadsheet provided in Table 6.6 and see whether you can reproduce those same results.

a. In what year do you first see a positive cash flow?

SOLN: Year 6

b. What is the present value of the cash flow in year 14?

SOLN:
$$P = \frac{F}{(1+d)^n} = \frac{F}{(1+d)^n} \quad P = \frac{F}{(1+d)^n} = \frac{\$126.21}{(1+0.05)^{14}} = \$63.75$$

c. What NPV do you get when you set the discount rate to be the same as the IRR?

SOLN: NPV = 0

d. Eliminate the down payment and see what happens to the IRR. Then try paying for the whole system with cash. Again, compare IRR and comment.

SOLN: Base case IRR = 7.37%;
 Without down payment IRR = 11.63%
 With no loan, IRR = 4.02%

... the less of your own money you put in the better your IRR.

e. Find the NPV and IRR for a 4-kW system in an area with 5.5 kWh/m²-day insolation, using a derating of 0.72. Keep everything else the same.

SOLN: IRR = 6.86%; NPV = \$533.77

6.6 A grid-connected PV array consisting of sixteen 150-W modules can be arranged in a number of series and parallel combinations: (16S, 1P), (8S, 2P), (4S, 4P), (2S, 8P), (1S, 16P). The array delivers power to a 2500-W inverter. The key characteristics of modules and inverter are given below.

INVERTER		MODULE	
Maximum AC power	2500 W	Rated power P _{dc,STC}	150 W
Input voltage range for MPP	250 V - 550 V	Voltage at MPP	34 V
Maximum input voltage	600 V	Open-circuit voltage	43.4 V
Maximum input current	11 Amp	Current at MPP	4.40 A
		Short-circuit current	4.8 A

Table P 6.6

Using the input voltage range of the inverter maximum power point tracker and the maximum input voltage of the inverter as design constraints, what

series/parallel combination of modules would best match the PVs to the inverter? Check the result to see whether the inverter maximum input current is satisfied. For this simple check, you don't need to worry about temperatures.

SOLN: The PV modules have $V_{OC} = 43.4V$ and $V_R = 34V$. The SB2500 has an MPPT range of 250-550V and a maximum input voltage of 600V.

(16S, 1P) has $V_{OC} = 16 \times 43.4 = 694 V$ which is too high X

(8S, 2P) has $V_{OC} = 347 V$, which is OK. $V_R = 8 \times 34 = 272 V$ which is OK

(4S, 4P) has $V_R = 4 \times 34 = 136 V$ which is too low X

(2S, 8P) and (1S, 16P) also are below the MPPT range X

Therefore (8S, 2P) is the best arrangement for the array. Checking its current 2P means 9.6A max, which fits under the 11A max. So it is fine with current.

6.7 Redo Example 6.5 using the 85.7-W CdTe modules described below and the same Sunny-Boy 5-kW inverter to supply about 5,000 kWh/yr in an area with 5.32 kWh/m²/day of insolation. Assume the same 0.75 derate factor.

Module characteristics:

Peak power:	87.5 W
Rated voltage V_{MPP} :	49.2 V
Open-circuit voltage V_{OC} :	61 V
Short-circuit current I_{SC} :	1.98 A
Temperature coefficient of power:	-0.25%/K
Temperature coefficient of V_{OC} :	-0.27%/K
Temperature coefficient of I_{SC} :	-0.04%/K
NOCT:	45°C

a. What dc-STC power (kW) would be required to provide 5,000 kWh/yr?

SOLN:

$$P_{dc} \text{ (kW)} = \frac{\text{kWh/yr}}{\text{h/day @ 1 kW/m}^2 \times 365 \text{ day/yr} \times \text{derate}}$$

$$= \frac{5000}{5.32 \times 365 \times 0.75} = 3.43 \text{ kW}$$

b. Before worrying about the inverter, how many modules would be required?

SOLN:

$$\text{modules} = \frac{3.43 \text{ kW}}{87.5 \text{ W/mod}} \times \frac{1000 \text{ W}}{\text{kW}} = 39.2$$

c. Using the 600-V maximum allowable voltage on a residential roof, what is the maximum number of modules in a single string if the coldest temperature expected is -5°C? Using this constraint, what is the best number of strings and modules per string?

SOLN: With nominal $V_{OC} = 61V$, and 0.27%/K change per degree below 25°C

$$V_{oc}(\max) = 61V[1 - 0.0027(-5 - 25)] = 65.9 \text{ V/module}$$

$$\text{Max \# modules/string} = \frac{600V}{65.9V/\text{module}} = 9.1$$

- d. Use the coldest ambient temperature to help determine the maximum number of modules per string need to be sure the MPP stays below the 480-V that the inverter needs for proper tracking.

SOLN: MPP is given as 49.2 V with a 0.27% increase per degree below 25°C:

$$V_{oc}(\max) = 49.2V[1 - 0.0027(-5 - 25)] = 53.2 \text{ V/module}$$

$$\text{Max \# modules/string} = \frac{480V}{53.2V/\text{module}} = 9.0$$

- e. What is the minimum number of modules per string to satisfy the inverter constraint that says it needs at least 250 V to properly maintain maximum power point tracking. Assume the hottest ambient temperature is 40°C.

$$T_{cell} = T_{amb} + \left(\frac{NOCT - 20}{0.8} \right) S = 40 + \left(\frac{45 - 20}{0.8} \right) 1 = 71.25^\circ C$$

$$V_{MPP}(hot) = 49.2[1 - 0.0027(71.24 - 25)] = 43.06V$$

$$\text{min \# modules/string} = \frac{250 \text{ V}}{43.06 \text{ V/module}} = 5.8$$

So we need at least 6 modules per string.

- f. The maximum allowable DC input current for the inverter is 21A. What is the maximum number of strings that would be allowed?

SOLN:
$$\text{max\# strings} = \frac{21A}{1.98A/\text{string}} = 10.6$$

- g. Using all of these constraints, choose the best combination of strings and number of modules per string to satisfy the design.

SOLN: From (c) and (d) the maximum modules/string = 9

From (e) the minimum modules/string = 6

From (f) the maximum number of strings is 10

We need about 40 modules.

4 strings of 10 modules WON'T WORK

5 strings of 8 modules WORKS

6 strings of 7 modules WORKS, but provides more than needed

7 strings of 6 modules WORKS, but provides more than needed

The best combination is 5 strings of 8 modules. It is closer to the design goals and it uses more modules per string, which raises voltages and reduces line losses.

6.8 You have four PV modules with identical $I-V$ curves ($I_{sc} = 1\text{ A}$, $V_{oc} = 20\text{ V}$) as shown. There are three ways you could wire them up to deliver power to a dc-motor (which acts like a $10\text{-}\Omega$ load):

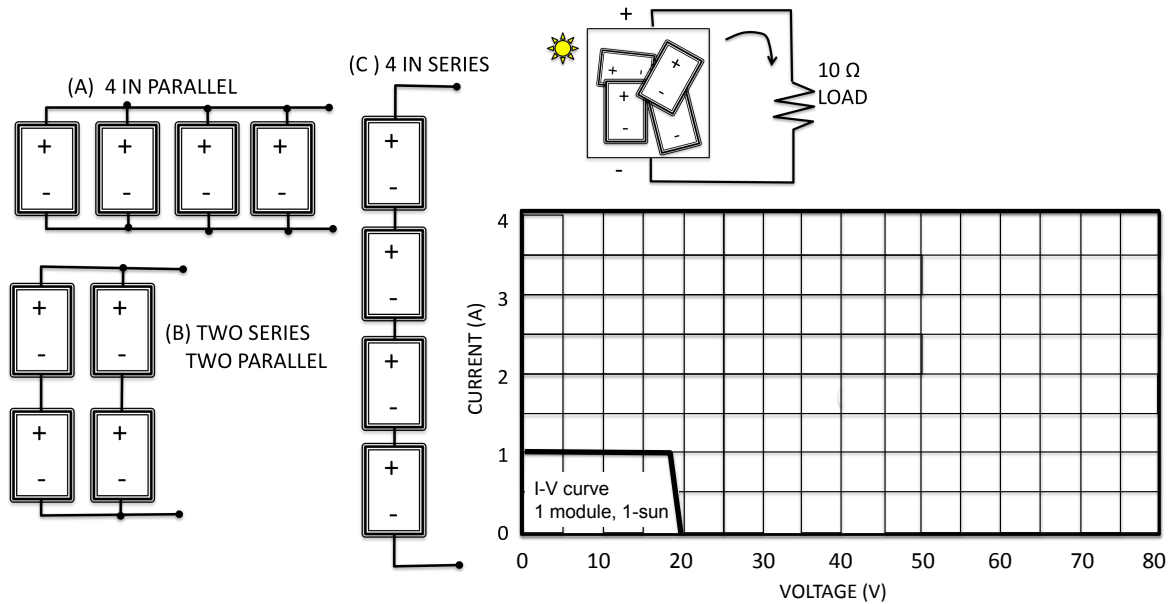


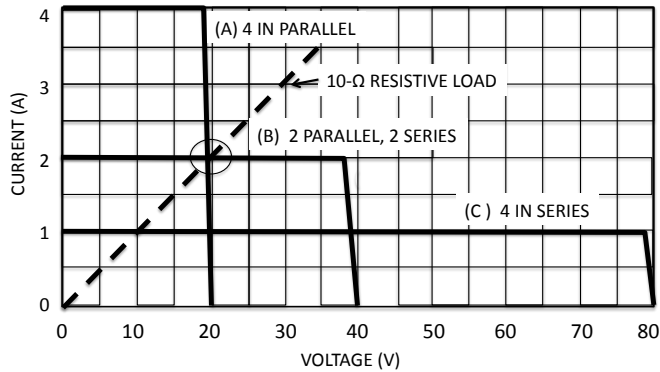
Figure P 6.8

Draw the I-V curves for all three combinations on the same graph. Which wiring system would be best? Briefly explain your answer.

SOLN: (A) 4 IN PARALLEL IS BEST

Why: At 1 sun A and B look to be about equal... each delivering 2A at about 20V, with perhaps a slight edge for B since at higher temperatures it will be less affected by the drop in V_{oc} . B also looks slightly better at 1-sun given the tiny bit of curvature around the knee.

However: Over the course of a day, A is far better since at less than 1-sun A is not affected until you get less than 0.5 Sun ($500\text{W}/\text{m}^2$) of insolation, and by then B is really terrible.



6.9 The summer TOU rate structure shown in Table 6.7 includes an off-peak energy charge of \$0.0846/kWh for usage up to 700 kWh/mo and \$0.166/kWh for usage above that base. During the peak demand period it is a flat \$0.27/kWh.

- a. What will the customer's bill be for 1000 kWh used off peak and 800 kWh on peak?

SOLN:

$$\text{Off Peak} = 700 \text{ kWh} \times \$0.0846/\text{kWh} + 300 \text{ kWh} \times \$0.166/\text{kWh} = \$109$$

$$\text{On Peak} = 800 \text{ kWh} \times \$0.27/\text{kWh} = \$216$$

$$\text{Total} = \$325/\text{mo}$$

- b. Suppose the customer signs up for the TOU + CPP rate structure. During three days, a critical peak pricing period is announced during which time electricity costs \$0.75/kWh. If they use 100 of their 800 peak period kilowatt-hours during that time, what will their bill be that month.

$$\text{Off Peak} = 700 \times \$0.0721 + 300 \times \$0.1411 = \$93$$

$$\text{On Peak (non CPP)} = 700 \times \$0.27 = \$189$$

$$\text{On Peak (CPP)} = 100 \text{ kWh} \times \$0.75/\text{kWh} = \$75.$$

$$\text{Total Bill} = \$93 + 189 + 75 = \$357/\text{mo}$$

That's a bit more than their standard bill of \$325.

- c. Suppose the customer shuts off their power during those CPP periods, what would now be the utility bill?

$$\text{Off Peak} + \text{On Peak (non CPP)} = \$93 + \$189 = \$282$$

$$\text{That's a savings of } \$325 \text{ (base)} - \$282 = \$43$$

6.10 A small office building that uses 40,000 kWh/month during the summer has a peak demand of 100 kW. An 80-kW photovoltaic system is being proposed that will provide 20,000 kWh/mo. The before and after demand curves are shown in Figure P 6.10.

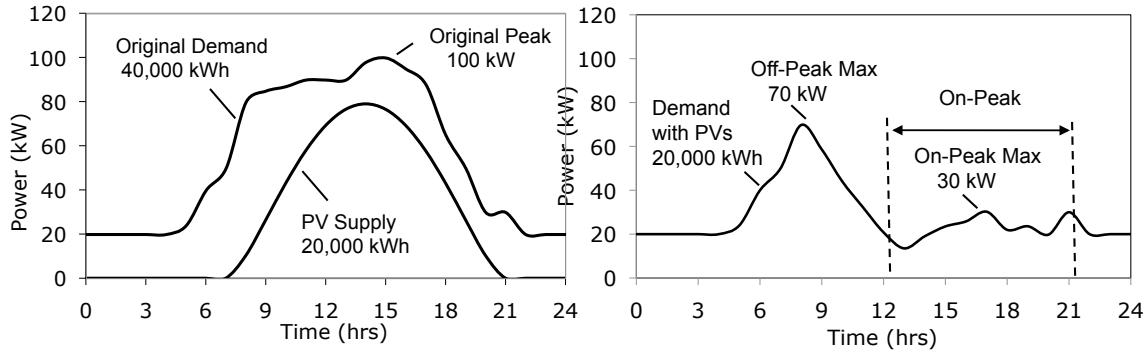


Figure P 6.10

The utility rate schedule includes demand charges that vary depending on whether the customer has signed up for time-of-use rates or not. And the TOU rates have a demand charge that varies with on or off peak periods.

	Non TOU Rates	TOU Rates	
Energy Charge (\$/kWh)	\$ 0.06	\$ 0.05	
Demand Charge (\$/mo/kWp)	\$ 12.00	\$ 14.00	on peak
		\$ 5.00	off peak

Table P 6.10

- a. What would be the utility bill without the PVs when the non-TOU rate schedule has been chosen?

SOLN: Non-TOU, no PVs:

$$\text{Bill} = 40,000 \text{ kWh} \times \$0.06/\text{kWh} + 100 \text{ kW} \times \$12/\text{mo-kW} = \$3600/\text{mo}$$

- b. Which rate structure would be the best for the customer if they install the PVs. How much money would the PVs save with that rate structure?

SOLN: Non-TOU with PV: $\text{Bill} = 20,000 \text{ kWh} \times \$0.06 + 70 \text{ kW} \times \$12/\text{kW} = \$2040/\text{mo}$

TOU with PV:

$$\begin{aligned} &= \$20,000 \text{ kWh} \times \$0.05/\text{kWh} + \$14/\text{mo-kW} \times 30 \text{ kW} + \$5/\text{mo-kW} \times 70 \text{ kW} \\ &= \$1770/\text{mo} \end{aligned}$$

Better to use the TOU rates with PV, which means the

$$\text{PVs save } \$3600 - \$1770 = \$1830/\text{mo}$$

6.11 A 100-kW PV system is being proposed for a commercial building in an area with 5.5 kWh/m²-day insolation. The before tax-credit cost of the system is \$5/Wp. Assuming a 0.72 derate factor:

- a. What is the annual electricity production that might be expected?

SOLN: $\text{Energy} = 100\text{kW} \times 5.5 \text{ h/d} \times 365 \text{ d/yr} \times 0.72 = 144,540 \text{ kWh/yr}$

- b. What is the MACRS depreciable basis for the system after a 30% tax credit has been taken?

SOLN: The system cost is $100 \text{ kW} \times \$5000/\text{kW} = \$500,000$

tax credit is $30\% \times 100 \text{ kW} \times \$5000/\text{kW} = \$150,000$, Half of that is $\$75,000$

So the depreciable basis is $\$500,000 - 75,000 = \$425,000$

- c. For a corporate tax rate of 38% and a discount rate of 7%, find the present value of the MACRS depreciation.

SOLN:

Investment		\$500,000		
30% Tax credit		\$150,000.0		
Depreciable basis		\$425,000.0		
Corporate tax rate		38%		
Corporate discount rate		7%		
Year	MACRS	Depreciation	Tax Savings	Present Value
1	20%	\$85,000.0	\$32,300.0	\$ 30,186.92
2	32%	\$136,000.0	\$51,680.0	\$ 45,139.31
3	19.20%	\$81,600.0	\$31,008.0	\$ 25,311.76
4	11.52%	\$48,960.0	\$18,604.8	\$ 14,193.51
5	11.52%	\$48,960.0	\$18,604.8	\$ 13,264.97
6	5.76%	\$24,480.0	\$9,302.4	\$ 6,198.58
Totals	100%	\$ 425,000.00	\$ 161,500.00	\$ 134,295.05

- d. What is the effective net system cost after the tax credit and MACRS depreciation. Effective \$/kW cost of the system?

Effective cost = $\$500,000 - \$150,000 - \$134,295 = \$215,705$

Cost per watt = $\$215,705/100,000\text{W} = \$2.157 /\text{Wp}$ Wow..

6.12 In Example 6.9, the time-of-day (TOD) factors for a PPA were worked out for a PV array that faces due south. Since those TOD factors favor afternoon generation, consider the following clear-sky hourly insolation values for those same collectors now facing toward the southwest.

Solar Time	Insolation (W/m2)
6	60
7	92
8	282
9	484
10	670
11	821
12	923
1	967
2	947
3	862
4	716
5	516
6	272

Compare the revenue generated by the south-facing versus southwest-facing collectors under the following conditions:

- a. One hour at solar noon on a summer weekday.

SOLN: At noon on a weekday, TOD = 3.13 so

$$\$/\text{day} = 1000 \text{ kW} \times 0.923 \text{ h of full sun} \times 0.75 \times 0.10 \text{ \$/kWh} \times 3.13 = \$216.67$$

(versus \$228.41 for south-facing)

- b. One week at solar noon in the summer.

SOLN: On weekends, TOD = 0.75

$$\$/\text{day} = 1000 \text{ kW} \times 0.923 \text{ h of full sun} \times 0.75 \times 0.10 \text{ \$/kWh} \times 0.75 = \$51.92$$

$$\text{One week SW} = 5 \text{ d} \times \$216.67/\text{day} + 2 \text{ day} \times \$51.92/\text{day} = \$1187.19$$

South facing

$$\text{One week} = 5 \times \$225.36 + 2 \times \$54.00 = \$1239.80 \text{ (a bit more than SW)}$$

- c. One week at 3 pm in the summer.

SOLN: Southwest:

$$5 \text{ days} = 1000 \text{ kW} \times 0.862 \text{ h} \times 0.75 \times 0.10 \text{ \$/kWh} \times 3.13 \times 5 = \$1011.77$$

$$2 \text{ days} = 1000 \text{ kW} \times 0.862 \times 0.75 \times 0.10 \times 0.75 \times 2 = \$96.98$$

$$1 \text{ week} = \$1011.77 + \$96.98 = \$1108.74$$

SOUTH: (from the Example)

$$\$/\text{week} = 5 \times \$161.04 + 2 \times \$38.15 = \$881.50$$

$$\text{SW beats it by } \$1108.74 - \$881.50 = \$227/\text{week}$$

- d. Create a spreadsheet to work out the entire month's revenue to compare to the \$32,334 that a south-facing collector will generate. What is the average PPA price paid per kWh generated?

SOLN: Revenue = \$34,713/mo vs \$32,334

$$\text{Avg PPA} = \$0.203/\text{kWh} \text{ vs } \$0.189/\text{kWh}$$

Pdc, stc	1000	kW		Lat 40 Tilt 30		
# days	22	Weekdays		Azimuth (SW, AZ = - 45°)		
#days	8	Weekends				
DeRate	0.750					
PPA	\$ 0.10	\$/kwh base rate				
			Weekdays		Weekends	
Solar Time	Insolation (W/m2)	kWh/day del	TOD X	Revenue \$/day	TOD X	Revenue \$/day
6	60	45	0.75	\$ 3.38	0.75	\$ 3.38
7	92	69	0.75	\$ 5.18	0.75	\$ 5.18
8	282	212	1.35	\$ 28.55	0.75	\$ 15.86
9	484	363	1.35	\$ 49.01	0.75	\$ 27.23
10	670	503	1.35	\$ 67.84	0.75	\$ 37.69
11	821	616	1.35	\$ 83.13	0.75	\$ 46.18
12	923	692	3.13	\$ 216.67	0.75	\$ 51.92
1	967	725	3.13	\$ 227.00	0.75	\$ 54.39
2	947	710	3.13	\$ 222.31	0.75	\$ 53.27
3	862	647	3.13	\$ 202.35	0.75	\$ 48.49
4	716	537	3.13	\$ 168.08	0.75	\$ 40.28
5	516	387	3.13	\$ 121.13	0.75	\$ 29.03
6	272	204	1.35	\$ 27.54	0.75	\$ 15.30
Totals		5709		\$ 1,422.16		\$ 428.18
REVENUE	\$ 34,713	\$/mo				
Energy	171,270	kWh/mo				
Avg	\$ 0.203	\$/kWh				

6.13 Suppose a 12-V battery bank rated at 200 Ah under standard conditions needs to deliver 600 Wh over a 12-hour period each day. If they operate at -10°C, how many days of use would they be able to supply?

SOLN:

$$\text{Discharge rate} = \frac{600 \text{ VAh/d}}{12\text{V} \times 12 \text{ h/d}} = 4.16 \text{ A}$$

$$\text{Discharge time} = \frac{200 \text{ Ah}}{4.16 \text{ A}} = 48 \text{ h}$$

From Figure 6.28, the C/48 discharge rate at T = - 10°C is 90% of the 200 Ah standard rating. So, with 80% maximum discharge the hours of storage is

$$\text{Hrs of usable storage} = \frac{200 \text{ Ah} \times 0.90(\text{temp}) \times 0.80(\text{depth})}{4.16 \text{ A}} = 34.6 \text{ h}$$

Discharging them for 12 hours per day means they can provide

$$\text{Days of storage} = \frac{34.6 \text{ h}}{12 \text{ h/day}} = 2.9 \text{ days}$$

6.14 Consider the design of a small PV-powered light-emitting-diode (LED) flashlight. The PV array consists of 8 series cells, each with rated current 0.3A @ 0.6V. Storage is provided by three series AA batteries that each store 2Ah at 1.2V when fully charged. The LED provides full brightness when it draws 0.4A @ 3.6V.

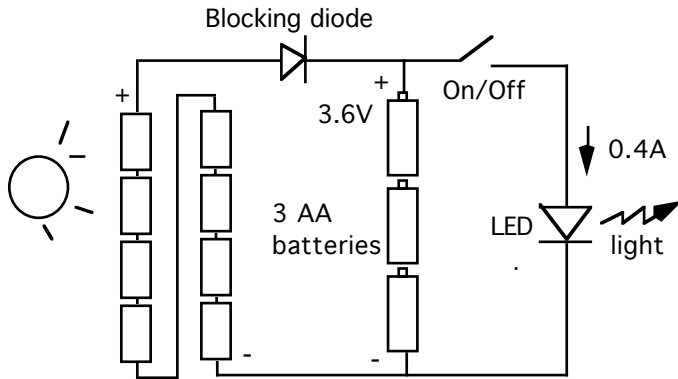


Figure P6.14

The batteries have a Coulomb efficiency of 95% and for maximum cycle life can be discharged by up to 80%. Assume the PVs have a 0.90 derating due to dirt and aging.

- a. How many hours of light could this design provide each evening if the batteries are fully charged during the day?

SOLN: 2 Ah x 0.80 = 1.6 Ah available. LED needs 0.4 A so 1.6/0.4 = 4 h of light

- b. How many kWh/m²-day of insolation would be needed to provide the amount of light found in (a)?

SOLN: Using (6.34)

$$\text{Ah battery} = I_R \times (\text{h @ full sun}) \times \text{PV derate} \times \text{Coulomb}$$

$$1.6 = 0.3 \text{ A} \times (\text{kWh/m}^2\text{-day}) \times 0.90 \times 0.95$$

$$\text{kWh/m}^2\text{-day} = \frac{1.6}{0.3 \times 0.90 \times 0.95} = 6.24 \dots \text{ a good sunny day}$$

- c. With 14%-efficient cells, what PV area would be required?

SOLN: Each 14%-efficient cell produces 0.3A x 0.6V = 0.18 W at STC

$$P_R (\text{W}) = 1,000 \text{ W/m}^2 \times A (\text{m}^2) \times \eta$$

$$A = \frac{0.18\text{W/cell} \times 8 \text{ cell}}{1000 \times 0.14} = 0.01028 \text{ m}^2 \text{ (about 16 in}^2\text{)}$$

- 6.15** 4. You are to design a 24-volt, all-dc, stand-alone PV system to meet a 2.4 kWh/day demand for a small, isolated cabin. You want to size the PV array to meet the load in a month with average insolation equal to 5.0 kWh/m²-day.

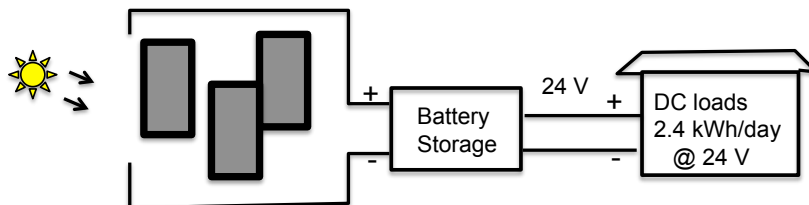


Figure P6.15

Your chosen PVs have their 1-sun maximum power point at $V_R = 18V$ and $I_R = 5A$. Assume a 0.80 derate factor for dirt, wiring, module mismatch (i.e. 20% loss). You'll use 200-Ah, 12-V batteries with 100% Coulomb efficiency.

- a. How many PV modules are needed (you may need to round up or down)? Sketch your PV array.

SOLN:

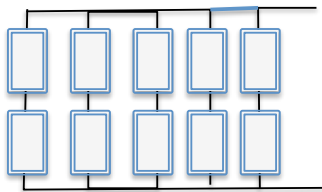
$$\text{Demand} = 2400 \text{ V-A-hr/day} / 24 \text{ V} = 100 \text{ Ah/day}$$

Need 2 modules in series to get above 24V for the batteries, each string provides 5A.

$$\text{Need Supply} = 5A/\text{string} \times 5 \text{ hr/day} \times N \text{ strings} \times 0.80 = 20 N \text{ Ah/day}$$

$$\text{Strings} = N = 100/20 = 5 \text{ in parallel}$$

$$\text{Total number of modules} = 2 \times 5 = 10 \text{ modules}$$



- b. How many 200-Ah, 12-V, deep-cycle batteries would be required to cover three days of no sun if their maximum discharge depth is 75%? Show how you would wire them up.

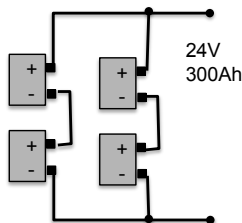
SOLN: Demand = 100 Ah/day x 3 days = 300 Ah @ 24V

Each battery has a usable storage of 200 Ah x 0.75 = 150 Ah @ 12V

Need 300 Ah/150Ah -per-string = 2 strings in parallel so AH add.

Need two 12-V batteries in series in each string to provide 24-V system voltage.

Total 2 strings x 2 batteries/string = 4 batteries.



6.16 Analyze the following simple PV/battery system, which includes four PV modules each with rated current and voltage as shown, and four 160-Ah, 6-V batteries with 90% Coulomb efficiency. An 85%-efficient inverter feeds the AC load. Notice there is no maximum power point tracker (MPPT).

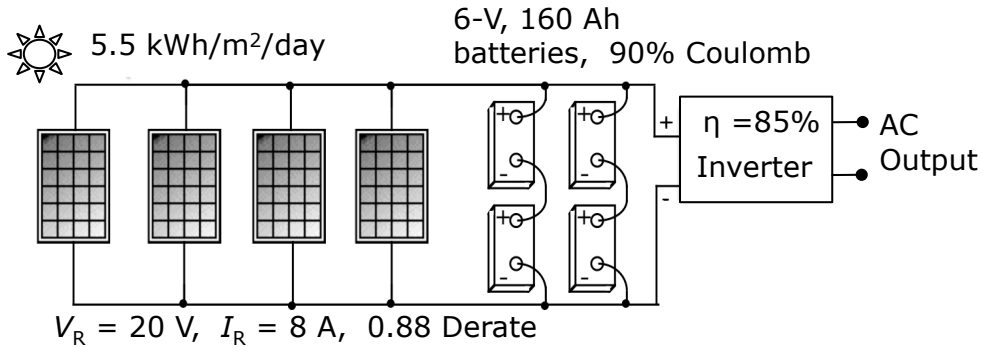


Figure P6.16

- a. With 5.5 kWh/m²-day of insolation, and 12% module loss due to dirt, wiring losses, etc, estimate the kWh/day that would reach the loads (assume all PV current passes through the batteries before it reaches the inverter).

SOLN: Using the peak hours approach with rated current from the modules:

$$\begin{aligned} \text{Wh/d} &= (\text{h/d @ full sun}) \times I_R (\text{A/module}) \times n (\text{modules}) \times \text{Dirt} \times \text{Coulomb} \times \text{Inverter} \\ &\quad \times \text{Battery voltage} \\ &= 5.5 \text{ hr/d} \times 8.0 \text{ A/module} \times 4 \text{ modules} \times 0.88 \times 0.90 \times 0.85 \times 12 \text{ V} = 1422 \text{ Wh/d} \\ &= 1.422 \text{ kWh/day reaching the load} \end{aligned}$$

- b. If the load requires 1 kWh/day, for how many cloudy days in a row can previously fully-charged batteries supply the load with no further PV input? Assume 80% of their Ah capacity is available.

SOLN:

$$\begin{aligned} \text{Batteries deliver} &= 2 \times 6\text{V} \times 160 \text{ Ah/string} \times 2 \text{ strings} \times 0.80 \text{ discharge} \times 0.85 \text{ inverter} \\ &= 2611 \text{ Wh} = 2.611 \text{ kWh available} \end{aligned}$$

At 1 kWh/day load, this is 2.61 days of battery-provided energy

6.17 Suppose the system in Problem 6.16 is redesigned to include a maximum power point tracker and a 97%-efficient charge controller. Assuming a typical 80% round-trip energy efficiency for the batteries, and a module derate of 0.88, how many kWh/day would reach the load with the same 5.5 kWh/m²-day insolation?

SOLN: With four 160-W modules, the energy delivered to the batteries is

$$\text{To batteries} = 4 \times 0.160 \text{ kW} \times 5.5 \text{ h/d} \times 0.88 \times 0.97 = 3.01 \text{ kWh/day}$$

With 80% round trip battery efficiency and an 85%-efficient inverter, the energy reaching the loads is

$$\text{To Loads} = 3.01 \text{ kWh/d} \times 0.80 \times 0.85 = 2.04 \text{ kWh/day}$$

This is 44% more than the non-MPPT system would deliver.

6.18 Following the guidelines given in Figure 6.34 for the design of off-grid PV systems, design a system that will deliver an average of 10 kWh/day of ac energy for a house located in a region with average insolation equal to 5 kWh/m²-day.

a. How many kW_{dc,STC} of photovoltaics should be used?

SOLN: Using the factors from PV to load given in Fig. 6.34

$$10 \text{ kWh} = kW_p \times 0.88(\text{derate}) \times 1(\text{MPP}) \times 0.97(\text{charge}) \times 0.80(\text{batt}) \times 0.85(\text{inv}) \times 5.0 \text{ h/d}$$

$$PV \text{ kW}_p = \frac{10 \text{ kWh/d}}{0.88 \times 0.97 \times 0.80 \times 0.85 \times 5 \text{ h/d}} = 3.45 \text{ kW}$$

b. For a 48-V battery system, how many amp-hours of storage would be needed to provide three days worth of energy with no sun? Assume maximum discharge of 80%.

$$\text{SOLN: Storage} = \frac{3 \text{ days} \times 10,000 \text{ VAh/day}}{48\text{V} \times 0.80(\text{depth})} = 781 \text{ Ah}$$

c. Using the cost guidelines provided in Example 6.16, what would be the capital cost of this system?

$$\text{SOLN: PV array @ } \$2/\text{W}_p = 3450 \text{ W} \times \$2/\text{W} = \$6900$$

$$\text{Battery @ } \$150/\text{kWh} = 781 \text{ Ah} \times 48\text{V} \times \$0.15/\text{Wh} = \$5623$$

$$\text{BOS @ } \$2/\text{W}_p = 3450 \times \$2 = \$6900$$

$$\text{Total hardware cost} = \$6900 + \$5623 + \$6900 = \$19,423$$

$$\text{BOS non hardware @ } 30\% \text{ of hardware} = 0.30 \times \$19,423 = \$5826$$

$$\text{Total cost} = \$19,423 + \$5,826 = \$25,249$$

d. Assuming a 4%, 20-yr loan, what would be the cost per kWh?

SOLN:

$$\text{CRF}(4\%, 20\text{yr}) = \frac{0.04 \times (1.04)^{20}}{(1.04)^{20} - 1} = 0.07358/\text{yr}$$

$$\text{Cost per kWh} = \frac{\$25,249 \times 0.07358/\text{yr}}{10 \text{ kWh/d} \times 365\text{d/yr}} = \$0.509 = 50.94\text{¢/kWh}$$

e. Suppose the diesel generator with efficiency curve shown in Fig. 6.35 operates at 70% of rated power while burning diesel that costs \$4.50/gallon. What is its cost per kWh generated?

SOLN: From the figure, it looks like it delivers about 8 kWh/gallon, which works out to

$$\text{Cost (70\%load)} = \frac{\$4.50/\text{gal}}{8 \text{ kWh/gal}} = \$0.563/\text{kWh}$$

6.19 Consider a directly-coupled PV-pump system with PV I - V curves and pump/system H - Q curves as shown below. Notice the start-up characteristics of the pump motor as it tries to overcome static friction before it can actually start pumping.

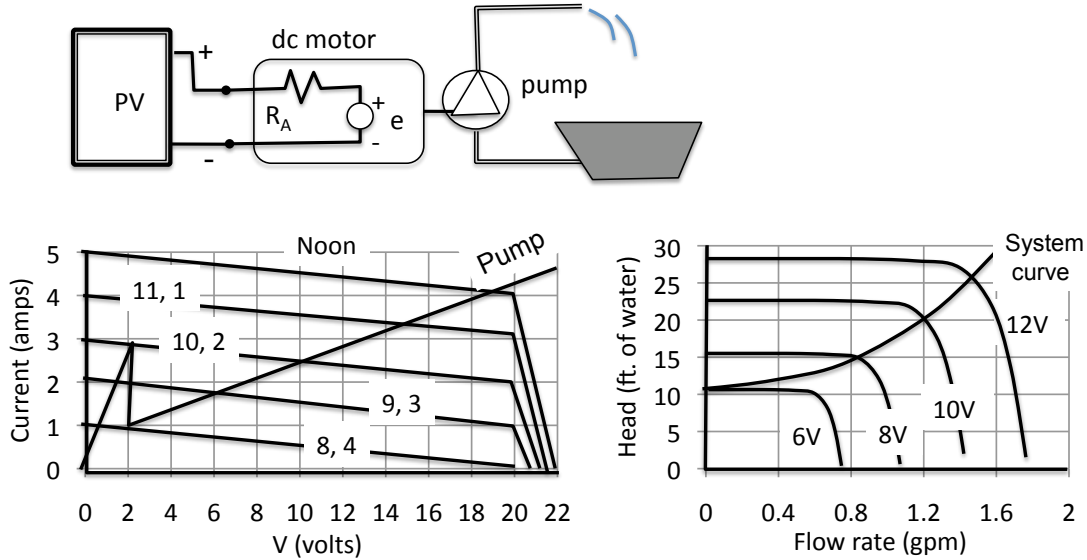


Figure P 6.19

a. At what time in the morning will the water start to flow?

SOLN: 10 am

b. What will the flow rate be a little after 10:00 am?

SOLN: 1.2 gpm

c. At what time in the afternoon will the flow stop?

SOLN: 3 pm (drops below 6V)

d. Assuming the equivalent circuit shown above for the pump motor, what is the motor's armature resistance, R_A ?

SOLN: $R = V/I = 1/\text{slope of initial } I\text{-}V \text{ rise} = 2V/3A = 0.67 \text{ ohms}$

6.20 A single PV module is directly connected to a dc water pump. The module has 41 cells, each of which has a parallel resistance of 9Ω . The I - V curves for the dc pump motor and the PV module under 1-sun of insolation are shown below. Also shown are the hydraulic Q - H curves for the pump and its load.

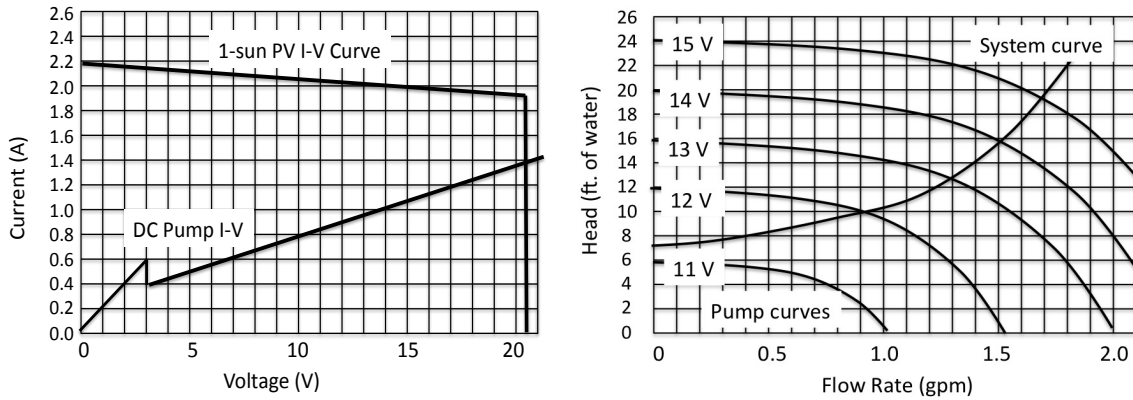


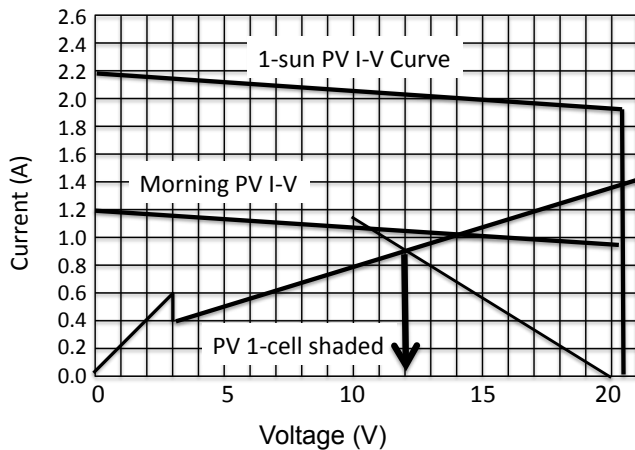
Figure P6.20

- a. At some time in the morning the pump is delivering 1.5 gpm of flow. At that time sketch the PV I-V curve. What must the insolation have been at that time?

SOLN: From the pump Q-H curves, at 1.5 gpm the voltage is 14 V. On the pump I-V curve, at 14V the current is 1 A, so the PV I-V curve has to shift down to that intersection (1A, 14V). The PV I-V curve shifts down by 1 A, making $I_{SC} = 1.2A$.

I_{SC} dropping from 2.2 A to 1.2A is a drop of $(2.2 - 1.2)/2.2 = 45.5\%$. That means insolation also dropped 45.5% (455 W/m²). So insolation is now

$$\text{Insolation} = 1000 - 455 = 545 \text{ W/m}^2$$



- b. At that time in the morning, what will the flow rate drop to if one cell is completely shaded?

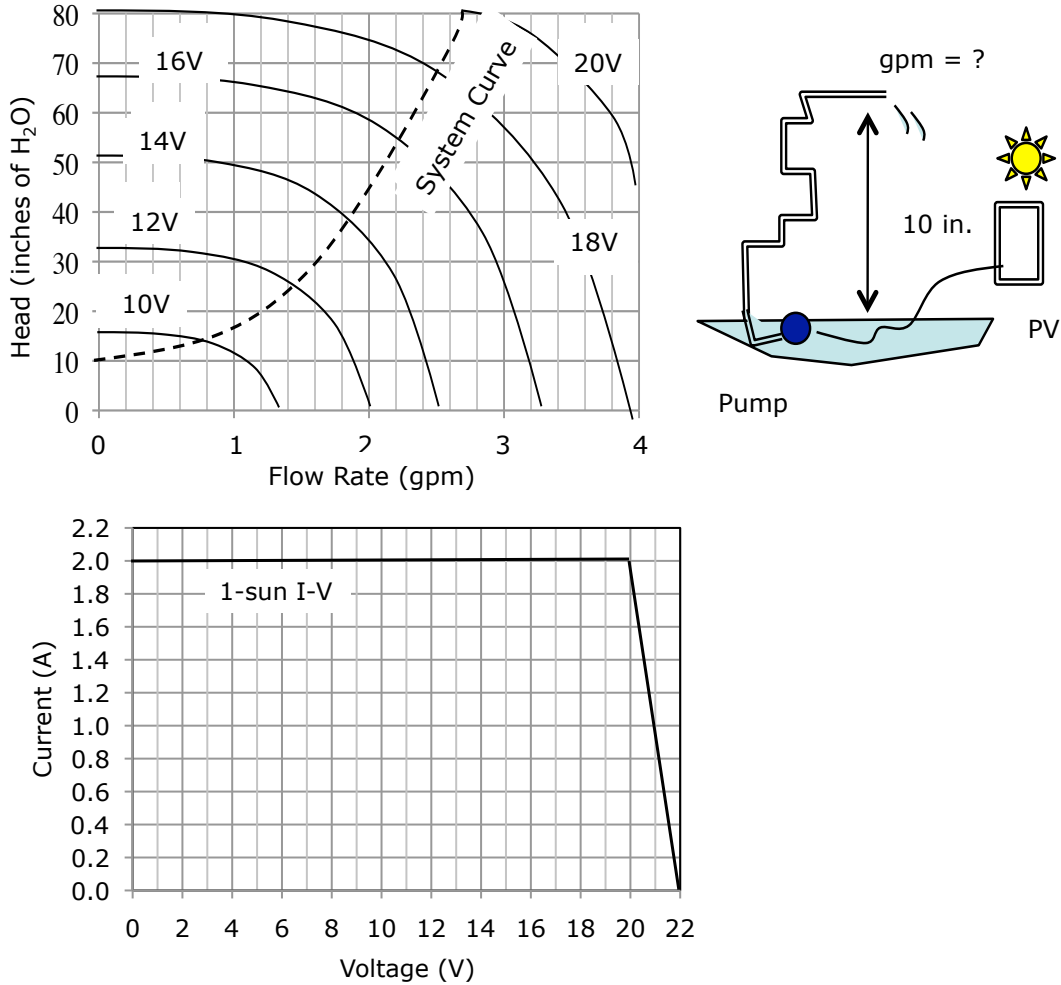
SOLN: At any given current, the PV I-V curve drops in voltage by

$$\Delta V = I \text{ (A)} \times 9 \Omega + 0.5$$

So, for example, at 1A $\Delta V = 9.5V$ and V is now about $20.5 - 9.5 = 11V$, which leads to the line drawn above. The intersection of pump and PV curves is

now at about 12V, which, from the hydraulic curves, means the flow rate is now about 0.9 gpm (quite a drop from the 1.6 gpm before).

6.21 Suppose you are setting up a little fountain for a pond using a PV-powered dc pump. Shown below are pump curves for various voltages along with a system curve including a static head of 10 inches. The PV I-V curve and hourly insolation are also shown.

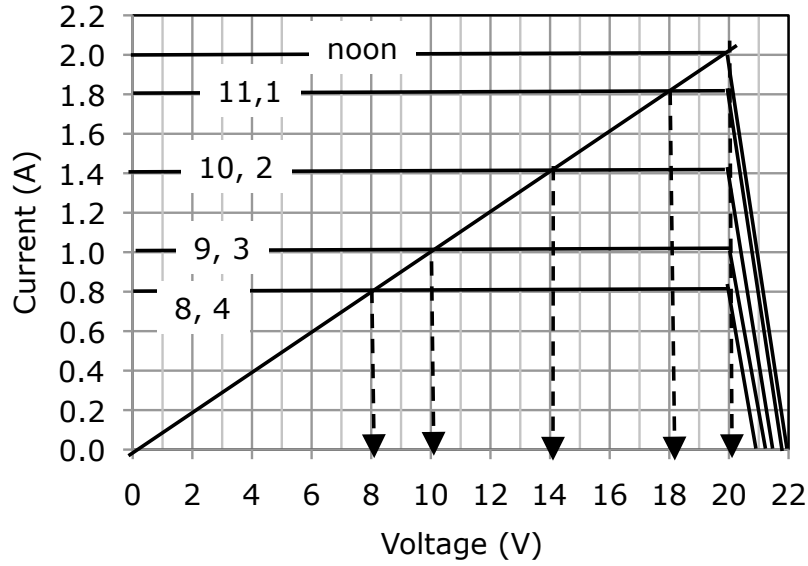


Time	8 am	9	10	11	12	1	2	3	4 pm
Insolation (kW/m ²)	0.4	0.5	0.7	0.9	1.0	0.9	0.7	0.5	0.4
gpm = ?									

Figure P 6.21

Suppose the electrical characteristics of the pump can be modeled as a simple 10-Ω resistance. Find the hourly flow rates (gpm) and estimate the total gallons pumped in one day (assume insolation is constant over each one-hour interval).

SOLN: I_{sc} is proportional to insolation, which establishes each hourly I-V curve. For example at 10 am, insolation is 0.7, so I_{sc} is $0.7 \times 2.0 = 1.4$ A. The intersection then is at 14 V, which from the pump curves gives about 1.8 gpm.



Time	8 am	9	10	11	12	1	2	3	4 pm
Insolation (kW/m ²)	0.4	0.5	0.7	0.9	1.0	0.9	0.7	0.5	0.4
voltage	8	10	14	18	20	18	14	10	8
gpm	0	0.8	1.8	2.5	2.7	2.5	1.8	0.8	0

Gallons/day = (2 x 0.8 + 2 x 1.8 + 2 x 2.5 + 2.7) x 60 min/h = 774 gallons/day