Chapter 8 PROBLEMS

8.1 A perfect Carnot heat engine receives 1000 kJ/s of heat from a high temperature source at 600°C and rejects heat to a cold temperature sink at 20°C.

a. What is the thermal efficiency of this engine?

b. What is the power delivered by the engine in watts?

c. At what rate is heat rejected to the cold temperature sink?

d. What are the entropy changes of the source and the sink?

SOLN:

a. What is the thermal efficiency of this engine?

Since it is a perfect heat engine, (8.6) gives

$$\eta = 1 - \frac{T_C}{T_H} = 1 - \frac{(20+273.15)}{(600+273.15)} = 0.6643 = 66.43\%$$

b. How many kW of output power would be delivered by this engine?

$$P = \eta\, P_{in} = 0.6643 \cdot 1000\ kJ/s \cdot 1\ kW/(kJ/s) = 664.3\ kW$$

c. At what rate is heat rejected to the cold temperature sink?

$$Q_C = (1-\eta)Q_H = (1-0.6643)\cdot 1000 kJ/s = 335.7 kJ/s$$

d. What is the entropy change of the source and sink?

$$\Delta S(source) = \frac{Q_H}{T_H} = \frac{1000 kJ/s}{(600+273.15)K} = 1.145 kJ/s \cdot K \text{ loss}$$

$$\Delta S(\text{sink}) = \frac{Q_C}{T_C} = \frac{335.7 kJ/s}{(20+273.15)K} = 1.145 kJ/s \cdot K \text{ gain}$$

As expected, they the net change is zero.

8.2 Suppose an ocean thermal energy conversion system (OTEC) uses the difference in temperature between 25°C water at the ocean's surface and 5°C water several hundred meters below the surface to power a Rankine cycle engine.

a. What is the maximum theoretical efficiency of the system ?

SOLN: From (8.6)

$$\eta_{\max} = 1 - \frac{T_C}{T_H} = 1 - \frac{5+273.15}{25+273.15} = 0.067 = 6.7\%$$

b. If the system runs at 40% of the Carnot efficiency, what flow rate of water through the system would be needed to generate 1 MW of electrical power (the density of seawater is 10^3 kg/m^3 with a specific heat of 4.2 x 10^3 Jkg^{-1}K^{-1})?

SOLN: The heat source must provide

$$\text{Thermal input} = \frac{1\text{ MW}}{0.40 \text{ x } 0.067} = 37.3\text{ MW}_t$$

It does so by dropping the temperature by 20°C.

$$Q_H = \rho c \dot{m} \Delta T$$

$$Q_H = 37.3x10^6 W \cdot \frac{1J/s}{W} = \frac{10^3 kg}{m^3} \cdot \frac{4.2x10^3 J}{kgK} \cdot \dot{m}\left(\frac{m^3}{s}\right) \cdot 20K$$

$$\dot{m} = \frac{Q_H}{\rho c \Delta T} = \frac{37.3x10^6}{10^3 x 4.2x10^3 x20} = 0.444 m^3 s = 7.0x10^3 gpm$$

That is a pretty substantial flow rate.

8.3 A solar pond consists of a thin layer of fresh water floating on top of a denser layer of salt water. When the salty layer absorbs sunlight it warms up and much of that heat is held there by the insulating effect of the fresh water above it (without the fresh water, the warm salt water would rise to the surface and dissipate its heat to the atmosphere).

a. What is the maximum efficiency of a heat engine operating between the 90°C salty layer of a pond and the 20°C freshwater layer?

SOLN:

$$\text{Carnot } \eta = 1 - \frac{T_C}{T_H} = 1 - \frac{20 + 273.15}{90 + 283.15} = 0.214 = 21.4\%$$

b. If a real engine is able to achieve 20% of the efficiency of a Carnot engine, how many kilowatt hours of electricity could be generated per day from a 100-m x 100-m pond that captures and stores 30% of the 6 kWh/m² solar radiation striking the surface?

SOLN:

$$\text{Energy} = 0.20 \text{ x } 0.214 \text{ x } \left(100 \text{ x } 100\text{m}^2\right) \text{x } 0.30 \text{ x } 6 \text{ kWh/m}^2 = 772 \text{ kWh/d}$$

8.4 A portion of the wave energy conversion performance (kW) of the 750-kW Pelamis system is shown in Fig. P8.4. Suppose this machine is located in the vicinity of a buoy having the wave scatter diagram shown in Table 8.5.

	Peak Wave Period T_P (s)												
H_S (m)	3	4	5	6	7	8	9	10	11	12	14	17	20
3.0	0	91	180	246	402	424	417	369	343	331	229	144	93
2.5	0	7	93	171	279	342	351	320	274	230	174	100	65
2.0	0	0	66	109	199	219	225	205	195	162	112	64	41
1.5	0	0	26	62	112	141	143	129	110	91	63	36	23
1.0	0	0	11	27	50	62	64	57	49	41	28	0	0
0.5	0	0	0	0	0	0	0	0	0	0	0	0	0

Figure P8.4

a. What is the energy available (kWh/m/yr) in the waves at that location that have a 2-m significant wave height and an 8-s peak wave period?

SOLN: From Table 8.5, there are 272 hours of 2-m, 8-s waves. From (8.10) the power they contain is

$$P = 0.42\left(H_S\right)^2 T_P = 0.42x2^2x8 = 13.44kW\ /\ m$$

So their energy content is 13.44 kW/m x 272 h/yr = 3656 kWh/m/yr (which agrees with Table 8.6).

b. Assuming power take-off losses of 25%, how much energy (kWh/yr) would be delivered by the Pelamis machine from those same 2-m, 8-s waves?

SOLN: From Table P8.4, this machine delivers 219 kW in these waves, so

Energy = 219 kW x 272 h/yr x 0.75 = 4.47 x 10^3 kWh/yr

c. This Pelamis (sea snake) is 180-m long. What fraction of the energy in the waves that impact this machine is actually delivered?

SOLN: $$\eta = \frac{\text{delivered}}{\text{available}} = \frac{4.47\text{x}10^3\ \text{kWh/yr}}{3656\ \text{kWh/m/yr x 180m}} = 0.0068 = 0.68\%$$

8.5 The following table (P8.5) is an hours-per-month wave scatter diagram for the month of August for the same buoy described in Table 8.5 in the chapter.

				Peak Wave Period T_P (s)									
H_S (m)	3	4	5	6	7	8	9	10	11	12	14	17	20
3.0	0	0	0	0	0	2	1	0	0	0	0	0	0
2.5	0	0	0	0	3	9	3	2	0	0	0	0	0
2.0	0	0	0	4	19	42	12	7	4	2	1	0	0
1.5	0	0	2	24	52	74	32	19	9	6	9	15	1
1.0	1	4	10	30	37	68	42	28	16	16	47	40	5
0.5	1	1	1	3	3	9	5	3	2	5	9	4	0
0.0	0	0	0	0	0	0	0	0	0	0	0	0	0

Table P8.5

a. Compare the average power (kW/m) in waves with 3-m significant wave height versus 2-m height if both have a peak wave period of 9 s.

SOLN: From (8.10)

$$P(3m) = 0.42\left(H_S\right)^2 T_P = 0.42 \text{ x } 3^2 \text{x } 9 = 34.0 \text{ kW/m}$$

$$P(2m) = 0.42\left(H_S\right)^2 T_P = 0.42 \text{ x } 2^2 \text{x } 9 = 15.1 \text{ kW/m}$$

That's a ratio of 34/15.1 = 2.25

b. How much energy (kWh/m) would waves with significant wave heights between 2.25 m and 2.75 m have during this month?

SOLN: From the table there are 3 h @ 7s, 9 h @ 8s, 3h @ 9s and 2 h@ 10s. Since all have the same H_S = 2.5 m, it is easy to use (8.10) to give

$$\text{Energy} = 0.42\left(H_S\right)^2 T_P \text{ kW/m x h/mo}$$

$$= 0.42\text{x}\left(2.5\right)^2\left[3hx7 + 9hx8 + 3hx9 + 2hx10\right] = 367.5 \text{ kWh/m/mo}$$

c. How much energy would the 1-MW turbine in Table 8.7 deliver in this month for those 2.5-m waves? Assume 95% availability, an 85% directionality factor, and a 90%-efficient generator.

SOLN: With 3 h at 7s and 231 W, 9h @ 8s and 269 W, 3h @9s and 325 W, and 2h @10s and 401 W:

Shaft = 231W x 3h + 269W x 9h + 325W x 3 h + 401W x 2h

= 4891 Wh/mo = 4.89 kWh/mo

Delivered = 4.89 x 0.95 x 0.85 x 0.90 = 3.55 kWh/mo/m for these 2.5 m waves.

8.6 An in-stream tidal power system consisting of an underwater 11-m-diameter, horizontal axis turbine that delivers its rated power of 300 kW in 2.4 m/s currents.

a. What is the efficiency of this system at that current speed?

SOLN: First find power in the flow from (8.11)

$$P = \frac{1}{2}\rho A v^3 = 0.5x1025x\frac{\pi}{4}x11^2x2.4^3 = 673x10^3W = 673kW$$

so, the efficiency is $\eta = \dfrac{320 \text{ kW}}{673 \text{ kW}} = 0.48 = 48\%$

b. In those currents what will be the blade rpm and the blade tip speed (m/s and mph) if it operates with a TSR = 5.0?

SOLN: Since TSR = tip speed/water speed,

Tip speed = 5 x 2.4 m/s = 12 m/s = 26.8 mph

And the rpm would be

$$\frac{\text{rev}}{\text{min}} = \frac{12 \text{ m/s x } 60 \text{ s/min}}{\pi \text{ D (m/rev)}} = \frac{12x60}{\pi x11} = 20.8 \text{ rev/min}$$

c. If placed in a region with sinusoidal current oscillations with peak speed of 2.5 m/s, what would be the average power that the water would provide to this turbine?

SOLN: Using (8.15)

$$P_{avg,water} = \frac{2}{3\pi}\rho A V_m^3 = \frac{2}{3\pi}x1025x\frac{\pi}{4}x11^2x2.5^3 = 323x10^6W = 323kW$$

d. If we assume the average turbine efficiency is 60% of its value at rated power, how much energy would the turbine deliver and what would be its capacity factor?

SOLN: At an average efficiency of 0.60 x 48% = 28.8% this machine would deliver

Energy = 0.288 x 323 kW x 8760 h/yr = 0.815 x 10^6 kWh/yr

$$CF = \frac{\text{Actual}}{\text{Possible}} = \frac{0.815\text{x}10^6 \text{ kWh/yr}}{300 \text{ kW x } 8760 \text{ h/yr}} = 0.31$$

8.7 Suppose a 300-kW tidal power system is located in the current regime described below (Fig. P8.7). Also shown are data for the power curve of the turbine.

Tidal Speed (m/s)	Probability	Power (kW)
0.0	0.000	0
0.5	0.243	0
1.0	0.278	25
1.5	0.222	90
2.0	0.150	220
2.5	0.077	300
3.0	0.030	300
3.5	0.000	300

Table P8.7

a. Estimate the annual energy that would be delivered by 1.5 m/s tidal speeds (actually, between 1.25 and 1.75 m/s).

SOLN: Hours @ 1.5 m/s = 0.222 x 8760 h/yr = 1944 h/yr

Energy = 1944 h/yr x 90 kW = 175,025 kWh/yr

b. Estimate the total annual energy delivered and the overall capacity factor for the turbine.

SOLN: Easier to do on a spreadsheet:

Tidal Speed (m/s)	Probability	Power (kW)	h/yr @ speed	Energy (kWh/yr)
0.0	0.000	0	-	-
0.5	0.243	0	2,129	-
1.0	0.278	25	2,435	60,882
1.5	0.222	90	1,945	175,025
2.0	0.150	220	1,314	289,080
2.5	0.077	300	675	202,356
3.0	0.030	300	263	78,840
3.5	0.000	300	-	-
Totals =	1.00		8760	806,183

$$CF = \frac{\text{delivered}}{\text{possible}} = \frac{806{,}183 \text{ kWh/yr}}{300 \text{ kW x } 8760 \text{ h/yr}} = 0.307$$

8.8 Suppose 200 gpm of water is taken from a creek and delivered through 800 ft of 3-in diameter PVC pipe to a turbine 100 ft lower than the source. If the turbine/generator has an efficiency of 40%,

a. Find the electrical power that would be delivered by the generator.

SOLN: From Fig. 8.34, at 200 gpm, 3-in. PVC loses about 6 ft of head for every 100 ft of length. Since we have 800 ft of pipe, the friction loss is

Friction Loss = 800 ft x 6 ft/100 ft = 48 ft of head loss

That is 48 percent of the 100 feet of available elevation head.

From (8.20) power delivered from the system will be

$$P(kW) = \frac{\eta Q(gpm) H_N(ft)}{5300} = \frac{0.40x200x(100-48)}{5300} = 0.785 kW$$

b. What diameter pipe would be needed to keep the flow speed around 5 ft/s or less?

SOLN:

To keep flow to less than 5 ft/s

$$A = \frac{Q}{v} = \frac{200 \text{ gal/min}}{5 \text{ ft/s x } 60 \text{ s/min x } 7.4805 \text{ gal/ft}^3} = 0.08912 \text{ ft}^2 = \frac{\pi}{4} D^2$$

$$D = \sqrt{\frac{4x0.08912 ft^2}{\pi} x \frac{144 in^2}{ft^2}} = 4.04 \text{ in}$$

Choose 4-in PVC pipe (speed will be slightly higher than the guideline)

c. Assuming locally available PVC pipe comes in 1-in diameter increments (2-in, 3-in, etc), pick a pipe size closest to the above suggested diameter and find the power delivered by the generator with this pipe.

SOLN: From Fig. (8.34), friction loss will now be about 1.8 ft/100 ft

Friction loss = 800 ft x 1.8 ft/100 ft = 14.4 ft

Power delivered would now be

$$P(kW) = \frac{\eta Q(gpm) H_N(ft)}{5300} = \frac{0.40x200x(100-14.4)}{5300} = 1.29 kW$$

... an increase of about 65% compared with the 0.785 kW using 3-in pipe

8.9 Equations (8.21) and (8.22) for a rectangular weir are based on water height *h* above the notch being at least 2 inches while the notch width *W* must be at least 3*h*. Design a notch (width and height) that will be able to measure the maximum flow when the minimum flow is estimated to be 200 gpm. What maximum flow rate could be accommodated?

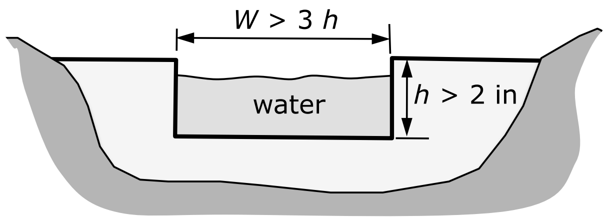

Figure P8.9

SOLN: To keep the water height at least 2 in above the notch at the lowest flow rate, (8.22) suggests

$$W(in) \le \frac{Q}{2.9h^{1.5}} + 0.2h = \frac{200}{2.9 \text{ x } 2^{1.5}} + 0.2 \text{ x } 2 = 24.8 \text{ in}$$

The narrower the notch, the higher the water will rise above the weir. To keep the weir from overflowing at high water rates, we want the widest weir possible. So choose W = 24.8 in.

To satisfy the W > 3h constraint ,

h < W/3 = 24.8/3 = 8.27 in

The flow rate that would correspond to h = 8.27-in with W = 24.8-in is

$$Q = 2.9(W - 0.2h)h^{3/2} = 2.9(24.8 - 0.2x8.27)\cdot 8.27^{3/2} = 1595\, gpm$$

8.10 Suppose 300-m of elevation separate the upper and lower reservoirs of a pumped hydro system. Each has an average surface area of 10 hectares (100,000 m^2) and their surfaces are allowed to vary in elevation by 1 m. If the penstock efficiency is 90% and the turbine/generator efficiency is 80% what is the average power that could be delivered over a 12-h period?

SOLN: From (8.23), daily energy available from the 100,000 m^2 upper reservoir when it drops 1 m is

$$E = \frac{\rho(kg/m3)\, g(m/s^2)\, \Delta h(m) x A(m^2) x H(m)}{3.6x10^6\, J/kWh}$$

$$E = \frac{1000x9.81x1x100x10^3x300}{3.6x10^6} = 81{,}750\, kWh$$

Accounting for the penstock and turbine/generator efficiencies and averaging over a day

$$P_{avg} = \frac{81{,}750\, kWh/day x 0.80 x 0.90}{12 h/day} = 4905\, kW$$

8.11 Consider the design of an off-grid, pumped-hydro system for a photovoltaic-powered small cabin located next to a pond. . Suppose the demand is estimated to be 10 kWh/day with a peak demand of 3 kW. A tank will be placed on a nearby hilltop 100 ft above the turbine/generator. Connecting pipe runs to and from the tank are each 800 ft.

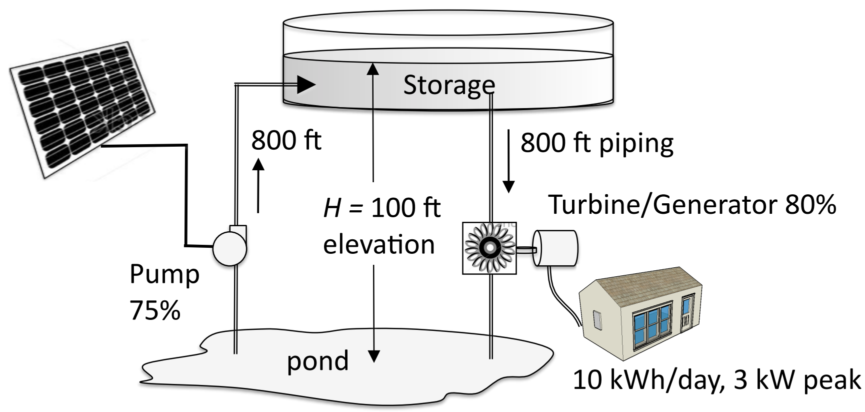

Figure P8.11

a. Assuming an average of 15% head losses in the pipeline and 80% conversion efficiency for the turbine/generator, how many gallons of water needs to flow from the upper tank to the pond in a day's time?

SOLN: Using the basic potential energy = weight x height (or 8.23)

$$E(kWh) = 10kWh = \frac{\text{V(gal) x 8.34 lb/gal x 100 ft}}{2.655\text{x}10^{6}\text{ft-lb/kWh}}\text{x}0.85\text{x}0.80 = 2.14\text{x}10^{-4}V$$

$$V = \frac{10}{2.14\text{x}10^{-4}} = 46{,}815 \text{ gal}$$

b. Size the upper storage tank (gallons) to provide a full day's worth of back-up energy when the renewables don't supply any power. Assume maximum allowed tank drainage is 75%.

SOLN: Tank size = 46,815 gal / 0.75 = 62,420 gal

c. Assuming piping losses are 20% during peak demand what flow rate from the tank would be needed to supply the 3 kW peak?

SOLN: Using (8.20) with 20% head losses and 80% conversion efficiency

$$Q(gpm) = \frac{5300 x P(kW)}{\eta \cdot H_G(ft)} = \frac{5300 x 3}{0.80 x (100 - 20)} = 248 \text{ gpm}$$

d. Using the choices given in Fig. 8.34, what size pipe would keep losses to around 20% at peak demand?

SOLN: With the assumed head loss of 20 ft, the loss is 20 ft/800 ft = 2.5 ft per 100 ft of piping at 250 gpm. From the figure, it looks like 4-in PVC at 250 gpm loses about 3 ft/100-ft, so to meet our goals you should probably move up to the 5-in PVC (with 1.5 ft/100 ft loss).

e. How much energy would a photovoltaic system have to provide to meet the average daily demand for energy. Assume 15% piping losses in each piping run and a 75%-efficient pump.

SOLN: The house needs 10 kWh/day. The pump is 75% efficient, the piping losses over both the supply and return lines totals 30%, the turbine/generator is 80% efficient, which makes the round-trip efficiency only

Round trip efficiency = 0.75 x 0.70 x 0.80 = 0.42 = 42%

Total demand = 10 kWh/d / 0.42 = 23.8 kWh/day

f. In an area with 5.5 kWh/m^2/day insolation, size a photovoltaic system to supply the 10 kWh/day needed by the household. Assume a derate factor of 0.75.

SOLN: Using (6.7)

23.8 kWh/d = $P_{dc,stc}$(kW) x h/day full sun x derate = kWp x 5.5 x 0.75

Peak power dc, stc rating = kWp = 5.77 kW