Review of Single-Phase AC Sinusoidal Circuits

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Root Mean Square (rms) Values and Phasors

- Goal of phasor analysis is to simplify the analysis of constant frequency ac systems

$$v(t) = V_{\max}\cos(\omega t + \theta_v)$$
$$i(t) = I_{\max}\cos(\omega t + \theta_i)$$

- Root Mean Square (RMS) voltage of sinusoid

$$\sqrt{\frac{1}{T}\int_0^T v(t)^2 dt} = \frac{V_{\max}}{\sqrt{2}}$$

The RMS, cosine-referenced voltage phasor is:

$$\begin{aligned}
V &= |V| e^{j\theta_V} = |V| \angle \theta_V \\
v(t) &= \operatorname{Re} \sqrt{2}\, V e^{j\omega t} e^{j\theta_V} \\
V &= |V| \cos\theta_V + j|V| \sin\theta_V \\
I &= |I| \cos\theta_I + j|I| \sin\theta_I
\end{aligned}$$

Impedance of linear elements

Device	**Time Analysis**	**Phasor**
Resistor	$v(t) = Ri(t)$	$V = RI$
Inductor	$v(t) = L\dfrac{di(t)}{dt}$	$V = j\omega LI$
Capacitor	$\dfrac{1}{C}\int_0^t i(t)dt + v(0)$	$V = \dfrac{1}{j\omega C}I$

$$\text{Z} \;=\; \text{Impedance} = R + jX = |Z|\angle\phi$$

$$\text{R} \;=\; \text{Resistance}$$

$$\text{X} \;=\; \text{Reactance}$$

$$|\text{Z}| \;=\; \sqrt{R^2 + X^2} \qquad \phi = \arctan\left(\frac{\text{X}}{R}\right)$$

Instantaneous and Average Power, Power Factor

Instantaneous Power

$$p(t) = v(t)\, i(t)$$

$$\mathrm{v(t)} = V_{\max} \cos(\omega t + \theta_V)$$

$$i(\mathrm{t}) = I_{\max} \cos(\omega t + \theta_I)$$

$$p(t) = \frac{1}{2} V_{\max} I_{\max} [\cos(\theta_V - \theta_I) + \cos(2\omega t + \theta_V + \theta_I)]$$

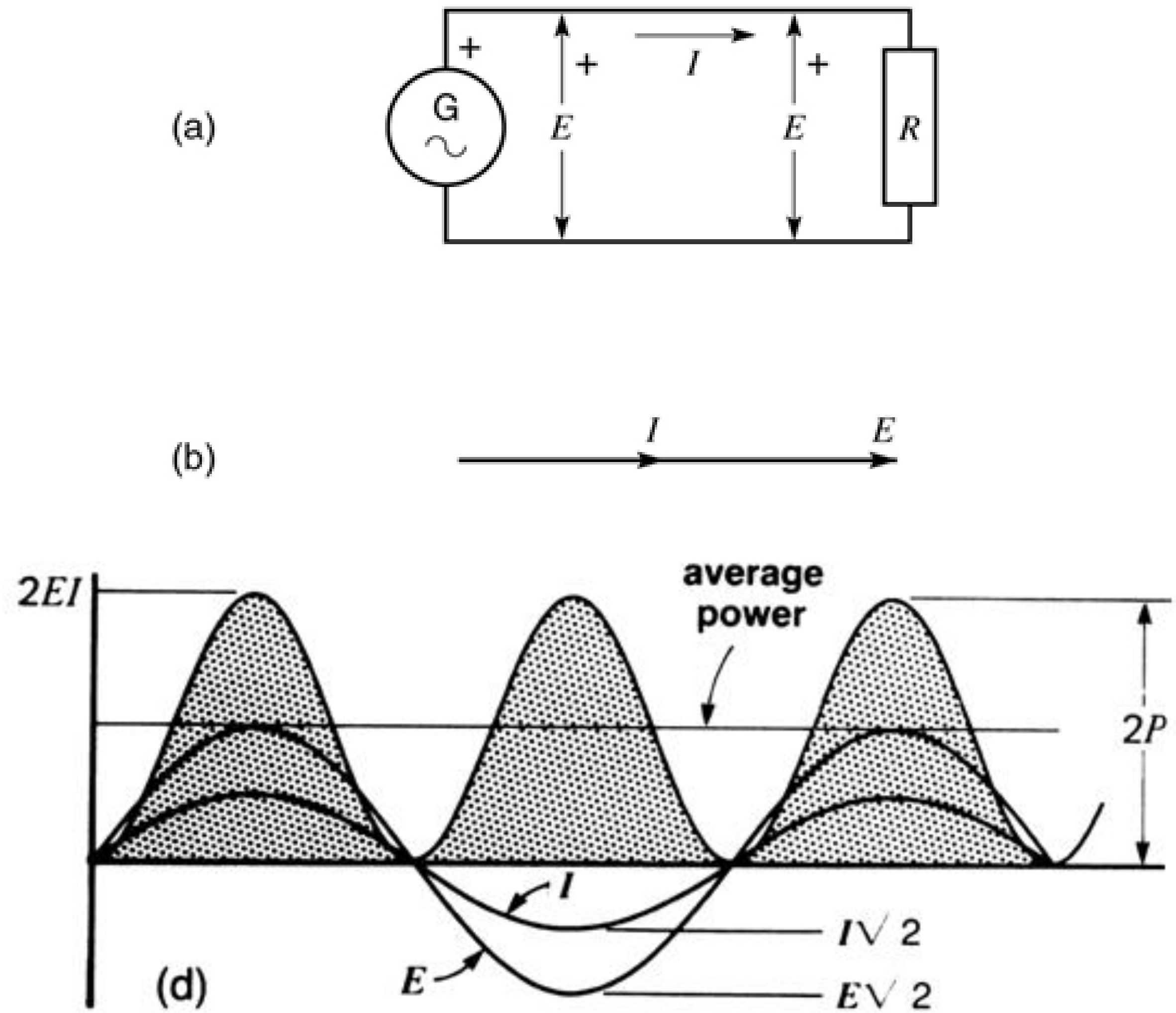
$$P_{avg} = \frac{1}{T} \int_0^T p(t)\,dt$$

$$= \frac{1}{2} V_{\max} I_{\max} \cos(\theta_V - \theta_I)$$

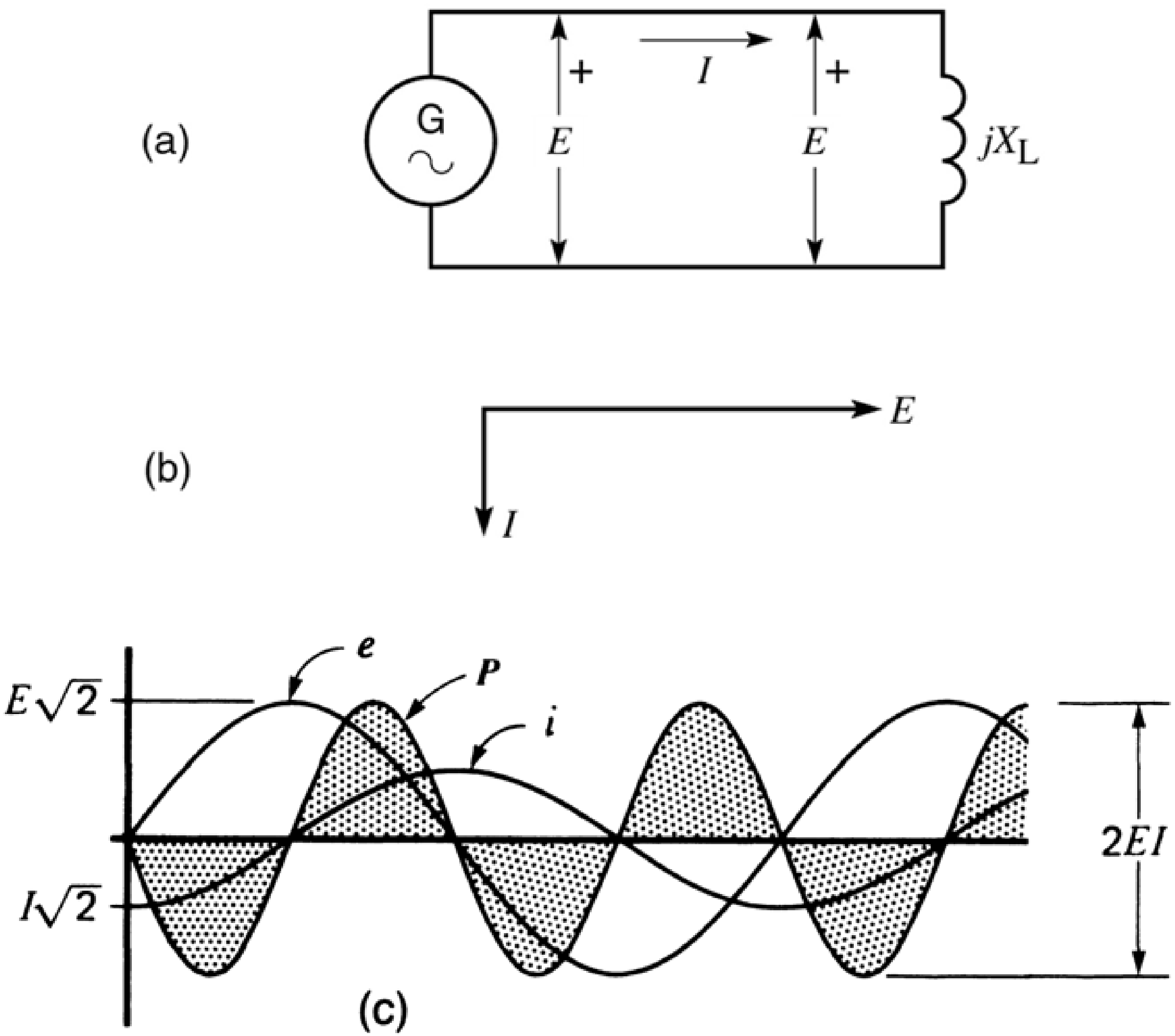
$$= |V||I| \cos(\theta_V - \theta_I)$$

Power Factor Angle $= \ \phi = \theta_V - \theta_I$

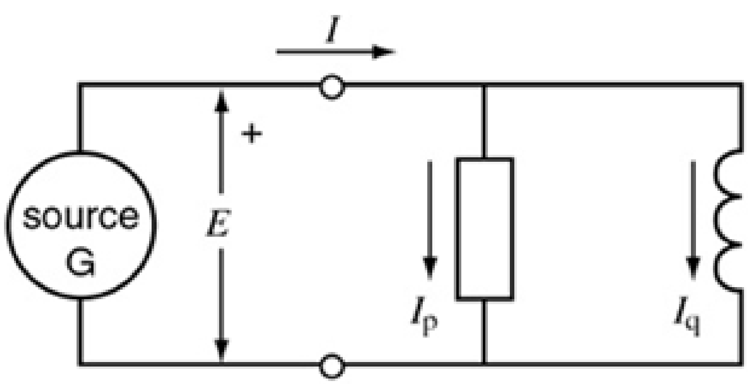
Pure Resistive Load

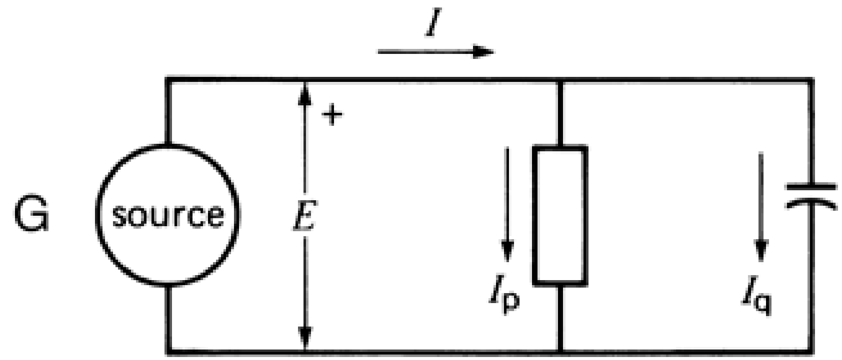


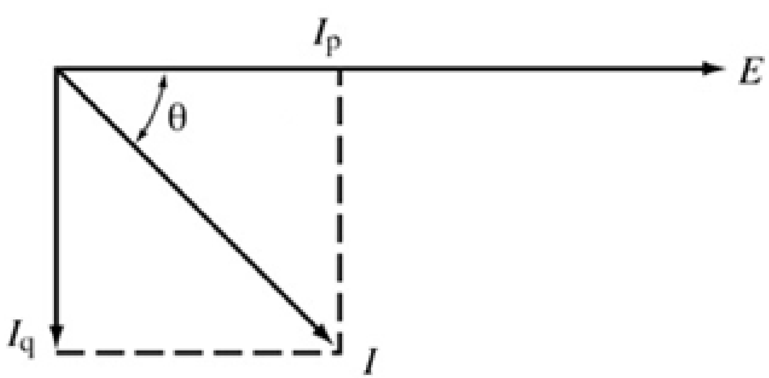
Pure Inductive Load

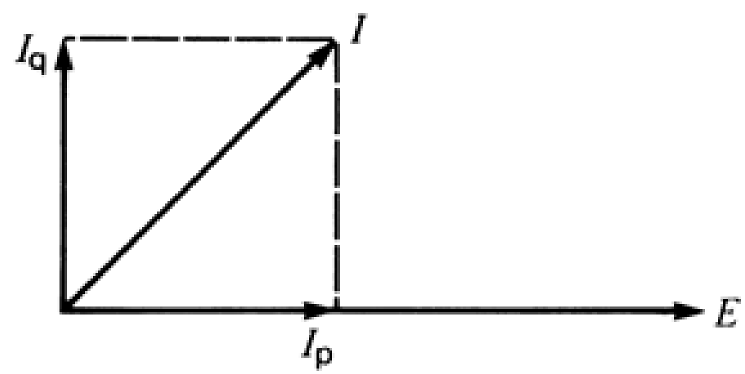


Resistive-Inductive and resistive-capacitive Load



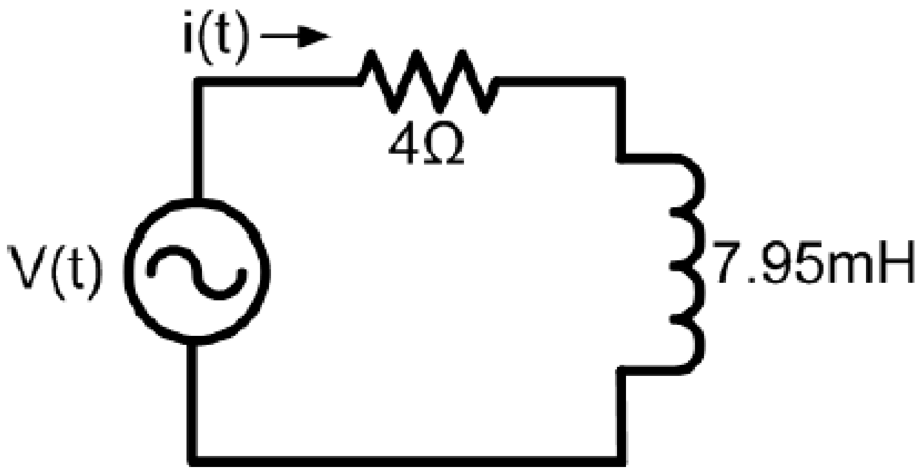






Example: analyzing AC circuits with phasors

Find the instantaneous current i(t)



$$
\begin{aligned}
V(t) &= \sqrt{2}\,100\cos(\omega t + 30^\circ) \\
f &= 60\text{Hz} \\
\text{R} &= 4\Omega \qquad X = \omega L = 3 \\
|Z| &= \sqrt{4^2+3^2} = 5 \qquad \phi = 36.9^\circ \\
I &= \frac{V}{Z} = \frac{100\angle 30^\circ}{5\angle 36.9^\circ} \\
&= 20\angle -6.9^\circ \text{ Amps} \\
\text{i(t)} &= 20\sqrt{2}\cos(\omega t - 6.9^\circ)
\end{aligned}
$$

Complex Power, Apparent Power and Reactive Power

$$S = |V||I|[\cos(\theta_V - \theta_I) + j\sin(\theta_V - \theta_I)]$$
$$= P + jQ$$
$$= V\,I^*$$

(Note: S is a complex number but not a phasor)

P = Real Power (W, kW, MW)

Q = Reactive Power (var, kvar, Mvar)

S = Complex power (VA, kVA, MVA)

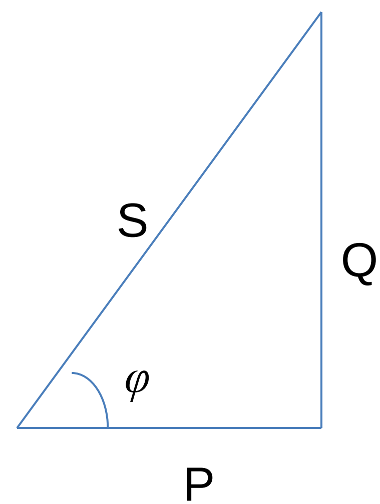
Power Factor (pf) = $\cos\phi$

If current leads voltage then pf is leading

If current lags voltage then pf is lagging

$$P = |S|\cos\phi$$

$$Q = |S|\sin\phi = \pm|S|\sqrt{1 - pf^2}$$



Power Triangle

$|S| = |V||I| = (P^2+Q^2)^{1/2}$ = Apparent Power

Power Consumption of Linear Circuit Elements

- Resistors only consume real power

$$\mathrm{P}_{\mathrm{Resistor}} = \left| I_{\mathrm{Resistor}} \right|^2 R$$

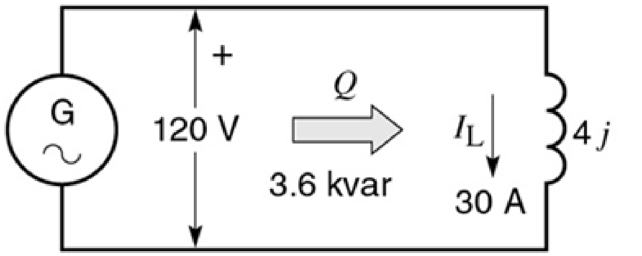
- Inductors only consume reactive power

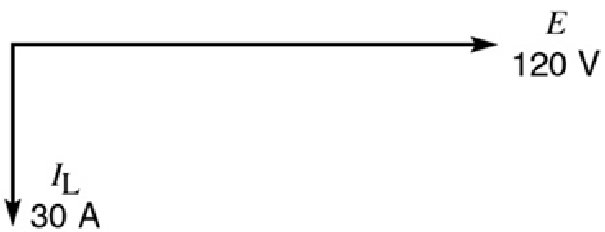
$$\mathrm{Q}_{\mathrm{Inductor}} = \left| I_{\mathrm{Inductor}} \right|^2 X_{\mathrm{L}}$$

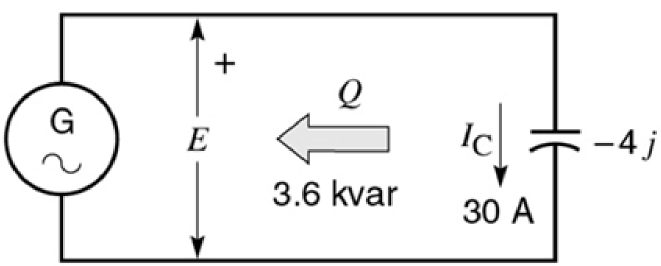
- Capacitors only generate reactive power

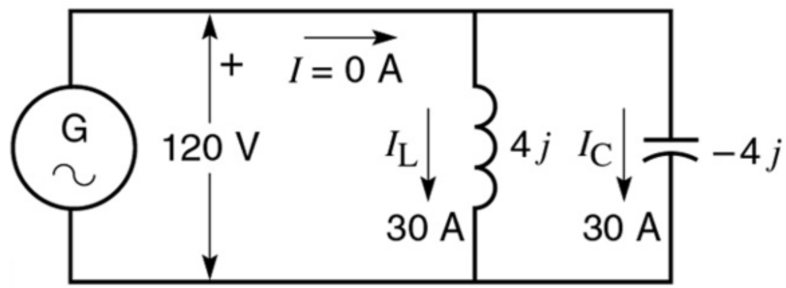
$$\mathrm{Q}_{\mathrm{Capacitor}} = -\left| I_{\mathrm{Capacitor}} \right|^2 X_{\mathrm{C}} \qquad X_C = \frac{1}{\omega C}$$

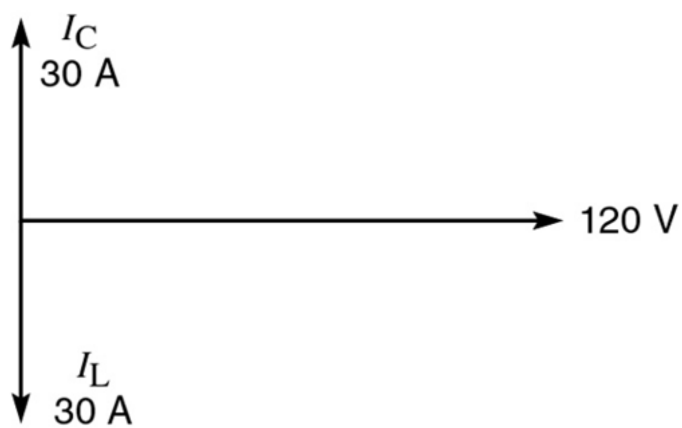
Power in inductive and capacitive circuits











Example: Active, Reactive, Apparent Power

Example: A load draws 100 kW with a leading pf of 0.85. What are ϕ (power factor angle), Q and $|S|$?

$$\phi = -\cos^{-1} 0.85 = -31.8^\circ$$

$$|S| = \frac{100kW}{0.85} = 117.6 \text{ kVA}$$

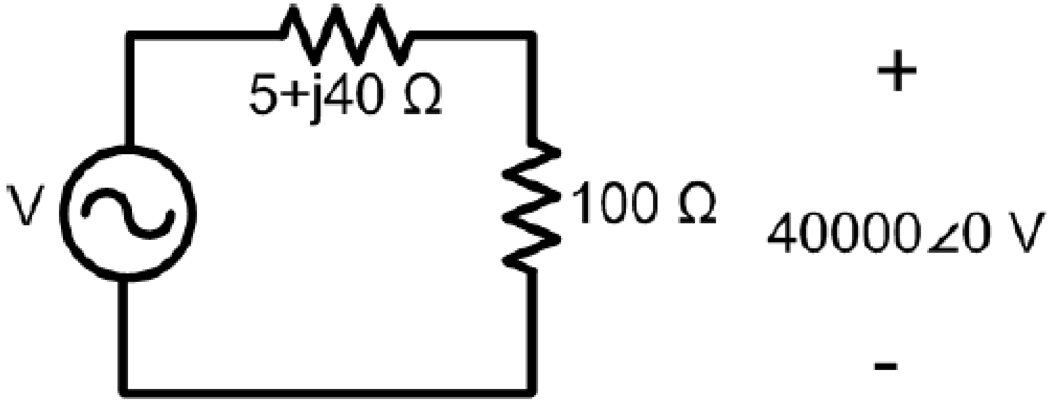
$$Q = 117.6 \sin(-31.8^\circ) = -62.0 \text{ kVar}$$

Conservation of Power

- At every node (bus) in the system
  - Sum of real power into node must equal zero
  - Sum of reactive power into node must equal zero
- This is a direct consequence of Kirchhoff's current law, which states that the total current into each node must equal zero.
  - Conservation of power follows since $S = VI^*$

Example: active and reactive power generation

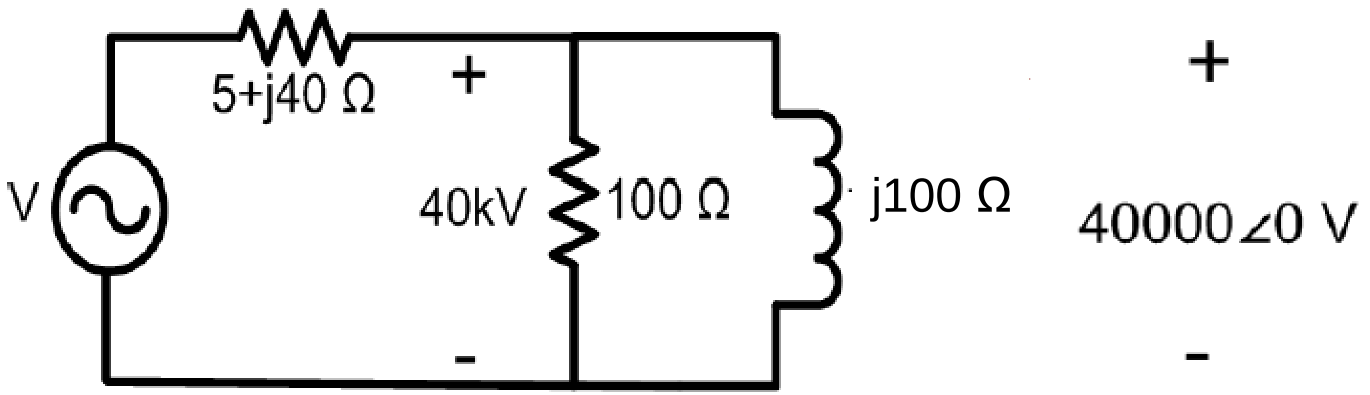
Find the real and reactive power generated by the source



$$
\begin{aligned}
I &= \frac{40000\angle 0^\circ\, V}{100\angle 0^\circ\, \Omega} = 400\angle 0^\circ \text{ Amps} \\
V &= 40000\angle 0^\circ + (5 + j40)\, 400\angle 0^\circ \\
&= 42000 + j16000 = 44.9\angle 20.8^\circ \text{ kV} \\
\text{S} &= VI^{*} = 44.9\text{k}\angle 20.8^\circ \times 400\angle 0^\circ \\
&= 17.98\angle 20.8^\circ \text{ MVA} \quad = 16.8 + j6.4 \text{ MVA}
\end{aligned}
$$

Example: active and reactive power generation

Find the real and reactive power generated by the source



$$Z_{Load} = 70.7\angle 45^\circ \qquad pf = 0.7 \text{ lagging}$$

$$I = 564\angle -45^\circ \text{ Amps}$$

$$V = 59.7\angle 13.6^\circ \text{ kV}$$

$$S = 33.7\angle 58.6^\circ \text{ MVA} \quad = 17.6 + j28.8 \text{ MVA}$$

Capacitors in Power Systems

Capacitors are used extensively in power systems to generate reactive power locally in order to correct the power factor, reduce the source current, regulate the voltage and improve system efficiency.

Example: power factor correction

Assume we have 100 kVA load with pf = 0.8 lagging, and would like to correct the pf to 0.95 lagging

$$S \;=\; 80 + j60 \text{ kVA} \qquad \phi \;=\; \cos^{-1} 0.8 = \; 36.9°$$

$$\text{pf of } 0.95 \text{ requires } \phi_{\text{desired}} \;=\; \cos^{-1} 0.95 = 18.2°$$

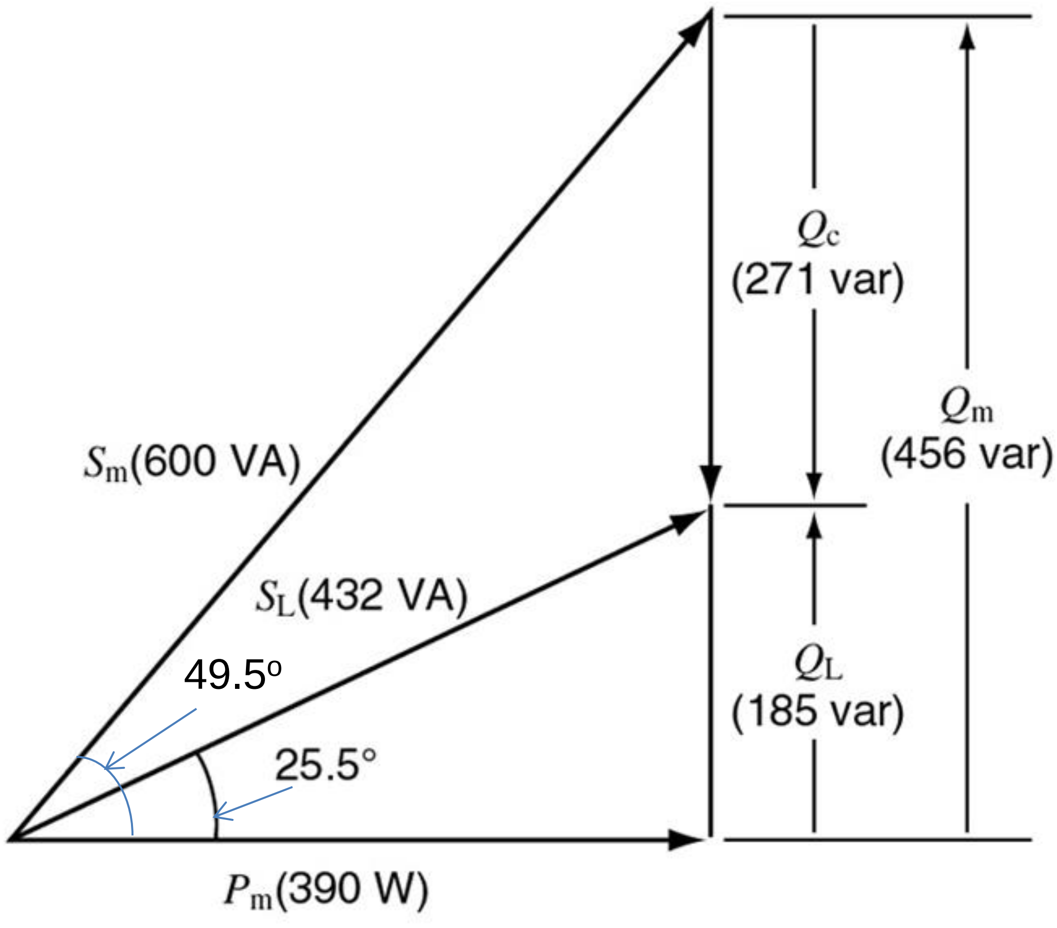
$$S_{\text{new}} = 80 + j(60 - Q_{\text{cap}})$$

$$\frac{60 - Q_{\text{cap}}}{80} = \tan 18.2° \;\Rightarrow\; 60 - Q_{\text{cap}} = 26.3 \text{ kvar}$$

$$Q_{\text{cap}} = 33.7 \text{ kvar}$$

Example: Partial Power Factor Correction

The power triangle below shows that the power factor is corrected by a shunt capacitor from 65% up to 90.26%



Practice Problem 1

You are asked to connect a pool pump to a 240 V, 60 Hz circuit that is protected by a 15 A circuit breaker (i.e., the breaker will trip if the current exceeds 15 A). Assume that this motor draws 3 kW with a power factor of 75% (lag).

1. Will the circuit breaker trip when connecting the motor? Explain. If so, make some suggestions on how to fix this in a sentence or two.

   source current = 16.67 A …. Circuit breaker will trip.

1. The equivalent impedance of the motor is nearly equal to
   – 19.2 Ω in series with j21.77 Ω --- 19.2 Ω in parallel with j15.5 Ω -- 10.8 Ω in parallel with j9.52 Ω ---- None of these (answer: 10.8 + j 9.5 Ω, or 19.2 Ω //j21.77 Ω)

2. A shunt capacitor size (in terms of its VARs rating) that results in a source current that is equal to the breaker current rating (i.e., 15 A) is nearly equal to
   – 1,990 VAR --- 402 VAR ---- 655 VAR --- 800 VAR

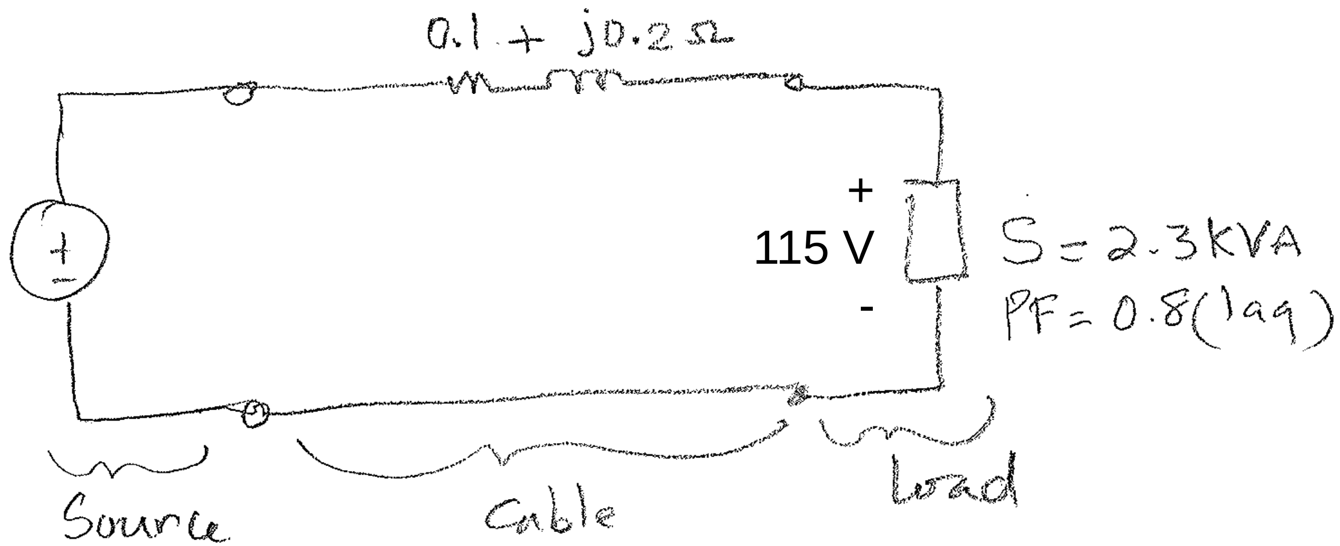
3. The installation of a capacitor bank that is rated at 1.5 kVAR will result in an overall power factor of nearly
   – 90.8% lag --- 93.4% lag ---- 95.3% lag ---- 97.2% lag

4. The optimal shunt capacitor size (in µF) that results in minimum source current is nearly equal to
   – 48 µF -- 75 µF --- 82 µF --- 122 µF

5. Suppose the optimal shunt capacitor in 5) above is placed across the motor terminals. Which of the following loads will trip the circuit breaker when added to the circuit?
   – A pure resistive load with R = 100 Ω
   – A load that draws 500 W and 800 VAR
   – A load that has an apparent power of 800 VA with power factor of 50% lag
   – A pure resistive load with R = 90 Ω

Practice Problem 2

Consider the circuit below. The source (or load) current is 20 A, the load apparent power and power factor are 2.3 KVA, and 80% (lag), respectively. Compute the following:



1. Source current 20 A
2. Voltage across the cable impedance 4.47 V
3. Source voltage magnitude and phase (relative to load voltage) 119 V, +0.96 deg.
4. Real and reactive power supplied by the source 1,880 W and 1,460 VAR

Homework Assignment # 1

Consider a single-phase circuit where a 14 kV, 60 Hz voltage source feeds an electric load through a feeder whose impedance is 1.5 + j 3 Ω. The load is known to consume 4 MW and 3 MVAR. Let V and δ represent the rms voltage and its phase angle (relative to the source voltage) at the load terminals. Two equations are needed to solve for V and δ.

1) Derive two analytical expressions in terms of V and δ that need to be solved.
2) Determine the values of V and δ (hint: numerical solution is required) .
3) Determine the active and reactive powers supplied by the source.