Handout #1 Domain of Elementary Functions:

1. $\boxed{f = \frac{u}{v}} \Longrightarrow \boxed{v \neq 0}$

2. $\boxed{f = \sqrt[2k]{u}} \Longrightarrow \boxed{u \geq 0}$ $\quad$ (k = 1,2,3, ...)

3. $\boxed{f = log_v u} \Longrightarrow \boxed{u > 0, \quad v > 0, \quad v \neq 1}$

4. $\boxed{f = \tan u} \Longrightarrow \boxed{u \neq \frac{\pi}{2} + k\pi \quad (k = 0, \pm 1, \pm 2, \ldots)}$

5. $\boxed{f = \cot u} \Longrightarrow \boxed{u \neq k\pi \quad (k = 0, \pm 1, \pm 2, \ldots)}$

6. $\boxed{f = \arcsin u} \Longrightarrow \boxed{-1 \leq u \leq 1)}$

7. $\boxed{f = \arccos u} \Longrightarrow \boxed{-1 \leq u \leq 1)}$

8. $\boxed{f = u^v} \Longrightarrow \boxed{u > 0}$ $\quad$ $v \neq$ constant or $v =$ irrational constant

Important fact: All elementary functions are continuous in their domains except at the isolated points.

Handout #2 Fundamental Formulas for Integration:

1. $\displaystyle\int u^n u' dx = \int u^n du = \frac{u^{n+1}}{n+1} + C \quad (n \neq -1)$

2. $\displaystyle\int \frac{1}{u} u' dx = \int \frac{1}{u} du = ln|u| + C$

3. $\displaystyle\int a^u u' dx = \int a^u du = \frac{a^u}{\ln a} + C \quad (a > 0, a \neq 1)$

4. $\displaystyle\int \sin u \ u' dx = \int \sin u \ du = -\cos u + C$

5. $\displaystyle\int \cos u \ u' dx = \int \cos u \ du = \sin u + C$

6. $\displaystyle\int \frac{1}{\cos^2 u} u' dx = \int \frac{1}{\cos^2 u} du = \tan u + C$

7. $\displaystyle\int \frac{1}{\sin^2 u} u' dx = \int \frac{1}{\sin^2 u} du = -\cot u + C$

8. $\displaystyle\int \frac{u' dx}{u^2 + a^2} = \int \frac{du}{u^2 + a^2} = \frac{1}{a} \arctan \frac{u}{a} + C_1 = -\frac{1}{a} \operatorname{arccot} \frac{u}{a} + C_2$

9. $\displaystyle\int \frac{u' dx}{u^2 - a^2} = \int \frac{du}{u^2 - a^2} = \frac{1}{2a} \ln |\frac{u-a}{u+a}| + C$

10. $\displaystyle\int \frac{u' dx}{\sqrt{u^2 \pm a^2}} = \int \frac{du}{\sqrt{u^2 \pm a^2}} = \ln |u + \sqrt{u^2 \pm a^2}| + C$

11. $\displaystyle\int \frac{u' dx}{\sqrt{a^2 - u^2}} = \int \frac{du}{\sqrt{a^2 - u^2}} = \arcsin \frac{u}{a} + C_1 = -\arccos \frac{u}{a} + C_2$

12. $\displaystyle\int \frac{u' dx}{u\sqrt{u^2 - a^2}} = \int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a} \operatorname{arcsec} \frac{|x|}{a} + C$

13. $\displaystyle\int \ln u \ du = u \ln |u| - v$

Handout #3 Some basic Formulas, definitions and equations:

Basic trigonometric equations:

$$\boxed{\sin x = a \quad (-1 \le a \le 1)} \Leftrightarrow \boxed{x = (-1)^n \arcsin a + n\pi \quad (n = 0, \pm 1, \pm 2, \dots)}$$

$$\boxed{\sin x = 0} \Leftrightarrow \boxed{x = n\pi \quad (n = 0, \pm 1, \pm 2, \dots)}$$

$$\boxed{\sin x = 1} \Leftrightarrow \boxed{x = \frac{\pi}{2} + 2k\pi \quad (k = 0, \pm 1, \pm 2, \dots)}$$

$$\boxed{\sin x = -1} \Leftrightarrow \boxed{x = \frac{-\pi}{2} + 2k\pi \quad (k = 0, \pm 1, \pm 2, \dots)}$$

$$\boxed{\cos x = a \quad (-1 \le a \le 1)} \Leftrightarrow \boxed{x = \pm \arccos a + 2k\pi \quad (k = 0, \pm 1, \pm 2, \dots)}$$

$$\boxed{\cos x = 0} \Leftrightarrow \boxed{x = \frac{\pi}{2} + n\pi \quad (n = 0, \pm 1, \pm 2, \dots)}$$

$$\boxed{\cos x = 1} \Leftrightarrow \boxed{x = 2k\pi \quad (k = 0, \pm 1, \pm 2, \dots)}$$

$$\boxed{\cos x = -1} \Leftrightarrow \boxed{x = (2k+1)\pi \quad (k = 0, \pm 1, \pm 2, \dots)}$$

$$\boxed{\tan x = a} \Leftrightarrow \boxed{x = \arctan a + k\pi \quad (k = 0, \pm 1, \pm 2, \dots)}$$

$$\boxed{\cot x = a} \Leftrightarrow \boxed{x = \operatorname{arccot} a + k\pi \quad (k = 0, \pm 1, \pm 2, \dots)}$$

Definitions of inverse trigonometric functions:

$$\boxed{\arcsin a = b \quad (-1 \le a \le 1)} \Leftrightarrow \boxed{a \sin b \quad \text{and} \quad -\frac{\pi}{2} \le b \le \frac{\pi}{2}}$$

$$\boxed{\arccos a = b \quad (-1 \le a \le 1)} \Leftrightarrow \boxed{a \cos b \quad \text{and} \quad 0 \le b \le \pi}$$

$$\boxed{\arctan a = b} \Leftrightarrow \boxed{a = \tan b \quad \text{and} \quad -\frac{\pi}{2} \le b \le \frac{\pi}{2}}$$

$$\boxed{\operatorname{arccot} a = b} \Leftrightarrow \boxed{a = \cot b \quad \text{and} \quad 0 \le b \le \pi}$$

Some formulas:

$$\boxed{\sqrt{a^2} = |a|} \qquad \boxed{e^{a \ln b} = b^a} \qquad \boxed{\log_b a = \frac{\ln a}{\ln b}} \qquad \boxed{e^{i\theta} = \cos\theta + i \sin\theta}$$

$$\boxed{\cos\theta = \frac{e^{i\theta} + e^{-i\theta}}{2}} \qquad \boxed{\sin\theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}} \qquad \boxed{\cosh\theta = \frac{e^{\theta} + e^{-\theta}}{2}} \qquad \boxed{\sinh\theta = \frac{e^{\theta} - e^{-\theta}}{2}}$$

$$\boxed{\sqrt{A \pm \sqrt{B}} = \sqrt{\frac{A + \sqrt{A^2 - B}}{2}} \pm \sqrt{\frac{A - \sqrt{A^2 - B}}{2}} \qquad \Big((A^2 - B) > 0\Big)}$$

$$\boxed{\log z = \ln|z| + i\theta, \qquad \theta = \arg z}$$

Handout #4 Derivatives:

1. $f'(x) = \lim_{z \to x} \frac{f(z) - f(x)}{z - x} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

2. $c' = 0$
3. $x' = 1$
4. $(u^n)' = nu^{n-1}u'$
5. $(\sqrt{u})' = \frac{u'}{2\sqrt{u}}$
6. $(\sin u)' = (\cos u)u'$
7. $(\cos u)' = (-\sin u)u'$
8. $(\tan u)' = (\sec^2 u)u'$
9. $(\cot u)' = (-\csc^2 u)u'$
10. $(u \pm v)' = u' \pm v'$
11. $(uv)' = u'v + v'u$
12. $(cu)' = cu'$
13. $\left(\frac{u}{c}\right)' = \frac{u'}{c}$
14. $\left(\frac{c}{u}\right)' = \frac{-c}{u^2}u'$
15. $\left(\frac{u}{v}\right)' = \frac{u'v - v'u}{v^2}$
16. $\left(\frac{c}{u^n}\right)' = \frac{-cn}{u^{n+1}}u'$
17. $(e^u)' = e^u u'$
18. $(a^u)' = a^u(\ln a)u'$
19. $(\ln u)' = \frac{1}{u}u'$
20. $(\log_a u)' = \frac{1}{u \ln a}u'$
21. $(\arcsin u)' = \frac{1}{\sqrt{1-u^2}}u'$
22. $(\arccos u)' = \frac{-1}{\sqrt{1-u^2}}u'$
23. $(\arctan u)' = \frac{1}{1+u^2}u'$
24. $(\operatorname{arccot} u)' = \frac{-1}{1+u^2}u'$

Handout #5.1 Trigonometric identities:

$$\sin\alpha\cos\beta = \frac{1}{2}\sin(\alpha-\beta) + \frac{1}{2}\sin(\alpha+\beta)$$

$$\cos\alpha\cos\beta = \frac{1}{2}\cos(\alpha-\beta) + \frac{1}{2}\cos(\alpha+\beta)$$

$$\sin\alpha\sin\beta = \frac{1}{2}\cos(\alpha-\beta) - \frac{1}{2}\cos(\alpha+\beta)$$

$$\cos^2\alpha = \frac{1+\cos 2\alpha}{2} \qquad \sin^2\alpha = \frac{1-\cos 2\alpha}{2}$$

$$\sin(\alpha\pm\beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$
$$\cos(\alpha\pm\beta) = \cos\alpha\cos\beta \pm \sin\alpha\sin\beta$$

$$\sin 2\alpha = 2\sin\alpha\cos\beta \qquad \cos 2\alpha = \cos^2\alpha - \sin^2\alpha$$

$$\tan(\alpha\pm\beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta} \qquad \tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha}$$

$$\sin\alpha \pm \sin\beta = 2\sin\left(\frac{\alpha\pm\beta}{2}\right)\cos\left(\frac{\alpha\mp\beta}{2}\right)$$

$$\cos\alpha + \cos\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

$$\cos\alpha - \cos\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\beta-\alpha}{2}\right)$$

How to exit trigonometry:

$$\tan\frac{x}{2} = y \Rightarrow \sin x = \frac{2y}{1+y^2}, \quad \cos x = \frac{1-y^2}{1+y^2}, \quad dx = \frac{2dy}{1+y^2}$$

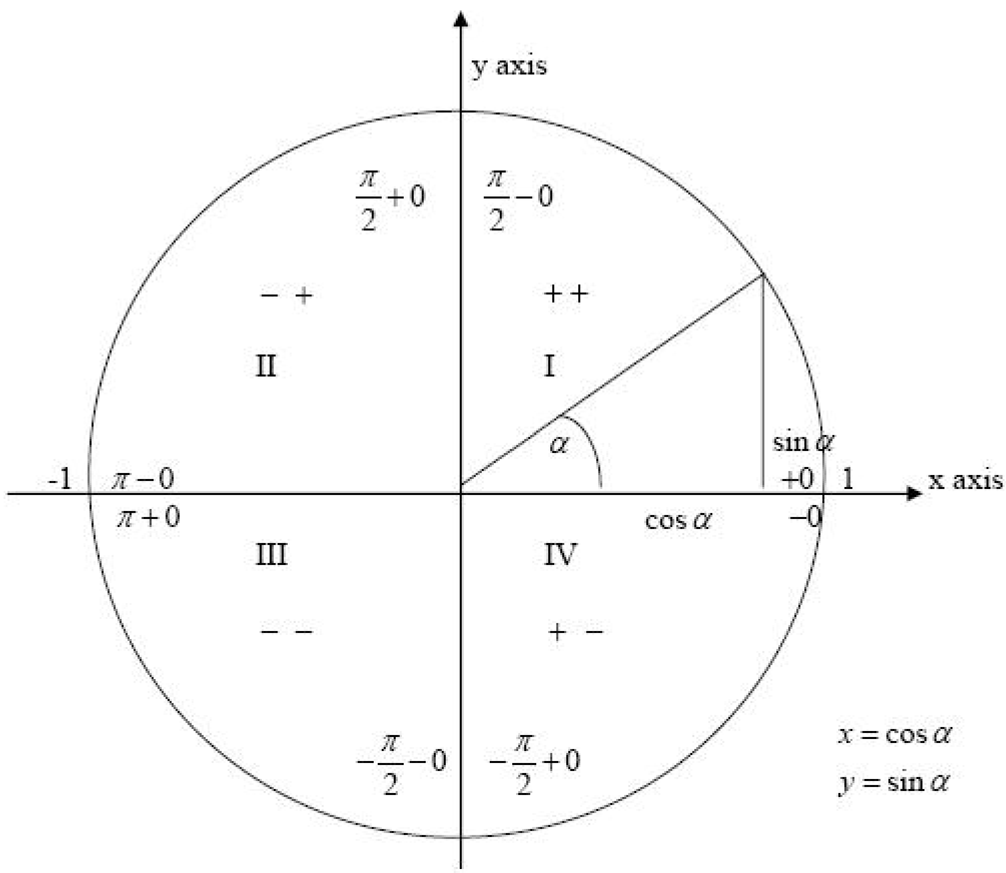
$$\tan x = y \Rightarrow \cos^2 x = \frac{1}{1+y^2}, \quad \sin^2 x = \frac{y^2}{1+y^2}, \quad \sin x\cos x = \frac{y}{1+y^2}, \quad dx = \frac{dy}{1+y^2}$$

Handout #5.2

Trigonometric chart

	$-\frac{\pi}{2}-0$	$-\frac{\pi}{2}+0$	-0	+0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}-0$	$\frac{\pi}{2}+0$	$\pi-0$	$\pi+0$
sin	-1	-1	-0	+0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	1	+0	-0
cos	-0	+0	1	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	+0	-0	-1	-1
tan	$+\infty$	$-\infty$	-0	+0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	$+\infty$	$-\infty$	-0	+0
cot	+0	-0	$-\infty$	$+\infty$	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	+0	-0	$-\infty$	$+\infty$

Unit circle



Handout #6 Power series for elementary functions:

1. $$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n \qquad -1 < x < 1$$

2. $$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \qquad x\epsilon(-\infty, +\infty)$$

3. $$\sin x = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^{2n-1}}{(2n-1)!} \qquad x\epsilon(-\infty, +\infty)$$

4. $$\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} \qquad x\epsilon(-\infty, +\infty)$$

5. $$\ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n} \qquad -1 < x \leq 1$$

6. $$\arctan x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1} \qquad -1 \leq x \leq 1$$

7. $$(1+x)^n = \sum_{n=0}^{\infty} \binom{\alpha}{n} x^n \qquad \binom{\alpha}{0} = 1, \qquad \binom{\alpha}{1} = \alpha, \ldots,$$

$$\binom{\alpha}{n} = \frac{\alpha(\alpha-1)(\alpha-2)\ldots(\alpha-(n-1))}{n!}, \qquad -1 < x < 1 \quad (\alpha \neq \mathit{integer})$$

8. $$\arcsin x = \sum_{n=0}^{\infty} \frac{(2n-1)!!}{(2n)!!} * \frac{x^{2n+1}}{2n+1} \qquad -1 \leq x \leq 1$$

$(2n-1)!! = (1)(3)(5)\ldots(2n-1)$, $(2n)!! = (2)(4)(6)\ldots(2n)$, $1!! = 1$, $2!! = 2$, $0!! = 1$, $(-1)!! = 1$

Handout #7 **Limits involving ∞, etc.**

Indeterminate cases:

$$\frac{\pm 0}{\pm 0} \qquad \frac{\pm\infty}{\pm\infty} \qquad (\pm\infty)(\pm 0) \qquad -\infty+\infty$$

$$1^{\pm\infty} \qquad (+0)^{\pm 0} \qquad (+\infty)^{\pm 0}$$

Other limits involving ∞ (examples):

$$\lim_{u\to\pm\infty}\left(1+\frac{1}{u}\right)^{u}=e \qquad \lim_{x\to 0}\frac{\sin x}{x}=1 \qquad \ln(+0)=-\infty \qquad \ln(+\infty)=+\infty$$

$$\arctan(+\infty)=\frac{\pi}{2}-0 \qquad \arctan(-\infty)=-\frac{\pi}{2}+0 \qquad \operatorname{arccot}(+\infty)=+0 \qquad \operatorname{arccot}(-\infty)=\pi-0$$

$$+\infty-7=+\infty \qquad -\infty+7=-\infty \qquad (+\infty)(-\infty)=-\infty \qquad (\pm\infty)^2=+\infty$$

$$\frac{-7}{+\infty}=-0 \qquad \frac{-7}{-0}=+\infty \qquad (-7)(+\infty)=-\infty \qquad \frac{+\infty}{-0}=-\infty$$

$$\frac{+0}{-\infty}=-0 \qquad (-\infty)^3=-\infty \qquad \sqrt{+\infty}=+\infty \qquad \sqrt[3]{-\infty}=-\infty$$

$$(2)^{+\infty}=+\infty \qquad (3)^{-\infty}=+0 \qquad (0.5)^{+\infty}=+0 \qquad (0.7)^{-\infty}=+\infty$$

$$(+\infty)^{-\infty}=+0 \qquad \tan\left(\frac{\pi}{2}-0\right)=+\infty \qquad \tan\left(\frac{\pi}{2}+0\right)=-\infty \qquad \tan\left(-\frac{\pi}{2}+0\right)=-\infty$$

$$\cot(+0)=+\infty \qquad \cot(\pi-0)=-\infty \qquad \cot(-0)=-\infty \qquad \cot(\pi+0)=+\infty$$

$\cos\infty, \sin\infty, \tan\infty, and \cot\infty$ do not exist.

created by Jia Tse using LaTeX
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