Sequences and Series

1 Partial Sum:

$$S_n = \sum_{i=1}^{n} a_i = a_1 + a_2 + a_3 + ... + a_n$$

if the sequence S_n is convergent and

$$\lim_{n\to\infty} S_n = S \; and \; S \in \Re$$

then $S =$ Sum of series

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2 Geometric Series:

$$\sum_{n=1}^{\infty} ar^{n-1} = a + ar + ar^2 + ar^3 + ...$$

Convergent if $\mid r \mid < 1$, and

$$\sum_{n=1}^{\infty} ar^{n-1} = \frac{a}{1-r}, \quad \mid r \mid < 1$$

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3 Theorem

If a series

$$\sum_{n=1}^{\infty} a_n$$

is convergent, then

$$\lim_{n\to\infty} a_n = 0$$

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4 Divergence Test

If

$$\lim_{n\to\infty} a_n = DNE \quad \text{or} \quad \lim_{n\to\infty} a_n \neq 0$$

Then

$$\sum_{n=1}^{\infty} a_n \quad \text{is divergent}$$

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5 Integral Test

Function $f(x)$ is continuously decreasing, and $a_n = f(n)$

i. If $\int_1^\infty f(x)$ converges, then $\sum_{n=1}^\infty a_n$ converges

ii. If $\int_1^\infty f(x)$ diverges, then $\sum_{n=1}^\infty a_n$ diverges

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6 P-Series

$$\sum_{n=1}^{\infty} \frac{1}{n^p} \quad \begin{cases} \text{converges} & \text{if } p > 1 \\ \text{diverges} & \text{if } p \leq 1 \end{cases}$$

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7 Comparison Test

$\sum a_n, \sum b_n$ have positive terms.
If $\sum b_n$ converges and $a_n \leq b_n, \forall n \Rightarrow \sum a_n$converges
If $\sum b_n$ diverges and $a_n \geq b_n, \forall n \Rightarrow \sum a_n$diverges

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8 Limit Comparison Test

$\sum a_n, \sum b_n$ have positive terms.

IF

$$\lim_{n\to\infty} \frac{a_n}{b_n} = c, \quad 0 < c < \infty$$

THEN

$$Conv\left(\sum a_n \wedge \sum b_n\right) \bigvee Div\left(\sum a_n \wedge \sum b_n\right)$$

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9 Alternating Series Test

$$\sum_{n=1}^{\infty}(-1)^{n-1}b_n$$

If: $b_{n+1} \leq b_n, \quad \forall n$ and $\lim_{n\to\infty} b_n = 0$
Then the series converges

10 Absolute Convergence (AC)

$$\sum a_n \text{ is AC if } \sum |a_n| \text{ is convergent}$$

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11 Conditionally Convergent (CC)

$\sum a_n$ is CC if it converges but not AC (i.e. Alternating Harmonic Series)
If $\sum a_n$ is AC, Then $\sum a_n$ converges.

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12 Ratio Test

i. If

$$\lim_{n\to\infty} |\frac{a_{n+1}}{a_n}| = L < 1$$

Then

$$\sum_{n=1}^{\infty} a_n \text{ is Absolutely Convergent}$$

ii. If

$$\lim_{n\to\infty} |\frac{a_{n+1}}{a_n}| = L > 1 \text{ or } L = \infty$$

Then

$$\sum_{n=1}^{\infty} a_n \text{ is divergent}$$

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13 Root Test

i. If

$$\lim_{n\to\infty} \sqrt[n]{|a_n|} = L < 1$$

Then

$$\sum_{n=1}^{\infty} a_n \text{ is Absolutely Convergent}$$

ii. If

$$\lim_{n\to\infty} \sqrt[n]{|a_n|} = L > 1 \text{ or } L = \infty$$

Then

$$\sum_{n=1}^{\infty} a_n \text{ is divergent}$$

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14 Power Series

$$\sum_{n=0}^{\infty} c_n(x-a)^n \text{ , there are 3 cases}$$

i. Series converges only when $x = a$

ii. Series converges $\forall x$

iii. $\exists R > 0$ such that
if $|x-a| < R$ then the series converges
if $|x-a| > R$ then the series diverges

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15 Theorem:

$$f(x) = \sum_{n=0}^{\infty} c_n(x-a)^n \quad , \quad |x-a| < R$$

$$c_n = \frac{f^{(n)}(a)}{n!}$$

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16 Taylor Series

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$$

**Maclaurin Series = Taylor Series with $a = 0$

created by Jia Tse using LaTeX
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