1. True or False. [5 points each]
   (a) ______ A good programmer should never use linear search.
   (b) ______ The height of a binary tree with \( n \) nodes is \( O(\log n) \).
   (c) ______ \( 3^n = O(2^n) \).
   (d) ______ The height of a balanced binary search tree with \( n \) items is \( O(\log n) \).
   (e) ______ \( \log \log n = O(\log n) \).
   (f) ______ A stack is an example of a search structure.
   (g) ______ \( n^{\frac{3}{2}} = O(n \log n) \).
   (h) ______ An item can always be inserted into a min-heap of size \( n \) in \( O(\log n) \) time.
   (i) ______ A treap is an example of a priority queue.

2. Fill in the blanks (5 points each blank, except for 2c, which is 10 points)
   (a) Probing is used in __________________ hashing. (Open or Closed?)
   (b) A __________________ hash table is designed so that the size of the table is exactly the number of data and there are no collisions.
   (c) [10 points] A planar graph with 100 edges must have at least ______ vertices. (Exact answer, please.)
   (d) A condition that is
      i. True for the first iteration of a loop, and
      ii. true at the end of any iteration of that loop, provided it is true at the beginning of that iteration,
      is called a __________________ of that loop. [2 word answer]

3. The three kinds of priority queues we have discussed in class are:
   (a) ____________________________
   (b) ____________________________
4. [30 points] For each application, name the kind of search structure you would use.

(a) __________________________________________. Items will be inserted and deleted frequently. You expect there to be many items.

(b) __________________________________________. The expected number of items in the structure is only two, but there is a small probability that it will be as large as ten.

(c) __________________________________________. You will execute find many times, but all items will be inserted at once at the beginning of the program, and there will never be another insert.

5. For each of the following blanks, the correct answer is stack, queue, array, heap, or search structure. (5 points each blank.)

(a) pop is an operator of __________.

(b) find is an operator of __________.

(c) You would use a __________ to do breadth first search of a graph.

(d) You would use a __________ to hold the records of the customers of a business.

(e) You would use an __________ to keep track of the number of times each student in a class asks a question.

(f) You would use a __________ to keep track of patients in the waiting area of the emergency room of a hospital.

(g) You would use a __________ to match left with right parentheses in an algebraic expression.

(h) You would use a __________ to store your unpaid bills, if every time you get money, you pay as many bills as possible, in order of urgency.

6. [20 points] Build a binary search tree, starting from an empty tree, inserting the following items one at a time: Moe Abe Joe Nan Ted Kim Sam Ron Dan Sue Zed. Once an item is inserted into the tree, it is not moved.

7. State the asymptotic complexity of each of the following code fragments in terms of \( n \), where \( n \) is a positive integer. Use \( \Theta \) notation.

(a) [10 points]

\[
\text{for(int i=0; i<n; i++)}
\text{for(int j=n+1; j>i; j=j-2)}
\text{cout \ll \"Hello world\ll endl;}
\]

(b) [10 points]
for(int i=1; i<n; i++)
    for(int j=0; j*j<i; j++)
        cout << "Hello world" << endl;

8. [20 points]
   By drawing pictures and writing words, define the loop invariant of the partition phase of quicksort.

9. [15 points] Describe the simple version of heapsort that we discussed in class, not the more complex version that you find on the internet, for example.


12. [20 points] Suppose that the items of a queue are A, H, K, B, T in that order, where A is the front item.
   (a) Sketch the appearance of a circular linked list implementation of that queue.
   (b) Insert the item L into that queue. Show the steps. (You should draw at least two additional figures.)

13. [20 points] Consider the following binary tree T.

   ![Binary Tree Image]

   List the nodes of T in preorder, inorder, postorder, and level order.

14. [30 points] Consider the heap of 12 elements, implemented as an array as illustrated below. Illustrate the implementation after insertion of the letter E. (You are not required to show steps, but it doesn’t hurt.)

   ![Heap Array Image]

   12
15. [30 points] Consider the treap illustrated below, where the heap key is a randomly chosen integer in the range 0…99. A new item, “Fay,” is inserted, and the heap key “12” is randomly chosen. Show the treap after that insertion, and show the intermediate steps.

![Treap Diagram]

16. [10 points] Write a topological ordering of the nodes of the weighted directed graph shown in the figure below.

![Directed Graph]

17. [20 points] You are building a cuckoo hash table for the following data set consisting of 8 items. The indices of your hash table are \{0, 1, …, 9\}. The two hash values for each item are listed in the first and second columns of the array below.

Walk through the steps of inserting the items, in the order given in the array.
18. [20 points] $A$ is a $4\times 8\times 6$ three dimensional array. The first value of each index is 0, as in C++. If $A$ is stored in the memory in column-major order with base address 1024, and if each array item takes 2 spaces in the memory, where is $A[2][5][3]$ stored?

19. [20 points] Use the union-find data structure to find the components of the graph whose vertices are the integers in the range $0 \ldots 4$ and which has three edges, namely $\{0, 1\}$, $\{2, 3\}$ and $\{1, 4\}$.

20. [20 points] $A$ is a two-dimensional triangular array. $A[i][j]$ exists if $0 \leq i \leq j < 4$. Each entry of $A$ takes up one memory location. If $A$ is stored in main memory in row-major order, and if the base address is 1024, where would $A[1][2]$ be stored?